Week 4

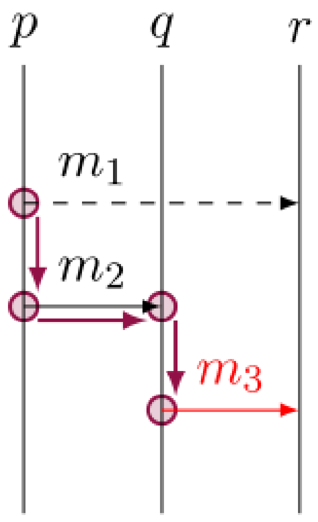
Définition 2.3.3 (Exécution causalement ordonnée [Charron-Bost et al., 1996]). Soit un MSC $\mu = (Ev, \lambda, \prec_{po}, \prec_{src})$, μ admet une exécution causalement ordonnée si, pour deux messages $m, m' \in \mathbb{V}$, tels que $\mathbf{m} = \{s, r\}$ et $\mathbf{m'} = \{s', r'\}$:

$$(\mathsf{proc}_{\mathsf{R}}(\mathbf{m}) = \mathsf{proc}_{\mathsf{R}}(\mathbf{m'})) \wedge (s \prec s') \implies r \prec r' \tag{2.2}$$

Causally ordered MSCs An MSC is causally ordered if all the messages sent to the same process are received in an order which is consistent with the causal ordering of the corresponding send events. More formally, for an MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ we define an additional binary relation $\blacktriangleleft_M \subseteq \mathcal{E} \times \mathcal{E}$ that represents a constraint under the causal ordering semantics. In particular, given two receive events f_1 and f_2, we have that $f_1 \blacktriangleleft_M f_2$ if both the following hold:

- $\lambda(f_1) \in Rec(\_, q, \_)$, $\lambda(f_2) \in Rec(\_, q, \_)$
- $e_1 \lhd f_1$ and $e_2 \lhd f_2$ for some $e_1, e_2 \in \mathcal{E}$, such that $e_1 \leq_M e_2$.

We let $\lessdot_M = (\rightarrow \cup \lhd \cup \blacktriangleleft_M)^*$. Note that $\leq_M \subseteq \lessdot_M$. We call $M \in \mathsf{MSC}$ a *causally ordered (CO) MSC* if $\lessdot_M$ is a partial order. The set of causally ordered MSCs $M \in \mathsf{MSC}$ is denoted by $\mathsf{MSC}_{\mathsf{co}}$.



(a)

This should **not** be causally ordered…
but it is with this definition

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$$(\mathsf{proc}_{\mathsf{R}}(\mathbf{m}) = \mathsf{proc}_{\mathsf{R}}(\mathbf{m}')) \wedge (s \prec s') \implies r \prec r' \tag{2.2}$$

Definition 2.1 (Causally ordered MSC). An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is *causally ordered* if, for any two send events s and s', such that $\lambda(s) = \mathit{Send}(\_, q, \_)$, $\lambda(s') = \mathit{Send}(\_, q, \_)$, and $s \leq_M s'$, we have either:

- $s, s' \in \mathit{Matched}(M)$ and $r \rightarrow^+ r'$, where r and r' are two receive events such that $s \lhd r$ and $s' \lhd r'$.
- $s' \in \mathit{Unm}(M)$.

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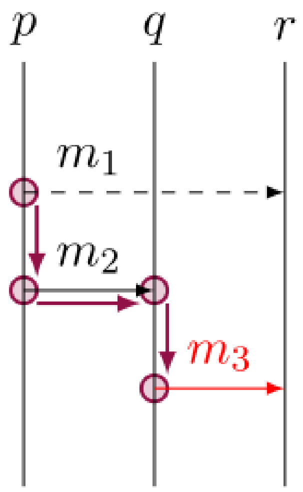

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- $s' \in Unm(M)$.

Lemma 2.1. *Every prefix of a causally ordered MSC is a causally ordered MSC.*

Proof. Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}_{\mathsf{co}}$ and let $M_0 = (\mathcal{E}_0, \rightarrow_0, \lhd_0, \lambda_0)$ be a prefix of M. By contradiction, suppose that M_0 is not a causally ordered MSC. There must be two distinct $s, s' \in \mathcal{E}_0$ such that $\lambda(s) = Send(\_, q, \_)$, $\lambda(s') = Send(\_, q, \_)$, $s \leq_{M_0} s'$ and either (i) $r' \rightarrow^+ r$, where r and r' are two receive events such that $s \lhd r$ and $s' \lhd r'$, or (ii) $s \in Unm(M_0)$ and $s' \in Matched(M_0)$. In both cases, M would also not be a causally ordered MSC, since $\mathcal{E}_0 \subseteq \mathcal{E}$, $\rightarrow_0 \subseteq \rightarrow$, and $\lhd_0 \subseteq \lhd$. This is a contradiction, thus M_0 has to be causally ordered. $\square$

Definition 2.1 (Causally ordered MSC). An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is *causally ordered* if, for any two send events s and s', such that $\lambda(s) = Send(\_, q, \_)$, $\lambda(s') = Send(\_, q, \_)$, and $s \leq_M s'$, we have either:

- $s, s' \in Matched(M)$ and $r \rightarrow^+ r'$, where r and r' are two receive events such that $s \lhd r$ and $s' \lhd r'$.
- $s' \in Unm(M)$.

Proposition 2.1. The set $\mathsf{MSC}_{\mathsf{co}}$ of causally odered MSCs is MSO-definable.

Proof. Given an MSC M, it is causally ordered if it satisfies the MSO formula

$$\varphi_{\mathsf{co}} = \neg\exists s.\exists s'. \left(\bigvee_{\substack{q \in \mathbb{P} \\ a,b \in Send(\_,q,\_)}} \lambda(s) = a \ \wedge \ \lambda(s') = b \ \wedge \ s \leq_M s' \ \wedge \ (\psi_1 \vee \psi_2) \right)$$

where ψ_1 and ψ_2 are

$$\psi_1 = \exists r.\exists r'. \begin{pmatrix} s \lhd r & \wedge \\ s' \lhd r' & \wedge \\ r' \rightarrow^+ r & \end{pmatrix} \qquad \psi_2 = (\neg matched(s) \wedge matched(s'))$$

$$matched(x) = \exists y. x \lhd y$$

The property φ_{co} says that there cannot be two send events s and s', with the same recipient, such that $s \leq_M s'$ and either (i) their corresponding receive events r and r' happen in the opposite order, i.e. $r' \rightarrow^+ r$, or (ii) s is unmatched and s' is matched. The set $\mathsf{MSC}_{\mathsf{co}}$ of causally ordered MSCs is therefore MSO-definable as $\mathsf{MSC}_{\mathsf{co}} = L(\varphi_{\mathsf{co}})$. $\square$

2.4.3 Existentially k causally ordered bounded MSCs

Definition 2.2. Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}$ and $k \in \mathbb{N}$. A linearization $\rightsquigarrow$ of M is called *k-bounded* if, for all $e \in \mathit{Matched}(M)$, with $\lambda(e) = \mathit{send}(p, q, m)$, we have

$$\#_{\mathit{Send}(p,q,\_)}(\rightsquigarrow, e) - \#_{\mathit{Rec}(p,q,\_)}(\rightsquigarrow, e) \leq k.$$

Recall that $\#_{\mathit{Send}(p,q,\_)}(\rightsquigarrow, e)$ denotes the number of send events from p to q that occured before e, according to $\rightsquigarrow$.

Definition 2.3. An MSC is said to be *existentially p2p bounded* ($\exists k$-$p2p$-bounded) if it has a k-bounded linearization.

Definition 2.4. An MSC is said to be *existentially k causally ordered bounded* ($\exists k$-co-bounded) if it is causally ordered and it has a k-bounded linearization.

Note that every existentially k causally ordered bounded MSC is an existentially k-p2p-bounded MSC.

Definition 2.2. Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}$ and $k \in \mathbb{N}$. A linearization $\rightsquigarrow$ of M is called *k-bounded* if, for all $e \in \mathit{Matched}(M)$, with $\lambda(e) = \mathit{send}(p, q, m)$, we have

$$\#_{\mathit{Send}(p,q,\_)}(\rightsquigarrow, e) - \#_{\mathit{Rec}(p,q,\_)}(\rightsquigarrow, e) \leq k.$$

Definition 2.3. An MSC is said to be *existentially k causally ordered bounded* ($\exists k$-co-bounded) if it is causally ordered and it has a k-bounded linearization.

Proposition 2.4. For all $k \in \mathbb{N}$, the set of $\exists k$-co-bounded MSCs is MSO-definable and STW-bounded.

Proof. Let $\mathsf{MSC}_{\exists k\text{-}p2p\text{-}b}$ and $\mathsf{MSC}_{\exists k\text{-}\mathsf{co}\text{-}b}$ be the set of existentially k-p2p-bounded MSCs and the set of existentially k causally ordered bounded MSCs, respectively. $\mathsf{MSC}_{\exists k\text{-}p2p\text{-}b}$ was shown to be both MSO-definable (in [17]) and STW-bounded (in [3, Proposition 5.4, page 163]). $\mathsf{MSC}_{\exists k\text{-}\mathsf{co}\text{-}b}$ also has to be STW-bounded, since we have $\mathsf{MSC}_{\exists k\text{-}\mathsf{co}\text{-}b} \subseteq \mathsf{MSC}_{\exists k\text{-}p2p\text{-}b}$. Note that, by definition, $\mathsf{MSC}_{\exists k\text{-}\mathsf{co}\text{-}b} = \mathsf{MSC}_{\exists k\text{-}p2p\text{-}b} \cap \mathsf{MSC}_{\mathsf{co}}$. Since both $\mathsf{MSC}_{\exists k\text{-}p2p\text{-}b}$ and $\mathsf{MSC}_{\mathsf{co}}$ can be defined by an MSO formula, the latter according to Proposition 2.1, $\mathsf{MSC}_{\exists k\text{-}\mathsf{co}\text{-}b}$ is also MSO-definable[3]. □

Theorem 2.8. *The following problem is decidable: Given finite sets $\mathbb{P}$ and $\mathbb{M}$, a communicating system $\mathcal{S}$, and $k \in \mathbb{N}$, is every MSC in $L_{\mathsf{co}}(\mathcal{S})$ $\exists k$-co-bounded?*

Table 2: Summary of the decidability of the synchronizability problem in various classes

	P2P	Causal ordering	Mailbox
Weakly synchronous	Undecidable [Thm. 1.10]	Undecidable [Thm. 2.6]	EXPTIME [Thm. 1.9]
Weakly k-synchronous	Decidable [Thm. 2.7]		
Existentially k-bounded	Decidable	Decidable	Decidable

Définition 2.2.5 (Réalisabilité en boîte aux lettres). Soit $\mu = (Ev, \lambda, \prec_{po}, \prec_{src})$ un MSC. On dit alors que μ est mb-*réalisable* s'il existe une linéarisation $e = a_1 \cdots a_n$ avec un ordre total $<$ telle que, pour toute paire d'évènements $i < j$ telle que $a_i = \mathtt{s}(p, q, m)$ et $a_j = \mathtt{s}(p', q, m')$, soit a_j est non couplé, soit il existe i', j' tel que $a_i \vdash\!\dashv a_{i'}$, $a_j \vdash\!\dashv a_{j'}$ et $i' < j'$.

$$=$$

Definition 3.4 (Mailbox MSC). An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is a *mailbox MSC* if it has a linearization $\rightsquigarrow$ where, for any two send events s and s', such that $\lambda(s) = \mathit{Send}(\_, q, \_)$, $\lambda(s') = \mathit{Send}(\_, q, \_)$, and $s \rightsquigarrow s'$, we have either:

- $s, s' \in \mathit{Matched}(M)$ and $r \rightsquigarrow r'$, where r and r' are two receive events such that $s \lhd r$ and $s' \lhd r'$.
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Similar structure

Definition 2.1 (Causally ordered MSC). An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is *causally ordered* if, for any two send events s and s', such that $\lambda(s) = \mathit{Send}(\_, q, \_)$, $\lambda(s') = \mathit{Send}(\_, q, \_)$, and $s \leq_M s'$, we have either:

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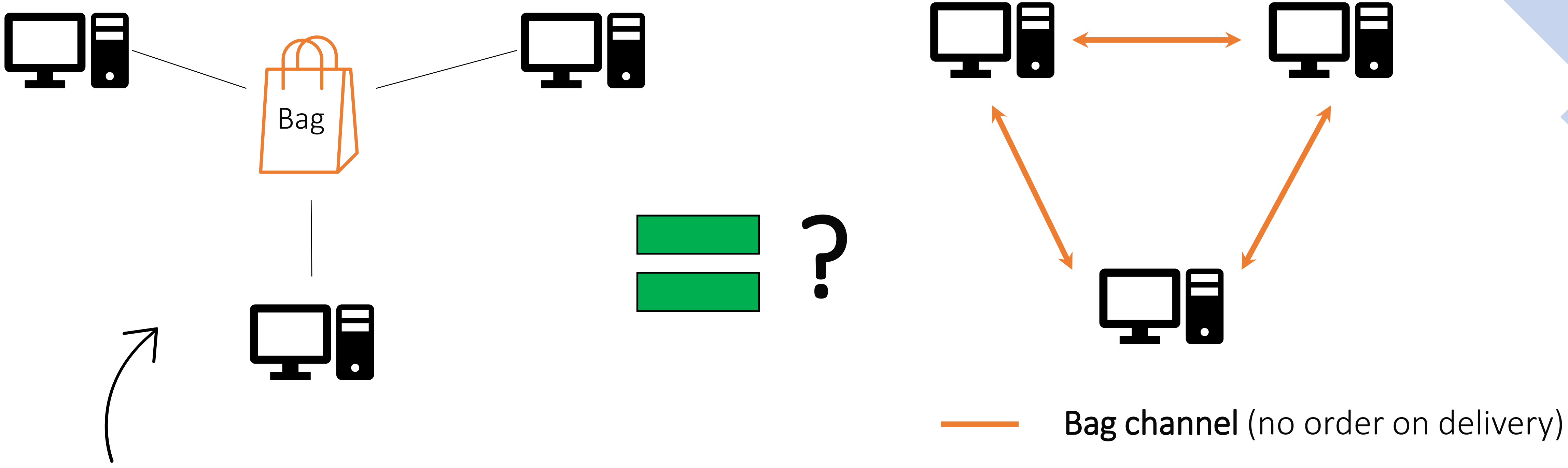
Reorganization of content, overview of communication architectures, hierarchy of MSCs

See “Sketches” section in report

Doubts



2.4.7. M_{async} *Fully Asynchronous Communication*

No order on message delivery is imposed. Messages can overtake others or be arbitrarily delayed. The implementation is usually modeled by a bag (or a set if messages are unique).