ASTRONOMY 598: MONTE CARLO METHODS HOMEWORK 1

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Contents

README

This directory contains the code that answers questions 1a, 1b, 2a, and 2b from homework 1 while this document answers the rest of the questions. Specifically, `poissonGaus.py` code implements the Poisson and Gaussian probability distribution functions and the corresponding cumulative distribution functions `CumulativePoisson` and `CumulativeGaussian`. The script `run_hw1.py` generates the figures for problems 1a and 2a, but I've commented out the lines that actually save the figures since they are supplied in this directory. To run, enter `python run_hw1.py`.

Running on Hyak

To run the code on Hyak, follow the instructions given below.

1) Create an interactive session by entering `qsub -I -l walltime=hr:min:sec` where hr = 03 is a safe amount of time

2) Find your favorite python distribution (2.7 for this code) using `module avail`

3) Load the python distribution via `module load (name of package found using module avail)`

4) Run the script by typing `python run_hw1.py`

Problem 1

The answers to part a and b are given in the accompanying code.

1c. CumulativePoisson$(\lambda, m) = e^{-\lambda} \sum_{m_i \leq m} \frac{\lambda^{m_i}}{m_i!}$ since for a discrete random variate with some probability distribution function $p(x)$, the cumulative distribution function is given by

$$F(x) = P(X \leq x) = \sum_{x_i \leq x} p(x_i). \tag{1}$$

Where $P(E)$ denotes the probability of event E occurring. Therefore, CumulativePoisson(λ, m_1) - CumulativePoisson$(\lambda, m_2) = P(m_1 < X \leq m_2)$ assuming $m_1 > m_2$. If $m_2 > m_1$, then it is equal to $-P(X \geq m_2)$.

1d.

$$\text{CumulativePoisson}(\lambda, \text{int}(100 \times \lambda)) = \sum_{m_i \leq \text{int}(100\times\lambda)} \frac{\lambda^{m_i}}{m_i!}. \tag{2}$$

Since this sum goes to integers $m \gg \lambda$, we know the denominator of the sum in the CumulativePoisson function will quickly become large as $m!$ grows much more quickly than λ^m for all m implying that m is vary likely to take smaller values. Additional m terms will contribute very little to the total probability $P(X \leq m)$ as the sum quickly converges to 1. Therefore, we expect CumulativePoisson$(\lambda, \text{int}(100 \times \lambda)) \approx 1$.

1e.

$$1 - \text{CumulativePoisson}(\lambda, m) = P(X > m). \tag{3}$$

1f. In the Poisson distribution, λ represents the event rate, the average number of events that occur in a given interval. It is also the expected value and the variance for this distribution.

Problem 2

The answers to part a and b are given in the accompanying code.

2c. For the Gaussian distribution, μ is the expected value (the mean), the median, and the mode of the distribution. The standard deviation, a measure of the width of the distribution, is given by σ. The variance of the distribution is given by σ^2.