Turbulent bottleneck

Reynolds Averaged $\hat{z}$ − momentum:

$$\partial_t \overline{\rho v_z} + \partial_z \left(\overline{\rho}\, \overline{v_z}^2 + \overline{\rho}\overline{v_z'^2} + 2\overline{\rho' v_z'}\overline{v_z} + \overline{\rho' v_z'^2} + \overline{P} \right) = 0$$

Steady state:

$$\overline{\rho}\, \overline{v_z}^2 + \overline{\rho}\overline{v_z'^2} + 2\overline{\rho' v_z'}\overline{v_z} + \overline{\rho' v_z'^2} + \overline{P} = \mathrm{const}\,.$$

Absorb asymptotic $\overline{P}$ into const.

$$\overline{\rho}\, \overline{v_z}^2 + \overline{\rho}\overline{v_z'^2} + 2\overline{\rho' v_z'}\overline{v_z} + \overline{\rho' v_z'^2} + \Delta\overline{P} = \overline{\rho}\, \overline{v_z}^2 + \overline{P}_{\mathrm{eff}}$$

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Absorb asymptotic $\bar{P}$ into const.

$$\bar{\rho}\,\overline{v_z}^2 + \bar{\rho}\overline{v_z'^2} + 2\overline{\rho' v_z'}\,\overline{v_z} + \overline{\rho' v_z'^2} + \Delta\bar{P} = \bar{\rho}\,\overline{v_z}^2 + \bar{P}_{\rm eff}$$

Something like this gives the right scaling

$$\overline{v_z^2} = \frac{\bar{P}_{\rm eff}}{\bar{\rho}} \sim \frac{\Delta P}{P}\frac{P_{\rm eff}}{\rho}, \text{ if } \frac{\Delta P}{P} \sim \frac{t_{\rm mix}}{t_{\rm cool}} \propto \frac{h/v}{t_{\rm cool}},$$

and $P_{\rm eff}/\rho \propto v^2$, then we can plug everything, including $h = (vLt_{\rm cool})^{1/2}$

Gives $\overline{v_z} = \left(\frac{Lv^3}{t_{\rm cool}}\right)^{\frac{1}{4}}$, which is the same

as $v_z \sim (v_{\rm cool}v_{\rm turb})^{1/2}$ where $v_{\rm cool} = h/t_{\rm cool}$