Week 5

Overview

- New definition of mailbox existentially/universally bounded using the general notation of k-bounded linearization
- MSO logic over asynchronous MSCs
- MSO definability for existentially and universally k-bounded MSCs

MSO-definability

MSO definability	Raw	Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	Y	Y	Y	Y	Y
FIFO 1-1 (p2p)	Y	Y	Y	Y	Y
Causally ordered	Y	Y	Y	Y	Y
FIFO n-1 (mailbox)	Y	Y	Y	Y	Y
FIFO 1-n					
FIFO n-n					
RSC					

= new

= changed

= informal/idea

MSO-definability of raw MSCs

MSO definability	Raw	Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	Y	Y	Y	Y	Y
FIFO 1-1 (p2p)	Y	Y	Y	Y	Y
Causally ordered	Y	Y	Y	Y	Y
FIFO n-1 (mailbox)	Y	Y	Y	Y	Y
FIFO 1-n					
FIFO n-n					
RSC					

= new

= changed

= informal/idea

MSO-definability of ∃k-bounded

MSO definability	Raw	Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	Y	Y	Y	Y	Y
FIFO 1-1 (p2p)	Y	Y	Y	Y	Y
Causally ordered	Y	Y	Y	Y	Y
FIFO n-1 (mailbox)	Y	Y	Y	Y	Y
FIFO 1-n					
FIFO n-n					
RSC					

= new

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= informal/idea

3.5 Existentially bounded MSCs

Definition 3.5. Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}$ and $k \in \mathbb{N}$. A linearization $\rightsquigarrow$ of M is called *k-bounded* if, for all $e \in \mathit{Matched}(M)$, with $\lambda(e) = \mathit{send}(p, q, m)$, we have

$$\#_{\mathit{Send}(p,q,\_)}(\rightsquigarrow, e) - \#_{\mathit{Rec}(p,q,\_)}(\rightsquigarrow, e) \leq k.$$

Recall that $\#_{\mathit{Send}(p,q,\_)}(\rightsquigarrow, e)$ denotes the number of send events from p to q that occured before e, according to $\rightsquigarrow$. Intuitively, a linearization is k-bounded if, at any moment in time, there are no more than k messages in any channel.

Definition 3.6 (Existentially bounded MSC)**.** Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}_{\mathsf{asy}}$ and $k \in \mathbb{N}$. We call M *existentially k-bounded* if it has a k-bounded linearization.

Let $\mathsf{MSC}_{\exists k\text{-}b}$ be the set of existentially k-bounded MSCs, for a given $k \in \mathbb{N}$.

Definition 3.7. An MSC M is *p2p existentially k-bounded* (p2p-$\exists k$-bounded) if it is a p2p MSC and it is also existentially k-bounded.

Definition 3.8. An MSC M is *causally orderered existentially k-bounded* (co-$\exists k$-bounded) if it is a causally ordered MSC and it is also existentially k-bounded.

When moving on to mailbox MSCs, the definition of mailbox existentially k-bounded MSC should require that there exists a k-bounded linearization that is also a mailbox linearization, not just any linearization. Recall that an MSC is a mailbox MSC if it has at least one mailbox linearization, which represents a sequence of events that can be executed by a mailbox system. Following this intuition, we want one of these mailbox linearizations to be k-bounded, because *non*-mailbox linearizations cannot be executed by a mailbox system.

Definition 3.9. An MSC M is *mailbox existentially k-bounded* (mb-$\exists k$-bounded) if it has a k-bounded mailbox linearization.

It should be noted that, for a k-bounded mailbox linearization, it is not necessarily true that at any time we have at most k messages in each channel. Recall that in the mailbox communication architecture every process has a single incoming channel, but the Definition 3.5 of k-bounded linearization considers the number of pending messages between each pair (p, q) of processes. Let n be the number of processes. We can say that, for a k-bounded mailbox linearization, we have at most $k(n-1)$ messages in each channel at any moment (because each process can have at most k pending messages coming from any of the other $n-1$ processes).

MSO-definability of ∀k-bounded

MSO definability	Raw	Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	Y	Y	Y	Y	Y
FIFO 1-1 (p2p)	Y	Y	Y	Y	Y
Causally ordered	Y	Y	Y	Y	Y
FIFO n-1 (mailbox)	Y	Y	Y	Y	Y
FIFO 1-n					
FIFO n-n					
RSC					

= new

= changed

= informal/idea

3.6 Universally bounded MSCs

Definition 3.10 (Universally bounded MSC)**.** Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}_{\mathsf{asy}}$ and $k \in \mathbb{N}$. We call M *universally k-bounded* if all of its linearizations are k-bounded.

Let $\mathsf{MSC}_{\forall k\text{-}b}$ be the set of universally k-bounded MSCs.

Definition 3.11. An MSC M is *p2p universally k-bounded* (p2p-$\forall k$-bounded) if it is a p2p MSC and it is also universally k-bounded.

Let $\mathsf{MSC}_{p2p\text{-}\forall k\text{--}b}$ be the set of p2p universally k-bounded MSCs.

Definition 3.12. An MSC M is *causally universally orderered k-bounded* (co-$\forall k$-bounded) if it is a causally ordered MSC and it is also universally k-bounded.

Let $\mathsf{MSC}_{\mathsf{co}\text{-}\forall k\text{--}b}$ be the set of causally ordered universally k-bounded MSCs.

Definition 3.13. An MSC M is *mailbox universally k-bounded* (mb-$\forall k$-bounded) if all of its mailbox linearizations are k-bounded.

Let $\mathsf{MSC}_{\mathsf{mb}\text{-}\forall k\text{--}b}$ be the set of mailbox universally k-bounded MSCs.

3.6.1 Hierarchy

In this section we will investigate the relations between the various classes of universally k-bounded MSCs that we introduced. From their definition, it is quite straightforward to see that $\mathsf{MSC}_{\mathsf{co}\text{-}\forall k\text{-}b} \subseteq \mathsf{MSC}_{p2p\text{-}\forall k\text{-}b} \subseteq \mathsf{MSC}_{\forall k\text{-}b}$. The set of mailbox universally k-bounded MSCs, however, does not fit in this hierarchy. Recall that an MSC is mb-$\forall k$-bounded if all of its mailbox linearizations are k-bounded, but the definition does not say anything about non-mailbox linearizations. It can be the case that an MSC has a bound k on its mailbox linearization, but another bound k' on non-mailbox linearizations. Fig. 7 shows an MSC M which is mb-$\forall 1$-bounded, but not $\forall 1$-bounded. According to the mailbox semantics, a mailbox linearization of M has to respect the order $!m_1 \sqsubset_M !m_3 \sqsubset_M !m_4$. Note that all mailbox linearizations are 1-bounded, but we are able to find a non-mailbox linearization that is 2-bounded, such as $!m_1 \rightsquigarrow !m_4 \rightsquigarrow ?m_1 \rightsquigarrow !m_2 \rightsquigarrow ?m_2 \rightsquigarrow !m_3 \rightsquigarrow !m_4 \rightsquigarrow ?m_3 \rightsquigarrow ?m_4$.

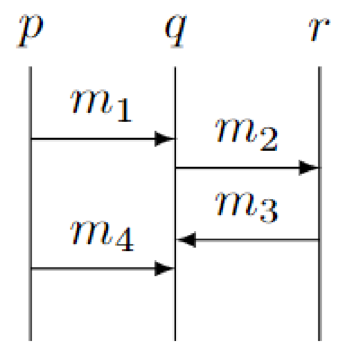
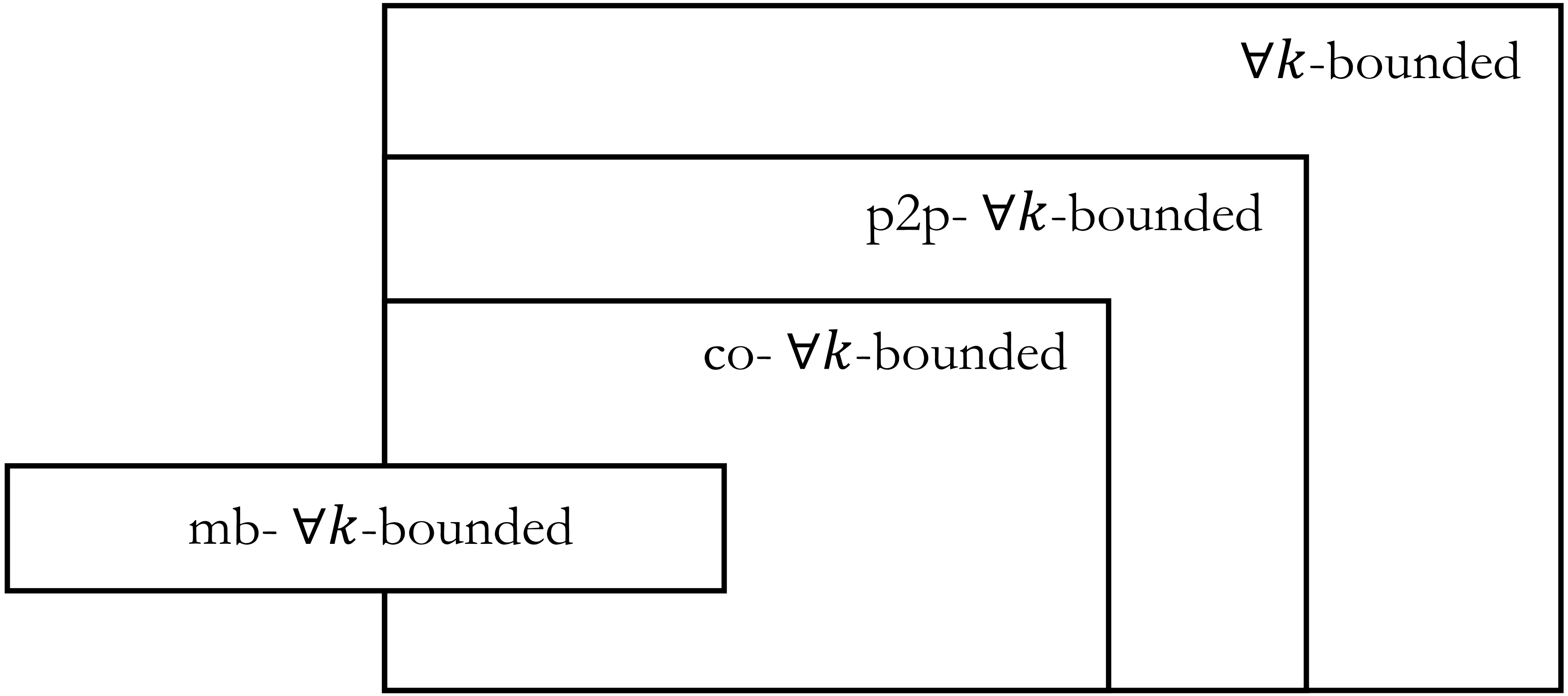


Figure 7: Example of MSC which is mailbox universally 1-bounded, but not universally 1-bounded (it is universally 2-bounded).

$\forall k$-bounded hierarchy (given k)



STW-boundness

Bounded STW	Raw	Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	N	N	Y	Y	Y
FIFO 1-1 (p2p)	N	N	Y	Y	Y
Causally ordered	N	N	Y	Y	Y
FIFO n-1 (mailbox)	N	Y	Y	Y	Y
FIFO 1-n					
FIFO n-n					
RSC					

= new

= informal/idea

MSO extra

Acyclicity Given a binary relation $\to$, the acyclicity of $\to$ can be expressed with an MSO formula. Recall that, given a binary relation $\to$, it is acyclic if and only if its transitive closure $\to^+$ is antisymmetric. The MSO formula of acyclicity directly follows from this definition:

$$\Phi_{acyclic} = \neg\exists x.\exists y.(x \to^+ y \wedge y \to^+ x).$$

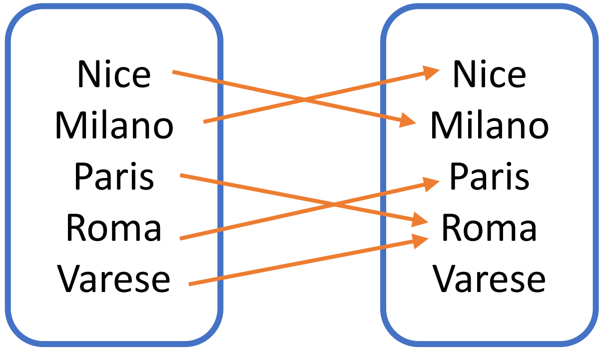
MSO extra

Transitive Closure Given a set Σ and binary relation $\rightarrow \subseteq \Sigma \times \Sigma$ that is irreflexive, antisymmetric, and transitive (i.e., $\rightarrow$ is a strict partial order), we can express its reflexive transitive closure $\rightarrow^*$ in MSO as

$$x \rightarrow^* y = \exists X.(x \in X \wedge y \in X \wedge \forall z.(z \in X \implies z = y \vee \exists k.(k \in X \wedge z \rightarrow k)))$$

Example (non-antisymmetric relation):

$a \rightarrow b =$ "there is a direct flight from a to b"

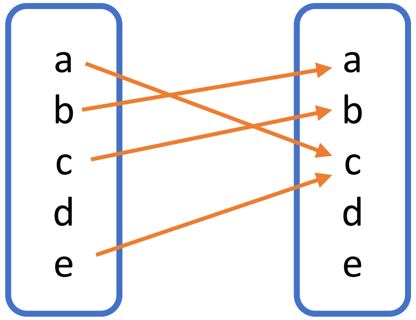


$Nice \ \rightarrow^* \ Varese$?

$X = \{Nice, Varese, Milano, Paris, Roma\}$

Example (non-transitive relation):

$a \rightarrow b$



$a \ \rightarrow^* \ d$?

$X = \{a, d, b, c\}$

Doubts

- MSO-definability of transitive closure
- Communication model and channels (On the Diversity of Asynchonous Communication - Chevrou, Hurault, Quéinnec)