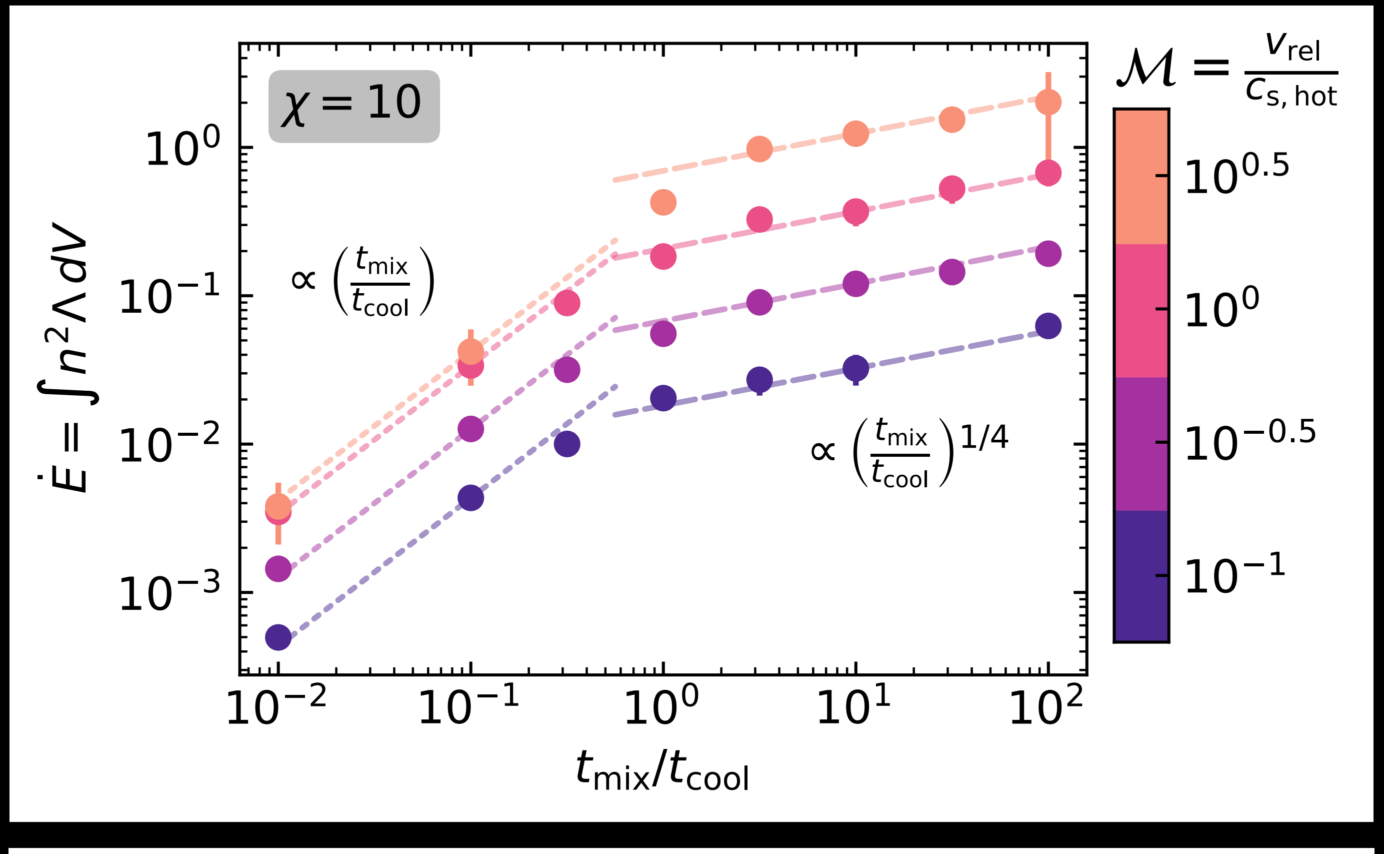
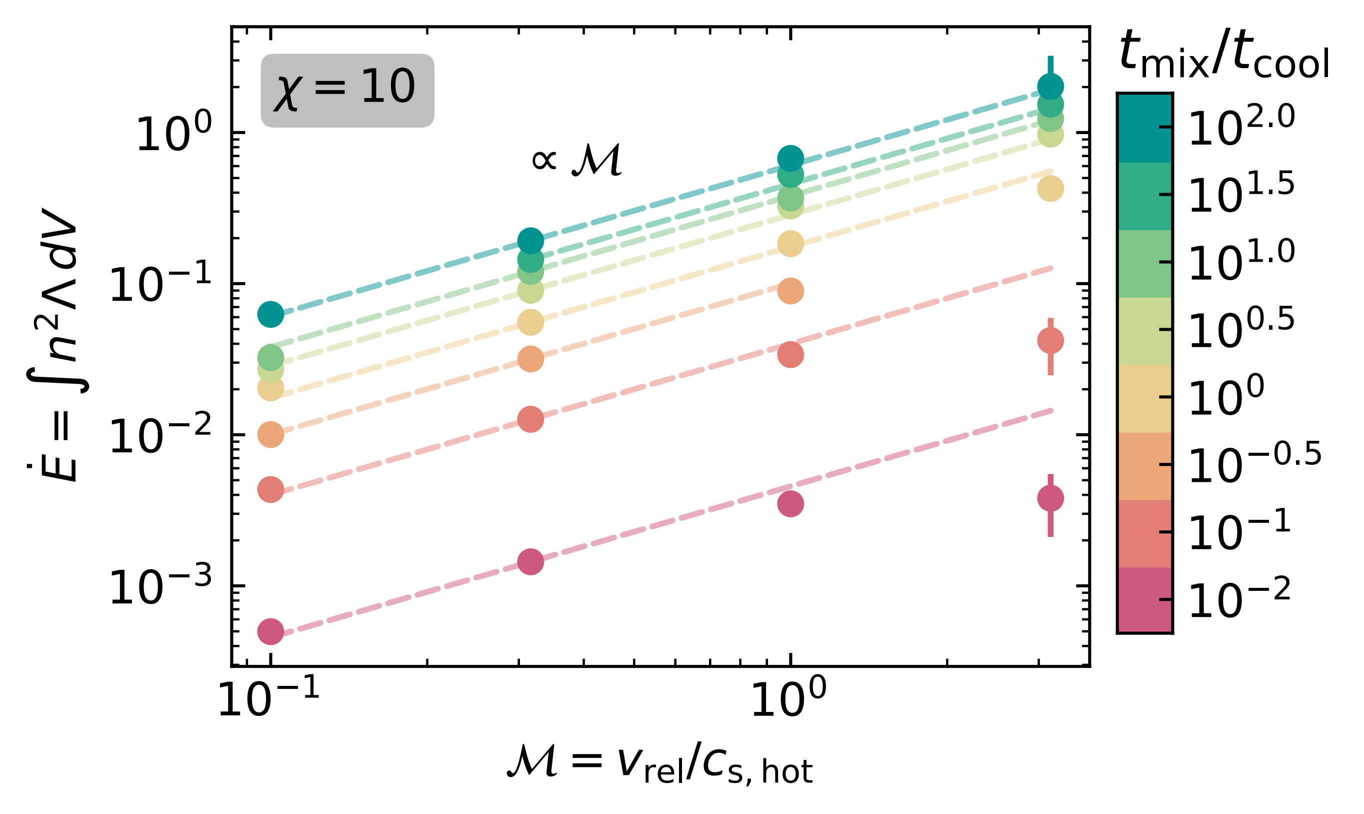
# Entrainment, acceleration, & cooling

Inflow of high enthalpy gas replenishes cooling losses:

$$\dot{E}_{cool} \propto v_z$$




In the slowly cooling limit, simple interpretation of the energy equation gives the right scaling:

$$\frac{\partial}{\partial z} \frac{\gamma}{\gamma - 1} P v_z \left(1 + \frac{\mathrm{Mach}^2}{2(\gamma - 1)}\right) = \dot{\varepsilon}_{cool} = \frac{1}{\gamma - 1} \frac{P}{t_{cool}}$$

Approximately reduces to

$$v_z \approx \frac{h}{t_{cool}},$$

which, since $t_{mix} < t_{cool}$, $h \sim \sqrt{v_{rel} L t_{mix}} \sim L$, gives

$$\dot{E}_{cool} \propto 1/t_{cool}$$

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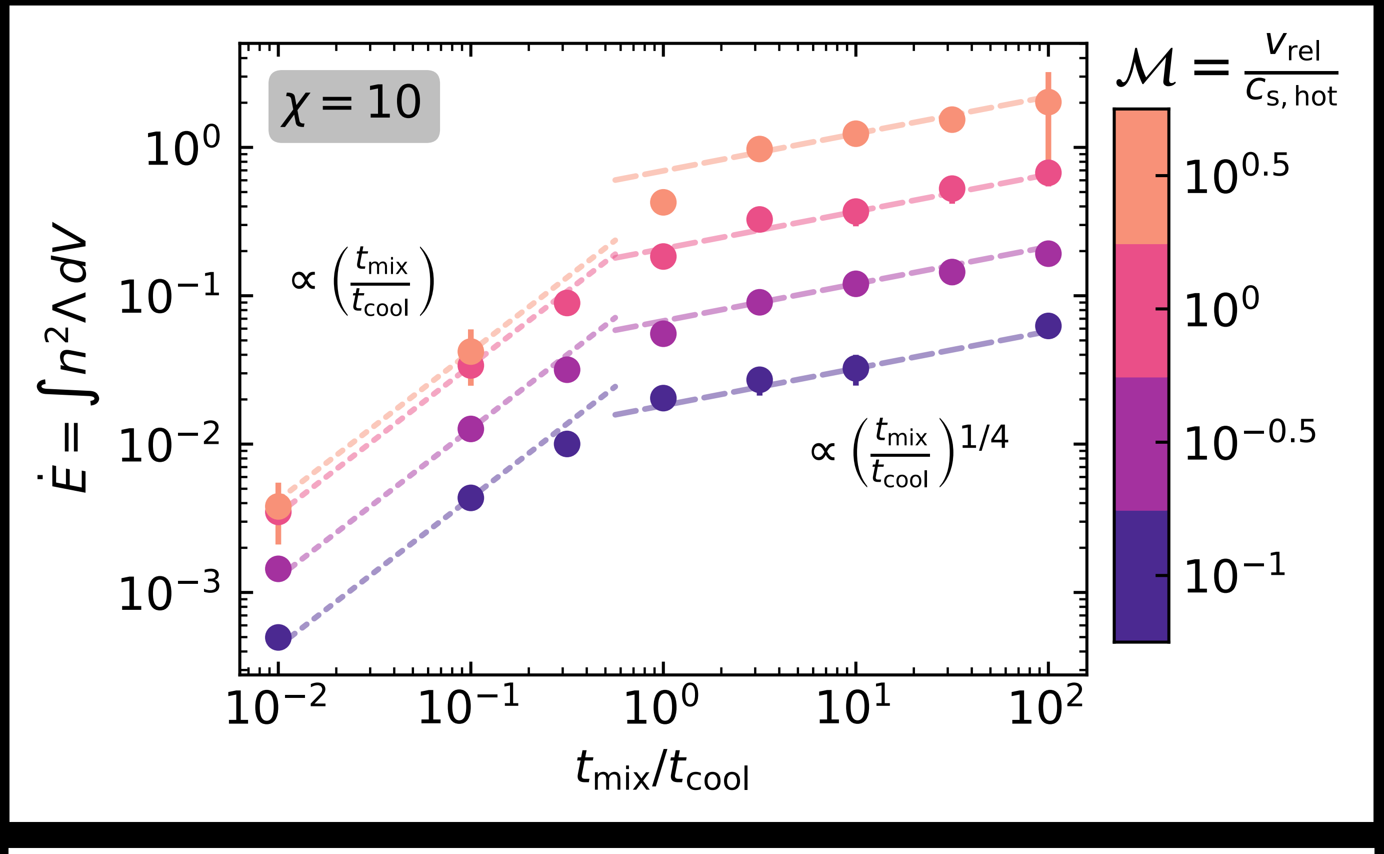
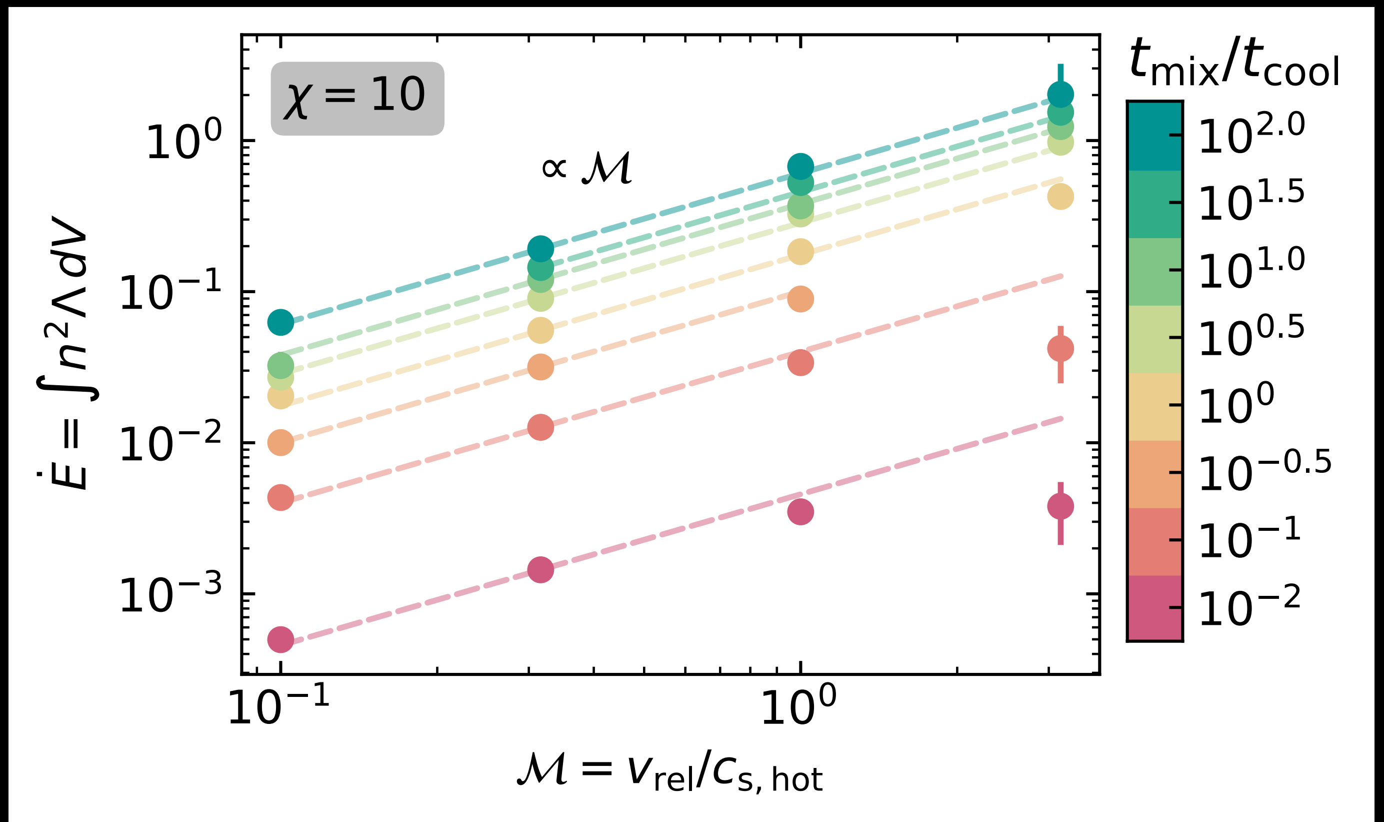
In the rapid cooling limit the total cooling is set by the amount that can be refilled




The naive use of the energy equation gives the wrong scaling because

$$\dot{E}_{\rm cool} = \int \frac{P}{t_{\rm cool}} dV \Big/ L^2 = \frac{P\,h}{t_{\rm cool}} \frac{\rm Area}{L^2} \neq \frac{P\,h}{t_{\rm cool}}$$

which would if $\mathrm{Area} = L^2$ predict $\dot{E}_{\rm cool} \propto \left(v_{\rm rel}/t_{\rm cool}\right)^{1/2}$ since $t_{\rm mix} > t_{\rm cool}$ and therefore $h \sim \sqrt{v_{\rm rel} L\, t_{\rm cool}}$

To recover the empirical result requires that

$$\frac{\rm Area}{L^2} \propto \left(v_{\rm rel}\, t_{\rm cool}\right)^{1/4}$$