SECOND FOLLOWAGE

Daniel Felipe Gómez Martínez
César Andrés García Posada
Juan Sebastian Pérez Salazar
Yhoan Alejandro Guzmán García

November 11 2020

Subject: Numerical Analysis
Professor in charge: Edwar Samir Posada Murillo
Semester: 6th
System name (project): SADA ANALYTICS
repository where we will work:
SADA_ANALYTICS

1 Methods test

for methods testing the following data is used

$$\mathbf{A} = \begin{bmatrix} 4 & -1 & 0 & 3 \\ 1 & 15.5 & 3 & 8 \\ 0 & -1.3 & -4 & 1.1 \\ 14 & 5 & -2 & 30 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \mathbf{Tol} = 1e{-7}, \mathbf{Nmax} = 100$$

Table =

x	**-1**	**0**	**3**	**4**
y	**15.5**	**3**	**8**	**1**

- **LU with simple Gaussian**
  1. $Matrix : A$
  2. $Constant\ vector : b$

  1. **Step 0:**

$$\begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 1.000000 & 15.500000 & 3.000000 & 8.000000 \\ 0.000000 & -1.300000 & -4.000000 & 1.100000 \\ 14.000000 & 5.000000 & -2.000000 & 30.000000 \end{bmatrix}$$

2. Step 1:

$$\begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & -1.300000 & -4.000000 & 1.100000 \\ 0.000000 & 8.500000 & -2.000000 & 19.500000 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.250000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 3.500000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

3. Step 2:

$$\begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & -3.752381 & 1.698413 \\ 0.000000 & 0.000000 & -3.619048 & 15.587302 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.250000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.082540 & 1.000000 & 0.000000 \\ 3.500000 & 0.539683 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & -3.752381 & 1.698413 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

4. Step 3:

$$\begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & -3.752381 & 1.698413 \\ 0.000000 & 0.000000 & 0.000000 & 13.949239 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.250000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.082540 & 1.000000 & 0.000000 \\ 3.500000 & 0.539683 & 0.964467 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & -3.752381 & 1.698413 \\ 0.000000 & 0.000000 & 0.000000 & 13.949239 \end{bmatrix}$$

5. **Solution:**

$$\mathbf{x} = \begin{bmatrix} 0.525109 \\ 0.255459 \\ -0.410480 \\ -0.281659 \end{bmatrix}$$

- **LU with partial pivoting**
  - $Matrix : A$
  - $Constant\ vector : b$

1. **Step 0:**

$$\mathbf{A} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 1.000000 & 15.500000 & 3.000000 & 8.000000 \\ 0.000000 & -1.300000 & -4.000000 & 1.100000 \\ 14.000000 & 5.000000 & -2.000000 & 30.000000 \end{bmatrix}$$

2. **Step 1:**

$$\mathbf{A} = \begin{bmatrix} 14.000000 & 5.000000 & -2.000000 & 30.000000 \\ 0.000000 & 15.142857 & 3.142857 & 5.857143 \\ 0.000000 & -1.300000 & -4.000000 & 1.100000 \\ 0.000000 & -2.428571 & 0.571429 & -5.571429 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.071429 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.285714 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 14.000000 & 5.000000 & -2.000000 & 30.000000 \\ 0.000000 & 15.142857 & 3.142857 & 5.857143 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.000000 & 0.000000 & 0.000000 & 1.000000 \\ 0.000000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 1.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

3. Step 2:

$$\mathbf{A} = \begin{bmatrix} 14.000000 & 5.000000 & -2.000000 & 30.000000 \\ 0.000000 & 15.142857 & 3.142857 & 5.857143 \\ 0.000000 & 0.000000 & -3.730189 & 1.602830 \\ 0.000000 & 0.000000 & 1.075472 & -4.632075 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.071429 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.085849 & 1.000000 & 0.000000 \\ 0.285714 & -0.160377 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 14.000000 & 5.000000 & -2.000000 & 30.000000 \\ 0.000000 & 15.142857 & 3.142857 & 5.857143 \\ 0.000000 & 0.000000 & -3.730189 & 1.602830 \\ 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.000000 & 0.000000 & 0.000000 & 1.000000 \\ 0.000000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 1.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

4. Step 3:

$$\mathbf{A} = \begin{bmatrix} 14.000000 & 5.000000 & -2.000000 & 30.000000 \\ 0.000000 & 15.142857 & 3.142857 & 5.857143 \\ 0.000000 & 0.000000 & -3.730189 & 1.602830 \\ 0.000000 & 0.000000 & 0.000000 & -4.169954 \end{bmatrix}$$

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.071429 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.085849 & 1.000000 & 0.000000 \\ 0.285714 & -0.160377 & -0.288316 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 14.000000 & 5.000000 & -2.000000 & 30.000000 \\ 0.000000 & 15.142857 & 3.142857 & 5.857143 \\ 0.000000 & 0.000000 & -3.730189 & 1.602830 \\ 0.000000 & 0.000000 & 0.000000 & -4.169954 \end{bmatrix}$$

$$\mathbf{P} = \begin{bmatrix} 0.000000 & 0.000000 & 0.000000 & 1.000000 \\ 0.000000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 1.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$$

5. **Solution:**

$$\mathbf{x} = \begin{bmatrix} 0.525109 \\ 0.255459 \\ -0.410480 \\ -0.281659 \end{bmatrix}$$

- **Doolitle Method**
  - $Matrix : A$
  - $Constant\ vector : b$

1. **Step 0:**

$$\mathbf{L} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 1.000000 & 15.500000 & 3.000000 & 8.000000 \\ 0.000000 & -1.300000 & -4.000000 & 1.100000 \\ 14.000000 & 5.000000 & -2.000000 & 30.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1.0000000 & 0.0000000 & 0.0000000 & 0.0000000 \\ 0.0000000 & 1.0000000 & 0.0000000 & 0.0000000 \\ 0.0000000 & 0.0000000 & 1.0000000 & 0.0000000 \\ 0.0000000 & 0.0000000 & 0.0000000 & 1.0000000 \end{bmatrix}$$

2. **Step 1:**

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.250000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 3.500000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

3. **Step 2:**

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.250000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.082540 & 1.000000 & 0.000000 \\ 3.500000 & 0.539683 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

4. **Step 3:**

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.250000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.082540 & 1.000000 & 0.000000 \\ 3.500000 & 0.539683 & 0.964467 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & -3.752381 & 1.698413 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

5. **Step 4:**

$$\mathbf{L} = \begin{bmatrix} 1.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.250000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & -0.082540 & 1.000000 & 0.000000 \\ 3.500000 & 0.539683 & 0.964467 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 0.000000 & 15.750000 & 3.000000 & 7.250000 \\ 0.000000 & 0.000000 & -3.752381 & 1.698413 \\ 0.000000 & 0.000000 & 0.000000 & 13.949239 \end{bmatrix}$$

6. **Solution:**

$$\mathbf{x} = \begin{bmatrix} 0.525109 \\ 0.255459 \\ -0.410480 \\ -0.281659 \end{bmatrix}$$

- **Crout**

  1. $Matrix : A$
  2. $Constant\, vector : b$

1. **Step 0:**

$$\mathbf{A} = \begin{bmatrix} 4.000000 & -1.000000 & 0.000000 & 3.000000 \\ 1.000000 & 15.500000 & 3.000000 & 8.000000 \\ 0.000000 & -1.300000 & -4.000000 & 1.100000 \\ 14.000000 & 5.000000 & -2.000000 & 30.000000 \end{bmatrix}$$

2. **Step 1:**

$$\mathbf{L} = \begin{bmatrix} 4.000000 & 0.000000 & 0.000000 & 0.000000 \\ 1.000000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 14.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1.000000 & -0.250000 & 0.000000 & 0.750000 \\ 0.000000 & 1.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

3. **Step 2:**

$$\mathbf{L} = \begin{bmatrix} 4.000000 & 0.000000 & 0.000000 & 0.000000 \\ 1.000000 & 15.750000 & 0.000000 & 0.000000 \\ 0.000000 & -1.300000 & 1.000000 & 0.000000 \\ 14.000000 & 8.500000 & 0.000000 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1.000000 & -0.250000 & 0.000000 & 0.750000 \\ 0.000000 & 1.000000 & 0.190476 & 0.460317 \\ 0.000000 & 0.000000 & 1.000000 & 0.000000 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

4. **Step 3:**

$$\mathbf{L} = \begin{bmatrix} 4.000000 & 0.000000 & 0.000000 & 0.000000 \\ 1.000000 & 15.750000 & 0.000000 & 0.000000 \\ 0.000000 & -1.300000 & -3.752381 & 0.000000 \\ 14.000000 & 8.500000 & -3.619048 & 1.000000 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1.000000 & -0.250000 & 0.000000 & 0.750000 \\ 0.000000 & 1.000000 & 0.190476 & 0.460317 \\ 0.000000 & 0.000000 & 1.000000 & -0.452623 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

5. **Step 4:**

$$\mathbf{L} = \begin{bmatrix} 4.000000 & 0.000000 & 0.000000 & 0.000000 \\ 1.000000 & 15.750000 & 0.000000 & 0.000000 \\ 0.000000 & -1.300000 & -3.752381 & 0.000000 \\ 14.000000 & 8.500000 & -3.619048 & 13.949239 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1.000000 & -0.250000 & 0.000000 & 0.750000 \\ 0.000000 & 1.000000 & 0.190476 & 0.460317 \\ 0.000000 & 0.000000 & 1.000000 & -0.452623 \\ 0.000000 & 0.000000 & 0.000000 & 1.000000 \end{bmatrix}$$

6. **Solution:**

$$\mathbf{x} = \begin{bmatrix} 0.525109 \\ 0.255459 \\ -0.410480 \\ -0.281659 \end{bmatrix}$$

- **Cholesky**

  1. $Matrix : A$
  2. $Constant\, vector : b$

  1. **Step 0:**

$$\begin{bmatrix} 4 & -1 & 0 & 3 \\ 1 & 15.5 & 3 & 8 \\ 0 & -1.3 & -4 & 1.1 \\ 14 & 5 & -2 & 30 \end{bmatrix}$$

  2. **Step 1:**

$$\mathbf{L} = \begin{bmatrix} 2.0 & 0 & 0 & 0 & \\ 0.5 & 0 & 0 & & 0 \\ 0.0 & 0 & 0 & 0 & \\ 7.0 & 0 & 0 & & 0 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 2.0 & -0.5 & 0.0 & 1.5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3. **Step 2:**

$$\mathbf{L} = \begin{bmatrix} 2.0 & 0 & 0 & 0 \\ 0.5 & 3.968626 & 0 & 0 \\ 0.0 & -0.327569 & 0 & 0 \\ 7.0 & 2.1417986 & 0 & 0 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 2.0 & -0.5 & 0.0 & 1.5 \\ 0 & 3.968626 & 0.7559289 & 1.826828 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

4. **Step 4:**

   At this stage the method fails because it does not support imaginary numbers

- **Jacobi**

  1. $Matrix : A$
  2. $Constant\, vector : b$
  3. $x0$
  4. Tol
  5. $Nmax$

$$\mathbf{T} = \begin{bmatrix} 0.000000 & 0.250000 & 0.000000 & -0.750000 \\ -0.064516 & 0.000000 & -0.193548 & -0.516129 \\ 0.000000 & -0.325000 & 0.000000 & 0.275000 \\ -0.466667 & -0.166667 & 0.066667 & 0.000000 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0.250000 & 0.064516 & -0.250000 & 0.033333 \end{bmatrix}$$

Spectral radius = 0.753517

Results

iter	E				
1	**3.6e-01**	0.250000	0.064516	−0.250000	0.033333
2	**1.5e-01**	0.241129	0.079570	−0.261801	−0.110753
3	**1.4e-01**	0.352957	0.156793	−0.306317	−0.109909
4	**7.5e-02**	0.371630	0.157759	−0.331183	−0.177933
5	**6.8e-02**	0.422890	0.196476	−0.350203	−0.188466
6	**4.0e-02**	0.440469	0.202287	−0.365683	−0.220108
7	**3.4e-02**	0.465653	0.220480	−0.376273	−0.230312
8	**2.2e-02**	0.477854	0.226172	−0.384992	−0.245803
9	**1.8e-02**	0.490895	0.235067	−0.391102	−0.253027
10	**1.3e-02**	0.498537	0.239137	−0.395979	−0.261002
11	**1.0e-02**	0.505536	0.243705	−0.399495	−0.265572
12	**7.3e-03**	0.510105	0.246292	−0.402236	−0.269834
13	**5.7e-03**	0.513948	0.248727	−0.404249	−0.272580
14	**4.2e-03**	0.516617	0.250286	−0.405796	−0.274914
15	**3.2e-03**	0.518757	0.251618	−0.406944	−0.276522
16	**2.4e-03**	0.520296	0.252532	−0.407819	−0.277819
17	**1.8e-03**	0.521498	0.253272	−0.408473	−0.278748
18	**1.4e-03**	0.522379	0.253801	−0.408969	−0.279476
19	**1.0e-03**	0.523057	0.254215	−0.409341	−0.280008
20	**7.7e-04**	0.523560	0.254518	−0.409622	−0.280418
21	**5.8e-04**	0.523943	0.254752	−0.409834	−0.280723
22	**4.4e-04**	0.524230	0.254925	−0.409993	−0.280954
23	**3.3e-04**	0.524447	0.255057	−0.410113	−0.281128
24	**2.5e-04**	0.524610	0.255156	−0.410204	−0.281259
25	**1.9e-04**	0.524733	0.255231	−0.410272	−0.281358
26	**1.4e-04**	0.524826	0.255287	−0.410323	−0.281432
27	**1.1e-04**	0.524896	0.255329	−0.410362	−0.281488
28	**8.0e-05**	0.524948	0.255361	−0.410391	−0.281530
29	**6.0e-05**	0.524988	0.255385	−0.410413	−0.281562
30	**4.6e-05**	0.525018	0.255403	−0.410430	−0.281586
31	**3.4e-05**	0.525040	0.255417	−0.410442	−0.281604
32	**2.6e-05**	0.525057	0.255427	−0.410452	−0.281618
33	**1.9e-05**	0.525070	0.255435	−0.410459	−0.281628
34	**1.5e-05**	0.525080	0.255441	−0.410464	−0.281636
35	**1.1e-05**	0.525087	0.255445	−0.410468	−0.281642
36	**8.3e-06**	0.525092	0.255448	−0.410471	−0.281646

37	**6.3e-06**		$\begin{bmatrix}0.525097 & 0.255451 & -0.410473 & -0.281649\end{bmatrix}$
38	**4.7e-06**		$\begin{bmatrix}0.525100 & 0.255453 & -0.410475 & -0.281652\end{bmatrix}$
39	**3.6e-06**		$\begin{bmatrix}0.525102 & 0.255454 & -0.410476 & -0.281654\end{bmatrix}$
40	**2.7e-06**		$\begin{bmatrix}0.525104 & 0.255455 & -0.410477 & -0.281655\end{bmatrix}$
41	**2.0e-06**		$\begin{bmatrix}0.525105 & 0.255456 & -0.410478 & -0.281656\end{bmatrix}$
42	**1.5e-06**		$\begin{bmatrix}0.525106 & 0.255457 & -0.410479 & -0.281657\end{bmatrix}$
43	**1.1e-06**		$\begin{bmatrix}0.525107 & 0.255457 & -0.410479 & -0.281658\end{bmatrix}$
44	**8.7e-07**		$\begin{bmatrix}0.525107 & 0.255457 & -0.410479 & -0.281658\end{bmatrix}$
45	**6.5e-07**		$\begin{bmatrix}0.525108 & 0.255458 & -0.410480 & -0.281658\end{bmatrix}$
46	**4.9e-07**		$\begin{bmatrix}0.525108 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
47	**3.7e-07**		$\begin{bmatrix}0.525108 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
48	**2.8e-07**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
49	**2.1e-07**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
50	**1.6e-07**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
51	**1.2e-07**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
52	**9.0e-08**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$

- **Gauss-Seidel**

  1. $Matrix : A$
  2. $Constant\, vector : b$
  3. $x0$
  4. Tol
  5. $Nmax$

$$\mathbf{T} = \begin{bmatrix}0.000000 & 0.250000 & 0.000000 & -0.750000\\ 0.000000 & -0.016129 & -0.193548 & -0.467742\\ 0.000000 & 0.005242 & 0.062903 & 0.427016\\ 0.000000 & -0.113629 & 0.036452 & 0.456425\end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix}0.250000 & 0.048387 & -0.265726 & -0.109113\end{bmatrix}$$

Spectral radius = 0.599488

Results

iter	E	
0		$\begin{bmatrix} 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$
1	**3.8e-01**	$\begin{bmatrix} 0.250000 & 0.048387 & -0.265726 & -0.109113 \end{bmatrix}$
2	**1.7e-01**	$\begin{bmatrix} 0.343931 & 0.150074 & -0.328780 & -0.174099 \end{bmatrix}$
3	**1.0e-01**	$\begin{bmatrix} 0.418093 & 0.191035 & -0.359964 & -0.217613 \end{bmatrix}$
4	**6.0e-02**	$\begin{bmatrix} 0.460969 & 0.216763 & -0.380292 & -0.243265 \end{bmatrix}$
5	**3.6e-02**	$\begin{bmatrix} 0.486640 & 0.232281 & -0.392389 & -0.258638 \end{bmatrix}$
6	**2.1e-02**	$\begin{bmatrix} 0.502049 & 0.241563 & -0.399633 & -0.267859 \end{bmatrix}$
7	**1.3e-02**	$\begin{bmatrix} 0.511285 & 0.247128 & -0.403978 & -0.273386 \end{bmatrix}$
8	**7.7e-03**	$\begin{bmatrix} 0.516822 & 0.250465 & -0.406582 & -0.276700 \end{bmatrix}$
9	**4.6e-03**	$\begin{bmatrix} 0.520141 & 0.252465 & -0.408143 & -0.278686 \end{bmatrix}$
10	**2.8e-03**	$\begin{bmatrix} 0.522131 & 0.253664 & -0.409079 & -0.279877 \end{bmatrix}$
11	**1.7e-03**	$\begin{bmatrix} 0.523324 & 0.254383 & -0.409640 & -0.280591 \end{bmatrix}$
12	**1.0e-03**	$\begin{bmatrix} 0.524039 & 0.254814 & -0.409977 & -0.281019 \end{bmatrix}$
13	**6.0e-04**	$\begin{bmatrix} 0.524467 & 0.255072 & -0.410179 & -0.281275 \end{bmatrix}$
14	**3.6e-04**	$\begin{bmatrix} 0.524724 & 0.255227 & -0.410299 & -0.281429 \end{bmatrix}$
15	**2.1e-04**	$\begin{bmatrix} 0.524879 & 0.255320 & -0.410372 & -0.281521 \end{bmatrix}$
16	**1.3e-04**	$\begin{bmatrix} 0.524971 & 0.255375 & -0.410415 & -0.281577 \end{bmatrix}$
17	**7.7e-05**	$\begin{bmatrix} 0.525026 & 0.255409 & -0.410441 & -0.281610 \end{bmatrix}$
18	**4.6e-05**	$\begin{bmatrix} 0.525059 & 0.255429 & -0.410457 & -0.281630 \end{bmatrix}$
19	**2.8e-05**	$\begin{bmatrix} 0.525079 & 0.255441 & -0.410466 & -0.281642 \end{bmatrix}$
20	**1.7e-05**	$\begin{bmatrix} 0.525091 & 0.255448 & -0.410472 & -0.281649 \end{bmatrix}$
21	**1.0e-05**	$\begin{bmatrix} 0.525098 & 0.255452 & -0.410475 & -0.281653 \end{bmatrix}$
22	**6.0e-06**	$\begin{bmatrix} 0.525103 & 0.255455 & -0.410477 & -0.281656 \end{bmatrix}$
23	**3.6e-06**	$\begin{bmatrix} 0.525105 & 0.255456 & -0.410479 & -0.281657 \end{bmatrix}$
24	**2.1e-06**	$\begin{bmatrix} 0.525107 & 0.255457 & -0.410479 & -0.281658 \end{bmatrix}$
25	**1.3e-06**	$\begin{bmatrix} 0.525108 & 0.255458 & -0.410480 & -0.281659 \end{bmatrix}$
26	**7.7e-07**	$\begin{bmatrix} 0.525108 & 0.255458 & -0.410480 & -0.281659 \end{bmatrix}$
27	**4.6e-07**	$\begin{bmatrix} 0.525109 & 0.255458 & -0.410480 & -0.281659 \end{bmatrix}$
28	**2.8e-07**	$\begin{bmatrix} 0.525109 & 0.255458 & -0.410480 & -0.281659 \end{bmatrix}$
29	**1.7e-07**	$\begin{bmatrix} 0.525109 & 0.255458 & -0.410480 & -0.281659 \end{bmatrix}$
30	**9.9e-08**	$\begin{bmatrix} 0.525109 & 0.255458 & -0.410480 & -0.281659 \end{bmatrix}$

- **SOR**

  1. $Matrix : A$
  2. $Constant\, vector : b$
  3. $x0$
  4. Tol
  5. $Nmax$

$$\mathbf{T} = \begin{bmatrix} -0.500000 & 0.375000 & 0.000000 & -1.125000 \\ 0.048387 & -0.536290 & -0.290323 & -0.665323 \\ -0.023589 & 0.261442 & -0.358468 & 0.736845 \\ 0.335544 & -0.102283 & 0.036734 & 0.527515 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 0.375000 & 0.060484 & -0.404486 & -0.268070 \end{bmatrix}$$

Spectral radius = 0.631208

Results

iter	E	
0		$\begin{bmatrix} 0.000000 & 0.000000 & 0.000000 & 0.000000 \end{bmatrix}$
1	**6.2e-01**	$\begin{bmatrix} 0.375000 & 0.060484 & -0.404486 & -0.268070 \end{bmatrix}$
2	**3.2e-01**	$\begin{bmatrix} 0.511760 & 0.341976 & -0.450049 & -0.304696 \end{bmatrix}$
3	**1.5e-01**	$\begin{bmatrix} 0.590144 & 0.235228 & -0.390336 & -0.308594 \end{bmatrix}$
4	**1.1e-01**	$\begin{bmatrix} 0.515306 & 0.281526 & -0.444371 & -0.271236 \end{bmatrix}$
5	**7.4e-02**	$\begin{bmatrix} 0.528060 & 0.243909 & -0.383605 & -0.283362 \end{bmatrix}$
6	**4.3e-02**	$\begin{bmatrix} 0.521218 & 0.255125 & -0.424458 & -0.279399 \end{bmatrix}$
7	**2.1e-02**	$\begin{bmatrix} 0.524387 & 0.258003 & -0.403799 & -0.282252 \end{bmatrix}$
8	**1.1e-02**	$\begin{bmatrix} 0.527091 & 0.252514 & -0.412630 & -0.282229 \end{bmatrix}$
9	**6.9e-03**	$\begin{bmatrix} 0.523655 & 0.258137 & -0.410946 & -0.281073 \end{bmatrix}$
10	**5.5e-03**	$\begin{bmatrix} 0.526181 & 0.253697 & -0.409146 & -0.282129 \end{bmatrix}$
11	**4.2e-03**	$\begin{bmatrix} 0.524441 & 0.256380 & -0.411790 & -0.281318 \end{bmatrix}$
12	**2.8e-03**	$\begin{bmatrix} 0.525405 & 0.255085 & -0.409503 & -0.281846 \end{bmatrix}$
13	**1.7e-03**	$\begin{bmatrix} 0.525031 & 0.255513 & -0.411073 & -0.281584 \end{bmatrix}$
14	**8.8e-04**	$\begin{bmatrix} 0.525084 & 0.255547 & -0.410197 & -0.281673 \end{bmatrix}$
15	**4.4e-04**	$\begin{bmatrix} 0.525171 & 0.255337 & -0.410569 & -0.281674 \end{bmatrix}$
16	**2.7e-04**	$\begin{bmatrix} 0.525049 & 0.255562 & -0.410493 & -0.281637 \end{bmatrix}$
17	**2.2e-04**	$\begin{bmatrix} 0.525153 & 0.255389 & -0.410431 & -0.281679 \end{bmatrix}$
18	**1.7e-04**	$\begin{bmatrix} 0.525083 & 0.255497 & -0.410532 & -0.281646 \end{bmatrix}$
19	**1.1e-04**	$\begin{bmatrix} 0.525122 & 0.255443 & -0.410441 & -0.281667 \end{bmatrix}$
20	**6.8e-05**	$\begin{bmatrix} 0.525106 & 0.255461 & -0.410504 & -0.281656 \end{bmatrix}$

21	**3.6e-05**		$\begin{bmatrix}0.525108 & 0.255462 & -0.410469 & -0.281660\end{bmatrix}$
22	**1.8e-05**		$\begin{bmatrix}0.525111 & 0.255454 & -0.410484 & -0.281660\end{bmatrix}$
23	**1.1e-05**		$\begin{bmatrix}0.525107 & 0.255463 & -0.410481 & -0.281659\end{bmatrix}$
24	**8.4e-06**		$\begin{bmatrix}0.525111 & 0.255456 & -0.410479 & -0.281660\end{bmatrix}$
25	**6.6e-06**		$\begin{bmatrix}0.525108 & 0.255460 & -0.410482 & -0.281659\end{bmatrix}$
26	**4.5e-06**		$\begin{bmatrix}0.525110 & 0.255458 & -0.410479 & -0.281660\end{bmatrix}$
27	**2.7e-06**		$\begin{bmatrix}0.525109 & 0.255459 & -0.410481 & -0.281659\end{bmatrix}$
28	**1.5e-06**		$\begin{bmatrix}0.525109 & 0.255459 & -0.410480 & -0.281659\end{bmatrix}$
29	**7.2e-07**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410481 & -0.281659\end{bmatrix}$
30	**4.2e-07**		$\begin{bmatrix}0.525109 & 0.255459 & -0.410480 & -0.281659\end{bmatrix}$
31	**3.3e-07**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
32	**2.6e-07**		$\begin{bmatrix}0.525109 & 0.255459 & -0.410480 & -0.281659\end{bmatrix}$
33	**1.8e-07**		$\begin{bmatrix}0.525109 & 0.255458 & -0.410480 & -0.281659\end{bmatrix}$
34	**1.1e-07**		$\begin{bmatrix}0.525109 & 0.255459 & -0.410480 & -0.281659\end{bmatrix}$
35	**5.9e-08**		$\begin{bmatrix}0.525109 & 0.255459 & -0.410480 & -0.281659\end{bmatrix}$

- **Vandermonde**

  1. *Table*

  Vandermonde matrix = $\begin{bmatrix}-1.000000 & 1.000000 & -1.000000 & 1.000000\\ 0.000000 & 0.000000 & 0.000000 & 1.000000\\ 27.000000 & 9.000000 & 3.000000 & 1.000000\\ 64.000000 & 16.000000 & 4.000000 & 1.000000\end{bmatrix}$

  Coefficients of the polynomial =
  $\begin{bmatrix}-1.141667 & 5.825000 & -5.533333 & 3.000000\end{bmatrix}$

  Polynomial:

  $$-1.141667x^3 + 5.825000x^2 - 5.533333x + 3.000000$$

- **Newton**

  1. *Table*

  divided differences = $\begin{bmatrix}15.500000 & 0.000000 & 0.000000 & 0.000000\\ 3.000000 & -12.500000 & 0.000000 & 0.000000\\ 8.000000 & 1.666667 & 3.541667 & 0.000000\\ 1.000000 & -7.000000 & -2.166667 & -1.141667\end{bmatrix}$

  Coefficients of the Newton polynomial =
  $\begin{bmatrix}15.500000 & -12.500000 & 3.541667 & -1.141667\end{bmatrix}$

Newton polynomial:

$15.500000 - 12.500000(x+1) + 3.541667(x+1)x - 1.141667(x+1)x(x-3)$

- **Lagrange**

  1. $Table$

Lagrange interpolating polynomials	
$\mathbf{-0.050000x^3} + 0.350000x^2 - 0.600000x - 0.000000$	$\mathbf{L_0}$
$\mathbf{0.083333x^3} - 0.500000x^2 - 0.416667x + 1.000000$	$\mathbf{L_1}$
$\mathbf{-0.083333x^3} + 0.250000x^2 + 0.333333x - 0.000000$	$\mathbf{L_2}$
$\mathbf{0.050000x^3} - 0.100000x^2 - 0.150000x + 0.000000$	$\mathbf{L_3}$

Polynomial:

$$15.5L_0 + 3L_1 + 8L_2 + L_3$$

- **Linear Spline**

  1. $Table$

$$\textbf{Spline coefficients} = \begin{bmatrix} -12.500000 & 3.000000 \\ 1.666667 & 3.000000 \\ -7.000000 & 29.000000 \end{bmatrix}$$

Splines:

$$-12.500000x + 3.000000$$

$$1.666667x + 3.000000$$

$$-7.000000x + 29.000000$$

- **Quadratic Spline**

  1. $Table$

$$\textbf{Spline coefficients} = \begin{bmatrix} 0.000000 & -12.500000 & 3.000000 \\ 4.722222 & -12.500000 & 3.000000 \\ -22.833333 & 152.833333 & -245.000000 \end{bmatrix}$$

Splines:

$$0.000000x^2 - 12.500000x + 3.000000$$

$$4.722222x^2 - 12.500000x + 3.000000$$

$$-22.833333x^2 + 152.833333x - 245.000000$$

- **Cubic Spline**

  1. $Table$

  $$\textbf{Spline coefficients} = \begin{bmatrix} 2.533333 & 7.600000 & -7.433333 & 3.000000 \\ -1.522222 & 7.600000 & -7.433333 & 3.000000 \\ 2.033333 & -24.400000 & 88.566667 & -93.000000 \end{bmatrix}$$

  Splines:

  $$2.533333x^3 + 7.600000x^2 - 7.433333x + 3.000000$$

  $$-1.522222x^3 + 7.600000x^2 - 7.433333x + 3.000000$$

  $$2.033333x^3 - 24.400000x^2 + 88.566667x - 93.000000$$

- **Gaussian elimination for tridiagonal matrices**

  1. $Matrix : A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

  2. $Constant\,vector : b = \begin{bmatrix} 124 \\ 4 \\ 14 \end{bmatrix}$

$$\textbf{Final matrix} = \begin{bmatrix} 2.0000000 & -1.0000000 & 0.0000000 \\ 0.0000000 & 1.5000000 & -1.0000000 \\ 0.0000000 & 0.0000000 & 1.3333333 \end{bmatrix}$$

$$\textbf{Solution} = \begin{bmatrix} 98.5000000 & 73.0000000 & 43.5000000 \end{bmatrix}$$

- **Gaussian elimination with stepped pivoting**

  1. **Step 0:**

  $$\textbf{Initial Matriz} = \begin{bmatrix} 2.11 & -4.21 & 0.921 \\ 4.01 & 10.2 & -1.12 \\ 1.09 & 0.987 & 0.831 \end{bmatrix}, \qquad \textbf{b vector} = \begin{bmatrix} 2.01 \\ -3.09 \\ 4.21 \end{bmatrix}$$

  2. **Step 1:**

  $$\textbf{Ab} = \begin{bmatrix} 1.0900000 & 0.987 & 0.831 & 4.21 \\ 0.0000000 & 6.5689266 & -4.1771651 & -18.5781651 \\ 0.0000000 & -6.1206147 & -0.687633 & -6.139633 \end{bmatrix}$$

  3. **Step 2:**

  $$\textbf{Ab} = \begin{bmatrix} 1.0900000e+00 & 9.8700000e-01 & 8.3100000e-01 & 4.2100000e+00 \\ 0.0000000e+00 & -6.1206147e+00 & -6.8763303e-01 & -6.1396330e+00 \\ 0.0000000e+00 & 0.0000000e+00 & -4.9151647e+00 & -2.5167503e+01 \end{bmatrix}$$

  4. **Solution:**

  $$\textbf{x} = \begin{bmatrix} -0.4287342 \\ 0.4278478 \\ 5.1203784 \end{bmatrix}$$

- **Crout for tridiagonal matrices**

  1. $Matrix : A = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$

  2. $Constant\,vector : b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

1. **Step 0:**

$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

2. **Step 1:**

$$\mathbf{L} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 1.5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1 & -0.5 & 0.0 & 0 \\ 0 & 1 & -0.666666 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. **Step 2:**

$$\mathbf{L} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ -1 & 1.5 & 0 & 0 \\ 0 & -1 & 1.333333 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{U} = \begin{bmatrix} 1 & -0.5 & 0.0 & 0 \\ 0 & 1 & -0.666666 & 0 \\ 0 & 0 & 1 & -0.749999 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4. **Solution:**

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

- **Neville's method**

1. $Table =$

x	**2**	**2.2**	**2.3**
y	**0.693147**	**0.788457**	**0.832909**

For this example, Y Values are calculated using Ln(x)

2. $X\,to\,interpolate = 2.1$

1. **Solution:**

   Result matrix $\begin{bmatrix} 0.693147 & 0 & 0 \\ 0.788457 & 0.740802 & 0 \\ 0.832909 & 0.744005 & 0.7418697 \end{bmatrix}$

   $Y\ interpolated\ = 0.7418697$

   Real Y (from the function Ln(x)) = 0.741937

- **Hermite interpolation**

  1. *Table*

  Method failed to run. You can see it here Click here