Turbulent bottleneck

Reynolds Averaged $\hat{z}$ – momentum:

$$\partial_t \overline{\rho \mathrm{v}_z} + \partial_z \left(\bar{\rho}\,\overline{\mathrm{v}_z}^2 + \bar{\rho}\,\overline{\mathrm{v}_z'^2} + 2\overline{\rho' \mathrm{v}_z'}\,\overline{\mathrm{v}_z} + \overline{\rho' \mathrm{v}_z'^2} + \bar{\mathrm{P}} \right) = 0$$

Steady state:

$$\bar{\rho}\,\overline{\mathrm{v}_z}^2 + \bar{\rho}\,\overline{\mathrm{v}_z'^2} + 2\overline{\rho' \mathrm{v}_z'}\,\overline{\mathrm{v}_z} + \overline{\rho' \mathrm{v}_z'^2} + \bar{\mathrm{P}} = \mathrm{const}\,.$$

Absorb asymptotic $\bar{\mathrm{P}}$ into const.

$$\bar{\rho}\,\overline{\mathrm{v}_z}^2 + \bar{\rho}\,\overline{\mathrm{v}_z'^2} + 2\overline{\rho' \mathrm{v}_z'}\,\overline{\mathrm{v}_z} + \overline{\rho' \mathrm{v}_z'^2} + \Delta\bar{\mathrm{P}} = \bar{\rho}\,\overline{\mathrm{v}_z}^2 + \bar{\mathrm{P}}_{\mathrm{eff}}$$

Something like this gives the right scaling

$$\overline{\mathrm{v}_z^2} = \frac{\bar{\mathrm{P}}_{\mathrm{eff}}}{\bar{\rho}} \sim \frac{\Delta \mathrm{P}}{\mathrm{P}} \frac{\mathrm{P}_{\mathrm{eff}}}{\rho}, \text{ if } \frac{\Delta \mathrm{P}}{\mathrm{P}} \sim \frac{t_{\mathrm{mix}}}{t_{\mathrm{cool}}} \propto \frac{h/v}{t_{\mathrm{cool}}},$$

and $\mathrm{P}_{\mathrm{eff}}/\rho \propto \mathrm{v}^2$, then we can plug everything, including $h = (\mathrm{v}L t_{\mathrm{cool}})^{1/2}$

Gives $\overline{\mathrm{v}_z} = \left(\frac{L\mathrm{v}^3}{t_{\mathrm{cool}}} \right)^{\frac{1}{4}}$, which is the same as $\mathrm{v}_z \sim (\mathrm{v}_{\mathrm{cool}} \mathrm{v}_{\mathrm{turb}})^{1/2}$ where $\mathrm{v}_{\mathrm{cool}} = h/t_{\mathrm{cool}}$

τ_{zz} Reynolds Stress at fixed t_{cool}

$$\partial_z\left(\bar{\rho}\bar{v}_z^2 + \bar{\rho}\overline{v_z'^2} + 2\overline{\rho' v_z'}\bar{v}_z + \overline{\rho' v_z'^2} + \bar{P}\right) = 0$$