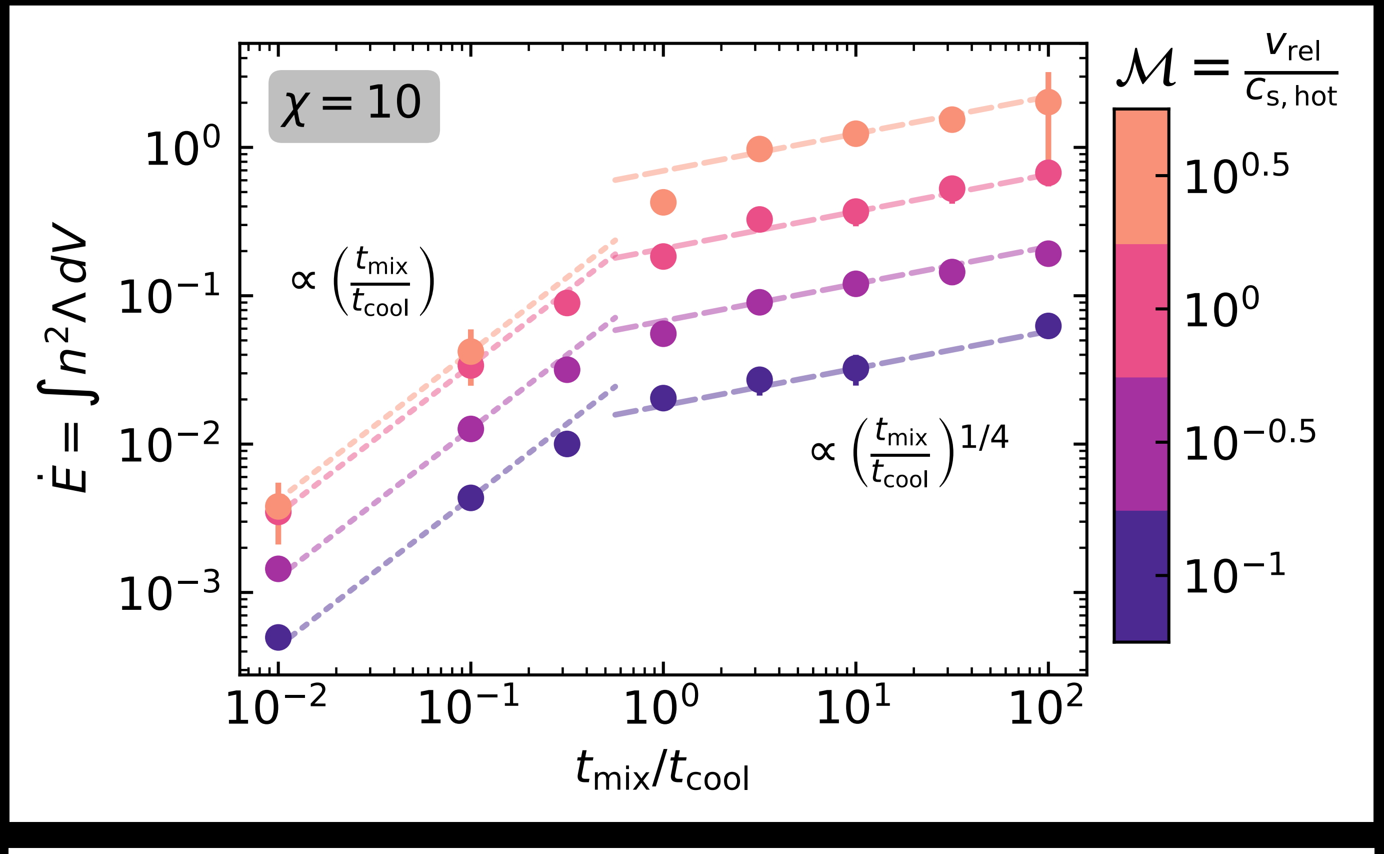
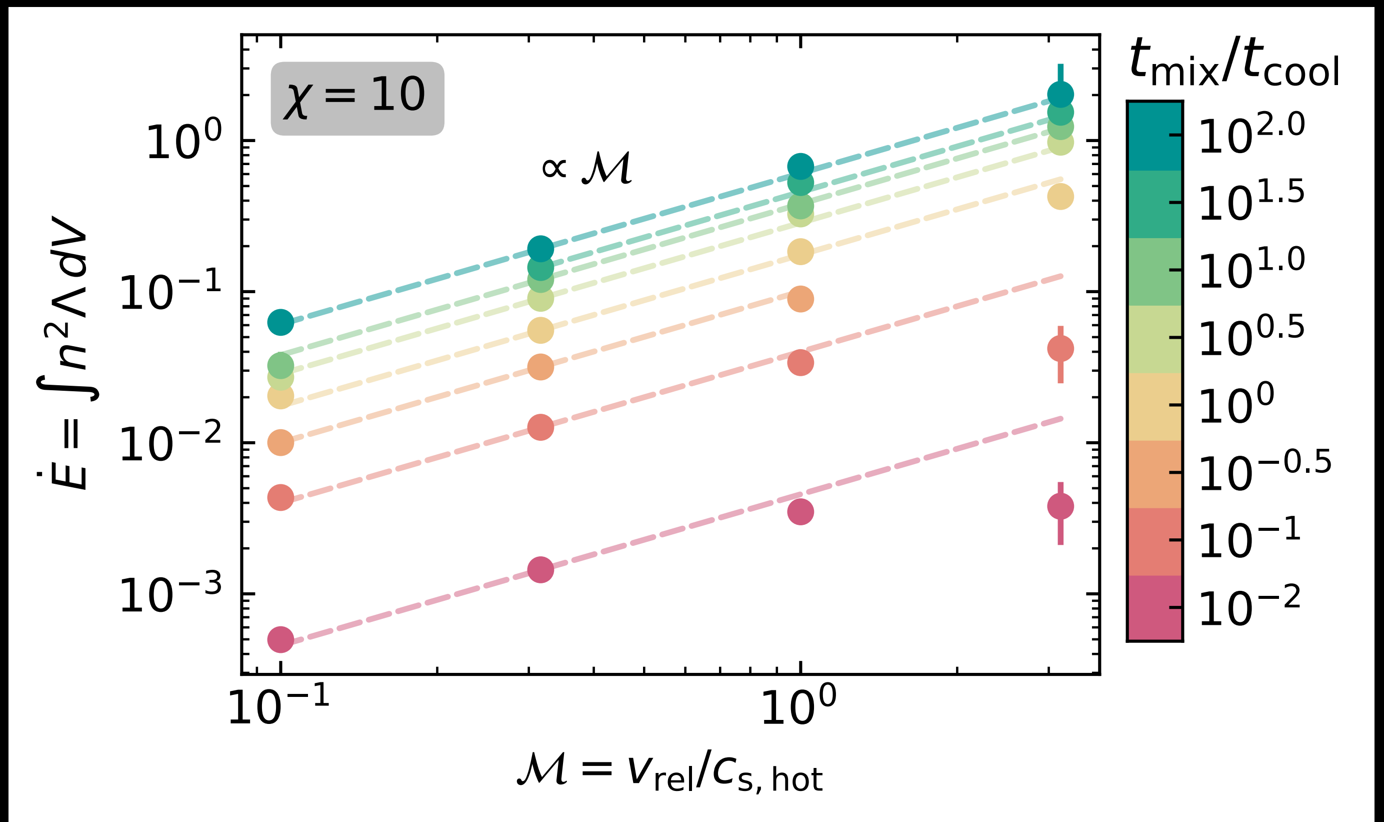
Entrainment, acceleration, & cooling

In the rapid cooling limit the total cooling is set by the amount that can be refilled





The naive use of the energy equation gives the wrong scaling because

$$\dot{E}_{\rm cool} = \int \frac{P}{t_{\rm cool}} dV \Big/ L^2 = \frac{P\,h}{t_{\rm cool}} \frac{\rm Area}{L^2} \neq \frac{P\,h}{t_{\rm cool}}$$

which would if $\mathrm{Area} = L^2$ predict $\dot{E}_{\rm cool} \propto \left(v_{\rm rel}/t_{\rm cool}\right)^{1/2}$ since $t_{\rm mix} > t_{\rm cool}$ and therefore $h \sim \sqrt{v_{\rm rel} L\, t_{\rm cool}}$

To recover the empirical result requires that

$$\frac{\rm Area}{L^2} \propto \left(v_{\rm rel}\, t_{\rm cool}\right)^{1/4}$$

Turbulent bottleneck

Reynolds Averaged $\hat{z}$ − momentum:

$$\partial_t \overline{\rho v_z} + \partial_z \left(\overline{\rho}\, \overline{v_z}^2 + \overline{\rho}\, \overline{v_z'^2} + 2 \overline{\rho' v_z'}\, \overline{v_z} + \overline{\rho' v_z'^2} + \overline{P} \right) = 0$$

Steady state:

$$\overline{\rho}\, \overline{v_z}^2 + \overline{\rho}\, \overline{v_z'^2} + 2 \overline{\rho' v_z'}\, \overline{v_z} + \overline{\rho' v_z'^2} + \overline{P} = \mathrm{const}\,.$$

Absorb asymptotic $\overline{P}$ into const.

$$\overline{\rho}\, \overline{v_z}^2 + \overline{\rho}\, \overline{v_z'^2} + 2 \overline{\rho' v_z'}\, \overline{v_z} + \overline{\rho' v_z'^2} + \Delta \overline{P} = \overline{\rho}\, \overline{v_z}^2 + \overline{P}_{\mathrm{eff}}$$