



Week 6

Overview

- $1 - n$ MSCs:
 - Two alternative definitions
 - MSO-definability
 - Relation between causally ordered and $1 - n$ MSCs
- Existentially and universally k -bounded MSCs...
are they really MSO-definable? (week 5 fail)

1 – n MSC - definition

Moving on to the 1– n semantics, we say that M is a 1– n MSC if there is a 1– n system that can exhibit the behaviour described by M .

Definition 3.5 (1– n MSC). A p2p MSC $M = (\mathcal{E}, \rightarrow, \triangleleft, \lambda)$ is a 1– n MSC if it has a linearization \rightsquigarrow where, for any two send events s and s' , such that $\lambda(s) = \text{Send}(p, _, _)$, $\lambda(s') = \text{Send}(p, _, _)$, and $s \rightarrow^+ s'$ (which implies $s \rightsquigarrow s'$), we have either:

- $s, s' \in \text{Matched}(M)$ and $r \rightsquigarrow r'$, where r and r' are two receive events such that $s \triangleleft r$ and $s' \triangleleft r'$.
- $s' \in \text{Unm}(M)$.

Such a linearization will be referred to as a 1– n *linearization*. Note that the definition is very similar to the mailbox case, but here s and s' are two send events executed by the same process. Let MSC_{1-n} denote the set of all the 1– n MSCs over two given sets \mathbb{P} and \mathbb{M} . By definition, every 1– n MSC is a p2p MSC. Conversely, Example ?? shows a p2p MSC which is not a 1– n MSC. It follows that $\text{MSC}_{\text{mb}} \subset \text{MSC}_{\text{p2p}}$. We show here that each 1– n MSC is also a causally ordered MSC, which is not as intuitive as the mailbox case.

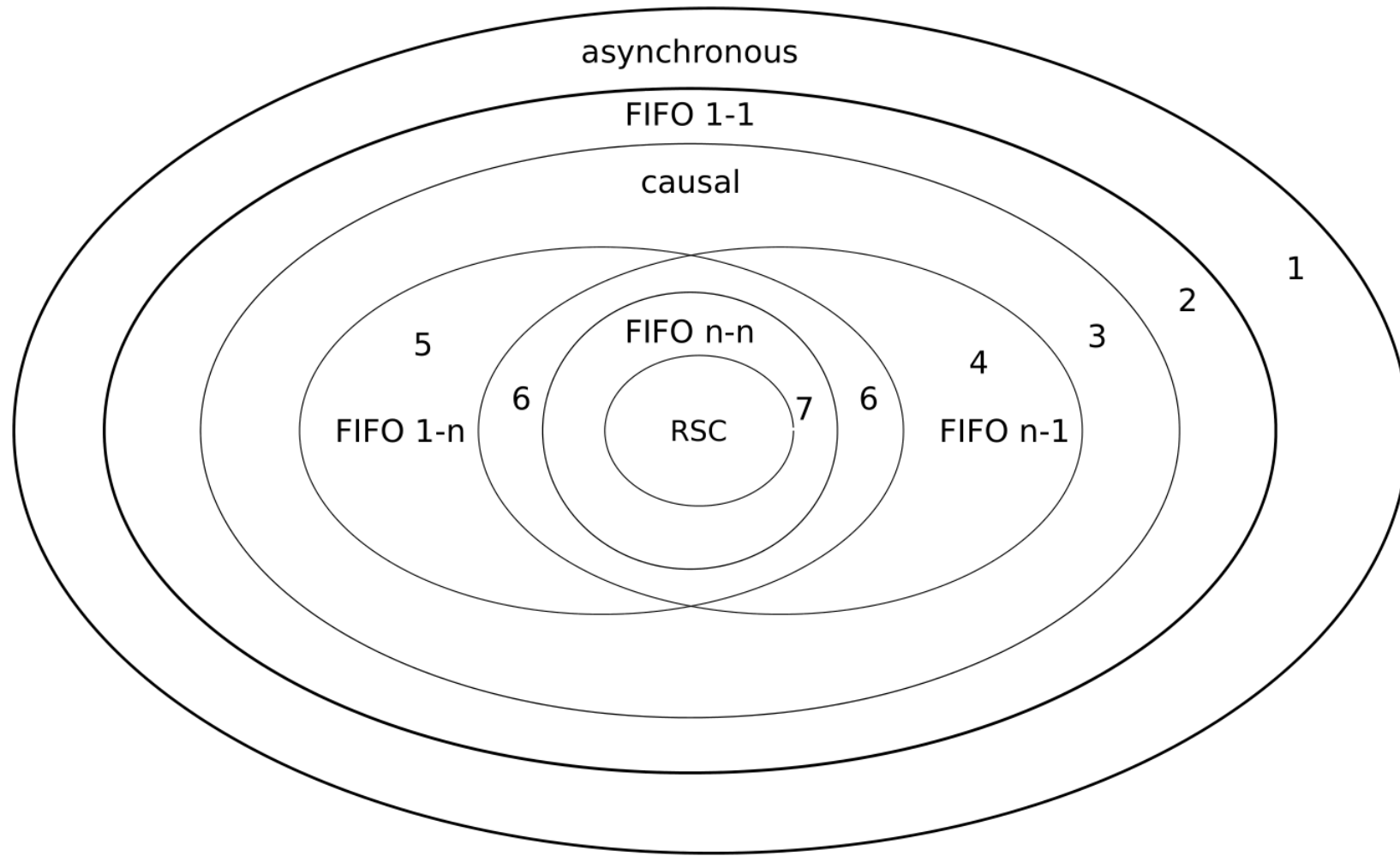
1 – n MSC - alternative definition

Definition 3.6 (1– n alternative). For a p2p MSC $M = (\mathcal{E}, \rightarrow, \triangleleft, \lambda)$, we define an additional binary relation that represents a constraint under the 1– n semantics, which ensures that messages sent from the same process are received in the same order. Let $\blacktriangleleft_M \subseteq \mathcal{E} \times \mathcal{E}$ be defined as $e_1 \blacktriangleleft_M e_2$ if there are two events e_1 and e_2 , and $p \in \mathbb{P}$ such that either:

- $\lambda(e_1) \in \text{Send}(p, _, _)$, $\lambda(e_2) \in \text{Send}(p, _, _)$, $e_1 \in \text{Matched}(M)$, and $e_2 \in \text{Unm}(M)$, or
- $\lambda(e_1) \in \text{Rec}(p, _, _)$, $\lambda(e_2) \in \text{Rec}(p, _, _)$, $s_1 \triangleleft e_1$ and $s_2 \triangleleft e_2$ for some $s_1, s_2 \in \mathcal{E}_p$, and $s_1 \rightarrow^+ s_2$.

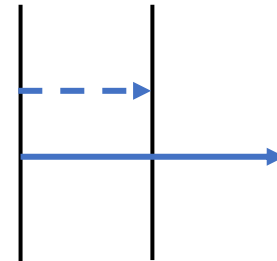
We let $\leq_M = (\rightarrow \cup \triangleleft \cup \blacktriangleleft_M)^*$. Note that $\leq_M \subseteq \triangleleft_M$. We call $M \in \text{MSC}_{\text{p2p}}$ a 1– n MSC if \leq_M is a partial order.

Hierarchy of executions



Relation between mailbox and $1 - n$ MSCs

- Can we find an MSC that is $1 - n$, but not mailbox?
- $1 - n \subset \text{mailbox}$?
- If we do not consider unmatched messages, do we have $1 - n = \text{mailbox}$?



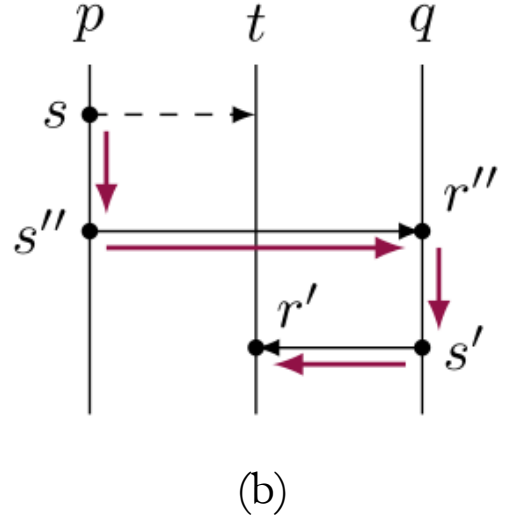
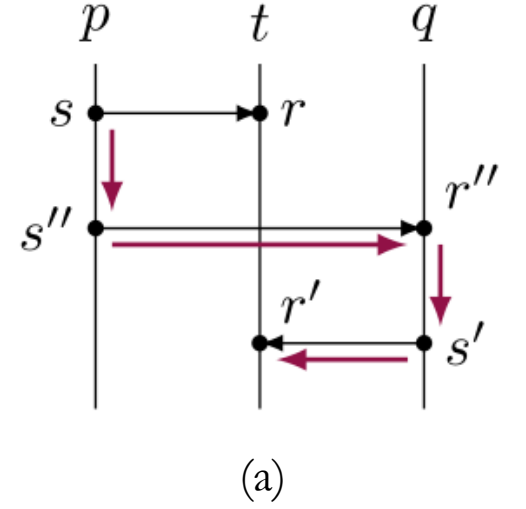
mailbox, but not $1 - n$

Proposition 3.2. Every 1- n MSC is a causally ordered MSC.

Proof. By contradiction. Suppose that M is a 1- n MSC, but not a causally ordered MSC. Since M is not causally ordered, there must be two send events s and s' such that $\lambda(s) = \text{Send}(_, q, _)$, $\lambda(s') = \text{Send}(_, q, _)$, $s \leq_M s'$, and we have either:

1. $s, s' \in \text{Matched}(M)$ and $r' \rightarrow^* r$, where r and r' are two receive events such that $s \triangleleft r$ and $s' \triangleleft r'$.
2. $s \in \text{Unm}(M)$ and $s' \in \text{Matched}(M)$.

We need to show that both of these scenarios lead to a contradiction. (1) Suppose s and s' are executed by the same process. Since M is a 1- n MSC, there must be a linearization \rightsquigarrow such that $r \rightsquigarrow r'$, but this is clearly impossible since we have $r' \rightarrow^* r$. Suppose now that s and s' are executed by two different processes p and q . We know by hypothesis that $s \leq_M s'$, i.e. there is a causal path of events $P = s \sim a \sim \dots \sim s' \sim r'$ from s to r' , where \sim is either \rightarrow or \triangleleft . Refer to the first example in Figure 6 for a visual representation (P is drawn in purple). To have a causal path P , there must be a send event s'' that is executed by p after s and that is part of P , along with its receipt r'' (i.e. $P = s \leq_M s'' \triangleleft r'' \leq_M s' \triangleleft r'$). We clearly have $r'' \rightsquigarrow r'$ for any linearization of M , because $r'' \leq_M r'$ (they are both in the causal path P and r'' happens before r). Since M is a 1- n MSC, there has to be a linearization \rightsquigarrow where $r \rightsquigarrow r''$, because s and s'' are send events executed by the same process. It follows that M should have a linearization where $r \rightsquigarrow r'' \rightsquigarrow r'$, but this is not possible because of the hypothesis that $r' \rightarrow^* r$. This is a contradiction. (2) Suppose s and s' are executed by the same process. It is trivial to see, by definition, that M cannot be a 1- n MSC. Suppose now that s and s' are executed by two different processes p and q , and consider the same send event s'' as before (executed by p). Refer to the second example in Figure 6 for a visual representation. Since s'' is matched, we have two events s and s'' , sent by the same process p , that are unmatched and matched, respectively. Clearly, M cannot be a 1- n MSC. \square



MSO-definability of $1 - n$ MSCs

$1-n$ **MSCs** Given an MSC M , it is a $1-n$ MSC if it satisfies the MSO formula

$$\varphi_{1-n} = \varphi_{p2p} \wedge \neg \exists x. \exists y. (\neg(x = y) \wedge x \prec_M y \wedge y \prec_M x)$$


Here, $x \prec_M y$ is the MSO-definable transitive closure of the union of the MSO-definable relations \rightarrow , \triangleleft , and \blacktriangleleft_M . In particular, we can define $x \blacktriangleleft_M y$ as

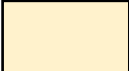
$$x \blacktriangleleft_M y = \left(\bigvee_{\substack{p \in \mathbb{P} \\ a, b \in \text{Send}(p, _, _)}} (\lambda(x) = a \wedge \lambda(y) = b) \wedge \text{matched}(x) \wedge \neg \text{matched}(y) \right) \vee \left(\bigvee_{\substack{p \in \mathbb{P} \\ a, b \in \text{Rec}(p, _, _)}} (\lambda(x) = a \wedge \lambda(y) = b) \wedge \exists x'. \exists y'. (x' \triangleleft x \wedge y' \triangleleft y \wedge x' \rightarrow^+ y') \right)$$

The MSO formula for $x \blacktriangleleft_M y$ closely follows Definition 3.6. The set MSC_{1-n} of mailbox MSCs is therefore MSO-definable as $\text{MSC}_{1-n} = L(\varphi_{1-n})$.

MSO-definability

<i>MSO definability</i>	Raw	Weakly sync	Weakly k-sync	$\exists k$ bounded	$\forall k$ bounded
Asynchronous	Y	Y	Y	?	?
FIFO 1-1 (p2p)	Y	Y	Y	Y	Y
Causally ordered	Y	Y	Y	Y	Y
FIFO n-1 (mailbox)	Y	Y	Y	Y	Y
FIFO 1-n	Y				
FIFO n-n					
RSC					

 = new

 = changed

Doubts

- Should I provide a clear semantics for CFSA with bag channels (FIFO is assumed in the classic description)?