Week 10

Overview

- Decidability of synchronizability:
  - Preliminary results for $1 - n$ and $n - n$
- MSO-definability $\exists k$-bounded and $\forall k$-bounded asynchronous MSC
- RSC communication model:
  - Definition
  - MSO-definability
  - STW-boundness (*idea*)

MSO - definability

MSO definability		Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	✓	✓	✓	✓	✓
FIFO 1-1 (p2p)	✓	✓	✓	✓	✓
Causally ordered	✓	✓	✓	✓	✓
FIFO n-1 (mailbox)	✓	✓	✓	✓	✓
FIFO 1-n	✓	✓	✓	✓	✓
FIFO n-n	✓	✓	✓	✓	✓
RSC	✓	✓	✓	✓	✓

STW - boundness

Bounded STW		Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	✗	✗	✓	✓	✓
FIFO 1-1 (p2p)	✗	✗	✓	✓	✓
Causally ordered	✗	✗	✓	✓	✓
FIFO n-1 (mailbox)	✗	✓	✓	✓	✓
FIFO 1-n	✗	✓	✓	✓	✓
FIFO n-n	✗	✓	✓	✓	✓
RSC	✓	✓	✓	✓	✓

1-n and n-n preliminary results

Preliminary results

An important component of the decidability proof is the following lemma, which shows that we can reduce synchronizability wrt. an STW-bounded class to bounded model-checking.

▶ **Lemma 9.** *Let $\mathcal{S}$ be a communicating system,* com $\in \{\mathsf{p2p}, \mathsf{mb}\}$*, $k \in \mathbb{N}$, and $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-}\mathsf{stw}}$. Then, $L_{\mathrm{com}}(\mathcal{S}) \subseteq \mathcal{C}$ iff $L_{\mathrm{com}}(\mathcal{S}) \cap \mathsf{MSC}^{(k+2)\text{-}\mathsf{stw}} \subseteq \mathcal{C}$.*

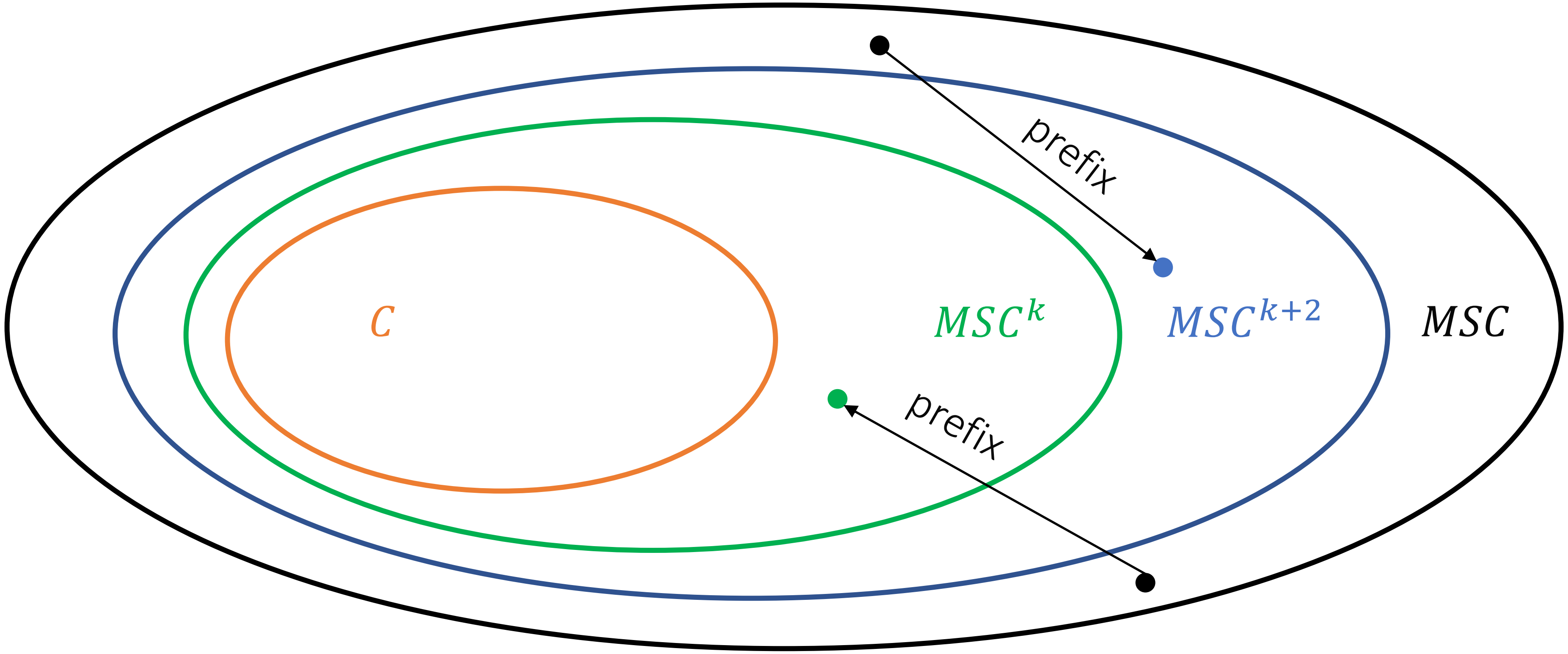
The result follows from the following lemma. Note that a similar property was shown in [18, Proposition 5.4] for the specific class of existentially k-bounded MSCs.

▶ **Lemma 10.** *Let $k \in \mathbb{N}$ and $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-}\mathsf{stw}}$. For all $M \in \mathsf{MSC} \setminus \mathcal{C}$, we have $(\mathit{Pref}(M) \cap \mathsf{MSC}^{(k+2)\text{-}\mathsf{stw}}) \setminus \mathcal{C} \neq \emptyset$.*

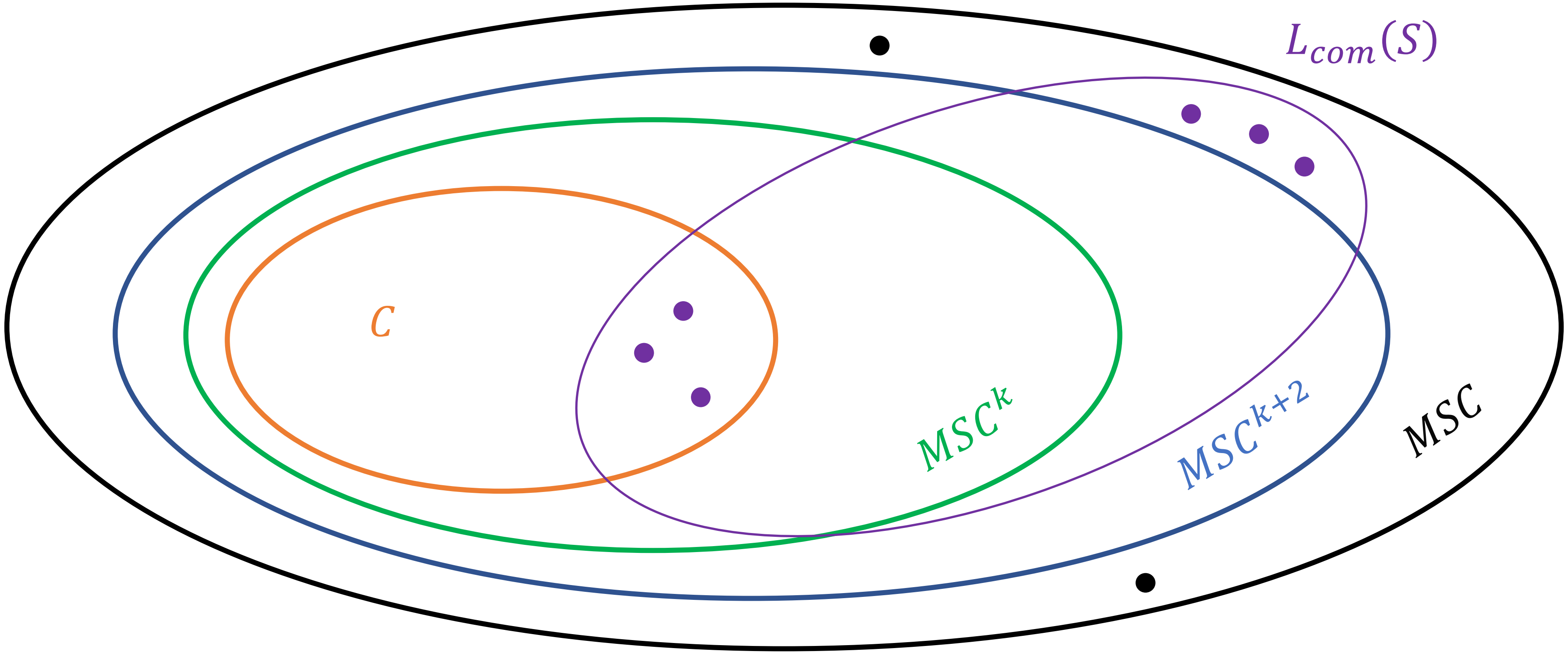
We now have all ingredients to state a generic decidability result for synchronizability:

▶ **Theorem 11.** *Fix finite sets $\mathbb{P}$ and $\mathbb{M}$. Suppose* com $\in \{\mathsf{p2p}, \mathsf{mb}\}$ *and let $\mathcal{C} \subseteq \mathsf{MSC}$ be an MSO-definable and STW-bounded class (over $\mathbb{P}$ and $\mathbb{M}$). The following problem is decidable: Given a communicating system $\mathcal{S}$, do we have $L_{\mathrm{com}}(\mathcal{S}) \subseteq \mathcal{C}$?*

▶ **Lemma 10.** *Let* $k \in \mathbb{N}$ *and* $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-stw}}$. *For all* $M \in \mathsf{MSC} \setminus \mathcal{C}$, *we have* $(\mathit{Pref}(M) \cap \mathsf{MSC}^{(k+2)\text{-stw}}) \setminus \mathcal{C} \neq \emptyset$.

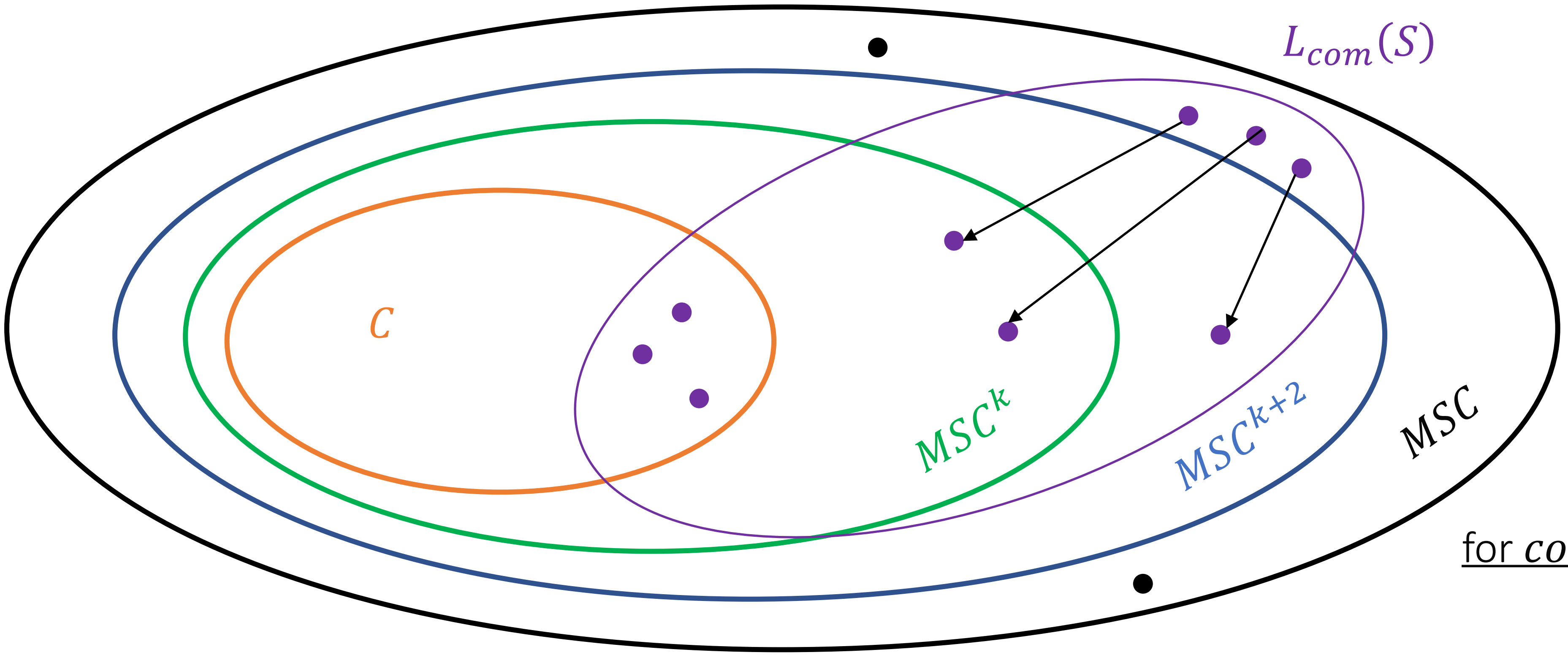


▶ **Lemma 9.** *Let $\mathcal{S}$ be a communicating system,* $\mathrm{com} \in \{\mathsf{p2p}, \mathsf{mb}\}$*,* $k \in \mathbb{N}$*, and* $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-stw}}$. *Then,* $L_{\mathrm{com}}(\mathcal{S}) \subseteq \mathcal{C}$ *iff* $L_{\mathrm{com}}(\mathcal{S}) \cap \mathsf{MSC}^{(k+2)\text{-stw}} \subseteq \mathcal{C}$.



Is this scenario possible?

▶ **Lemma 9.** *Let* $\mathcal{S}$ *be a communicating system,* $\mathrm{com} \in \{\mathsf{p2p}, \mathsf{mb}\}$*,* $k \in \mathbb{N}$*, and* $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-stw}}$. *Then,* $L_{\mathrm{com}}(\mathcal{S}) \subseteq \mathcal{C}$ *iff* $L_{\mathrm{com}}(\mathcal{S}) \cap \mathsf{MSC}^{(k+2)\text{-stw}} \subseteq \mathcal{C}$.



for $com \in \{p2p, mailbox\}$

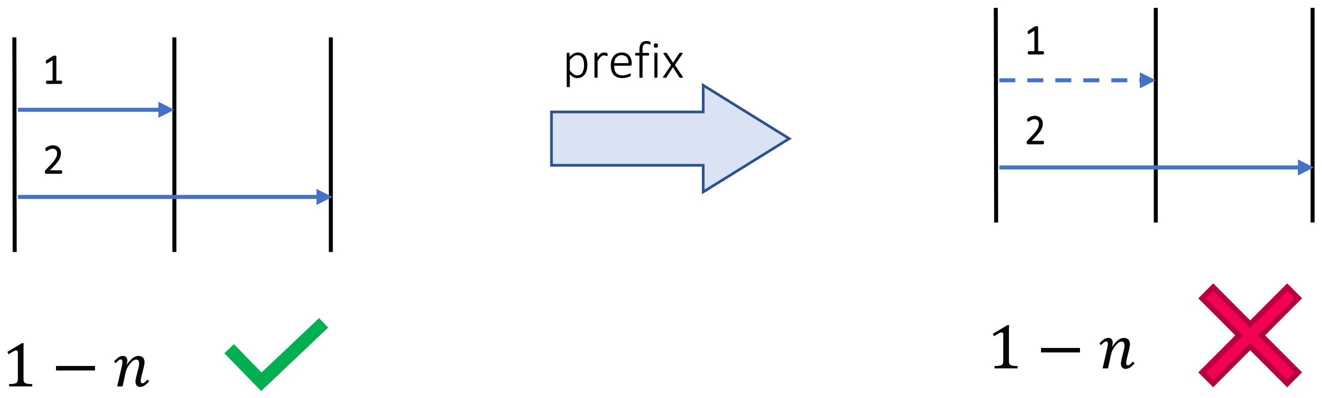
Is this scenario possible? **NO**, because of Lemma 10 and prefix-closure of $L_{com}(S)$

▶ **Lemma 9.** *Let $\mathcal{S}$ be a communicating system,* $\mathrm{com} \in \{\mathsf{p2p}, \mathsf{mb}\}$, $k \in \mathbb{N}$, *and* $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-}\mathsf{stw}}$. *Then,* $L_{\mathrm{com}}(\mathcal{S}) \subseteq \mathcal{C}$ *iff* $L_{\mathrm{com}}(\mathcal{S}) \cap \mathsf{MSC}^{(k+2)\text{-}\mathsf{stw}} \subseteq \mathcal{C}$.

Proof based on Lemma 10 and prefix-closure of $L_{com}(S)$

for $com \in \{p2p, mailbox\}$

Problem: $L_{1-n}(S)$ is not prefix-closed ☹



$1 - n$ prefixes

Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}$ and consider $E \subseteq \mathcal{E}$ such that E is $\leq_M$-*downward-closed*, i.e, for all $(e, f) \in \leq_M$ such that $f \in E$, we also have $e \in E$. Then, the MSC $(E, \rightarrow \cap (E \times E), \lhd \cap (E \times E), \lambda')$, where λ' is the restriction of $\mathcal{E}$ to E, is called a *prefix* of M. In particular, the empty MSC is a prefix of M. We denote the set of prefixes of M by $\mathit{Pref}(M)$. This is extended to sets $L \subseteq \mathsf{MSC}$ as expected, letting $\mathit{Pref}(L) = \bigcup_{M \in L} \mathit{Pref}(M)$.

Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}_{1-n}$ and consider $E \subseteq \mathcal{E}$ such that E is $\lessdot_M$-*downward-closed*, i.e, for all $(e, f) \in \lessdot_M$ such that $f \in E$, we also have $e \in E$. Then, the MSC $(E, \rightarrow \cap (E \times E), \lhd \cap (E \times E), \lambda')$, where λ' is the restriction of $\mathcal{E}$ to E, is called a *$1{-}n$ prefix* of M. We denote the set of $1{-}n$ prefixes of M by $\mathit{Pref}_{1-n}(M)$.

$1 - n$ preliminary results

Lemma 2.3. *Every $1{-}n$ prefix of a $1{-}n$ MSC is a $1{-}n$ MSC.*

Proof. Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}_{1-n}$ and let $M_0 = (\mathcal{E}_0, \rightarrow_0, \lhd_0, \lambda_0)$ be a $1{-}n$ prefix of M, where $\mathcal{E}_0 \subseteq \mathcal{E}$. Firstly, the $\lessdot_M$-downward-closeness of $\mathcal{E}_0$ guarantees that M_0 is still an MSC. We need to prove that it is a $1{-}n$ MSC. By contradiction, suppose that M_0 is not a $1{-}n$ MSC. Then, there are distinct $e, f \in \mathcal{E}_0$ such that $e \lessdot_{M_0} f \lessdot_{M_0} e$, where $\lessdot_{M_0} = (\rightarrow_0 \cup \lhd_0 \cup \blacktriangleleft_{M_0})^*$. As $\mathcal{E}_0 \subseteq \mathcal{E}$, we have that $\rightarrow_0 \subseteq \rightarrow$, $\lhd_0 \subseteq \lhd$, $\blacktriangleleft_{M_0} \subseteq \blacktriangleleft_M$. Clearly, $\lessdot_{M_0} \subseteq \lessdot_M$, so $e \lessdot_M f \lessdot_M e$. This implies that M is not a $1{-}n$ MSC, because $\lessdot_M$ is cyclic, which is a contradiction. Hence M_0 is a $1{-}n$ MSC. $\square$

Note that every $1{-}n$ prefix is also a prefix.

$1 - n$ prefix closure

Lemma 2.6. *$L_{1-n}(\mathcal{S})$ is $1-n$ prefix-closed: $Pref_{1-n}(L_{1-n}(\mathcal{S})) \subseteq L_{1-n}(\mathcal{S})$.*

Proof. Given a system $\mathcal{S}$, we have that $L_{1-n}(\mathcal{S}) = L_{\mathsf{p2p}}(\mathcal{S}) \cap \mathsf{MSC}_{1-n}$. Note that, because of how we defined a $1-n$ prefix, we have that $Pref_{1-n}(L_{1-n}(\mathcal{S})) = Pref(L_{1-n}(\mathcal{S})) \cap \mathsf{MSC}_{1-n}$. Moreover, $Pref(L_{1-n}(\mathcal{S})) \subseteq Pref(L_{\mathsf{p2p}}(\mathcal{S}))$, and $Pref(L_{1-n}(\mathcal{S})) \subseteq L_{\mathsf{p2p}}(\mathcal{S})$ for Lemma 1.2. Putting everything together, $Pref_{1-n}(L_{1-n}(\mathcal{S})) \subseteq L_{\mathsf{p2p}}(\mathcal{S}) \cap \mathsf{MSC}_{1-n} = L_{1-n}(\mathcal{S})$. □

$1 - n$ preliminary results

Lemma 2.12. *Let $k \in \mathbb{N}$ and $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-stw}}$. For all $M \in \mathsf{MSC}_{1-n} \setminus \mathcal{C}$, we have $(\mathit{Pref}_{1-n}(M) \cap \mathsf{MSC}^{(k+2)\text{-stw}}) \setminus \mathcal{C} \neq \emptyset$.*

Proof. Let k and $\mathcal{C}$ be fixed, and let $M \in \mathsf{MSC}_{1-n} \setminus \mathcal{C}$ be fixed. If the empty MSC is not in $\mathcal{C}$, then we are done, since it is a valid $1-n$ prefix of M and it is in $\mathsf{MSC}^{(k+2)\text{-stw}} \setminus \mathcal{C}$. Otherwise, let $M' \in \mathit{Pref}_{1-n}(M) \setminus \mathcal{C}$ such that, for all $\lessdot_M$-maximal events e of M', removing e (along with its adjacent edges) gives an MSC in $\mathcal{C}$. In other words, M' is the "shortest" prefix of M that is not in $\mathcal{C}$. We obtain such an MSC by successively removing $\lessdot_M$-maximal events. Let e be $\lessdot_{M'}$-maximal and let $M'' = M' \setminus \{e\}$. Since M' was taken minimal in terms of number of events, $M'' \in \mathcal{C}$. The proof proceeds exactly as the proof of Lemma 2.11. $\square$

$1 - n$ preliminary results

Lemma 2.15. *Let $\mathcal{S}$ be a communicating system, $k \in \mathbb{N}$, and $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-}\mathsf{stw}}$. Then, $L_{1-n}(\mathcal{S}) \subseteq \mathcal{C}$ iff $L_{1-n}(\mathcal{S}) \cap \mathsf{MSC}^{(k+2)\text{-}\mathsf{stw}} \subseteq \mathcal{C}$.*

Proof. Follows from Lemma 2.6 and Lemma 2.12. $\square$

$n-n$ prefixes

Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}$ and consider $E \subseteq \mathcal{E}$ such that E is $\leq_M$-*downward-closed*, i.e, for all $(e, f) \in \leq_M$ such that $f \in E$, we also have $e \in E$. Then, the MSC $(E, \rightarrow \cap (E \times E), \lhd \cap (E \times E), \lambda')$, where λ' is the restriction of $\mathcal{E}$ to E, is called a *prefix* of M. In particular, the empty MSC is a prefix of M. We denote the set of prefixes of M by $Pref(M)$. This is extended to sets $L \subseteq \mathsf{MSC}$ as expected, letting $Pref(L) = \bigcup_{M \in L} Pref(M)$.

Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}_{n-n}$ and consider $E \subseteq \mathcal{E}$ such that E is $\bowtie_M$-*downward-closed*, i.e, for all $(e, f) \in \bowtie_M$ such that $f \in E$, we also have $e \in E$. Then, the MSC $(E, \rightarrow \cap (E \times E), \lhd \cap (E \times E), \lambda')$, where λ' is the restriction of $\mathcal{E}$ to E, is called a *$n-n$ prefix* of M. We denote the set of $n-n$ prefixes of M by $Pref_{n-n}(M)$.

$n-n$ preliminary results

Lemma 2.4. *Every $n{-}n$ prefix of a $n{-}n$ MSC is a $n{-}n$ MSC.*

Proof. Let $M = (\mathcal{E}, \rightarrow, \lhd, \lambda) \in \mathsf{MSC}_{n-n}$ and let $M_0 = (\mathcal{E}_0, \rightarrow_0, \lhd_0, \lambda_0)$ be a $n{-}n$ prefix of M, where $\mathcal{E}_0 \subseteq \mathcal{E}$. Firstly, the $\bowtie_M$-downward-closeness of $\mathcal{E}_0$ guarantees that M_0 is still an MSC. We need to prove that it is a $n{-}n$ MSC. By contradiction, suppose that M_0 is not a $n{-}n$ MSC. Then, there are distinct $e, f \in \mathcal{E}_0$ such that $e \bowtie_{M_0} f \bowtie_{M_0} e$. As $\mathcal{E}_0 \subseteq \mathcal{E}$, we have that $\rightarrow_0 \subseteq \rightarrow$, $\lhd_0 \subseteq \lhd$, $\ltimes_0 \subseteq \ltimes$. Clearly, $\bowtie_{M_0} \subseteq \bowtie_M$, so $e \bowtie_M f \bowtie_M e$. This implies that M is not a $n{-}n$ MSC, because $\bowtie_M$ is cyclic, which is a contradiction. Hence M_0 is a $n{-}n$ MSC. $\square$

Note that every $n{-}n$ prefix is also a prefix.

$n - n$ prefix closure

Lemma 2.7. *$L_{n-n}(\mathcal{S})$ is $n{-}n$ prefix-closed: $Pref_{n-n}(L_{n-n}(\mathcal{S})) \subseteq L_{n-n}(\mathcal{S})$.*

Proof. Given a system $\mathcal{S}$, we have that $L_{n-n}(\mathcal{S}) = L_{\mathsf{p2p}}(\mathcal{S}) \cap \mathsf{MSC}_{n-n}$. Note that, because of how we defined a $n{-}n$ prefix, we have that $Pref_{n-n}(L_{n-n}(\mathcal{S})) = Pref(L_{n-n}(\mathcal{S})) \cap \mathsf{MSC}_{n-n}$. Moreover, $Pref(L_{n-n}(\mathcal{S})) \subseteq Pref(L_{\mathsf{p2p}}(\mathcal{S}))$, and $Pref(L_{n-n}(\mathcal{S})) \subseteq L_{\mathsf{p2p}}(\mathcal{S})$ for Lemma 1.2. Putting everything together, $Pref_{n-n}(L_{n-n}(\mathcal{S})) \subseteq L_{\mathsf{p2p}}(\mathcal{S}) \cap \mathsf{MSC}_{n-n} = L_{n-n}(\mathcal{S})$. $\square$

$n-n$ preliminary results

Lemma 2.13. *Let* $k \in \mathbb{N}$ *and* $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-stw}}$*. For all* $M \in \mathsf{MSC}_{n-n} \setminus \mathcal{C}$*, we have* $(\mathit{Pref}_{n-n}(M) \cap \mathsf{MSC}^{(k+2)\text{-stw}}) \setminus \mathcal{C} \neq \emptyset$*.*

Proof. Let k and $\mathcal{C}$ be fixed, and let $M \in \mathsf{MSC}_{n-n} \setminus \mathcal{C}$ be fixed. If the empty MSC is not in $\mathcal{C}$, then we are done, since it is a valid $n-n$ prefix of M and it is in $\mathsf{MSC}^{(k+2)\text{-stw}} \setminus \mathcal{C}$. Otherwise, let $M' \in \mathit{Pref}_{n-n}(M) \setminus \mathcal{C}$ such that, for all $\bowtie_M$-maximal events e of M', removing e (along with its adjacent edges) gives an MSC in $\mathcal{C}$. In other words, M' is the "shortest" prefix of M that is not in $\mathcal{C}$. We obtain such an MSC by successively removing $\bowtie_M$-maximal events. Let e be $\bowtie_{M'}$-maximal and let $M'' = M' \setminus \{e\}$. Since M' was taken minimal in terms of number of events, $M'' \in \mathcal{C}$. The proof proceeds exactly as the proof of Lemma 2.11. $\square$

$n-n$ preliminary results

Lemma 2.16. *Let $\mathcal{S}$ be a communicating system, $k \in \mathbb{N}$, and $\mathcal{C} \subseteq \mathsf{MSC}^{k\text{-}\mathsf{stw}}$. Then, $L_{n-n}(\mathcal{S}) \subseteq \mathcal{C}$ iff $L_{n-n}(\mathcal{S}) \cap \mathsf{MSC}^{(k+2)\text{-}\mathsf{stw}} \subseteq \mathcal{C}$.*

Proof. Follows from Lemma 2.7 and Lemma 2.13. $\square$

MSO-definability of $\exists k$ and $\forall k$-bounded asynchronous MSCs

$\exists k$ and $\forall k$-bounded

Following the approach taken in Lohrey and Muscholl 2002, we introduce a binary relation $\longmapsto_k$ ($\rightsquigarrow_b$ in their work) associated with a given bound k and an MSC M. Let $k \geq 1$ and M be a fixed MSC: we have $r \longmapsto_k s$ if, for some $i \geq 1$ and some channel (p,q)[9]:

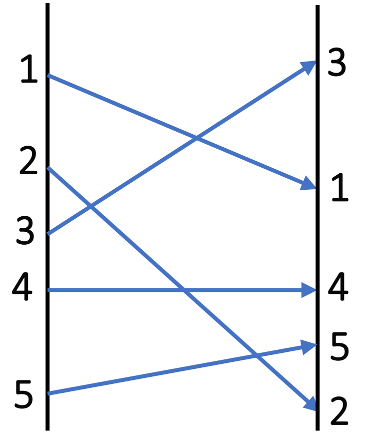
1. r is the i-th receive event (executed by q).
2. s is the $(i+k)$-th send event (executed by p).

Note that, for any two events s and r such that $r \longmapsto_k s$, every linearization of M in which r is executed after s cannot k-bounded. Intuitively, we can read $r \longmapsto_k s$ as "r has to be executed before s in a k-bounded linearization". A linearization $\rightsquigarrow$ that respects $\longmapsto_k$ (i.e. $\longmapsto_k \subseteq \rightsquigarrow$) is k-bounded.

$\exists k$ and $\forall k$-bounded

In Lohrey and Muscholl 2002 it was shown that an MSC is $\exists k$-bounded if and only if the relation $\leq_M \cup \longmapsto_k$ is acyclic[10]. Since $\leq_M$ and acyclicity are both MSO-definable, it suffices to find an MSO formula that defines the $\longmapsto_k$ relation to claim the MSO-definability of $\exists k$-bounded MSCs. Unfortunately, this relation is not MSO-definable for asynchronous MSCs, because MSO logic cannot be used to "count" for an arbitrary i. For this reason, we introduce another similar MSO-definable binary relation $\hookrightarrow_k$, and we show that with this new relation we still have that an MSC M is $\exists k$-bounded MSC iff $\leq_M \cup \hookrightarrow_k$ is acyclic and another condition holds. Let $k \geq 1$ and M be a fixed MSC: we have $r \hookrightarrow_k s$ if, for some $i \geq 1$ and some channel (p,q):

1. There are $k+1$ send events $(s_1, \ldots, s_k, s)$, where at least one is matched, such that $s_1 \rightarrow^+ \ldots \rightarrow^+ s_k \rightarrow^+ s$.
2. r is the first receive event among the receive events for the matched sends among $s_1, \ldots, s_k, s$.



$\exists$3-bounded
$\nexists$ 2-bounded

$!1, !2, !3, !4 \Rightarrow ?3 \hookrightarrow_3 !4$

$!1, !2, !3, !5 \Rightarrow ?3 \hookrightarrow_3 !5$

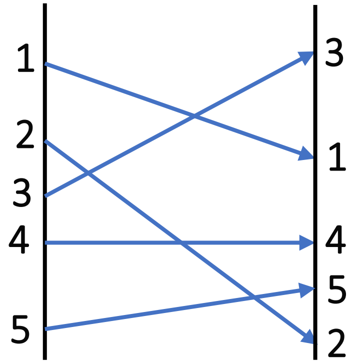
$!1, !2, !4, !5 \Rightarrow ?1 \hookrightarrow_3 !5$

$\leq \cup \hookrightarrow_3$ is acyclic

$!1, !2, !3 \Rightarrow ?3 \hookrightarrow_2 !3$

$\leq \cup \hookrightarrow_2$ is cyclic

$\exists k$ and $\forall k$-bounded



$\exists 3$-bounded
$\nexists 2$-bounded

$!1, !2, !3, !4 \Rightarrow ?3 \hookrightarrow_3 !4$

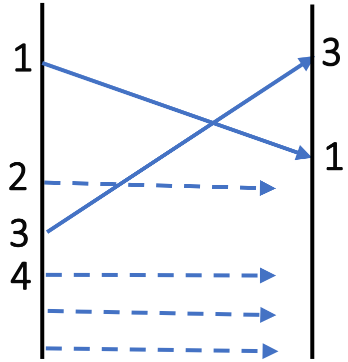
$!1, !2, !3, !5 \Rightarrow ?3 \hookrightarrow_3 !5$

$!1, !2, !4, !5 \Rightarrow ?1 \hookrightarrow_3 !5$

$\leq \cup \hookrightarrow_3$ is acyclic

$!1, !2, !3 \Rightarrow ?3 \hookrightarrow_2 !3$

$\leq \cup \hookrightarrow_2$ is cyclic



$\nexists 3$-bounded

$!1, !2, !3, !4 \Rightarrow ?3 \hookrightarrow_3 !4$

$!1, !2, !3, !5 \Rightarrow ?3 \hookrightarrow_3 !5$

$!1, !2, !3, !4 \Rightarrow ?3 \hookrightarrow_3 !6$

$\leq \cup \hookrightarrow_2$ is acyclic, but the MSC is not $\exists 3$-bounded

Proposition 3.9. An MSC M is $\exists k$-bounded if and only if $\leq_M \cup \hookrightarrow_k$ is acyclic and, for each channel (p,q), there are at most k unmatched send events.

Proof. ($\Rightarrow$) Suppose M is $\exists k$-bounded, which by definition means there is at least one $\exists k$-bounded linearization $\rightsquigarrow$. Firstly, notice that every MSC that has more than k unmatched send events in any channel cannot be an $\exists k$-bounded MSC. We already know that $\leq_M \subseteq \rightsquigarrow$, and we will show show that also $\hookrightarrow_k \subseteq \rightsquigarrow$. This implies that $\leq_M \cup \hookrightarrow_k$ is acyclic, otherwise we would not be able to find a linearization $\rightsquigarrow$ that respects both $\leq_M$ and $\hookrightarrow_k$. Suppose, by contradiction, that $\hookrightarrow_k \not\subseteq \rightsquigarrow$, i.e. there are two events r and s such that $r \hookrightarrow_k s$ and $s \rightsquigarrow r$. By definition of $\hookrightarrow_k$, there are k send events in a channel (p,q) that are executed before s, and whose respective receive events happens after r. If s is executed before r in the linearization, there will be $k+1$ messages in channel (i.e. $\rightsquigarrow$ is not a $\exists k$-bounded linearization). We reached a contradiction, hence $\hookrightarrow_k \subseteq \rightsquigarrow$ and $\leq_M \cup \hookrightarrow_k$ is acyclic.
($\Leftarrow$) Suppose $\leq_M \cup \hookrightarrow_k$ is acyclic and, for each channel (p,q), there are at most k unmatched send events. If $\leq_M \cup \hookrightarrow_k$ is acyclic, we are able to find at least one linearization $\rightsquigarrow$ for the partial order $(\leq_M \cup \hookrightarrow_k)^*$. We want to show that this linearization is $\exists k$-bounded. By contradiction, suppose $\rightsquigarrow$ is not $\exists k$-bounded, i.e. we are able to find $k+1$ send events $s_1 \rightarrow^+ \ldots \rightarrow^+ s_k \rightarrow^+ s$ on a channel (p,q), such that s is executed before any of the respective receive events takes place. There are two possible scenarios:

- Suppose all the $k+1$ send events are unmatched. This is impossible, since we supposed that there are at most k unmatched send events for any channel.
- Suppose there is at least one matched send event between the $k+1$ sends. Let the first matched send event be s_i and let r be the receive event that is executed first among the receive events for these $k+1$ sends. By hypothesis, $s \rightsquigarrow r$. However, according to the definition of $\hookrightarrow_k$, we must have $r \hookrightarrow_k s$. We reached a contradiction, since we cannot have that s happens before r in a linearization for the partial order $(\leq_M \cup \hookrightarrow_k)^*$, if $r \hookrightarrow_k s$.

$\square$

$\exists k$-bounded – MSO definability

According to Proposition 3.9, we can write the MSO formula the defines $\exists k$-bounded MSCs as

$$\Psi_{\exists k} = acyclic(\leq_M \cup \hookrightarrow_k) \ \wedge \ \neg \left(\exists s_1 \dots s_{k+1}. s_1 \rightarrow^+ \dots \rightarrow^+ s_{k+1} \ \wedge \ allSends\_p\_q \ \wedge \ allUnm\right)$$

$$allSends\_p\_q = \bigvee_{p \in \mathbb{P}, q \in \mathbb{P}} \ \bigwedge_{s \in s_1, \dots, s_{k+1}} \ \bigvee_{a \in Send(p,q,\_)} (\lambda(s) = a)$$

$$allUnm = \bigwedge_{s \in s_1, \dots, s_{k+1}} (\neg matched(s))$$

where $acyclic(\leq_M \cup \hookrightarrow_k)$ is an MSO formula that checks the acyclicity of the $\leq_M \cup \hookrightarrow_k$ relation, and the $\hookrightarrow_k$ relation can be defined as

$$r \hookrightarrow_k s = \exists s_1 \dots s_{k+1}. \left(\begin{array}{l} s_1 \rightarrow^+ \dots \rightarrow^+ s_{k+1} \ \wedge \ allSends\_p\_q \ \wedge \\ \exists r. (\bigvee_{s \in s_1, \dots, s_{k+1}} s \lhd r) \ \wedge \ \bigwedge_{e \in s_1, \dots, s_{k+1}} (\exists f. e \lhd f \implies r \rightarrow^* f) \end{array} \right)$$

RSC communication model

RSC MSC - definition

Definition 3.9 (RSC MSC). An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is a *RSC MSC* if it has no unmatched send events and there is a linearization $\rightsquigarrow$ where any matched send event is immediately followed by its respective receive event.

In order to characterize S-computations more precisely, we shall now look at a definition given by Bougé [10]: "*A system has synchronous communications if no message of a given type can be sent along a channel before the receiver is ready to receive (that is, in a state where the next action may be a reception of) a message of this type on the channel. For an external observer, the transmission then looks instantaneous and atomic. Sending and receiving a message correspond in fact to the same event.*"

RSC MSC – alternative definition

Following the characterization given in Charron-Bost, Mattern, and Tel 1996, Theorem 4.4, we give an alternative but equivalent definition of RSC MSC, which is based on a property of the conflict graph.

Definition 3.10 (RSC alternative). An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is a *RSC MSC* if and only if it has no unmatched send events and if in $CG(M)$ there is no cycle, of length ≥ 2, whose edges are all SR.

$\prec = \leq$

Theorem 4.4 (Crown criterion). *A computation C is RSC if and only if C contains no crown.*

Definition 4.1 (Crown). Let C be a computation. A crown (of size k) in C is a sequence $\langle (s_i, r_i), i \in \{1, \ldots, k\} \rangle$ of pairs of corresponding send and receive events such that

$$s_1 \prec r_2,\ s_2 \prec r_3,\ \ldots,\ s_{k-1} \prec r_k,\ s_k \prec r_1.$$

RSC MSC – MSO definability

Definition 3.10 (RSC alternative)**.** An MSC $M = (\mathcal{E}, \rightarrow, \lhd, \lambda)$ is a *RSC MSC* if and only if it has no unmatched send events and if in $CG(M)$ there is no cycle, of length ≥ 2, whose edges are all SR.

RSC **MSCs** Following Definition 3.10, an MSC M is a RSC MSC if and only if it satisfies the LCPDL formula

$$\Phi_{\mathsf{RSC}} = \neg \mathsf{Loop}\langle \xrightarrow{SR} \cdot (\xrightarrow{SR})^* \rangle$$

Recall that every LCPDL-definable property is also MSO-definable. In particular, this is one way to write the same property using MSO:

$$\Phi_{\mathsf{RSC}} = \neg \exists s_1 . \exists s_2 . s_1 \rightarrow_{SR} s_2 \ \wedge \ s_2 \rightarrow^*_{SR} s_1$$

where $\rightarrow_{SR}$ is defined as

$$s_1 \rightarrow_{SR} s_2 = \bigvee_{e in Send(\_,\_,\_)(\lambda(s_1)=e)} \wedge \exists s_2 . \exists r_2 . (s_1 \rightarrow^+ r_2 \ \wedge \ s_2 \lhd r_2)$$

RSC MSC – STW boundess

Idea: The class of RSC MSC is k bounded, where k is the number of processes

Proof: color the first event of each process and then remove messages one by one