Problem E: Eulerian Inference

Advanced Algorithms for Programming Contests

Restrictions

Time: 2 seconds
Memory: 512 MB

Problem description

You are to guess n, a positive integer not exceeding 10^4, based on at most 42 queries asking for $\phi(\lfloor a/b \cdot n \rfloor)$ for some positive integers $a, b \leq 10^4$, where ϕ is Euler's totient function and $\lfloor \cdot \rfloor$ is the floor function. There are several testcases.

Hint: It might make sense to look up properties of Euler's totient function, e.g. in the lecture slides of Algo 1 topic 12 (number theory), which were uploaded to the "Supplementary Material" section of this course's Stud.IP.

Interaction Protocol

First you are given T ($1 \leq T \leq 100$) – the number of testcases you need to answer. For each testcase, the interaction works as follows:

To ask for $\phi(\lfloor a/b \cdot n \rfloor)$, simply output "? a b" and read the answer as an integer. To submit your guess m, output "! m". After submitting a guess you should move on to the next testcase or terminate if it was the last one. For technical reasons, most misbehavior yields the verdict `Run Error`.

Please note that, since $\phi(0)$ is undefined, if you pose a query "? a b" where $\lfloor a/b \cdot n \rfloor = 0$ (and a, b valid), there are no guarantees about what the answer will be (but it won't cause a `Run Error`).

Sample input and output

Input	Output
1 2 1	 ? 1 2 ? 1 3 ! 6