PortfolioRebalancing

April 27, 2015

```
In [17]: import numpy as np
         import cvxpy as cvx
         import matplotlib.pyplot as plt
         plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
         np.random.seed(123)

         %pylab inline

         #!wget http://stanford.edu/~boyd/cvxbook/cvxbook_additional_exercises/portf_weight_rebalance_d
         #!cat portf_weight_rebalance_data.py
```

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Populating the interactive namespace from numpy and matplotlib
```

```
In [18]: # data
         import numpy as np
         import cvxpy as cvx
         T = 100
         n = 5
         gamma = 8.0
         threshold = 0.001

         Sigma = np.array([[1.512e-02,  1.249e-03,  2.762e-04, -5.333e-03, -7.938e-04],
                           [ 1.249e-03,  1.030e-02,  6.740e-05, -1.301e-03, -1.937e-04],
                           [ 2.762e-04,  6.740e-05,  1.001e-02, -2.877e-04, -4.283e-05],
                           [-5.333e-03, -1.301e-03, -2.877e-04,  1.556e-02,  8.271e-04],
                           [-7.938e-04, -1.937e-04, -4.283e-05,  8.271e-04,  1.012e-02]])

         mu = np.array([ 1.02 , 1.028, 1.01 , 1.034, 1.017])
         kappa_1 = np.matrix([ 0.002, 0.002, 0.002, 0.002, 0.002])
         kappa_2 = np.matrix([ 0.004, 0.004, 0.004, 0.004, 0.004])

         ## generate a vector r of market returns
         generateReturns = lambda: np.random.multivariate_normal(mu,Sigma)
```

```
In [19]: def opt_port(mu, Sigma, gamma, trans_costs=False,
                      kappa=None, wt=None):
             """ Compuptes single period optimal portfolio
             """

             # initialize variable to optimize
             w = cvx.Variable(n)

             # coerce wt matrix to be a column vector
             wt = np.reshape(np.matrix(wt), (-1, 1))
```

```
    # objective function to maximize
    obj_func = w.T * mu - gamma / 2 * cvx.quad_form(w, Sigma)

    if trans_costs:
        obj_func -= kappa * cvx.abs(w - wt)

    # define constraints
    constraints = [cvx.sum_entries(w) == 1]

    # define convex optimization problem
    obj = cvx.Maximize(obj_func)
    prob = cvx.Problem(obj, constraints)

    # solve convex optimization problem
    prob.solve(solver=cvx.ECOS)

    return w.value
```

In [20]:
```
def run_multi_period_opt(mu, Sigma, gamma, kappa):
    """ Simulates multi-period portfolio optimization
    """
    w_star = opt_port(mu, Sigma, gamma).T

    # pre-trade weights
    wt = np.zeros((T, n))

    # post-trade weights
    ws = wt.copy()

    # trade weights
    us = ws.copy()

    # start at optimal
    ws[0, :] = w_star.copy()

    # multi-period rebalancing
    for t in xrange(1, T):

        # grow weights due to simulated market returns
        wt[t, :] = grow_weights(ws[t-1, :]).T

        # construct optimal portfolio
        ws[t, :] = opt_port(mu, Sigma, gamma,
                            trans_costs=True, kappa=kappa, wt=wt[t, :]).T

        # trades
        us[t, :] = ws[t, :] - wt[t, :]

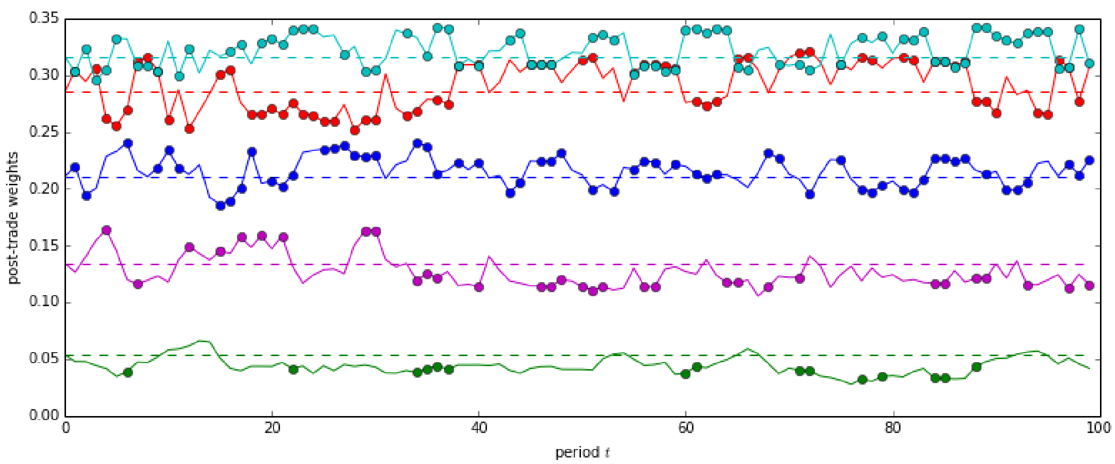
    return ws, us, np.array(w_star)[0], wt
```

In [21]:
```
def grow_weights(wt):
    # grows weights by simulated market returns
    wt = np.reshape(np.matrix(wt), (-1, 1))
    rt = generateReturns()
```

```
        return (np.diag(rt) * wt) / (rt * wt)
```

```
In [22]: def plot_weights(ws, us, w_star):
             """ Plotting code
             Provide three objects:
                 - ws: np.array of size T x n,
                       the post-trade weights w_t_tilde;
                 - us: np.array of size T x n,
                       the trades at each period: w_t_tilde - w_t;
                 - w_star: np.array of size n,
                       the "target" solution w_star.
             """
             colors = ['b','r','g','c','m']
             plt.figure(figsize=(13,5))
             for j in range(n):
                 plt.plot(range(T), ws[:,j], colors[j])
                 plt.plot(range(T), [w_star[j]]*T,  colors[j]+'--')
                 non_zero_trades = abs(us[:,j]) > threshold
                 plt.plot(np.arange(T)[non_zero_trades],
                     ws[non_zero_trades, j], colors[j]+'o')
             plt.ylabel('post-trade weights')
             plt.xlabel('period $t$')
```

```
In [23]: ws, us, w_star, wt = run_multi_period_opt(mu, Sigma, gamma, kappa=kappa_1)
         plot_weights(ws, us, w_star)
```




```
In [24]: def frac_tiny_trades(us):
             return 1.0 * np.sum(np.max(us, 1) <= 1e-3) / len(us)
```

```
In [25]: print "tiny trades ", frac_tiny_trades(us)
```

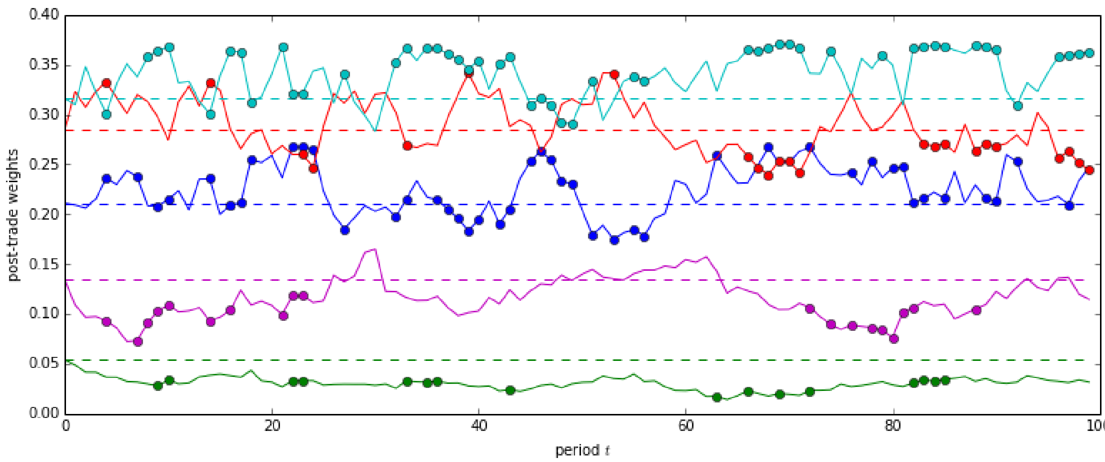
```
tiny trades  0.16
```

```
In [26]: def frac_no_trade_cond(wt, gamma, Sigma, kappa, w_star):
             no_trad_cond = (gamma * np.abs(np.dot(wt - w_star, Sigma)) <= kappa).all(axis=1)
             return 1.0 * np.sum(no_trad_cond) / len(no_trad_cond)
```

```
In [27]: frac_no_trade = frac_no_trade_cond(wt, gamma, Sigma, kappa_1, w_star)
         print "frac periods no trade cond",  frac_no_trade
```

```
frac periods no trade cond 0.06
```

```
In [29]: ws, us, w_star, wt = run_multi_period_opt(mu, Sigma, gamma, kappa=kappa_2)
         plot_weights(ws, us, w_star)
```




```
In [30]: # we see that for high transaction costs, 36% of periods negligible trades (too expensive!)
         print "tiny trades ", frac_tiny_trades(us)
```

```
tiny trades  0.41
```

```
In [31]: # with high transaction costs, the deviation of wt from
         # the target portfolio is infrequently less than the cost of tradding
         frac_no_trade = frac_no_trade_cond(wt, gamma, Sigma, kappa_1, w_star)
         print "frac periods no trade cond",  frac_no_trade
```

```
frac periods no trade cond 0.03
```