Week 7

Overview

- $1 - n$ MSCs:
  - Relation between causally ordered and $1 - n$ MSCs
  - Relation between $1 - n$ MSCs and mailbox, $1 - n \subset mailbox$
- Existentially and universally k-bounded MSCs… are they really MSO-definable? (week 5 fail)
- $n - n$ MSCs:
  - Definition
  - Relation between $1 - n$ and $n - n$ MSCs (conjecture)

$1-n$ MSC - definition

Moving on to the $1{-}n$ semantics, we say that M is a $1{-}n$ MSC if there is a $1{-}n$ system that can exhibit the behaviour described by M.

Definition 3.5 ($1{-}n$ MSC)**.** A p2p MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is a $1{-}n$ *MSC* if it has a linearization $\rightsquigarrow$ where, for any two send events s and s', such that $\lambda(s) = \mathit{Send}(p, \_, \_)$, $\lambda(s') = \mathit{Send}(p, \_, \_)$, and $s \rightarrow^{+} s'$ (which implies $s \rightsquigarrow s'$), we have either:

- $s, s' \in \mathit{Matched}(M)$ and $r \rightsquigarrow r'$, where r and r' are two receive events such that $s \lhd r$ and $s' \lhd r'$.
- $s' \in \mathit{Unm}(M)$.

Such a linearization will be referred to as a $1{-}n$ *linearization*. Note that the definition is very similar to the mailbox case, but here s and s' are two send events executed by the same process. Let MSC_{1-n} denote the set of all the $1{-}n$ MSCs over two given sets $\mathbb{P}$ and $\mathbb{M}$. By definition, every $1{-}n$ MSC is a p2p MSC. Conversely, Example **??** shows a p2p MSC which is not a $1{-}n$ MSC. It follows that $\mathsf{MSC}_{\mathsf{mb}} \subset \mathsf{MSC}_{\mathsf{p2p}}$. We show here that each $1{-}n$ MSC is also a causally ordered MSC, which is not as intuitive as the mailbox case.

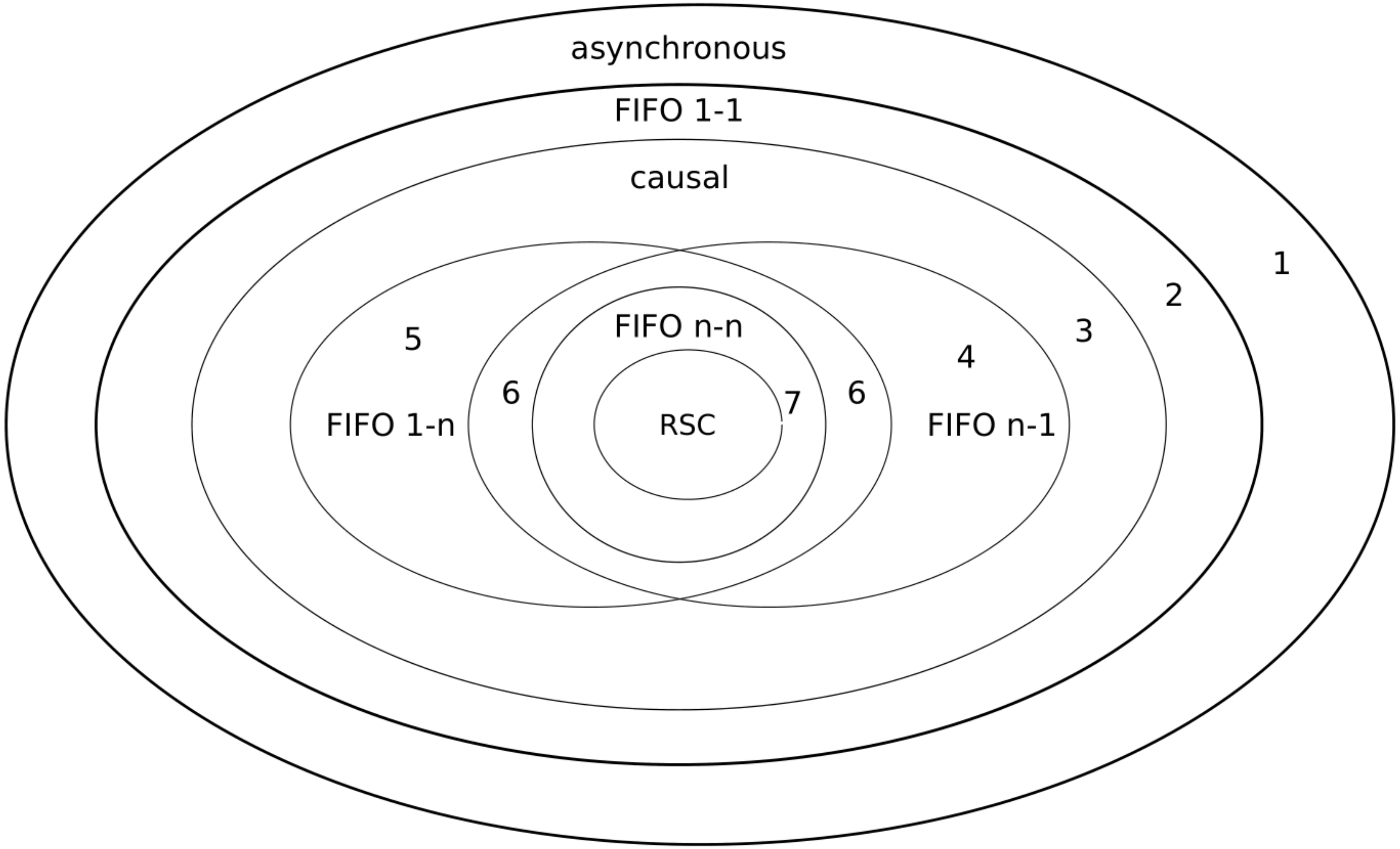
$1-n$ MSC - alternative definition

Definition 3.6 (1−n alternative)**.** For a p2p MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$, we define an additional binary relation that represents a constraint under the 1−n semantics, which ensures that messages sent from the same process are received in the same order. Let $\blacktriangleleft_M \subseteq \mathcal{E} \times \mathcal{E}$ be defined as $e_1 \blacktriangleleft_M e_2$ if there are two events e_1 and e_2, and $p \in \mathbb{P}$ such that either:

- $\lambda(e_1) \in \mathit{Send}(p, \_, \_)$, $\lambda(e_2) \in \mathit{Send}(p, \_, \_)$, $e_1 \in \mathit{Matched}(M)$, and $e_2 \in \mathit{Unm}(M)$, or
- $\lambda(e_1) \in \mathit{Rec}(p, \_, \_)$, $\lambda(e_2) \in \mathit{Rec}(p, \_, \_)$, $s_1 \lhd e_1$ and $s_2 \lhd e_2$ for some $s_1, s_2 \in \mathcal{E}_p$, and $s_1 \rightarrow^+ s_2$.

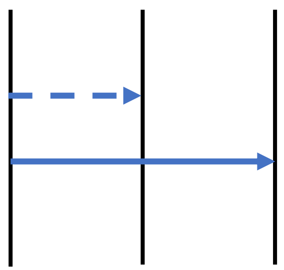
We let $\lessdot_M = (\rightarrow \cup \lhd \cup \blacktriangleleft_M)^*$. Note that $\leq_M \subseteq \lessdot_M$. We call $M \in \mathsf{MSC_{p2p}}$ a 1−n *MSC* if $\lessdot_M$ is a partial order.

Hierarchy of executions



Relation between mailbox and $\mathbf{1-n}$ MSCs

- Can we find an MSC that is $1-n$, but not mailbox?
- $1-n \subset mailbox$?
- If we do not consider unmatched messages, do we have $1-n = mailbox$?



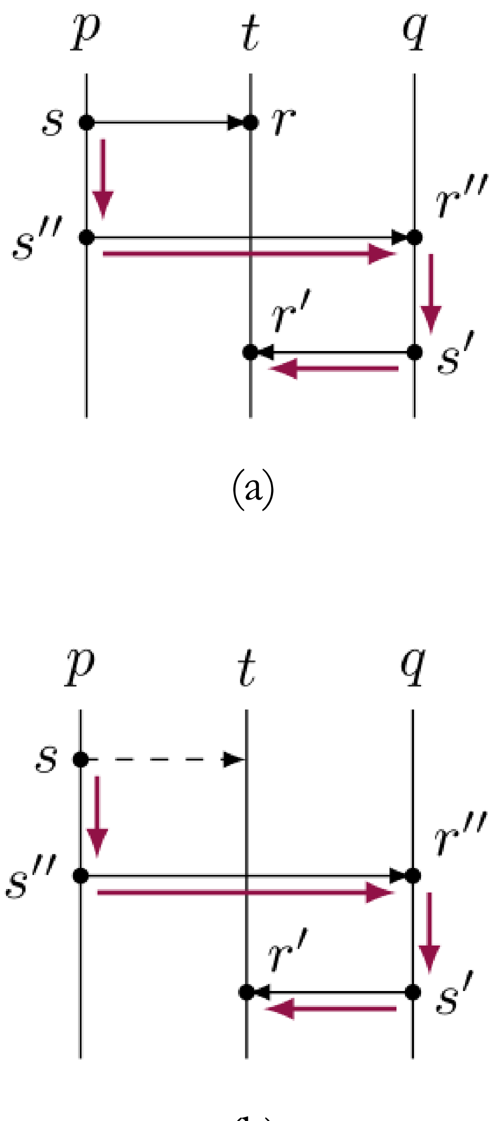
mailbox, but not $1-n$

Proposition 3.2. Every $1-n$ MSC is a causally ordered MSC.

Proof. By contradiction. Suppose that M is a $1-n$ MSC, but not a causally ordered MSC. Since M is not causally ordered, there must be two send events s and s' such that $\lambda(s) = Send(\_, q, \_)$, $\lambda(s') = Send(\_, q, \_)$, $s \leq_M s'$, and we have either:

1. $s, s' \in Matched(M)$ and $r' \rightarrow^* r$, where r and r' are two receive events such that $s \lhd r$ and $s' \lhd r'$.
2. $s \in Unm(M)$ and $s' \in Matched(M)$.

We need to show that both of these scenarios lead to a contradiction. (1) Suppose s and s' are executed by the same process. Since M is a $1-n$ MSC, there must be a linearization $\rightsquigarrow$ such that $r \rightsquigarrow r'$, but this is clearly impossible since we have $r' \rightarrow^* r$. Suppose now that s and s' are executed by two different processes p and q. We know by hypothesis that $s \leq_M s'$, i.e. there is a causal path of events $P = s \sim a \sim \cdots \sim s' \sim r'$ from s to r', where $\sim$ is either $\rightarrow$ or $\lhd$. Refer to the first example in Figure 6 for a visual representation (P is drawn in purple). To have a causal path P, there must be a send event s'' that is executed by p after s and that is part of P, along with its receipt r'' (i.e. $P = s \leq_M s'' \lhd r'' \leq_M s' \lhd r'$). We clearly have $r'' \rightsquigarrow r'$ for any linearization of M, because $r'' \leq_M r'$ (they are both in the causal path P and r'' happens before r). Since M is a $1-n$ MSC, there has to be a linearization $\rightsquigarrow$ where $r \rightsquigarrow r''$, because s and s'' are send events executed by the same process. It follows that M should have a linearization were $r \rightsquigarrow r'' \rightsquigarrow r'$, but this is not possible because of the hypothesis that $r' \rightarrow^* r$. This is a contradiction. (2) Suppose s and s' are executed by the same process. It is trivial to see, by definition, that M cannot be a $1-n$ MSC. Suppose now that s and s' are executed by two different processes p and q, and consider the same send event s'' as before (executed by p). Refer to the second example in Figure 6 for a visual representation. Since s'' is matched, we have two events s and s'', sent by the same process p, that are unmatched and matched, respectively. Clearly, M cannot be a $1-n$ MSC. $\square$



(a)

(b)

$1 - n$ vs mailbox

Proposition 3.3. Every $1{-}n$ MSC without unmatched messages is a mailbox MSC.

Proposition 3.4. Every $1{-}n$ MSC is a mailbox MSC.

Proofs can be found in the report...

MSO-definability of $1 - n$ MSCs

$1-n$ MSCs Given an MSC M, it is a $1-n$ MSC if it satisfies the MSO formula

$$\varphi_{1-n} = \neg\exists x.\exists y.(\neg(x = y) \wedge x \lessdot_M y \wedge y \lessdot_M x)$$

Here, $x \lessdot_M y$ is the MSO-definable transitive closure of the union of the MSO-definable relations $\rightarrow$, $\lhd$, and $\blacktriangleleft_M$. In particular, we can define $x \blacktriangleleft_M y$ as

$$x \blacktriangleleft_M y = \begin{array}{l} \left(\bigvee_{\substack{p \in \mathbb{P} \\ a,b \in Send(p,\_,\_)}} (\lambda(x) = a \ \wedge \ \lambda(y) = b) \ \wedge \ matched(x) \ \wedge \ \neg matched(y) \right) \ \vee \\ \left(\bigvee_{\substack{p \in \mathbb{P} \\ a,b \in Rec(p,\_,\_)}} (\lambda(x) = a \ \wedge \ \lambda(y) = b) \ \wedge \ \exists x'.\exists y'.(x' \lhd x \ \wedge \ y' \lhd y \ \wedge \ x' \rightarrow^{+} y') \right) \end{array}$$

The MSO formula for $x \blacktriangleleft_M y$ closely follows Definition 3.6. The set MSC_{1-n} of mailbox MSCs is therefore MSO-definable as $\mathsf{MSC}_{1-n} = L(\varphi_{1-n})$.

MSO-definability

MSO definability	Raw	Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	Y	Y	Y	?	?
FIFO 1-1 (p2p)	Y	Y	Y	Y	Y
Causally ordered	Y	Y	Y	Y	Y
FIFO n-1 (mailbox)	Y	Y	Y	Y	Y
FIFO 1-n	Y				
FIFO n-n					
RSC					

= new

= changed

$n-n$ MSC - definition

Definition 3.7 (n−n MSC)**.** An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is a n−n *MSC* if it has a linearization $\leadsto$ where, for any two send events s and s', we have either:

- $s, s' \in Matched(M)$ and $r \leadsto r'$, where r and r' are two receive events such that $s \lhd r$ and $s' \lhd r'$.
- $s' \in Unm(M)$.

Intuitively, with an n−n MSC we are always able to shedule events in such a way that messages are received in the same order as they were sent.

Relation between $n - n$ and $1 - n$ MSCs

Conjecture: $MSC_{1-n} = MSC_{n-n}$