

Inferring the Evolutionary Histories of Stars and Their Planets

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A dissertation
submitted in partial fulfillment of the
requirements for the degree of

Doctor of Philosophy

University of Washington

2020

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Program Authorized to Offer Degree:
Astronomy

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Abstract

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Abstract text.

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ACKNOWLEDGMENTS

Acknowledgement statement.

DEDICATION

To my wife, Anya

Chapter 1

INTRODUCTION

1.1 First Introduction Section Title

1.2 Second...

1.3 Third...

Chapter 2

RESONANT DYNAMICAL INTERACTIONS BETWEEN BINARY STARS AND CIRCUMBINARY DISKS

Chapter 3

COUPLED STELLAR-TIDAL EVOLUTION EJECTION OF CIRCUMBINARY EXOPLANETS

Portions of this chapter were originally published in collaboration with Rory Barnes, David E. Graham, Rodrigo Luger, and Thomas R. Quinn in the May 2018 edition of the Astrophysical Journal (Fleming et al., 2018, ApJ, Vol. 858, 2; 2018 ©American Astronomical Society), and are reproduced below with permission of the American Astronomical Society.

3.1 Abstract

We outline a mechanism that explains the observed lack of circumbinary planets (CBPs) via coupled stellar-tidal evolution of isolated binary stars. Tidal forces between low-mass, short-period binary stars on the pre-main sequence slow the stellar rotations, transferring rotational angular momentum to the orbit as the stars approach the tidally locked state. This transfer increases the binary orbital period, expanding the region of dynamical instability around the binary, and destabilizing CBPs that tend to preferentially orbit just beyond the initial dynamical stability limit. After the stars tidally lock, we find that angular momentum loss due to magnetic braking can significantly shrink the binary orbit, and hence the region of dynamical stability, over time impacting where surviving CBPs are observed relative to the boundary. We perform simulations over a wide range of parameter space and find that the expansion of the instability region occurs for most plausible initial conditions and that in some cases, the stability semi-major axis doubles from its initial value. We examine the dynamical and observable consequences of a CBP falling within the dynamical instability limit by running N-body simulations of circumbinary planetary systems and find that typically, at least one planet is ejected from the system. We apply our theory to the shortest period *Kepler* binary that possesses a CBP, Kepler-47, and find that its existence is consistent with our

model. Under conservative assumptions, we find that coupled stellar-tidal evolution of pre-main sequence binary stars removes at least one close-in CBP in 87% of multi-planet circumbinary systems.

3.2 Introduction

To date, 11 transiting circumbinary planets (CBPs) have been discovered by *Kepler*. The shortest period binary star system around which a CBP has been discovered is Kepler-47, with a binary period of 7.45 days (Orosz et al., 2012). The lack of CBPs around shorter period binaries is probably real given the thousands of short-period ($P_{bin} \lesssim 10$ days) eclipsing binaries discovered by the *Kepler* mission (Kirk et al., 2016) and observational biases that favor their detection (Muñoz and Lai, 2015). From a planet formation standpoint, there should not be a severe lack of CBPs: both Alexander (2012) and Vartanyan et al. (2016) show that circumbinary disks around binaries with semi-major axes $a \lesssim 1$ AU provide favorable conditions for planet formation. Bromley and Kenyon (2015) demonstrated that outside the inner region of the circumbinary disk, planet formation should occur similarly to planet formation in disks around single stars. From these results, Bromley and Kenyon (2015) concluded that circumbinary and single star planet occurrence rates should be similar, a claim bolstered by both Martin and Triaud (2014) and Armstrong et al. (2014) who find that the minimum CBP occurrence rate derived from *Kepler* data is of order 10% and increases with CBP inclination relative to the plane of the binary.

Although detecting CBPs via the transit method is more difficult than in the single star case (Welsh et al., 2014; Winn and Fabrycky, 2015), especially since many CBPs spend less than 50% of the time in a transiting configuration (Martin, 2017), CBPs have in general a higher transit probability than their single star counterparts (Martin and Triaud, 2015) making their detection feasible. Martin (2017) showed that the time-dependent chance of observing the transit of CBPs implies that continued future observations of the *Kepler* field could find up to 30 new CBPs, so where are the planets orbiting short-period binaries?

One explanation for the lack of transiting CBPs could simply be that most CBPs are not in a transiting configuration, perhaps due to dynamical interactions with the central binary. However,

Foucart and Lai (2013) show that natal circumbinary disks, and hence the planets themselves, should be nearly coplanar with the binary due to gravitational torques from the central binary on the disk, an effect that is especially pronounced for short-period binaries. Furthermore, in an analysis of the observed population of *Kepler* CBPs, Li et al. (2016) find that the observed coplanarity of CBPs and their host binaries is not due to a selection bias. From both theoretical arguments and analysis of *Kepler* data, it seems that an additional physical mechanism is required to explain the lack of discovered transiting CBPs in the *Kepler* field around short-period binaries.

Several recent studies have invoked the presence of a stellar tertiary companion to explain not only how short-period binaries could form but also to explain the lack of CBPs around short-period binaries. Fabrycky and Tremaine (2007) showed that secular interactions with a tertiary companion can drive Kozai-like oscillations that cause large eccentricity oscillations in the inner binary. The increased binary eccentricity leads to efficient tidal dissipation in the inner binary, shrinking the orbital period to of order a day from much longer periods. The comprehensive population synthesis study by Moe and Kratter (2018) support this finding and show that the combination of tidal dissipation and Kozai-like oscillations due to a tertiary companion can account for $\sim 40\%$ of binaries with periods $\lesssim 10$ days. Muñoz and Lai (2015), Martin et al. (2015), and Hamers et al. (2016) all show that these binary-tertiary interactions, in addition to shrinking the inner binary orbit, can lead to rich dynamical interactions that can drive many CBPs towards eccentric and inclined orbits, making detection more difficult and potentially leading to orbital instability. This mechanism provides a particularly compelling explanation for the lack of CBPs around short-period binaries given that in a survey of solar-type binaries, Tokovinin et al. (2006) find that 96% of binaries with periods $\lesssim 3$ days have a tertiary companion. However, no study to date has examined the lack of CBPs around isolated binaries, i.e., binaries without a tertiary companion. Not all close binaries have a companion, as Tokovinin et al. (2006) find that the tertiary companion fraction decreases to 34% for binaries with periods $\gtrsim 12$ days after correcting for observational biases, indicating that binaries with orbital periods $\gtrsim 3$ days are less likely to have a tertiary companion, and therefore the Kozai-like oscillations model cannot solely account for their lack of observed CBPs.

Short-period isolated binaries can form through a combination of fragmentation and dynamical

processing. Bonnell and Bate (1994) found that very low mass ($\lesssim 0.01M_{\odot}$) binaries with separations $\lesssim 1$ AU can form either in a protoplanetary disk orbiting an unstable protostellar core or from the unstable core itself, and must accrete mass to become a stellar binary. When close binaries do form, simulations by Bate (2000) find that they are likely to host circumbinary disks, necessary for CBP formation. Circumbinary disks play a major role in hardening the central binary and increasing its mass; Bate (2000) shows that accretion from a circumbinary disk is likely to shrink the binary separation, sometimes by up to 2 orders of magnitude. Simulations by Artymowicz and Lubow (1996) find that binaries can efficiently accrete mass from a circumbinary disk, indicating that shrinking the binary semi-major axis via accretion can readily occur. With the inclusion of a realistic treatment of magnetic fields in MHD simulations of protobinary stars, Zhao and Li (2013) find that binary orbital decay via accretion is significantly enhanced relative to simulations without magnetic fields. Additionally, gravitational torques between a circumbinary disk and the central binary shrink the binary semi-major axis (e.g. Artymowicz et al., 1991; Bate et al., 2002; Armitage and Natarajan, 2005; Fleming and Quinn, 2017), which, when coupled with accretion, can produce short-period, isolated binaries.

In this paper, we focus on isolated binaries with binary orbital periods in the regime $3 \leq P_{bin} \leq 7.45$ days, as these binaries are less likely to have a tertiary companion than binaries with $P_{orb} \leq 3$ days (Tokovinin et al., 2006). The upper limit of this range corresponds to the orbital period of Kepler-47, the shortest period CBP-hosting binary system. We also consider the full population of *Kepler* CBPs. As previously mentioned, in the *Kepler* sample there are no known CBPs orbiting the ~ 2000 eclipsing binaries with $P_{orb} \lesssim 7.45$ days (Kirk et al., 2016), highlighted in the red-shaded region in Fig. 3.1. Note that CBPs have been discovered by other means, such as microlensing (e.g. Bennett et al., 2016).

One intriguing characteristic of the observed population of *Kepler* CBPs is their tendency to orbit just exterior to the dynamical stability limit (Welsh et al., 2014; Winn and Fabrycky, 2015). The dynamical stability limit, referred to as the “critical semi-major axis” (a_{crit}), is the minimum semi-major axis for a CBP to remain dynamically stable (Dvorak et al., 1989; Holman and Wiegert, 1999). Holman and Wiegert (1999) derived an empirical formula for a_{crit} from an ensemble of

N-body simulations given by

$$a_{crit} = (1.60 + 5.1e - 2.22e^2 + 4.21\mu - 4.27e\mu - 5.09\mu^2 + 4.61e^2\mu^2)a \quad (3.1)$$

where a is the binary semi-major axis, e the binary eccentricity and $\mu = m_2/(m_1 + m_2)$ is the binary mass ratio. We note that the a_{crit} derived by Holman and Wiegert (1999) is not a hard limit as Eq. (3.1) has an error of about 3% - 6%. We plot the semi-major axis of observed *Kepler* CBPs normalized by a_{crit} (a_{CBP}/a_{crit}) as a function of host binary orbital period in Fig. 3.1 to demonstrate CBPs' tendency to orbit just exterior to a_{crit} . Using Nbody simulations, Quarles et al. (2018) found that some *Kepler* circumbinary systems could host an additional planet interior to the observed one. Clearly, however, the observed CBPs cluster near the stability limit as the distribution of the ratio of a_{CBP} to their host binary's a_{crit} has a minimum of $a_{cbp}/a_{crit} \approx 1.1$ and a median of $a_{cbp}/a_{crit} \approx 1.26$. Analyses by Martin and Triaud (2014) and Li et al. (2016) show that this clustering does not solely stem from an observational bias, suggesting a physical origin.

CBP migration in a protoplanetary disk provides a compelling physical explanation for the observed pile-up of planets near the dynamical stability limit around binary stars. Numerous studies of planet formation in circumbinary disks show that CBPs likely did not form *in situ* (e.g. Paardekooper et al., 2012; Meschiari, 2012a,b; Pelupessy and Portegies Zwart, 2013) and instead migrated to their present location. Simulations of CBPs embedded in a natal protoplanetary disk by Pierens and Nelson (2007) show that the planets migrate inward, halting in the region of stability just outside of the central disk cavity that is formed from binary gravitational truncation (Artymowicz and Lubow, 1994). Dunhill and Alexander (2013), Pierens and Nelson (2013), and Kley and Haghighipour (2014) all find that CBP migration can explain the observed properties of CBPs discovered by *Kepler*, depending on the precise mass and structure of the disk, suggesting that the pile-up of CBPs near the dynamical stability limit is indeed a real and expected effect. If these planets did in fact form farther out in the disk and migrate inward, they must have formed in the first few Myrs of the system's existence before the disk dispersed, given typical lifetimes of

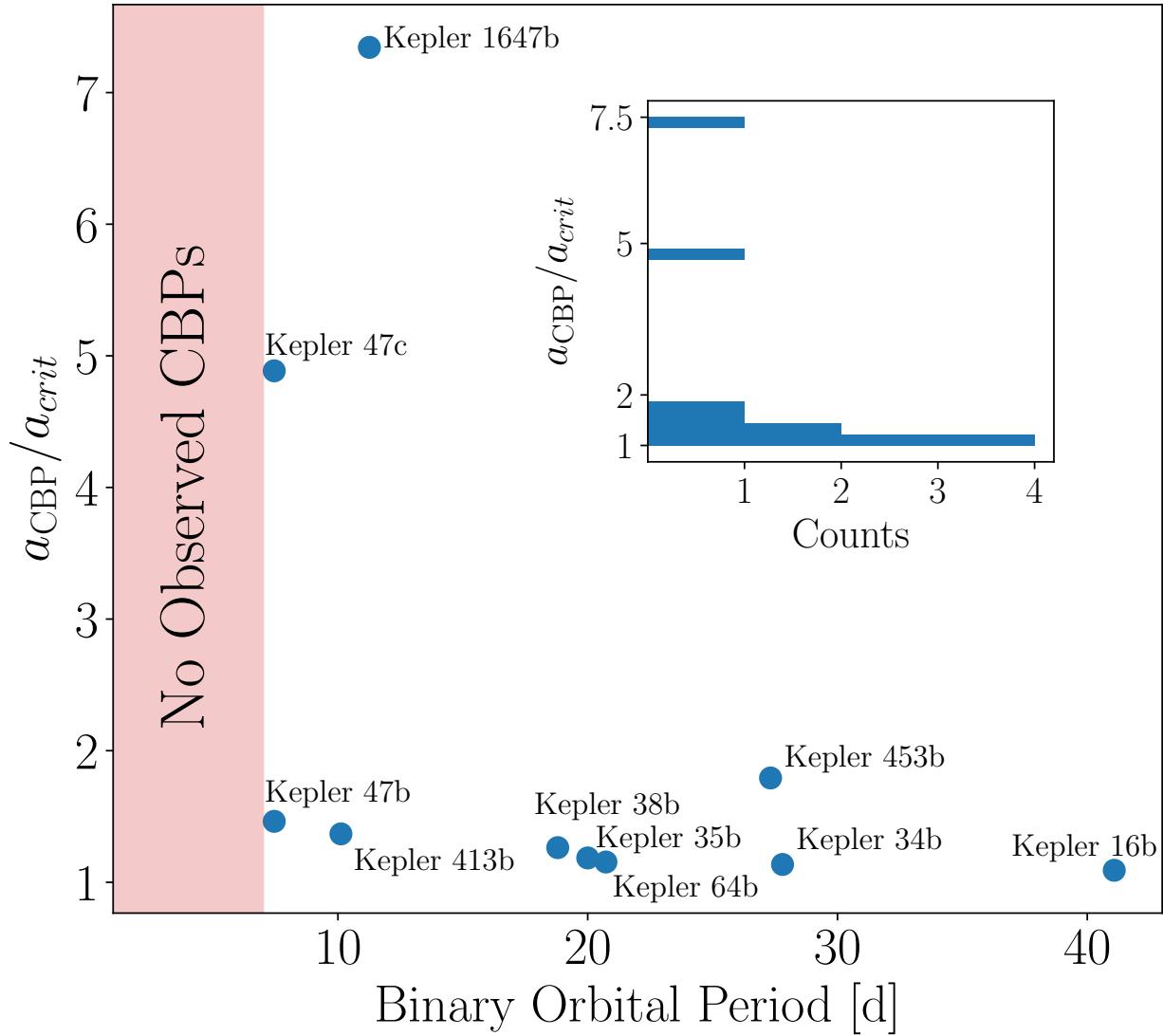


Figure 3.1: Semi-major axes of confirmed *Kepler* CBPs relative to the host binary's critical semi-major axis ($a_{\text{CBP}}/a_{\text{crit}}$) as a function of binary orbital period. Each point is annotated with the planet's name. The red-shaded region highlights the observational finding that, to date, no CBPs have been discovered orbiting binaries with binary orbital periods less than 7.45 days. (Welsh et al., 2014; Winn and Fabrycky, 2015). We neglect Kepler-47d as its orbital parameters are not well constrained. *Inset:* Histogram of observed *Kepler* CBPs' $a_{\text{CBP}}/a_{\text{crit}}$. The histogram demonstrates that most CBPs' semi-major axes cluster near the dynamical stability limit, a_{crit} .

protoplanetary disks (e.g. Haisch et al., 2001).

Whether due to migration or uncharacterized circumbinary disk physics, the pile-up of CBPs near a_{crit} seems to have a physical origin. The other important characteristic of the observed *Kepler* CBP population, the binary orbital period below which no CBPs are observed, 7.45 days (see Fig. 3.1), likely has a physical origin, as well. Curiously, this cutoff is within the range of binary orbital periods Zahn and Bouchet (1989) found that separates eccentric from circular binary systems, 7.2 – 8.5 days. The theoretical cutoff identified by Zahn and Bouchet (1989) in their study of coupled stellar-tidal physics effectively characterizes the approximate binary orbital period at which the influence of tides becomes important to the system’s evolution. In this case, it seems that in addition to stellar evolution, tidal processes in binary star systems may impact the observed circumbinary planet distribution.

In this work, we propose that the lack of CBPs around short-period binary stars is a natural outcome of coupled stellar-tidal binary evolution that we describe as follows: In young binary systems, tidal forces synchronize the stellar rotations to match the orbital period, transferring rotational angular momentum to the orbit, increasing the orbital semi-major axis, and finally, expanding the region of dynamical instability around the binary and engulfing CBPs. CBPs located just exterior to the initial dynamical stability limit enter the expanding instability region, become destabilized, and can be ejected from the system. We refer to this proposed mechanism, the Stellar-Tidal Evolution Ejection of Planets, as the STEEP process for notational convenience.

In § 4.3, we detail our computational methods and outline the mathematics of our theory. We outline our experimental scheme in § 4.4 and discuss the results of our simulations in § 4.5 and § 3.6. We apply our theory to the Kepler-47 system in § 3.7 and explore the implications our results and future prospects in § 4.6.

3.3 Methods

In this section, we outline how we simulate coupled stellar-tidal evolution using the code VPLanet (Barnes et al., 2016, Barnes *et al.*, *in prep*) and perform N-body simulations using the code REBOUND (Rein and Liu, 2012) to probe the stability of circumbinary planetary systems in which

the inner-most planet falls within a_{crit} as a result of coupled stellar-tidal evolution.

3.3.1 VPLanet

We simulate coupled stellar-tidal binary star evolution using the code `VPLanet`, a modular code that allows the user to specify which physical processes impact a given variable. Each physical process, here referred to as a module, is given by a set of nonlinear ordinary differential equations or explicit functions of time (see § 3.3.2 for the `STELLAR` module, § 3.3.3 for the `EQTIDE` module, and § 3.3.4 for additional coupling of `STELLAR` and `EQTIDE` for an in-depth description and their respective equations). `VPLanet` provides a framework in which equations from different modules are coupled such that different physical processes impact the evolution of a given variable by summing the time derivatives from each relevant module. For n modules impacting the evolution of the variable x , at each timestep `VPLanet` computes the time derivative of x as

$$\left(\frac{dx}{dt}\right)_{tot} = \left(\frac{dx}{dt}\right)_1 + \left(\frac{dx}{dt}\right)_2 + \dots + \left(\frac{dx}{dt}\right)_n. \quad (3.2)$$

This numerical setup allows `VPLanet` to simultaneously integrate an arbitrary number of coupled nonlinear ODEs to rapidly simulate a system in which numerous physical processes impact the system, such as tidally interacting pre-main sequence binaries.

`VPLanet` numerically integrates the equations using a fourth-order Runge-Kutta scheme with adaptive time-stepping. Our time-stepping algorithm chooses the timestep to resolve the evolution of the fastest changing variable for each simulation step in order to ensure that we completely resolve the evolution of the system. The timescale over which a given variable x changes is estimated by computing $|x|/|dx/dt|$ where dx/dt is the instantaneous derivative of the variable x computed by `VPLanet` via Eq. (3.2). Each simulation step, we calculate the timestep by computing, for each variable, its evolutionary timescale under each module and multiply the minimum value by a scale factor, η . We find that our simulations converge and approximately conserve both energy and angular momentum to $\sim 10^{-4}$ when $\eta \lesssim 10^{-3}$ (see § 3.3.5 and § 3.5.1).

3.3.2 Stellar Evolution

In our simulations, we track how a star’s rotation rate and radius change over time due to stellar evolution using a module called `STELLAR`. The two main stellar evolution processes that impact a star’s radius and rotation rate are stellar contraction/expansion and magnetic braking. In general, a star’s radius will contract during the pre-main sequence phase and expand slowly during the main sequence. Conservation of angular momentum dictates that as a star contracts, its rotation rate increases (and vice-versa). We model a star’s radius as a function of time using a cubic spline interpolation of the radius tracks for a star of a given mass from the stellar evolution models of Baraffe et al. (2015) for solar metallicity stars.

We derive the time derivative of a star’s rotation rate due to both magnetic braking and radius evolution under conservation of angular momentum. For simplicity, we model a star as a solid body with a given density profile parameterized by the radius of gyration, r_g , where the moment of inertia is given by $I = mr_g^2R^2$ for mass m and radius R . We assume solid body rotation for stars as the surface rotation evolution of low-mass ($\lesssim 1M_\odot$) stars can be reasonably approximated by assuming stellar solid-body rotation (Bouvier et al., 1997) and since adopting stellar solid-body rotation is common amongst studies examining stellar-tidal interactions (e.g. Dobbs-Dixon et al., 2004; Heller et al., 2011; Barnes et al., 2013; Repetto and Nelemans, 2014; Bolmont and Mathis, 2016; Bolmont et al., 2017). We neglect effects such as differential rotation and changes in r_g but perform sensitivity tests on r_g in § 3.5.3.

The rotational angular momentum for a star is simply $J = I\omega$, where ω is the rotational frequency. By conservation of angular momentum, the star’s rotation rate changes due to stellar radius evolution according to

$$\dot{\omega}_{contraction} = \frac{-2\dot{R}\omega}{R}. \quad (3.3)$$

A star loses angular momentum due to magnetic braking, decreasing ω . Magnetic braking is caused by the corotation of the stellar wind with the star’s magnetic field lines (see Parker, 1958; Mestel, 1968). The poloidal magnetic field of the star carries the corotating mass far away from the star, effectively removing angular momentum from the star. Even though mass loss rates for

sun-like stars are small (e.g., $\dot{M} \sim 10^{-14} M_{\odot}/\text{yr}$; Tarduno et al., 2014), a star's rotation rate can slow appreciably with time due to this effect (see Fig. 3.2).

Numerous models for stellar magnetic braking have been examined in the literature and here we consider two models. The first is from Reiners and Mohanty (2012), who derived their model in the context of relating stellar rotation to stellar magnetic field strength. The model is calibrated to reproduce observations of the Sun's current rotation period and the rotation-mass distribution of field stars that are a few Gyr-old. Reiners and Mohanty (2012) give the change in stellar angular momentum due to magnetic braking as

$$\begin{aligned}\frac{dJ_{\star}}{dt} &= -C \left[\omega \left(\frac{R^{16}}{m^2} \right)^{1/3} \right] \text{ for } \omega \geq \omega_{crit} \\ \frac{dJ_{\star}}{dt} &= -C \left[\left(\frac{\omega}{\omega_{crit}} \right)^4 \omega \left(\frac{R^{16}}{m^2} \right)^{1/3} \right] \text{ for } \omega < \omega_{crit},\end{aligned}\quad (3.4)$$

where the authors find a best fit of $C = 2.66 \times 10^3 (\text{gm}^5 \text{ cm}^{-10} \text{ s}^3)^{1/3}$, $\omega_{crit} = 8.56 \times 10^{-6} \text{ s}^{-1}$ for $m > 0.35 M_{\odot}$, and $\omega_{crit} = 1.82 \times 10^{-6} \text{ s}^{-1}$ for $m \leq 0.35 M_{\odot}$.

The second magnetic braking model we consider is presented in Repetto and Nelemans (2014) and is derived from the empirical relation for stellar spin-down of Sun-like stars empirically derived by Skumanich (1972). The change in angular momentum due to this spin-down law is given by

$$\frac{dJ_{\star}}{dt} = -\gamma m r_g^2 R^4 \omega^3 \quad (3.5)$$

where $\gamma = 5 \times 10^{-25} \text{ s m}^{-2}$ (Repetto and Nelemans, 2014). Assuming one of the magnetic braking laws for J_{\star} , the change in stellar rotation rate due to magnetic braking is

$$\dot{\omega}_{MB} = \frac{\dot{J}_{\star}}{I} \quad (3.6)$$

for a fixed stellar radius under conservation of angular momentum assuming negligible mass loss.

Both magnetic braking laws presented above are derived for spin-down rates of single stars,

while in this work, we apply them to the evolution of short-period stellar binaries. These magnetic braking laws, however, have successfully been used to model the evolution of short-period binary systems ranging from compact object-stellar binaries (e.g. Verbunt and Zwaan, 1981; Repetto and Nelemans, 2014), the formation of main sequence stellar contact binaries (e.g. Stepień, 1995; Andronov et al., 2006), and cataclysmic variable evolution (e.g. Ivanova and Taam, 2003). Therefore, our usage of magnetic braking laws derived for single stars is valid in this context.

We combine Eq. (3.3) and Eq. (3.6) to get the net change in stellar rotation rate due to magnetic braking and stellar radius evolution under conservation of angular momentum:

$$\dot{\omega} = \dot{\omega}_{\text{contraction}} + \dot{\omega}_{\text{MB}} = \frac{\dot{J}_\star}{I} - \frac{2\dot{R}\omega}{R}. \quad (3.7)$$

and \dot{J} is given by either Eq. (3.4) or Eq. (3.5).

In Fig. 3.2, we plot the stellar radius and rotation period evolution for solar metallicity low-mass stars assuming $r_g = 0.27$ using both magnetic braking laws to demonstrate the qualitative behavior of our stellar evolution model, STELLAR. In general, the stellar radii contract along the pre-main sequence and slowly expand on the main sequence, while the stellar rotations slow over time due to magnetic braking.

3.3.3 Tidal Evolution

For our tidal physics, we use a variant of the equilibrium tidal theory first introduced by Darwin (1880), the “Constant Phase Lag” (CPL) equilibrium tide theory as derived in Ferraz-Mello et al. (2008) in our module, EQTIDE. Equilibrium tidal theories predict that gravitational torques between the bodies and their respective tidal bulges drive a secular evolution in the eccentricity (e), semi-major axis (a) and the bodies’ spins (ω) and obliquities (ψ). The CPL model assumes the tidally-interacting bodies raise tidal bulges on each other that maintain a fixed phase with respect to the line connecting the bodies’ centers of mass. A tidal bulge is composed of a linear sum of discrete tidal lags each with their own respective frequency. Each tidal lag’s frequency is independent of any orbital or rotational forcing frequency and there is no coupling between tidal lags. In

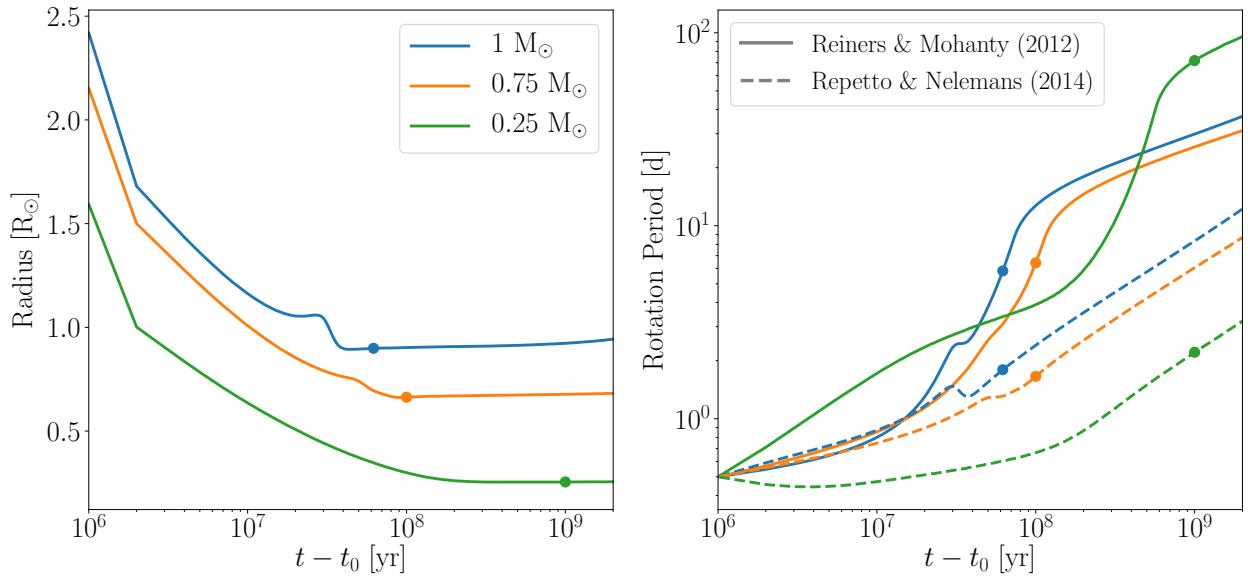


Figure 3.2: Stellar radius and rotation period evolution as computed by our stellar evolution model, STELLAR, for a $1 M_{\odot}$ (blue), $0.75 M_{\odot}$ (orange), and $0.25 M_{\odot}$ (green) star both with solar metallicity. The dots in both panels indicate the approximate time each star reaches the Zero Age Main Sequence. *Left:* Stellar radius as a function of time according to a cubic spline interpolation of the Baraffe et al. (2015) stellar models. *Right:* Stellar rotation period as a function of time computed via stellar radius evolution and both magnetic braking models under conservation of angular momentum (see Eq. (4.4)). The solid lines correspond to simulations using the Reiners and Mohanty (2012) magnetic braking law while the dashed lines use the Repetto and Nelemans (2014) magnetic braking law.

this formalism, the CPL model is akin to a driven, damped harmonic oscillator (Greenberg, 2009). This theory, accurate up to second order in e , has been used extensively in many previous studies (e.g. Leconte et al., 2010; Heller et al., 2011; Barnes et al., 2013) and has successfully reproduced the qualitative tidal evolution of Solar System bodies (e.g. Goldreich and Soter, 1966). Given the physical complexity of tidally interacting astrophysical bodies, linear equilibrium tidal models, such as the CPL model, are likely not valid at large e or inclinations as the linearity assumption breaks down (Ferraz-Mello et al., 2008; Greenberg, 2009). To maintain qualitative accuracy in our tidal evolution, we restrict the binary eccentricity to $e \lesssim 0.2$. We note that other equilibrium tidal theories exist, such as the “Constant Time Lag” (CTL) model (e.g. Hut, 1981), but we do not consider them here since in our adopted eccentricity regime, Leconte et al. (2010) showed that both the CPL and CTL model yield similar results.

Below we present a form of the CPL model tidal evolution given by Heller et al. (2011) with a modification for synchronous rotators from Ferraz-Mello et al. (2008). Note that in this work we set all obliquities to 0 and do not consider their evolution, however we include it in our model for completeness. The CPL equations for e and a evolution are:

$$\frac{de}{dt} = -\frac{ae}{8Gm_1m_2} \sum_{i=1}^2 Z'_i \left(2\varepsilon_{0,i} - \frac{49}{2}\varepsilon_{1,i} + \frac{1}{2}\varepsilon_{2,i} + 3\varepsilon_{5,i} \right) \quad (3.8)$$

$$\frac{da}{dt} = \sum_{i=1}^2 \frac{da_i}{dt} \quad (3.9)$$

where if the i^{th} body is tidally locked in a synchronous orbit,

$$\frac{da_{i,sync}}{dt} = -\frac{a^2}{Gm_1m_2} Z'_i (7e^2 + \sin^2(\psi_i)) \varepsilon_{2,i}, \quad (3.10)$$

otherwise

$$\begin{aligned} \frac{da_i}{dt} = & \frac{a^2}{4Gm_1m_2} Z'_i \left(4\varepsilon_{0,i} + e^2 \left[-20\varepsilon_{0,i} + \frac{147}{2}\varepsilon_{1,i} \right. \right. \\ & \left. \left. + \frac{1}{2}\varepsilon_{2,i} - 3\varepsilon_{5,i} \right] - 4\sin^2(\psi_i) [\varepsilon_{0,i} - \varepsilon_{8,i}] \right). \end{aligned} \quad (3.11)$$

The CPL equations for ψ and ω evolution are

$$\frac{d\psi_i}{dt} = \frac{Z'_i \sin(\psi_i)}{4m_i r_{g,i}^2 R_i^2 n \omega_i} ([1 - \xi_i] \epsilon_{0,i} + [1 + \xi_i] (\epsilon_{8,i} - \epsilon_{9,i})) \quad (3.12)$$

$$\begin{aligned} \frac{d\omega_i}{dt} = & -\frac{Z'_i}{8m_i r_{g,i}^2 R_i^2 n} (4\epsilon_{0,i} + e^2 [-20\epsilon_{0,i} + 49\epsilon_{1,i} + \epsilon_{2,i}] \\ & + 2 \sin^2(\psi_i) [-2\epsilon_{0,i} + \epsilon_{8,i} + \epsilon_{9,i}]) \end{aligned} \quad (3.13)$$

for the i^{th} body where G is Newton's gravitational constant, n is the binary's mean motion, and ϵ denote the signs of the tidal phase lags.

The intermediate variables Z'_i and ξ_i are given by

$$Z'_i = 3G^2 k_{2,i} m_j^2 (m_1 + m_2) \frac{R_i^5}{a^9} \frac{1}{n Q_i} \quad (3.14)$$

$$\xi_i = \frac{r_{g,i}^2 R_i^2 \omega_i a n}{G m_j} \quad (3.15)$$

where the j^{th} body is the i^{th} body's companion in the binary, $k_{2,i}$ is the i^{th} body's Love number of degree 2, and Q is the tidal quality factor (also referred to as the "tidal Q"). For all stars in all simulations, we assume $k_2 = 0.5$. This choice of k_2 does not impact our results as it is degenerate with the choice of tidal Q via the k_2/Q scaling in Eq. (3.14). We choose to vary stellar tidal Qs to probe how different tidal dissipation rates impact our results (see § 3.5.2).

The signs of the tidal phase lags for the i^{th} body are given by

$$\begin{aligned}\varepsilon_{0,i} &= \Sigma(2\omega_i - 2n) \\ \varepsilon_{1,i} &= \Sigma(2\omega_i - 3n) \\ \varepsilon_{2,i} &= \Sigma(2\omega_i - n) \\ \varepsilon_{5,i} &= \Sigma(n) \\ \varepsilon_{8,i} &= \Sigma(\omega_i - 2n) \\ \varepsilon_{9,i} &= \Sigma(\omega_i)\end{aligned}\tag{3.16}$$

where $\Sigma(x)$ gives returns 1 for positive x , -1 for negative x , or 0 otherwise.

As a system approaches a tidally locked state, the numerical integration of our tidal equations can become unstable due to the discrete nature of the CPL model and of the integration scheme itself. For example, if a simulation approaches a tidally locked and synchronous state, $\omega \approx n$, then the derivatives of the tidal equations become discontinuous. Numerical integration is inherently a discrete scheme, so solutions for systems near such a state will oscillate around a 1:1 spin-orbit resonance, causing $\varepsilon_{0,i}$ to rapidly switch signs and hence change how the systems evolve, leading to unstable, unphysical behavior. To rectify this issue, once a body's spin period is within 1% of the orbital period, we force the system into a tidally locked, synchronous state by setting $\omega = n$, following Barnes et al. (2013). In § 3.3.4, we derive equations that account for the coupled stellar-tidal evolution for tidally locked star(s) to conserve both energy and angular momentum and ensure that our model results in a physically realistic evolution.

Stars on eccentric orbits can enter a pseudo-synchronous rotation state or become trapped in a higher order spin orbit resonance when the system tidally locks, with a familiar example being Mercury's 3:2 spin-orbit resonance (Goldreich and Peale, 1966). Note that here the use of "spin-orbit resonance" does not mean this system is trapped in a dynamical resonance in the traditional sense, but instead enters into a spin-orbit commensurability in which the spin and rotational frequencies are integer multiples of each other; we use "spin-orbit resonance" for notational convenience. If a body tidally locks into a pseudo-synchronous rotation state, the rotational period is a continuous

function of both the orbital eccentricity and period (see Goldreich, 1966; Wisdom, 2008). The CPL model, however, only permits 2 rotation states for tidally locked bodies, a 1:1 and 3:2 spin-orbit resonance (Barnes, 2017). In the CPL model an orbit is trapped in a 3:2 spin-orbit resonance when the binary tidally locks with $e \geq \sqrt{1/19} \approx 0.229$ (Ferraz-Mello et al., 2008) and locks into synchronous rotation otherwise. In our adopted eccentricity regime, $e \lesssim 0.2$, stars tidally lock into a synchronous state with $\omega = n$ but in § 3.5.7 we probe how capture into a 3:2 spin-orbit-resonance for more eccentric binary star systems impacts our results.

In Fig. 3.3, we plot the tidal evolution of a $1 M_{\odot}$ – $1 M_{\odot}$ binary star system using the fiducial parameters given in Table 3.1 to demonstrate the qualitative behavior of our tidal evolution model, EQTIDE. For details of the numerical intergration of the simulation, see § 3.3.1. In general, tides transfer angular momentum from the stellar rotations into the orbit until the binary reaches a tidally locked, synchronous orbit after $\sim 10^6$ yr. The binary orbit circularizes after $\sim 5 \times 10^8$ yr.

3.3.4 Coupled Stellar-Tidal Evolution

The coupled stellar-tidal orbital evolution of binary systems has been extensively studied in the literature for systems ranging from star-star binaries (e.g. Huang, 1966; Mestel, 1968; van't Veer and Maceroni, 1988; Zahn and Bouchet, 1989; Li and Wickramasinghe, 1998; Khaliullin and Khaliullina, 2011) to star-planet binaries (e.g. Dobbs-Dixon et al., 2004; Barker and Ogilvie, 2009; Lanza and Mathis, 2016) to even star-compact object binaries (e.g. Verbunt and Zwaan, 1981; Repetto and Nelemans, 2014). In particular, the pioneering theoretical study of Zahn and Bouchet (1989) tracked the coupled stellar-tidal evolution of low-mass stellar binaries with a focus on the pre-main sequence evolution and outlined the general qualitative behavior that arises from this coupling. Although their study mainly focused on orbital circularization during the pre-main sequence, Zahn and Bouchet (1989) also showed that as the binary approached synchronization, the binary orbital period increases as tides transfer rotational angular momentum from the stellar rotations to the orbit. We reproduce this phenomenon in our simulations (see § 4.5).

Common to many of the aforementioned studies of coupled stellar-tidal evolution is that for tidally locked systems, any angular momentum lost from the star(s) is lost at the expense of the

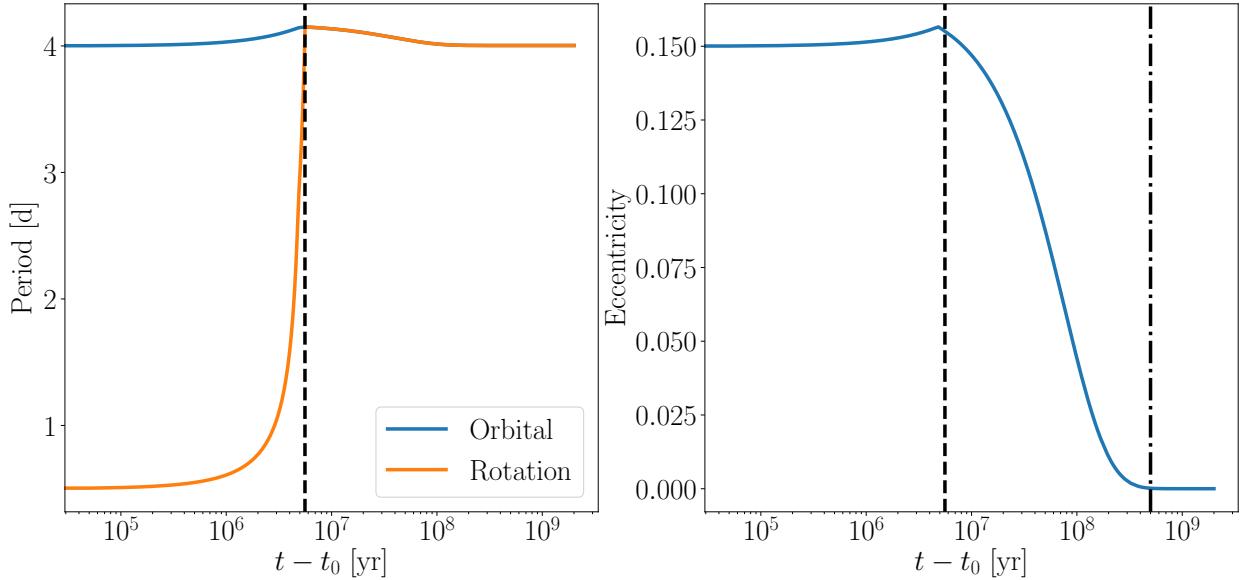


Figure 3.3: Tidal evolution of a binary star system with no stellar evolution under the CPL model (Ferraz-Mello et al., 2008; Heller et al., 2011) as computed by our tidal evolution model, EQTIDE. See the text for the system properties. *Left:* Binary orbital and stellar rotational period versus time. Tides transport angular momentum from the stellar rotations into the orbit until the binary becomes tidally locked and synchronous (denoted by the black dashed line). *Right:* Binary orbital eccentricity versus time. After the orbit synchronizes, the eccentricity decreases until the binary circularizes (denoted by the dash-dotted line).

orbit. For tidally locked synchronous rotators, for example, as magnetic braking slows the stellar rotations, tides speed up the stellar rotations to force the stars' spin periods to be equal to the orbital period. Tidal speed-up of stellar rotations to maintain synchronization removes angular momentum from the orbit, causing orbital semi-major axis decay and faster stellar rotations. A similar spin-orbit coupling occurs for stellar radius contraction and expansion for tidally locked stars. In this section, we derive equations for how the binary semi-major axis, a , changes due to both magnetic braking and stellar radius evolution when either one or both stars are in a tidally locked orbit. We assume constant mass, as mass loss is negligible for low-mass main sequence stars. These equations provide an additional change in the binary semi-major axis, denoted $\dot{a}_{coupled}$, such that the net change in the binary semi-major axis is $\dot{a}_{net} = \dot{a}_{tides} + \dot{a}_{coupled}$ where the \dot{a}_{tides} term comes from EQTIDE Eq. 3.9. We explore the dynamical and observational consequences for magnetic braking-driven semi-major axis decay in § 3.5.1 and § 3.5.8.

Case 1: One Tidally Locked Star

In the case of one tidally locked star, the angular momentum of the orbit and star are explicitly coupled as any angular momentum change in the star imparts a change in the orbit as mediated by tidal forces. In this case, we consider the following quantity

$$J = \mu_{tot} \sqrt{GMa(1 - e^2)} + m_1 r_{g,1}^2 R_1^2 \omega + J_{mb}, \quad (3.17)$$

where J is the total angular momentum, J_{mb} is the reservoir of angular momentum lost to space via magnetic braking, $\mu_{tot} = m_1 m_2 / (m_1 + m_2)$, $M = m_1 + m_2$ and the rotation rate ω is set by the star's tidally locked state, e.g. synchronous or a 3:2 spin-orbit-resonance. This net angular momentum quantity only includes the contributions from both the orbit and the tidally locked star as these are explicitly coupled by tides in this case. We assume $\dot{J} = 0$ as the total angular momentum is

conserved. By taking the time derivative of Eq. (3.17) and rearranging, we obtain

$$\dot{a}_{coupled}^{(1)} = \frac{-\dot{J}_{mb} - 2m_1 r_{g,1}^2 \omega R_1 \dot{R}_1 + \frac{\mu^2 G M a e}{J} \dot{e}}{\frac{\mu^2 G M (1-e^2)}{2J} - \frac{3\omega}{2a} m_1 r_{g,1}^2 R_1^2} \quad (3.18)$$

for the binary semi-major axis change due to the magnetic braking and stellar radius evolution for one tidally locked rotating star. Note that $\dot{J}_{mb} > 0$ as this term tracks the amount of angular momentum lost from the system and hence gains the amount lost from stars due to magnetic braking. Eq. (4.23) is given for the case when the primary star is tidally locked and is trivially altered for the case when the secondary star is tidally locked by exchanging indices.

Following the lead of previous works (e.g. Verbunt and Zwaan, 1981; Repetto and Nelemans, 2014), we assume magnetic braking and stellar radius evolution do not torque the orbit and hence cannot change e , only a . We leave the \dot{e} term in Eq. (4.23) and Eq. (4.24) for completeness.

Case 2: Two Tidally Locked Stars

When both stars are tidally locked, the angular momentum of the orbit and both stars is coupled. Just as in the one-tidally-locked-star case, the total angular momentum of the system

$$J = \mu_{tot} \sqrt{G M a (1 - e^2)} + m_1 r_{g,1}^2 R_1^2 \omega + m_2 r_{g,2}^2 R_2^2 \omega + J_{mb}, \quad (3.19)$$

where 1 and 2 denote the primary and secondary star, respectively, and $J = 0$. As before, we take the time derivative of Eq. (3.19), rearrange, and obtain

$$\dot{a}_{coupled}^{(2)} = \frac{-\dot{J}_{mb} - 2\omega \left(m_1 r_{g,1}^2 R_1 \dot{R}_1 + m_2 r_{g,2}^2 R_2 \dot{R}_2 \right) + \frac{\mu^2 G M a e}{J} \dot{e}}{\frac{\mu^2 G M (1-e^2)}{2J} - \frac{3\omega}{2a} \left(m_1 r_{g,1}^2 R_1^2 + m_2 r_{g,2}^2 R_2^2 \right)} \quad (3.20)$$

for the binary semi-major axis change due to the magnetic braking and stellar radius evolution.

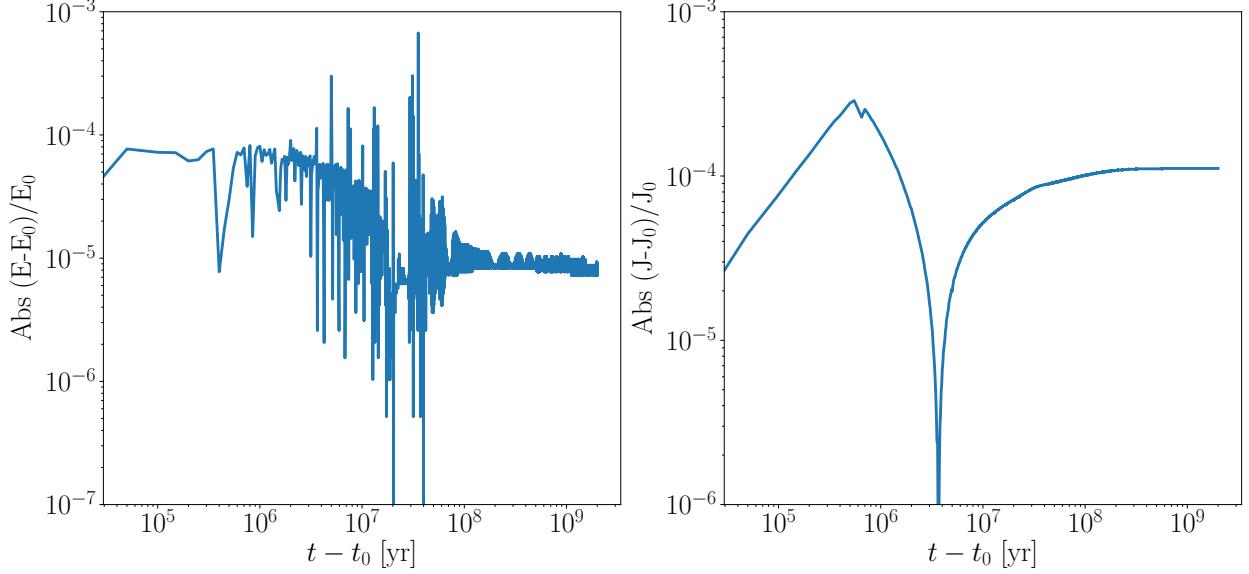


Figure 3.4: *Left Panel:* Absolute relative difference between the total system energy and initial total system energy as a function of time. *Right Panel:* Absolute relative difference of the total system angular momentum and initial total system angular momentum as a function of time. Both total energy and angular momentum are approximately conserved at the 10^{-4} level with high frequency oscillations caused by limitations of Runge-Kutta integrations.

Case 3: Free Rotators

We include either Eq. (4.23) or Eq. (4.24) in our numerical integration only when either one or both stars become tidally locked in our simulations. When neither star is tidally locked, the aforementioned mechanism does not apply, and hence we set $\dot{a}_{\text{coupled}} = 0$.

3.3.5 Energy and Angular Momentum Conservation

In Fig. 3.4, we plot the relative absolute change in both the total system energy and angular momentum as a function of time for our fiducial simulation (see Table 3.1, § 3.5.1) to demonstrate that both energy and angular momentum are approximately conserved in our VPLanet simulations. The secular drifts in total angular momentum stem from our usage of the CPL tidal model that extends to only second order in e and hence cannot exactly conserve angular momentum. We conclude our methodology satisfies the conservation laws to sufficient accuracy.

3.3.6 *N*-body Simulations of Circumbinary Planetary Systems

Here, we complement our VPLanet simulations of coupled stellar-tidal evolution with several suites of N-body simulations that probe the dynamical and observational impact of single and two-planet circumbinary planetary systems in which the inner-most planet falls within the a_{crit} . We simulate such systems using the N-body code REBOUND (Rein and Liu, 2012). We use REBOUND’s high-order, adaptive timestepping IAS15 integration scheme to integrate the gravitational forces in our simulations as it is accurate and flexible enough to handle close encounters and scattering events while conserving energy to a high precision (Rein and Spiegel, 2015). We outline the simulation setup and our choice of initial conditions in § 3.4.2.

We show that due to coupled stellar-tidal evolution, a CBP that initially resides near the dynamical stability limit can fall within the a_{crit} and go unstable, potentially disrupting the entire circumbinary planetary system. For single-planet circumbinary systems, the planet will likely be ejected, potentially leaving no observational signature that the binary ever hosted a planet. Multi-planet circumbinary systems in which the inner planet falls within a_{crit} , however, can potentially undergo richer dynamical evolution in which binary-planet scattering and planet-planet scattering can reshape the system resulting in multiple planetary ejections, but also leave one or more planets bound to the stars. Several previous studies have examined the impact of binary-planet and/or planet-planet scattering in circumbinary systems (e.g. Kratter and Shannon, 2014; Smullen et al., 2016; Sutherland and Fabrycky, 2016; Gong and Ji, 2017; Gong, 2017). No study yet has examined the case we consider in this paper in which the inner planet of a two-planet circumbinary planetary system initially resides interior to the dynamical stability limit. This calculation is important as it not only allows us to estimate how many planets are actually ejected from these systems but also to estimate how ejection and scattering events can impact the detectability of any surviving CBPs.

Note that in our simulations, we do not model planet-planet or star-planet collisions for computational speed and simplicity. This approach is justified as previous studies of planet-planet scattering and instabilities in circumbinary planetary systems all find that planetary ejection occurs far more often than collisions when a planet is lost from the system (see Smullen et al., 2016;

Table 3.1. Parameter Ranges

Parameter	Range	Fiducial
$M_\star [M_\odot]$	$0.5 - 1$	1
e	0.0 - 0.2	0.15
$P_{bin,init}$ [d]	$3 - 7.5$	4
$P_{rot,init}$ [d]	$0.1 - 2$	0.5
Q	$10^5 - 10^7$	10^6
r_g	$0.15 - 0.45$	0.27

Sutherland and Fabrycky, 2016). Also note we do not consider the coupled stellar-tidal-N-body evolution.

3.4 Simulations

Here we outline the details of both our VPLanet and REBOUND N-body simulations and justify our assumed initial conditions.

3.4.1 Coupled Stellar-Tidal Initial Conditions

We probe how model parameters and the underlying assumptions impact the ability for coupled stellar-tidal evolution to destabilize CBPs by running several sets of simulations varying just one parameter at a time and several suites of many simulations varying multiple parameters over the adopted ranges. Using VPLanet, we simulated coupled stellar-tidal evolution by simultaneously integrating the equations presented in § 3.3.2-3.3.4 for 2 Gyr for each simulation as described in § 4.3. In Table 3.1, we give the default values and range for our simulation initial conditions. Below, we justify our choice of parameter ranges and fiducial values and discuss the qualitative results of a simulation initialized with the fiducial values to elucidate how coupled stellar-tidal evolution drives changes in a_{crit} .

Stellar Tidal Qs

Binary stars' tidal Qs strongly determine the extent to which a short-period binary tidally evolves. Currently, the value of Q for low-mass stars is uncertain. From observations of tidally circularized binaries in clusters, tidal Qs for sun-like stars have been estimated to be of order $Q \sim 10^6$ (e.g. Meibom and Mathieu, 2005). Observations of orbital decay of hot Jupiters find tidal Qs for the host stars are of order $\sim 10^5 - 10^6$ (Jackson et al., 2009; Essick and Weinberg, 2016; Patra et al., 2017; Wilkins et al., 2017). Given these observations, we adopt $Q = 10^6$ as our fiducial value. Detailed studies of tidal dissipation in sun-like stars by Ogilvie and Lin (2007), however, have shown that a star's tidal Q has complicated dependencies on the viscous and hydrodynamical processes operating within the star and can strongly vary depending on the star's spin frequency and the orbital frequency of its companion. Additionally, Barker and Ogilvie (2009) find that the tidal Q likely varies even among stars in the same spectral class. In the case of pre-main sequence stars, Bolmont and Mathis (2016) find that efficient tidal dissipation yields tidal Qs of order $10^4 - 10^6$. Since our simple model uses a constant tidal Q and cannot capture more complex tidal Q evolution, in § 3.5.2 we vary the tidal Q amongst our simulations by two orders of magnitude to probe our model's sensitivity to various tidal Qs.

Stellar Rotations

The STEEP process requires that short-period binary stars form with rotation periods (P_{rot}) shorter than the orbital period. For short stellar rotational periods, $P_{rot} \lesssim 1$ day, the rotational angular momentum can be of order the orbital angular momentum, allowing for a significant increase in a_{crit} via tidal transfer of angular momentum. Here, we review observations of young single stars and binaries in open clusters to justify our stellar rotation assumptions.

In a study of about 250 stars in the ~ 1 Myr old Orion OB1c/d association, Stassun et al. (1999) find a flat distribution for $P_{rot} > 0.5$ days. Interestingly, Stassun et al. (1999) find several stars rotating near the break-up velocity, a rotational period of $P_{rot} \sim 0.25$ days, indicating that young stars can indeed be very rapid rotators. Given this finding, we set the lower limit for initial P_{rot}

in our study to be $0.1 - 0.2$ days. Rebull et al. (2006) examined the P_{rot} of about 900 stars in the roughly 1 Myr old Orion Molecular Cloud, finding most stars with $1 < P_{rot} < 10$ days, and a minority with $P_{rot} \lesssim 1$ day. A study of P_{rot} of young weak T Tauri star candidates in the Orion star forming region by Marilli et al. (2007) found a roughly flat distribution with a peak near $P_{rot} = 1.5$ days, similar to the aforementioned findings. Studies of older open clusters with ages $\sim 100 - 200$ Myr consistently find stellar rotation distributions with P_{rot} often as low as $P_{rot} = 0.5$ days and many stars with $P_{rot} = \lesssim 1$ day, although there is an appreciable spread in P_{rot} with some reaching up to $P_{rot} \sim 10$ days (see Marilli et al., 2007; Meibom et al., 2009, 2011). Young stars can readily have $P_{rot} \lesssim 1$ day.

For the P_{rot} of binary stars, Meibom et al. (2007) observed the ~ 150 Myr old open cluster M35 and measured the P_{rot} distributions of primary stars of close binaries and single stars. Meibom et al. (2007) found that the primary stars in binaries tended to have shorter P_{rot} than single stars with statistically significant differences in the means and medians of the two P_{rot} distributions of at least the 99.9% level after controlling for tidal effects. Stauffer et al. (2016) derive a similar result from observations of P_{rot} in young, low-mass binaries in the Pleiades. We conclude that a significant number of young binary stars form with $P_{rot} \lesssim 1$ days.

Stellar Radius of Gyration

Measuring the stellar r_g is, in general, quite a difficult task so here we rely on theoretical stellar evolution models to inform our choices. From the Baraffe et al. (2015) stellar evolution models for sun-like stars, $r_g \approx 0.45$ on the pre-main sequence, decreasing to $r_g \approx 0.27$, our fiducial value, on the main sequence. Our model utilizes a constant r_g , so we vary it in a series of simulations to gauge how strongly it impacts the evolution of a_{crit} over time in § 3.5.3. We consider our fiducial value of $r_g = 0.27$ to be a conservative estimate as we show in § 4.5 most of the a_{crit} evolution occurs while the stars reside on the pre-main sequence.

3.4.2 N-body Simulations

Each N-body simulation is comprised of two segments. First, we run long-term integration probing the dynamical stability of a one- or two-planet circumbinary system comprised of planets “b” and “c”, when applicable, to determine if any planets are ejected from the system. Once that run finishes, we run a short-term integration initialized with the final state of the long-term dynamical stability integration to perform a series of mock transit observations that explore how dynamical instabilities in such systems impact the detectability of the remaining planets. Splitting a given simulation into two segments allows us to both characterize the dynamics of circumbinary systems in which the inner planet falls within a_{crit} and to probe the resulting observational consequences. We outline the procedures for each part of the simulation below. We summarize and examine the results of these simulations in § 3.6.

N-body Simulations Initial Conditions

For each simulation, we initialize the binary with two $1 M_\odot$ stars with an orbital period of 7 days and e randomly uniformly sampled from $[0, 0.2]$ orbiting in the $x - y$ plane. This choice simplifies the system geometry such that the binary is always in a transiting configuration ($i_{bin} = 90^\circ$). The binary longitude of the ascending node (Ω), argument of pericenter (ω), and mean anomaly (M) are all randomly uniformly sampled from $[0, 2\pi]$.

With the binary parameters set, b’s semi-major axis is randomly uniformly sampled from $[0.94, 1]$ times a_{crit} . i.e. just within the a_{crit} (recall that the errors on Eq. (3.1) are at worst 6% (Holman and Wiegert, 1999)). In multi-planet simulations, planet c’s semi-major axis is constructed such that it is randomly uniformly sampled from $[5, 10]$ mutual Hill radii from b, where a mutual Hill radius is

$$R_{hill,mutual} = \left(\frac{m_b + m_c}{3M_{bin}} \right)^{\frac{1}{3}} \frac{a_b + a_c}{2}, \quad (3.21)$$

where m_b and m_c are the masses of planet b and c, respectively, M_{bin} is the total mass of the binary, and a_b and a_c are the semi-major axes of planets b and c, respectively (Chambers et al., 1996). CBP eccentricities are randomly uniformly sampled from $[0, 0.1]$ and Ω , ω and M are all randomly

uniformly sampled from $[0, 2\pi)$. We summarize the randomized orbital elements for both the binary and the planets in Table 3.2. All CBP orbital elements are initialized in Jacobi coordinates.

In single planet simulations, we examined three cases varying the mass of planet b, while in multi-planet simulations, we examined six cases in which we varied both the mass of planets b and c and their initial inclination relative to the plane of the binary to examine their impact on our results. For the planet masses, we consider three cases: Neptune-, Saturn-, and Jupiter-mass CBPs as these roughly span the observed masses for transiting CBPs. Both planet masses are randomly sampled from the same normal distributions with mean m_p and standard deviation $0.1m_p$, where m_p is equal to the mass of Neptune, Saturn, or Jupiter depending on the simulation suite. We chose to sample both planet masses from the same distribution for a given simulation to keep the planet mass ratio $m_b/m_c \sim 1$, which simplifies our results because varying CBP mass ratios can strongly impact the scattering process in such systems (Gong and Ji, 2017). For the planet inclinations relative to the binary, we randomly uniformly sample from $[0^\circ, 1^\circ]$ (the “low inclination” case) or $[0^\circ, 3^\circ]$ (the “high inclination” case). Both of these initial inclination distributions are broadly consistent with the observed trend of transiting circumbinary exoplanets to be nearly coplanar with their host binary (e.g. Li et al., 2016), but we stress that the true inclination distribution of transiting circumbinary exoplanets is unknown. We adopt these two simple distributions given our ignorance of the true underlying distribution and since the inclination distribution can significantly impact the observed transitability of circumbinary exoplanets (see Armstrong et al., 2014; Martin and Triaud, 2015). We run 1,000 simulations for each of the aforementioned cases for a total of 9,000 N-body simulations.

Dynamical Stability Integration

In the first part of a simulation, we integrate a one or two-planet circumbinary system for 10^5 binary orbital periods. This timescale, about an order of magnitude longer than the CBP dynamical stability simulations of Holman and Wiegert (1999) in terms of binary orbital periods, and is sufficiently long for the majority of dynamically unstable systems to go unstable. We classify a system as unstable when one or both of the planets is ejected from the system. A planet is considered

Table 3.2. N-body Simulation Initial Condition Ranges

Parameter	Distribution
e_{bin}	$U(0, 0.2)$
$e_{b,c}$	$U(0, 0.1)$
a_b [AU]	$U(0.94, 1.0) \times a_{crit}$
$nR_{hill,mutual}$	$U(5, 10)$
$\Omega_{bin,b,c}$	$U(0, 2\pi)$
$\omega_{bin,b,c}$	$U(0, 2\pi)$
$M_{bin,b,c}$	$U(0, 2\pi)$

ejected when its semi-major axis exceeds 50 AU from the system barycenter, a distance that is over an order of magnitude larger than any of the CBP’s initial semi-major axes. When a planet gets ejected, it is removed from the N-body simulation. At the end of the integration, we record the final architecture of the simulation, namely the remaining planet’s Cartesian positions and orbital elements relative to the binary barycenter.

Mock Transit Observation Integration

In the second part of a simulation, we integrate the remaining bodies in the system for 4 years, the approximate lifetime of the *Kepler* mission, and perform mock transit observations to estimate if any remaining CBPs transit and, if so, how frequently they transit. The geometry of our simulations is set up such that the binary orbits in the $x - y$ plane and the observer looks down the $+x$ axis towards the origin. In this simplified configuration, the binary is always in a transiting configuration and almost all of the CBPs are initially in a transiting configuration. A more realistic treatment would allow the binary to have an arbitrary inclination on the sky with respect to the observer and include more physically-motivated binary and CBP orbital parameter priors. For our purposes, however, this simple case permits a first order analysis of the dynamics of these unstable systems and their observational consequences. We leave a more robust treatment for future work and refer

the reader to Martin and Triaud (2014) for a detailed examination of the detectability of CBPs orbiting non-transiting binaries.

To perform mock transit observations, every ~ 2 simulation minutes we record if any of the remaining planets are transiting either of the two host stars. A planet is transiting if

$$d < r_{planet} + r_{star} \text{ and } x_{planet} > x_{star} \quad (3.22)$$

where r_i is the radius of the i^{th} body, d is the projected distance between the centers of mass of the planet and star under consideration, and x_i is the x Cartesian coordinate of the i^{th} body. We require $x_{planet} > x_{star}$ since in our simplified geometry, the observer lies along the $+x$ axis and looks towards the origin. This frequent sampling over the course of the *Kepler* timescale integration not only checks if any remaining planet transits, but also facilitates the calculation of the fraction of time transiting either or both of the binary stars (henceforth referred to as FTT). Given the inherent difficulty in detecting transiting CBPs (e.g. Welsh et al., 2014; Winn and Fabrycky, 2015), FTT is a useful quantity as CBPs that have larger FTTs spend more time transiting and should be more detectable. We discuss the results of this analysis in § 3.6.

3.4.3 Tying It All Together

In summary, we first run an expansive set of VPLanet simulations to examine how coupled-stellar tidal evolution affects a_{crit} . Our simulations not only probe how this evolution depends on parameters such as the stellar tidal Q, but also examine thousands of different initial states for diverse binary systems. These simulations reveal how the STEEP process forces CBPs within a_{crit} . We present the results of these simulations in § 4.5. To complete the theoretical picture given by the STEEP process, we then run an ensemble of N-body simulations with REBOUND to characterize how CBP systems evolve when one planet is interior to the stability limit. Finally, we follow up these simulations with mock transit observations to gain a crude understanding of how the STEEP process impacts the observability of any surviving CBPs. We present the results of these simulations in § 3.6.

3.5 Results: Coupled Stellar-Tidal Evolution Simulations

Here we present the results of simulations of coupled stellar-tidal evolution of binary star systems. In § 3.5.1 through § 3.5.9 we simulate coupled stellar-tidal evolution for binary star systems using VPLanet to quantify how the initial binary orbit, the initial stellar rotations, and the details of the tidal interactions impact the evolution of $a_{crit,init}$.

3.5.1 Fiducial Simulation

To demonstrate how binary star systems evolve due to coupled stellar-tidal physics, we present the full evolution of a $1 M_\odot - 1 M_\odot$ binary system using the fiducial parameter values given in Table 3.1 for the initial conditions (compare to Fig. 3.3). We present the results of this simulation in terms of a_{crit} , the orbital and rotation periods, and the e in Fig. 3.5, annotated with key simulation results and evolutionary regimes.

Initially, the rotation rate, ω , slows as tides transfer rotational angular momentum into the orbit causing the orbital period to grow. During this time, stellar contraction supplies additional rotational angular momentum, slowing the ω decay, while magnetic braking removes some rotational angular momentum from the system entirely. The system tidally locks after about 1 Myr as tides efficiently transport stellar rotational angular momentum into the orbit.

The early growth in the binary orbital period drives most of the a_{crit} growth with the small increases in e providing the rest. a_{crit} reaches its largest value just before tidal locking occurs due to the slight e growth. For $e = 0$, the binary would reach the peak a_{crit} precisely once the binary tidally locks. In the top panel of Fig. 3.5, we show in grey the difference between the maximum and initial critical semi-major axis, $a_{crit,max}/a_{crit,init} = 1.16$, a value likely large enough to destabilize some CBPs, see Fig. 3.1.

In all our simulations, we find that a binary reaches the maximum semi-major axis at about the time it becomes tidally locked. This occurs for two reasons. First, once both stars are tidally locked and synchronized, tides have already transferred as much angular momentum from the stellar rotations into the orbit as possible - any remaining tidal coupling will work to maintain

the tidal locking and will not further expand the binary orbit. Second, for these systems, we find that tidal locking occurs well in advance of orbital circularization via tides. Larger non-zero eccentricities maintain larger a_{crit} according to Eq. (3.1).

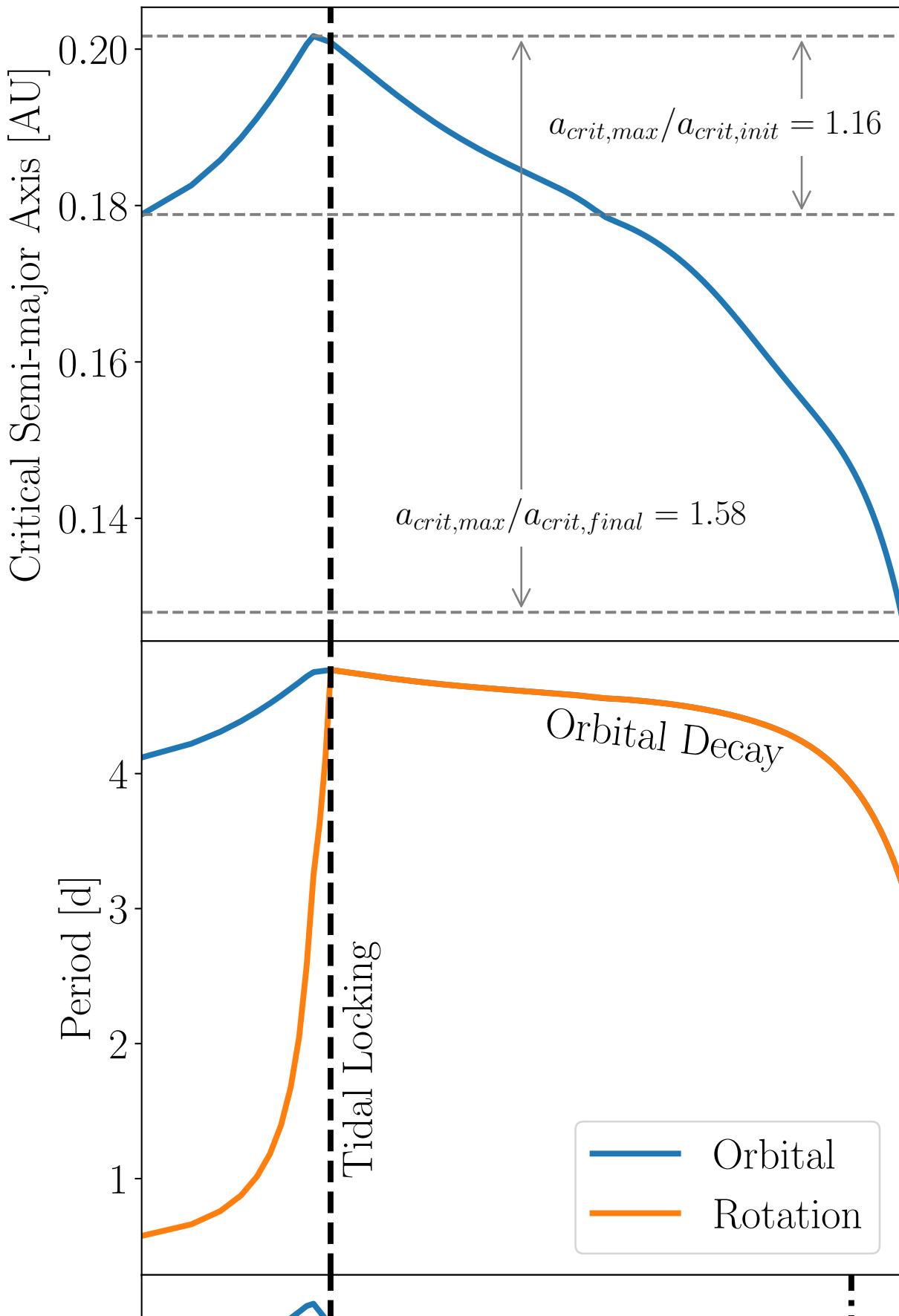
After tidal locking, the binary rotation and orbital periods perfectly mirror each other as tides keep the binary synchronous (see § 3.3.3 and § 3.5.7 for higher-order spin-orbit resonances at larger e). In this regime, e decreases until the orbit is circularized after about 1 Gyr, well after the stars are tidally locked, further decreasing a_{crit} .

Once the system is tidally locked, magnetic braking cannot slow stellar rotations, so instead it removes angular momentum from the orbit causing the orbital period and a_{crit} to decay dramatically with the orbital period dropping by almost 1 day per Gyr. This substantial orbital decay causes a_{crit} to drop by about a factor of 1.6 relative to its maximum value. The combination of orbital circularization and magnetic braking for tidally locked binaries causes significant observational consequences for short-period binaries: the orbit, and hence a_{crit} , observed today is likely much different than what it was in the past. For example if this system was observed at the end of the simulation, we might expect to find CBPs near a_{crit} based on the *Kepler* CBP discoveries. Coupled stellar-tidal evolution could have destabilized CBPs near a_{crit} early on in the system’s lifetime such that there would be no CBPs to detect. a_{crit} decay implies that the dynamical instability region around the binary was much larger in the past such that any surviving CBPs would necessarily have to be located at larger a_{CBP} relative to the central binary’s a and hence be harder to detect. We examine this effect further in § 3.5.8 with a particular focus on the impact of the initial binary orbital period and the details of the magnetic braking physics.

3.5.2 Varying Tidal Q

The value of the stellar tidal Q s is of primary importance to the STEEP process as it controls the timescale of tidal evolution and the rate of angular momentum transfer from stellar rotations to the orbit. In Fig. 3.6, we present the evolution of a $1 M_\odot - 1 M_\odot$ and a $1 M_\odot - 0.5 M_\odot$ binary with stellar tidal Q s ranging from 10^5 - 10^7 . For simplicity, we set both stars’ Q to the same value.

For each simulation, the binary orbital period increases by upwards of one day for smaller tidal



Q_s as angular momentum is tidally transferred to the orbit. Stellar contraction during the pre-main sequence phase speeds up the stellar rotations, providing additional angular momentum for transfer into the orbit. The increasing orbital period is accompanied by a modest increase in e from 0.15 to upwards of 0.18. The orbital period increases until the system tidally locks at which point the tidal transfer of rotational angular momentum to the orbit is complete. Tidal locking occurs at around 10^5 years for $Q = 10^5$ and 1 Myr and 10 Myr for $Q = 10^6$ and $Q = 10^7$, respectively. The increasing orbital period primarily drives the increase in a_{crit} , which peaks approximately at the same time as the orbital period. Once the system tidally locks, the orbital period decays as magnetic braking siphons angular momentum from the orbit. As seen in Fig. 3.6, magnetic braking drains a large amount of angular momentum from the orbit, often causing the orbital period to decrease by a little over a day in 100 Myr, a decrease of 25%.

We find that binary stars with lower tidal Q_s tend to reach larger $a_{crit,max}/a_{crit,init}$. Lower tidal Q_s lead to faster tidal evolution since the equations in our CPL model all have a $1/Q$ dependence (see § 3.3.3). When the tidal evolution proceeds more quickly, there is less time for magnetic braking to siphon rotational angular momentum away from the stars, allowing for tides to transport a larger amount of the rotational angular momentum into the orbit, increasing the period and $a_{crit,max}/a_{crit,init}$. Binary stars with $Q \lesssim 10^6$ readily reach $a_{crit,max}/a_{crit,init} \gtrsim 1.1$, a value large enough to destabilize some CBPs that form near the dynamical stability limit.

Orbital circularization via tides proceeds more quickly for binary stars with lower stellar tidal Q due to the $1/Q$ scaling in the CPL model equations (see § 3.3.3). The orbits of binary stars with $Q = 10^5$ circularize in around 10 Myr while those with $Q = 10^7$ can take longer than 2 Gyr to full circularize. In general, the tidal circularization timescale also depends on the binary orbital period as tighter binaries will circularize more rapidly. Binary star orbits with orbital periods $\gtrsim 10$ days are not likely to circularize (e.g. Zahn and Bouchet, 1989; Meibom and Mathieu, 2005; Raghavan et al., 2010; Lurie et al., 2017).

The unequal-mass binaries tend to have lower $a_{crit,max}/a_{crit,init}$ than their equal-mass counterparts because the lower-mass star tidally locks earlier than the more massive primary. At this point, magnetic braking cannot slow down the lower-mass star's rotation rate as tidal locking fixes

it to the mean motion, i.e. magnetic braking siphons angular momentum from the orbit, slowing the growth of the binary period (see § 3.3.4).

3.5.3 Varying Radius of Gyration

The parameter r_g strongly influences how this transfer occurs due to both tidal evolution, see Eq. (3.13), and stellar evolution for magnetic braking (Eq. (4.4)). In this subsection, we present the full evolution of simulations in which we vary r_g holding all other parameters constant, see Fig. 3.7.

As shown in Fig. 3.7, simulations with large r_g can reach orbital periods over 2 days larger than the initial value primarily due to the stars' rotational angular momentum scaling as r_g^2 . With more rotational angular momentum to transfer to the orbit, tidal locking occurs later, at around 10-100 Myr for stars with $r_g = 0.45$ as compared to around 10⁵ years for stars with $r_g = 0.15$. Interestingly, all systems in this set of simulations circularize after about 1 Gyr. Systems with larger r_g circularize much more rapidly once e begins to decrease. As before, the orbital period growth leads to a_{crit} growth with a peak when the binary tidally locks. Once tidal locking occurs for both stars, magnetic braking saps angular momentum from the orbit, rapidly decreasing the orbital period by over 1 day over 100 Myr.

Systems with unequal mass binaries tend to reach lower $a_{crit,max}/a_{crit,init}$ at a given r_g than their equal mass counterparts because the lower mass star tidally locks more quickly than the more massive primary. Once tidally locked, the secondary star cannot spin down via magnetic braking so angular momentum instead comes from the orbit, reducing the amount by which the orbital period can grow before the primary star tidally locks.

In general, we find that stars with larger r_g tend to reach larger $a_{crit,max}/a_{crit,init}$. At larger values of r_g for a given rotation rate, a star possesses more rotational angular momentum available for transfer into the orbit via tides allowing for larger $a_{crit,max}/a_{crit,init}$. For binary stars with $r_g \gtrsim 0.27$, the binaries attain $a_{crit,max}/a_{crit,init} \gtrsim 1.1$, large enough to destabilize and eject some CBPs.

For sun-like stars, the pre-main sequence lasts for ~ 50 Myr and up to 1 Gyr for lower-mass late M-dwarfs. As shown above, a significant portion of a_{crit} growth occurs while the stars reside

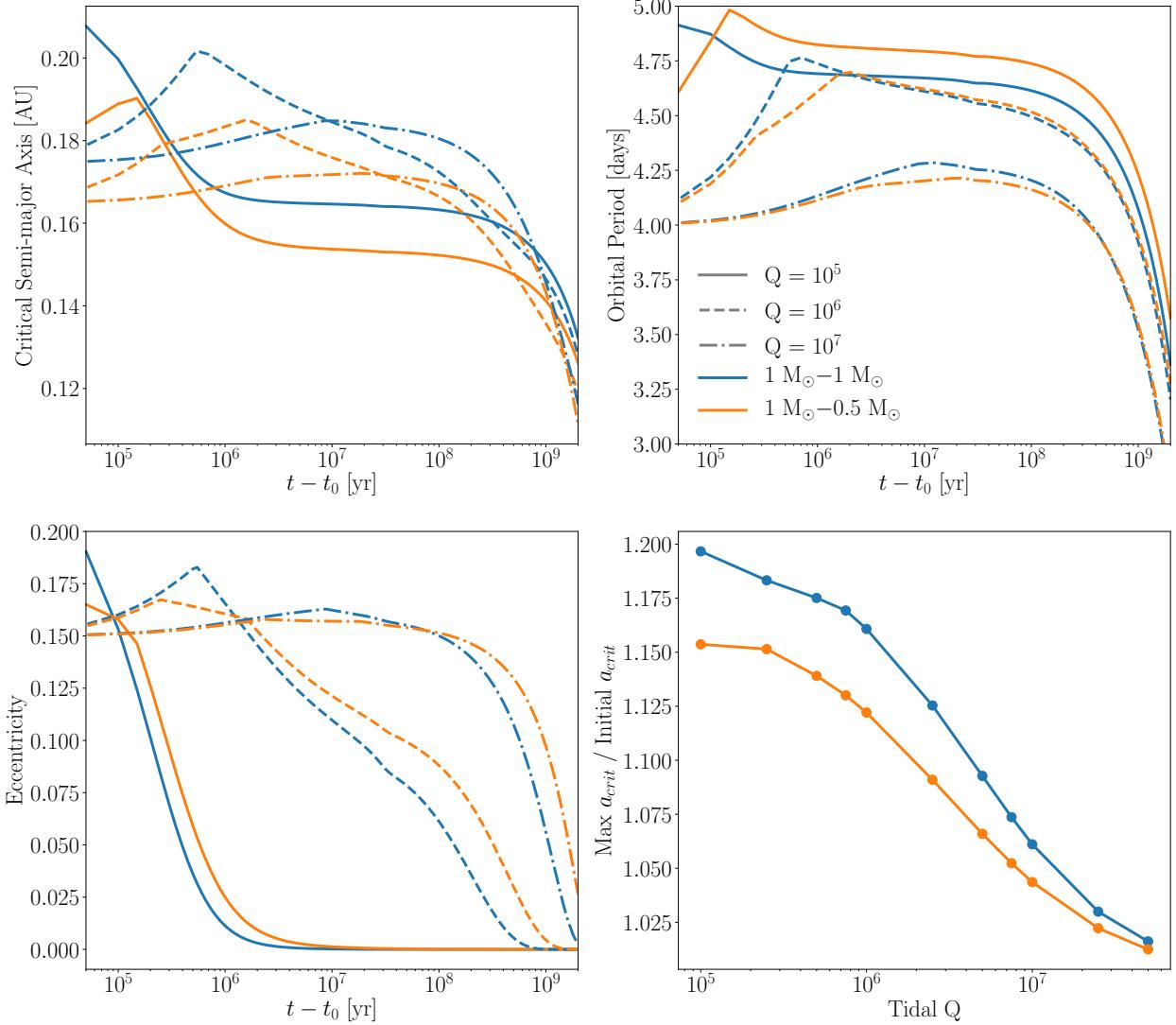


Figure 3.6: The role of tidal dissipation in binary star evolution. *Top Left:* a_{crit} vs. time. *Top Right:* Binary orbital period vs. time. *Bottom Left:* e vs. time. *Bottom Right:* $a_{\text{crit},\text{max}}/a_{\text{crit},\text{init}}$ vs. tidal Q . The blue and orange lines correspond to a $1 M_\odot - 1 M_\odot$ and a $1 M_\odot - 0.5 M_\odot$ binary, respectively. The solid, dashed, and dot-dashed lines correspond to tidal Q s of 10^5 , 10^6 , and 10^7 , respectively. Smaller tidal Q s drive large a_{crit} expansion.

on the pre-main sequence as radius contraction provides a substantial angular momentum reservoir for transfer into the orbit via tides. On the pre-main sequence, Baraffe et al. (2015) predicts that low mass stars have $r_g \sim 0.45$. Therefore during the pre-main sequence when a_{crit} tends to increase the most, the stellar radius of gyration is large allowing a_{crit} to grow significantly as seen in Fig. 3.7 yielding ratios upwards of $a_{crit,max}/a_{crit,init} \sim 1.4$. Once the stars reach the main sequence, r_g drops to ~ 0.27 slowing the a_{crit} evolution. Given that we adopted $r_g \sim 0.27$ as our fiducial value when it is likely much larger during the major period of a_{crit} growth, we consider our $a_{crit,max}/a_{crit,init}$ estimates to be conservative lower limits.

3.5.4 Varying Magnetic Braking Law

Magnetic braking removes angular momentum from stars, slowing their rotation rates. For the short-period binaries considered here, magnetic braking depletes the stellar rotational angular momentum reservoir that is available for tidal transfer in the orbit before tidal locking, reducing $a_{crit,max}/a_{crit,init}$. To probe the sensitivity of our results to our choice of magnetic braking law, we test run identical sets of simulations using the Reiners and Mohanty (2012) and Repetto and Nelemans (2014) magnetic braking models (see § 3.3.2) for binaries stars with various initial P_{rot} . Again for simplicity, in each simulation both stars start with the same given P_{rot} . The results of these simulations are shown in Fig. 3.8. Note that we examine the long-term evolutionary differences between these two magnetic braking models in § 3.5.8.

In terms of $a_{crit,max}/a_{crit,init}$, the differences between the two models are modest with the Repetto and Nelemans (2014) magnetic braking model yielding slightly larger values of $a_{crit,max}/a_{crit,init}$. In general, the Repetto and Nelemans (2014) model tends to remove less rotational angular momentum than the model of Reiners and Mohanty (2012). The differences between the two models in terms of $a_{crit,max}/a_{crit,init}$ decrease with increasing initial stellar P_{rot} as for longer P_{rot} there is less stellar angular momentum for magnetic braking to remove. The difference between the two models becomes more drastic in the late evolution of the binaries once both stars are tidally locked. In that case, the slow angular momentum depletion of the Repetto and Nelemans (2014) model causes a slight decay in the orbital period of about a quarter of a day while the Reiners and Mo-

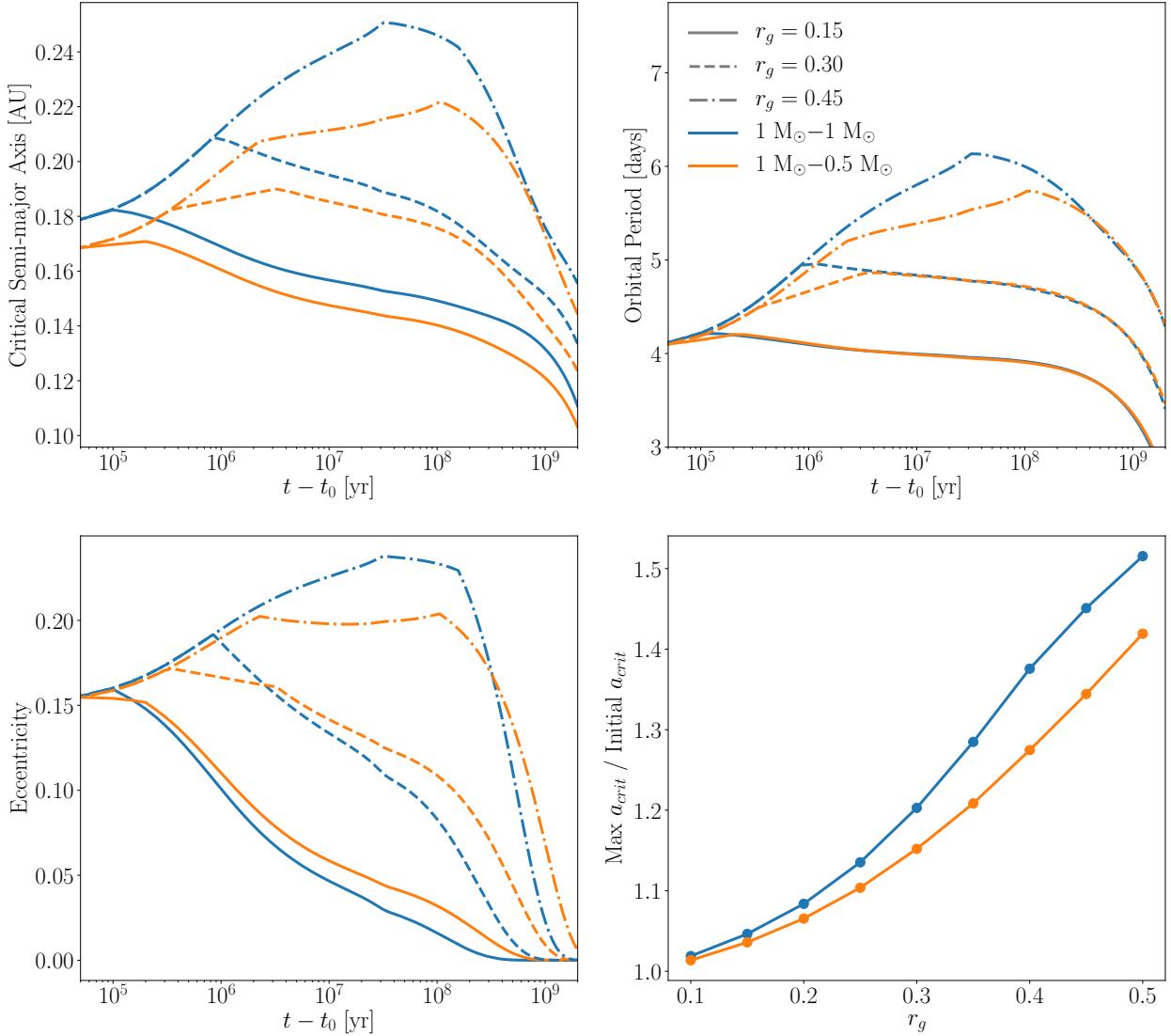


Figure 3.7: The role of the radius of gyration in binary star evolution. *Top Left:* a_{crit} vs. time. *Top Right:* Binary orbital period vs. time. *Bottom Left:* e vs. time. *Bottom Right:* $a_{crit,max}/a_{crit,init}$ vs. r_g . The blue and orange lines correspond to a $1 M_{\odot} - 1 M_{\odot}$ and a $1 M_{\odot} - 0.5 M_{\odot}$ binary, respectively. The solid, dashed, and dot-dashed lines correspond to an r_g of 0.15, 0.3, and 0.45, respectively. Larger r_g lead to increased a_{crit} growth.

hanty (2012) model produces a decay of 1-1.5 days. Overall, our results are moderately sensitive to the choice of magnetic braking model and we choose to use the Reiners and Mohanty (2012) model as our fiducial model as it tends to produce a more conservative estimate of $a_{crit,max}/a_{crit,init}$. For both magnetic braking models when the stars have initial $P_{rot} \sim 1$ day, the binary reaches $a_{crit,max}/a_{crit,init} \gtrsim 1.1$, large enough to destabilize CBPs.

3.5.5 Varying Rotation Periods

Previously, we posited that an initial $P_{rot} \lesssim 1$ day, a value consistent with observations of young stars (see § 3.4.1), should be sufficient for the STEEP process to operate. Here, we demonstrate that initial $P_{rot} \lesssim 1$ days do indeed lead to appreciable growth in a_{crit} through an expansive suite of simulations.

In Fig. 3.9, we present the results of 20,000 simulations in which we varied the initial P_{rot} for both stars over a grid of 0.2-1.5 days for a $1M_\odot - 1M_\odot$ binary for $e = 0.05$ and $e = 0.15$. For the other initial conditions, we adopt the default values given in Table 3.1. For both eccentricities, binaries in which both stars begin with $P_{rot} \lesssim 1$ days achieve $a_{crit,max}/a_{crit,init} \gtrsim 1.1$. When the stars have initial $P_{rot} \lesssim 0.5$ days $a_{crit,max}/a_{crit,init}$ can reach 1.3 – 1.4, large enough to potentially destabilize many *Kepler* CBPs as they typically reside at $a_{planet}/a_{crit} \lesssim 1.4$. The initial P_{rot} that lead to appreciable a_{crit} growth are entirely consistent with observations of young sun-like stars, demonstrating that the STEEP process can effectively destabilize some CBPs near the dynamical stability boundary.

3.5.6 Monte Carlo Simulations

In the previous sections, we ran simulations varying one or two parameters at a time to explore the sensitivity of the STEEP process to the initial conditions. We found that simulations with parameters roughly consistent with observations, such as initial stellar P_{rot} and parameters broadly consistent with theoretical expectations result in binaries whose coupled stellar and tidal evolution can effectively destabilize CBPs. However, the coupled nature of our model necessitates a broader

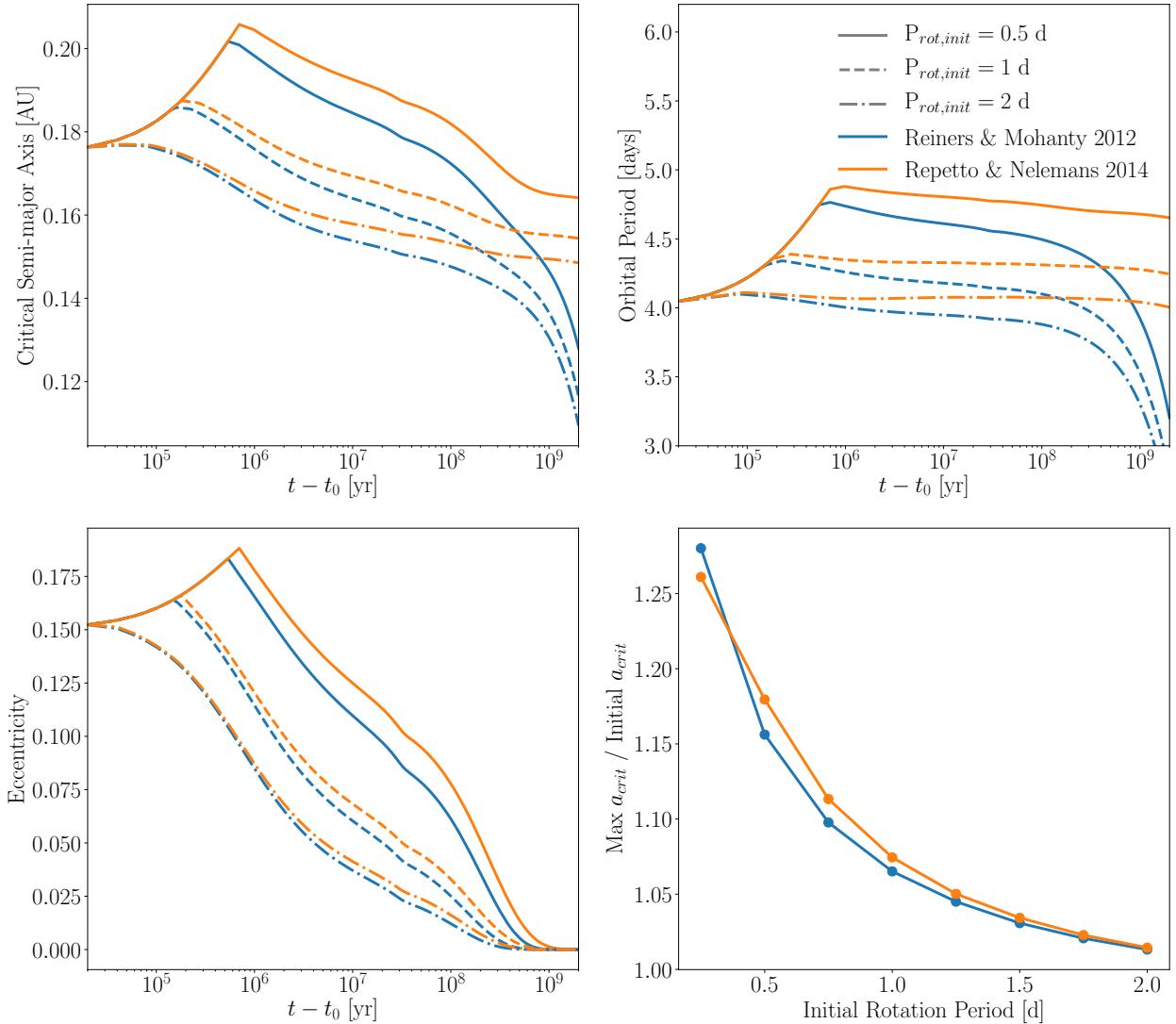


Figure 3.8: The role of the magnetic braking law and initial stellar P_{rot} in binary star evolution. *Top Left:* a_{crit} vs. time. *Top Right:* Binary orbital period vs. time. *Bottom Left:* e vs. time. *Bottom Right:* $a_{crit,max}/a_{crit,init}$ vs. initial stellar P_{rot} . The blue and orange lines correspond to simulations that use the stellar magnetic braking relation from Reiners and Mohanty (2012) and Repetto and Nelemans (2014), respectively. The solid, dashed, and dot-dashed lines correspond to a binary with stellar initial spin periods of 0.5 d, 1 d, and 2 d, respectively. We find that the Repetto and Nelemans (2014) relation leads to larger $a_{crit,max}/a_{crit,init}$ and stars with shorter initial spin periods lead to larger $a_{crit,max}/a_{crit,init}$ as well.

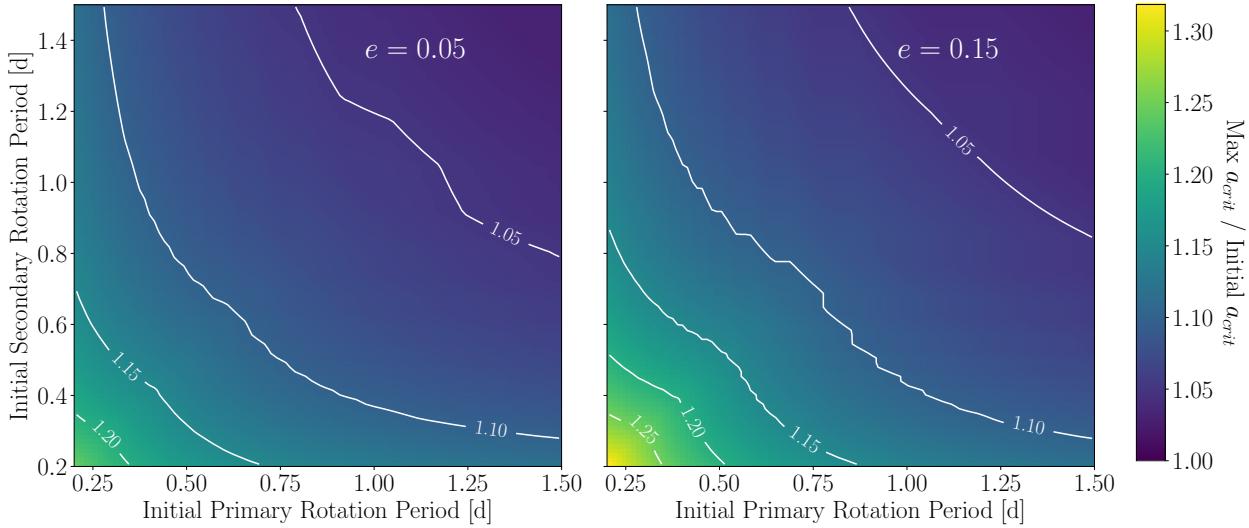


Figure 3.9: $a_{crit,max}/a_{crit,init}$ as a function of the initial stellar P_{rot} for a $1 M_\odot - 1 M_\odot$ binary for $e = 0.05$ (Left) and $e = 0.15$ (Right). The white lines are contours in $a_{crit,max}/a_{crit,init}$. When the stars start with $P_{rot} \lesssim 1$ days, $a_{crit,max}/a_{crit,init}$ readily grows to $\gtrsim 1.1$, large enough to destabilize some CBPs near the dynamical stability boundary.

study to test the STEEP process's robustness to combinations of the parameters.

We therefore perform a Monte Carlo study to identify regions in parameter space where large $a_{crit,max}/a_{crit,init}$ occur. We run 10,000 simulations where each star's mass, r_g , initial P_{rot} , initial binary orbital period and e are randomly sampled from uniform distributions from the ranges listed in Table 3.1. Each star's tidal Q was sampled randomly from a log-uniform distribution from the listed range. For each simulation, we compute $a_{crit,max}/a_{crit,init}$ from the full evolution of the system over 2 Gyr. The results of the simulations are displayed in Fig. 3.10 as a two-dimensional projection in terms of the initial orbital angular momentum, J_{orb} , and ratio of total initial stellar rotational angular momentum to the initial orbital angular momentum, $J_{rot,tot}/J_{orb}$.

A clear gradient emerges in $J_{rot,tot}/J_{orb}$ space. As $J_{rot,tot}/J_{orb}$ increases, on average so does $a_{crit,max}/a_{crit,init}$. This result is expected since when the initial rotational angular momentum content is comparable to the orbital angular momentum, tidal transfer to the orbit will on average increase a_{crit} more. We find that simulations with initial $J_{rot,tot}/J_{orb} \gtrsim 0.1$ yield $a_{crit,max}/a_{crit,init} \gtrsim 1.1$, large enough to destabilize CBPs near the dynamical stability boundary. Simulations with initial

$J_{rot,tot}/J_{orb} \approx 1$ can result in simulations with large $a_{crit,max}/a_{crit,init} \approx 1.4$, some upwards of $2 - 3$ for initial stellar rotation rates near the break-up velocity, a value that is almost certain to destabilize and eject any nascent circumbinary planetary system.

There is considerable scatter in $a_{crit,max}/a_{crit,init}$, however, as simulations with $J_{rot,tot}/J_{orb} \approx 1$ can result in little evolution in a_{crit} . We find that these cases correspond to simulations where one or both stars have tidal $Q_s \sim 10^7$. As discussed in § 3.5.2, large tidal Q_s result in much slower tidal evolution. When the tidal evolution proceeds more slowly, so too does the angular momentum transfer into the orbit (see Fig. 3.6). In this case, the slow tidal evolution allows for magnetic braking to efficiently siphon large amounts of angular momentum from the system, resulting in negligible a_{crit} evolution.

For tidal Q_s low enough to prevent significant angular momentum loss due to magnetic braking, i.e. $Q < 10^7$, young binary systems with short initial $P_{rot} \lesssim 1$ day can attain $J_{rot,tot}/J_{orb} \gtrsim 0.1$, resulting in large enough growth in a_{crit} to destabilize CBPs found near the dynamical stability boundary.

3.5.7 3:2 Spin-Orbit Resonance

To probe the impact of large e and capture into higher order spin-orbit resonances, we run two simulations using the default parameters in Table 3.1 but with $e = 0.15$ and $e = 0.3$. The results of the simulations are given in Fig. 3.11. The initially more eccentric binary tidally locks into a 3:2 spin-orbit resonance after about 1 Myr since $e > \sqrt{1/19}$ (see § 3.3.3), while the less eccentric binary tidally locks into and remains in a synchronous 1:1 spin-orbit state. Once e decays to $e < \sqrt{1/19}$ after about 100 Myr for the more eccentric binary, the system becomes trapped in the synchronous state, the only other allowed spin-orbit resonance for tidally locked systems under the CPL model.

The binary that tidally locks into a 3:2 spin-orbit resonance reaches a larger $a_{crit,max}/a_{crit,init}$ than the other synchronously rotating binary system. This result is surprising given that in a 3:2 spin-orbit resonance, $P_{rot} = 2/3P_{orb}$, so relative to the synchronous case, less stellar rotational angular momentum is transported into the orbit, reducing the increase in orbital period and hence a_{crit} .

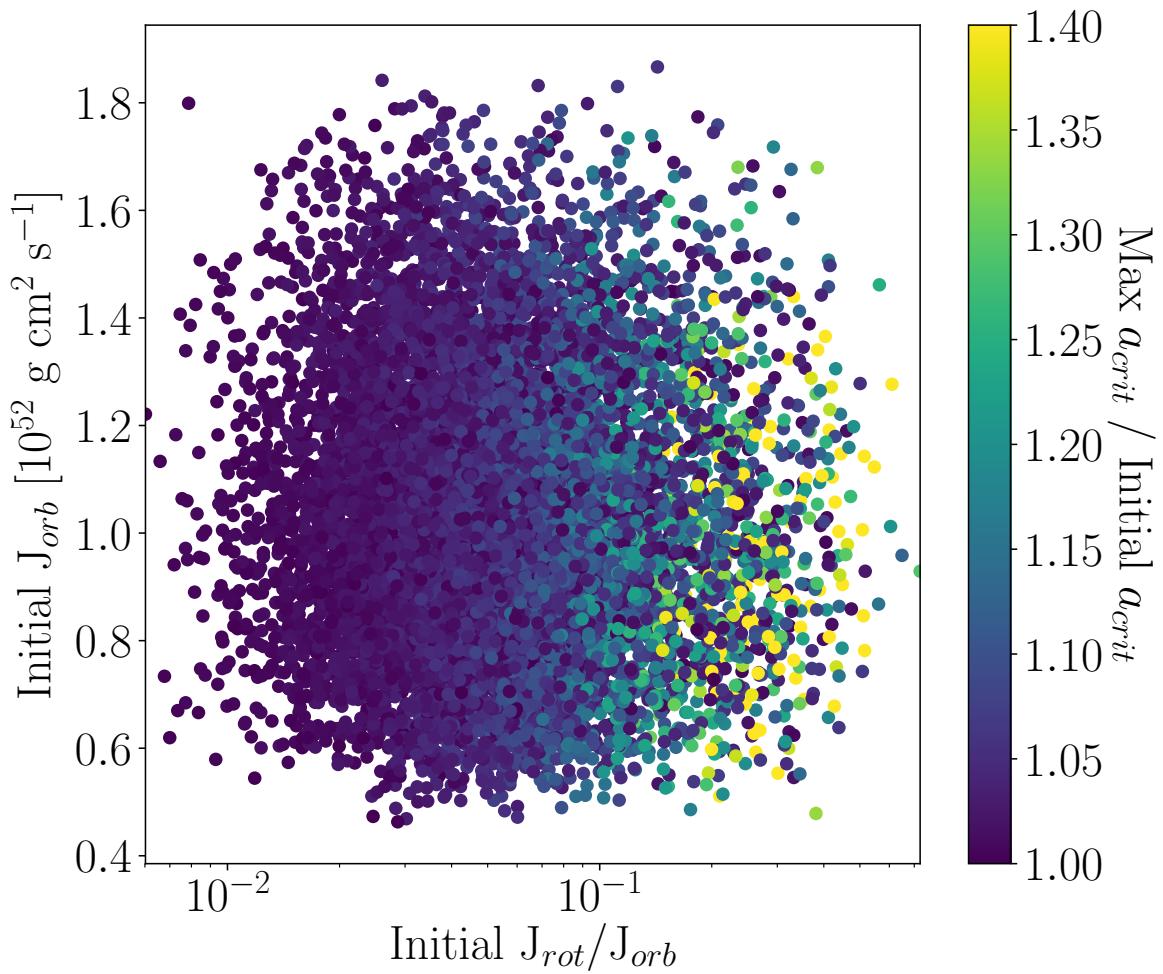


Figure 3.10: Scatter plot of the initial orbital angular momentum J_{orb} , versus the ratio of total initial stellar rotational angular momentum to the initial orbital angular momentum, $J_{rot,tot}/J_{orb}$. Each point is colored by the $a_{crit,max}/a_{crit,init}$ achieved in that simulation. Simulations with an initial angular momentum ratio of $\gtrsim 0.1$ tend to produce a large enough $a_{crit,max}/a_{crit,init}$ to destabilize a CBP.

However, larger e correspond to lower orbital angular momentum which scales as $\sqrt{1 - e^2}$. The more eccentric binary system has less orbital angular momentum than the other system so transfer of angular momentum from the stellar rotation to the orbit yields proportionally larger increases in a_{crit} , explaining the larger $a_{crit,max}/a_{crit,init}$. Therefore we anticipate that the STEEP process is effective for eccentric binary systems that tidally lock into higher order spin-orbit resonances.

3.5.8 Long-term a_{crit} Evolution

As shown in the above simulations, a_{crit} changes over time such that a binary's observed a_{crit} can differ significantly from past values. This effect is especially relevant for observed *Kepler* circumbinary systems as the orbit, and hence a_{crit} , observed today is much different than in the past and the magnitude of the difference depends on the age of the system. Most notably, the post-tidal locking a_{crit} decay implies that the dynamical instability region around the binary was likely larger in the past. Any CBPs that survived initial a_{crit} increases would necessarily appear to orbit on much larger a_{CBP} relative to the a_{crit} about the central binary and would therefore be harder to detect.

Specific Cases

To illustrate this effect and the observational consequences, we simulate 4 binary systems for 8 Gyr of evolution using the default parameters in Table 3.1 for binary orbital periods of 3, 4, 5, and 6 days using the magnetic braking formalisms of Reiners and Mohanty (2012) and Repetto and Nelemans (2014). The results of the simulations are shown in Fig. 3.12.

In left panel of Fig. 3.12, we see the full evolution of these systems and there is a stark difference between the two cases: the Reiners and Mohanty (2012) magnetic braking law causes significant orbital decay relative to the modest decay induced by the Repetto and Nelemans (2014) magnetic braking relation. The Reiners and Mohanty (2012) magnetic braking law removes a significant amount of angular momentum from the orbits driving orbital period decays of about 1 day per Gyr such that the binaries actually merge after a few Gyr as the stellar radii overlap. The or-

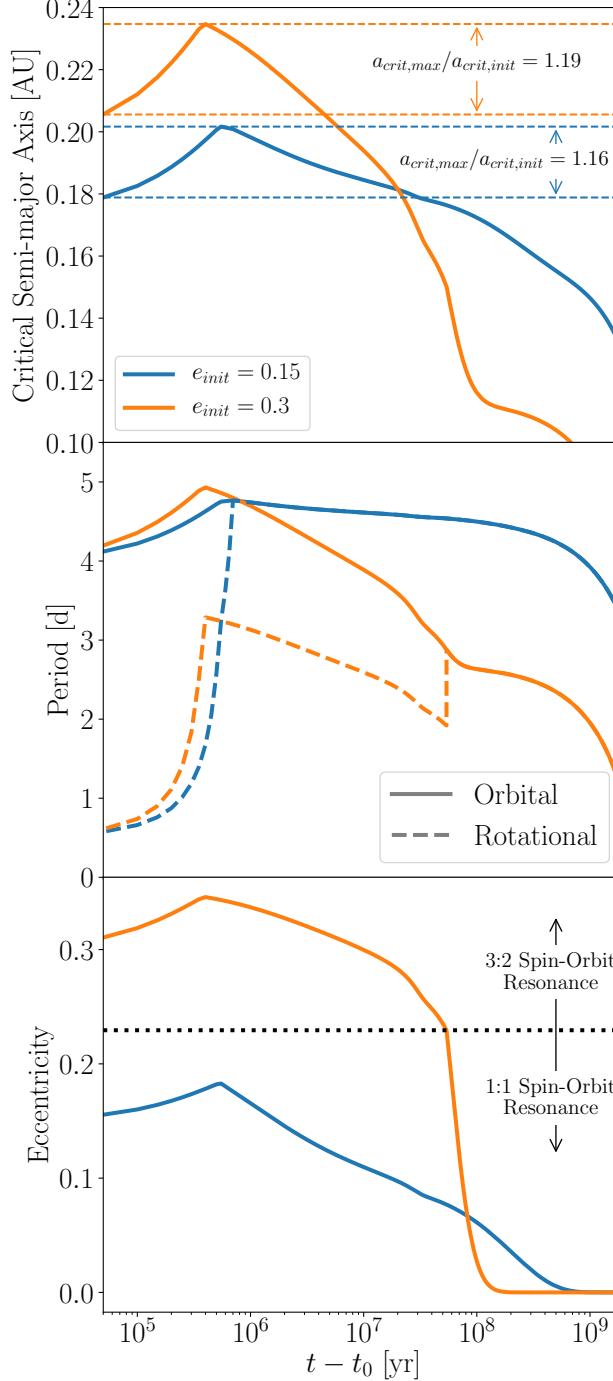


Figure 3.11: Full evolution of binary systems using our default parameters from Table 3.1, except with initial eccentricities of $e = 0.15$ (blue) and $e = 0.3$ (orange). *Top:* a_{crit} vs. time. The systems attain $a_{crit,max}/a_{crit,init} = 1.16, 1.19$ for binaries with initial eccentricities of $e = 0.15$ and $e = 0.3$, respectively. *Middle:* Orbital and stellar rotation period vs. time. Both systems tidally lock after ~ 1 Myr into 1:1 (blue) and 3:2 (orange) spin-orbit resonances. *Bottom:* e vs. time. The dotted line demarcates binaries that tidally locked into a 3:2 (above the line) and 1:1 (below the line) spin-orbit resonance.

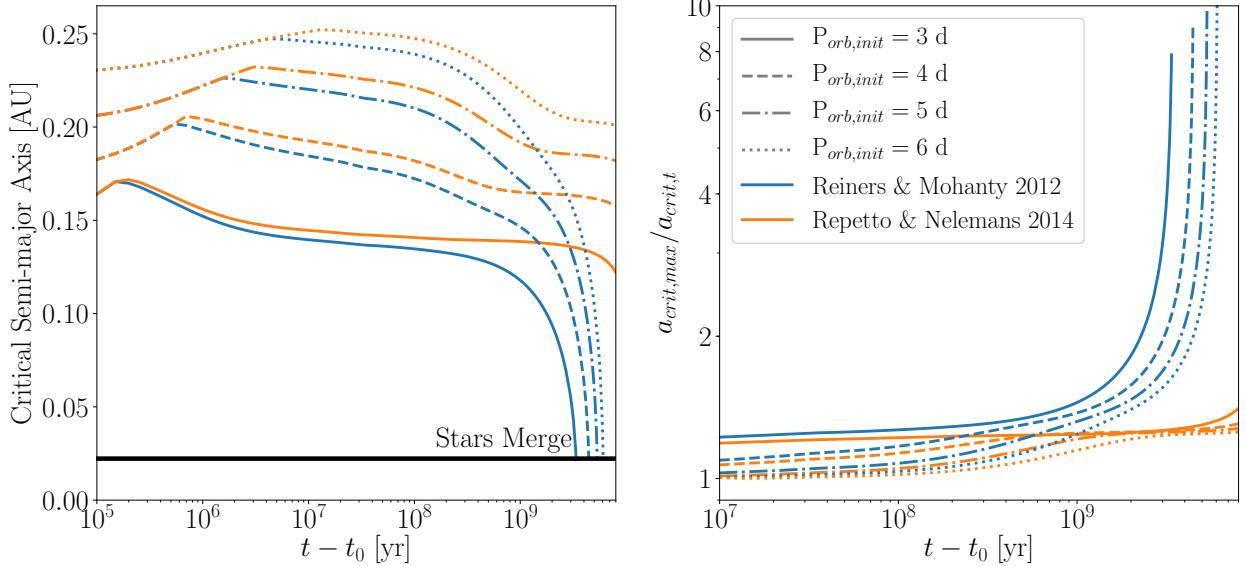


Figure 3.12: *Left:* a_{crit} as a function of time. The black horizontal line indicates when stellar radii overlap and the stars merge, halting the simulation. *Right:* Maximum critical semi-major axis divided by the critical semi-major axis observed at time t , $a_{crit,max}/a_{crit,t}$. Depending on the magnetic braking physics, the observed a_{crit} can differ from its maximum by an order of magnitude.

bital period decay is more modest for the Repetto and Nelemans (2014) magnetic braking law. The different outcomes predicted by these models could be used as an observational test to constrain which magnetic braking model is more suitable, however, we leave that analysis for future work.

In the right panel of Fig. 3.12, we plot $a_{crit,max}/a_{crit}$ over time. For the simulations using the Reiners and Mohanty (2012) magnetic braking law, a_{crit} can vary by a factor of 2 – 10 from its maximum value depending on the age of the system, while the simulations using the Repetto and Nelemans (2014) law vary by factors of order unity. This implies that the region of dynamical instability can appreciably shrink for tidally locked short orbital period binaries over time. Therefore when one observes these systems, the fact that the dynamical instability region was likely much larger in the past precludes any CBPs from orbiting near the observed a_{crit} . Any CBPs that survived the a_{crit} increase would then be orbiting at much larger a_{CBP}/a_{crit} ratios than what is observed for *Kepler* CBPs. If a CBP is discovered around a tidally locked binary in the future, its current state must be considered in the context of the host binary’s past coupled stellar-tidal evolution.

Monte Carlo Simulations

Here we examine how distributions of $a_{crit,max}/a_{crit,t}$ from our simulations from § 3.5.6 evolve as a function of time to examine the impact of different initial conditions. We plot the distribution of $a_{crit,max}/a_{crit,t}$ at four different times in Fig. 3.13. As the systems age, the distributions tend to shift towards larger $a_{crit,max}/a_{crit,t}$ values, indicating that for most initial states a , and hence a_{crit} , decays relative to its peak due to magnetic braking. At later times, the $a_{crit,max}/a_{crit,t}$ distributions tend to smear out and develop heavy tail indicating that even though $a_{crit,max}/a_{crit,t}$ tends to grow, the properties of the individual system, e.g. the stellar mass or tidal Qs, play an important role in determining how much a_{crit} evolves. For example, lower mass stars contract for longer periods of time, which for tidally locked stars, injects additional angular momentum in the orbit, slowing a_{crit} decay due to magnetic braking.

3.5.9 Relaxed Assumptions

In previously-discussed simulations, we have taken a rather conservative approach in selecting our initial conditions. Here we relax those assumptions and run five simulations with progressively more extreme-yet-plausible initial conditions to see how a_{crit} evolves. Case A has the same initial conditions as our fiducial simulation (see § 3.5.1) but with an initial orbital period of 5 days and an initial e of 0.1. Case B is the same as Case A but with both stellar tidal Qs set to 10^5 . Case C is the same as Case B but with both stellar radii of gyration set to $r_g = 0.45$. In Case D, the initial conditions are the same as Case C but with the initial stellar P_{rot} set to 0.25 days. Finally in Case E, we set the initial $e = 0.2$ and adopt the initial conditions of Case D for all other parameters.

For each individual simulations, see Fig. 3.14, the results proceed as expected: simulations with larger r_g and lower initial P_{rot} , e.g. Case C and Case D, respectively, result in larger $a_{crit,max}/a_{crit,init}$ with the ratios approaching 2. In all simulations, the a_{crit} growth peaks early on in the system at $\lesssim 100$ Myr. For binaries with $e > 0.3$, the model tends to break down and yield unrealistic results and merits further exploration with the CTL tidal model, which may be applicable at large e . Nevertheless, these simulations demonstrate that a_{crit} can grow up to twice the initial value

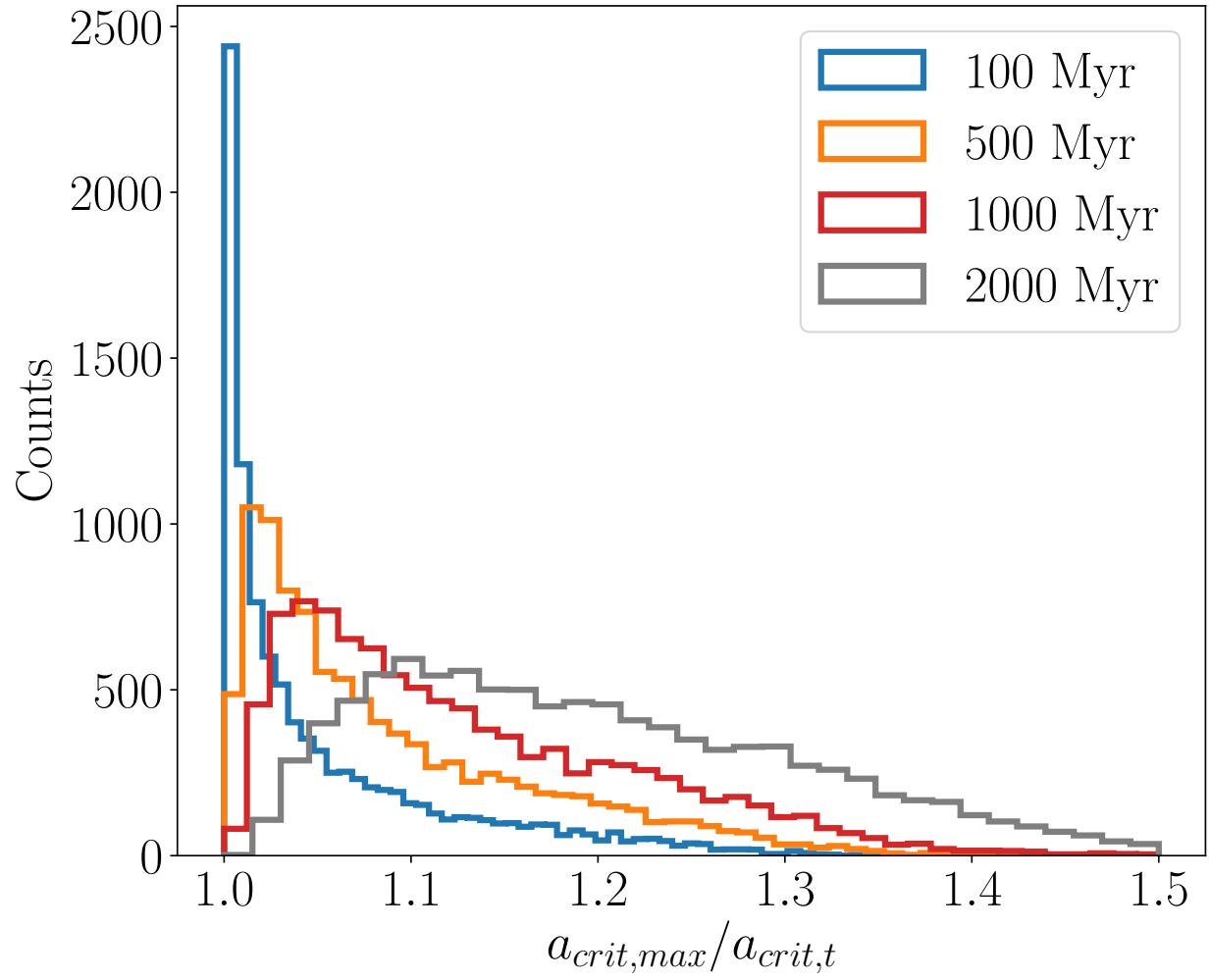


Figure 3.13: Histograms of the maximum a_{crit} divided by the critical semi-major axis observed at time t , $a_{crit,max}/a_{crit,t}$ for the binary systems simulated in § 3.5.6 observed at 100, 500, 1000, and 2000 Myr (blue, orange, red, and grey histograms, respectively). In general for older systems, $a_{crit,max}/a_{crit,t}$ increases as a_{crit} recedes due to angular momentum loss via magnetic braking in tidally locked systems. The distribution of $a_{crit,max}/a_{crit,t}$ smears out and develops an extended tail for older systems due to the different tidal and rotational properties of the binary systems.

depending on the initial conditions.

3.6 Results: N-body Simulations

Here we present the results of N-body simulations of circumbinary planetary systems using the N-body code REBOUND (Rein and Liu, 2012; Rein and Spiegel, 2015). The initial conditions and set-up are described in § 3.4.2. In § 3.6.1, we examine how orbital instabilities in circumbinary planetary systems stemming from the inner-most planet residing within a_{crit} affect the system architecture and lead to planetary ejections while in § 3.6.2, we examine the observational consequences of CBP ejections.

3.6.1 Dynamical Stability

Single Planet Circumbinary System

In Table 3.3, we display the fraction of simulations of a single planet circumbinary system that result in a stable or unstable planet for Neptune-, Saturn-, and Jupiter-mass planets. In our simulations, planet b, which initially resided within a_{crit} , is ejected 70% - 75% of the time confirming that the majority of CBPs that drift interior to a_{crit} are ejected from the system (c.f. Holman and Wiegert, 1999). From the definition of a_{crit} from Eq. (3.1), however, one expects *every* CBP that drifts interior to a_{crit} to go unstable and get ejected whereas only 70% - 75% of such planets are ejected in our simulations. The difference can be rectified by examining precisely how Holman and Wiegert (1999) computed a_{crit} . For each given a_{CBP} , Holman and Wiegert (1999) initialized 8 test particles on circular orbits that are equally spaced in mean anomaly. After the integration, Holman and Wiegert (1999) deemed the minimum semi-major axis in which all 8 test particles survive a_{crit} . Therefore a test particle, and hence a planet, within a_{crit} is likely, but not guaranteed, to go dynamically unstable and be ejected from the system, explaining our results.

We find that on average, the deeper within the region of dynamical instability a CBP is and the more eccentric its orbit, the more likely it is to be ejected from the system. There appears to be a weak dependence on ejection probability with planet mass with Neptune-mass CBPs getting

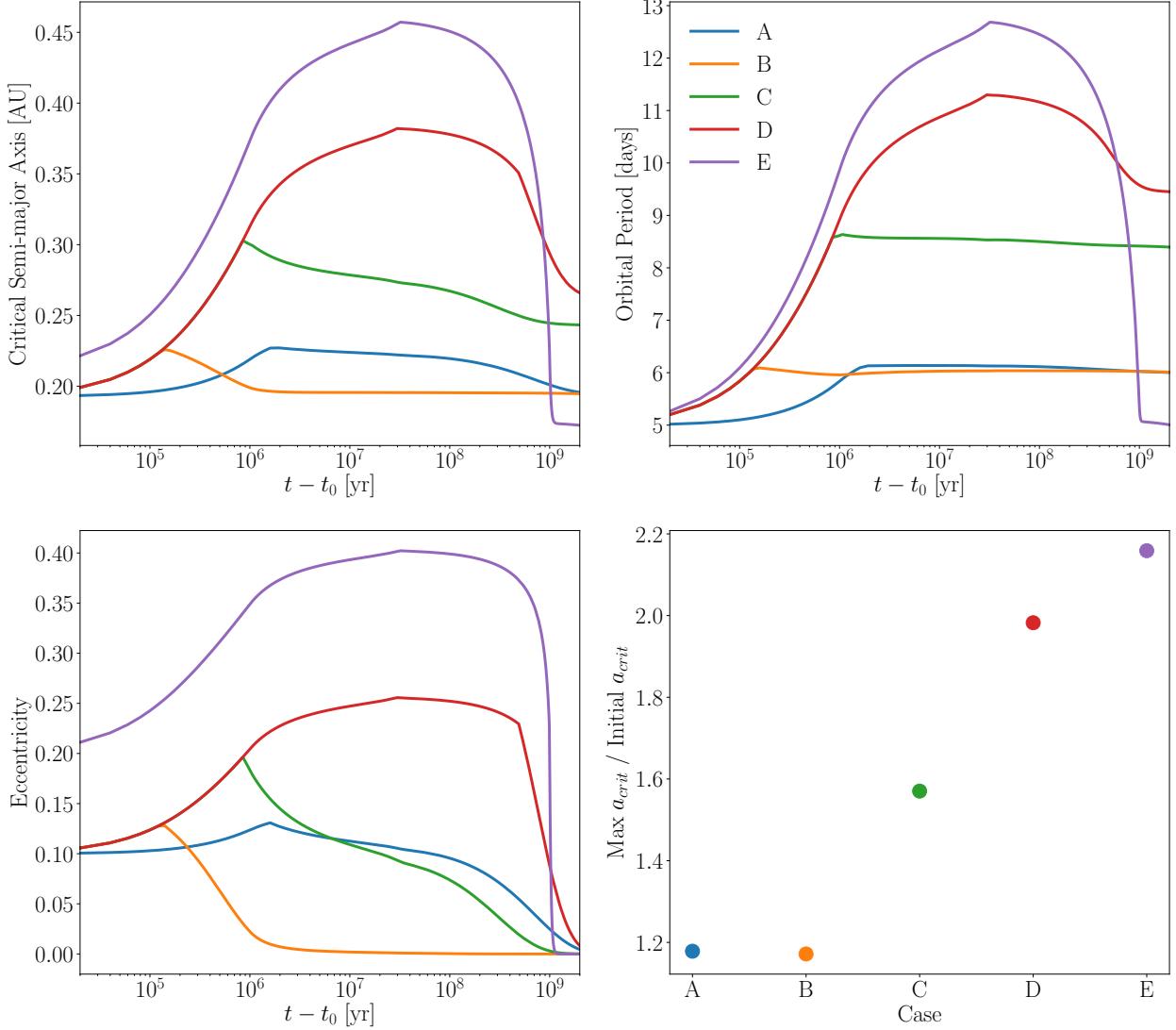


Figure 3.14: Binary orbital evolution for several plausible sets of initial conditions. *Top Left:* a_{crit} vs. time. *Top Right:* Binary orbital period vs. time. *Bottom Left:* e vs. time. *Bottom Right:* $a_{crit,max}/a_{crit,init}$ for the five cases. The blue, orange, green, red, and purple curves correspond to cases A, B, C, D, and E, respectively. In agreement with previously discussed simulations, we find that initial fast rotators with smaller tidal Qs achieve large $a_{crit,max}/a_{crit,init}$ up to of order 2 for plausible initial conditions.

Table 3.3. Single Planet System Outcome Fraction

Case	Stable	Unstable
Neptune Mass	0.253	0.747
Saturn Mass	0.282	0.718
Jupiter Mass	0.3	0.7

ejected $\sim 75\%$ of the time while more massive planets were slightly less likely to be ejected. We re-ran these simulations for 10^6 binary orbital periods, an order of magnitude longer, and found that our results did not significantly change.

Multiplanet Circumbinary System

In Table 3.6.1, we display the fraction of simulations of a two-planet circumbinary system that result in a stable or unstable planet as a function of initial planet orbital inclination distribution and planet mass. A clear result of our multi-planet circumbinary system dynamical stability simulations is that at least one planet is ejected from the system with this result occurring in 87% - 95% of simulations, a value broadly consistent with the results of previous studies of planet-planet scattering in circumbinary systems (e.g. Sutherland and Fabrycky, 2016; Smullen et al., 2016; Gong and Ji, 2017; Gong, 2017).

In simulations in which a planet is ejected, the typical result is planet c remaining stable while planet b is ejected from the system. This outcome occurs in 78% - 88% of simulations. Planet b remains stable 7% - 18% of the time where most instances in which b remains stable correspond to simulations in which both planets remain stable. Simulations in which both b and c remain stable typically correspond to initial conditions in which b and c are more widely separated with inter-planet separations near $10R_{Hill,mutual}$ and when b resides near to a_{crit} . The few simulations in which b remains stable while c is ejected correspond to scattering events where b scatters exterior to c while c scatters into the region of dynamical instability and is ejected from the system

soon thereafter. Planet b remains stable at the expense of c’s ejection in 2% - 6% of simulations. The spread in simulation outcomes stems from the planet(s)’ and binaries’ random initial orbital parameters (see Table 3.2).

The most infrequent result that occurs in 1% - 5% of simulations is when both b and c are ejected from the system, a value roughly consistent with Gong (2017) who find $\sim 10\%$ systems are destabilized due to planet-planet scattering near a_{crit} . Simulations in which both b and c are ejected tend to occur when b is initialized deeper within the region of dynamical instability and when b and c are more closely separated with separations near $5R_{Hill,mutable}$ in good agreement with the simulations of Kratter and Shannon (2014). Given that planet c survives in 89% - 97% of simulations, we anticipate that in systems with higher multiplicity, the farther out planets would likely remain stable.

Our results show clear dependences on both CBP orbital inclination relative to the binary’s and planet’s mass. Systems with more massive planets tend to remain stable. For a given planet mass, however, simulations with planets initialized using the “high inclination” distribution more frequently result in planet b going unstable as the larger mutual inclinations tend to result in more violent scattering events. This behavior is reflected in a depletion in the fraction of simulations in which b remains stable and an appreciable enhancement in the fraction of simulations in which planet c remains stable after b is ejected.

Our results indicate that systems in which the inner-most CBP falls within the region of dynamical instability, *e.g.* due to a_{crit} expansion resulting from coupled stellar-tidal evolution, likely lose at least one planet. Therefore for short-period binary systems where we expect coupled stellar-tidal evolution to increase a_{crit} and envelope CBPs that preferentially lie near the limit, at least one CBP is likely to be ejected from the system, potentially accounting for the lack of observed CBPs in such systems.

We note that our results are conservative given our assumptions that the inner-most CBP orbits just interior to a_{crit} to simulate the time right after the planet falls within a_{crit} due to the STEEP process. In practice, a CBP that falls within a_{crit} and is not promptly ejected will fall deeper into the dynamical instability region as coupled stellar-tidal evolution expands a_{crit} . As discussed above,

Table 3.4. Circumbinary planetary system N-body simulation outcome fractions.

Case	Low Inclination			High Inclination		
	Neptune Mass	Saturn Mass	Jupiter Mass	Neptune Mass	Saturn Mass	Jupiter Mass
b, c stable	0.064	0.088	0.132	0.053	0.09	0.106
b stable, c unstable	0.063	0.047	0.052	0.027	0.017	0.023
c stable, b unstable	0.825	0.843	0.784	0.881	0.879	0.849
b, c unstable	0.048	0.022	0.032	0.039	0.014	0.022

Note. — In “low inclination” simulations, both planet b and c’s inclination with respect to the binary orbital plane is sampled from $U(0^\circ, 1^\circ)$ while in “high inclination” simulations, both planet b and c’s inclination is sampled from $U(0^\circ, 3^\circ)$. The fractions are normalized such that each column sums to 1.

the deeper a CBP is within the dynamical instability region, the more likely it is to be ejected. Therefore if we relaxed our conservative assumptions and initialized CBPs more interior to a_{crit} , we would expect more ejections. Additionally it is possible that CBPs form with large mutual inclinations although the observed CBP population has low mutual inclinations with respect to the host binaries (Li et al., 2016). If we allowed for more inclined orbits, based off of our previous results we would anticipate more ejections based on the results of Chatterjee et al. (2008) and more CBPs scattered away from a transiting configuration.

3.6.2 Mock Transit Observations

Next we present the results of our mock transit observation simulations. In the single CBP system case, the result is trivial: if the planet is ejected, it does not transit while if it remains in the system, it most likely transits as each planet was initialized in a transiting configuration and interactions with the host binary tend to not excite large inclinations given the small initial mutual inclination. For the two-planet circumbinary system case, we focus on the detectability of planet c as we have shown that the dominant outcome of our dynamical stability N-body simulations is that planet b

is ejected while planet c remains in the system on a perturbed-yet-stable orbit. We summarize the results of our mock transit observations in Fig. 3.15 which depicts a histogram of the fraction of time transiting (FTT) for planet c for various initial masses and inclinations for the case in which both planets b and c remain stable (blue lines) and the case in which planet b gets ejected while planet c remains (orange lines). For reference, a Jupiter-sized exoplanet with $i = 90^\circ$ on a 30 day circular orbit, a typical orbital period for planets in our simulations, orbiting a Sun-like star spends about 1% of the time transiting for a FTT of 10^{-2} . Almost all of the CBPs in our simulations are initialized in a transiting configuration and typically have an initial FTT of order 10^{-2} .

We find that in most cases in which planet b is ejected while planet c remains stable, planet b's ejection does little to impact the “transitability” of the remaining planet. In the case of Neptune-mass planets in Fig. 3.15, the histograms for b unstable, c stable and b, c stable have similar shapes indicating that planet b's ejection does little to change planet c's FTT. For Saturn-mass planets, specifically those with an initially larger inclination, there is a small extended tail in the FTT distribution for the case when planet b is ejected while planet c remains stable indicating that more inclined, massive planets are scattered to slightly larger inclinations and eccentricities, reducing their FTT.

From Fig. 3.15, clear trends in both mass and initial inclination can be seen. After an instability event occurs in which planet b is ejected, the more massive the CBPs, the less time they spend transiting. For Neptune-mass CBPs after planet b is ejected, planet c's orbit does not appreciably change and retains a large FTT, see the left panel of Fig. 3.15. As the planet mass increases, planet b's chaotic evolution significantly perturbs planet c's orbit, scattering it to larger inclinations, reducing FTT. This effect is most notable in the extended tail and large peak at FTT near 0 for the orange lines in the right panel of Fig. 3.15 for Jupiter-mass planets. This demonstrates that planet b's ejection can readily scatter planet c away from a transiting configuration for massive planets, preventing its detection via the transit method. This effect is less pronounced for less massive CBPs.

The initial inclination significantly impacts the subsequent CBP transitability. The more inclined planet c initially is, the lower the FTT as planet c in general has a larger initial impact

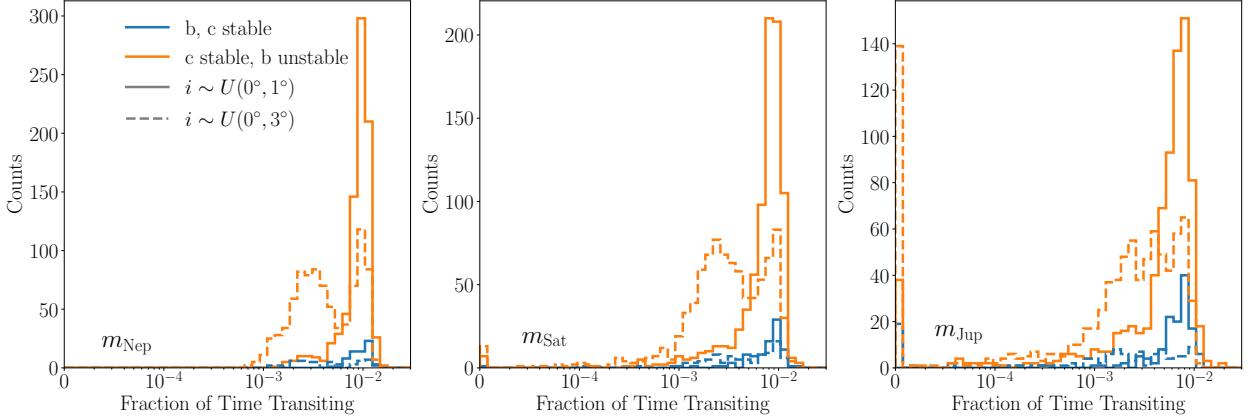


Figure 3.15: Histogram of planet c’s fraction of time spent transiting one of the binary stars for Neptune- (Left:), Saturn- (Middle:), and Jupiter-mass (Right:) exoplanets. The orange histograms correspond to when c remains stable while b goes unstable and the blue curve corresponds to when both b and c remain stable. The solid and dashed histograms correspond to cases where both planets’ initial inclinations are uniformly sampled from $[0^\circ, 1^\circ]$ and $[0^\circ, 3^\circ]$, respectively. Generally, the larger the mutual inclination, the less time c spends transiting.

parameter and can scatter more violently to higher inclinations when planet b is ejected from the system. This effect is most clearly seen in the right panel of Fig. 3.15 for Jupiter-mass CBPs as there is a prominent peak near FTT = 0 for the “high inclination” case (dashed line) that is much larger than the FTT = 0 peak for the “low inclination” case (solid line). This difference indicates that more highly-inclined Jupiter-mass CBPs are preferentially scattered away from a transiting configuration compared to less-inclined planets. In addition to the slight increase in inclination for some surviving planets reducing their FTT, we find that a_c also increases slightly after planet b’s ejection, reducing its FTT. This finding is in good agreement with Gong and Ji (2017).

Our results indicate that when the inner-most planet in a multi-planet circumbinary system is ejected after falling within a_{crit} , it is likely that there will be little impact on the remaining planet’s detectability. In the case of massive planets and/or large initial inclinations, however, the inner-most planet’s ejection can more readily scatter the remaining planet away from and potentially out of a transiting configuration. Even in this case, however, there is likely little to no change in the surviving planet’s FTT. Of course, more distant planets are in general less likely to transit, so the

STEEP process still reduces the total number of transiting CBPs.

One complicating factor is that we make our mock transit observations right after the dynamical stability integrations. From our coupled stellar-tidal VPLanet simulations, we showed that the maximum a_{crit} value is typically achieved well within 1 Gyr (see § 4.5) indicating that if a CBP will be enveloped by a_{crit} and fall within the region of dynamical instability, it will happen early on in the system’s lifetime. Our mock transit observations implicitly assume that we observe the system soon after a_{crit} has enveloped the inner-most planet in a given system, a time that likely occurs well within the 1st Gyr of the system’s lifetime. In real transit survey’s like the *Kepler* mission, the observed stars are not likely to have ages ≤ 1 Gyr like the systems in our mock observations. As shown in § 3.5.8 for tidally locked binary star systems, the older a system is, the more the binary semi-major axis and hence a_{crit} has decayed leaving the surviving CBPs to orbit at larger a_{CBP}/a_{crit} than they did in the past. This effect complicates the detection of CBPs via the transit method and requires that any CBPs discovered around short-period binaries must be understood in the context of the host binary’s past coupled stellar-tidal evolution.

3.7 Application to Kepler-47

We apply the STEEP process to Kepler-47, the shortest period planet-hosting binary system (Orosz et al., 2012). Kepler-47 is a nearly circular G and M dwarf binary with an orbital period of about 7.45 days. Kepler-47 is the only known multi-planet hosting binary with three CBPs, all with nearly co-planar, low-eccentricity orbits (Orosz et al., 2012; Welsh et al., 2015). Kepler-47 appears to have undergone appreciable tidal evolution as the primary star’s P_{rot} is about 4% longer than the orbital period, an indication that the binary is near tidal synchronization (Orosz et al., 2012). Kepler-47 is a suitable candidate for the STEEP process so we seek to examine how its planets could have survived a potential destabilization from the coupled stellar-tidal evolution of the binary.

The planets in the Kepler-47 system are all of order Neptune mass or less, so we can draw comparisons with our two-planet CBP system N-body simulations of Neptune mass planets in § 3.6 (see also Fig. 3.15). As shown previously, the ejection of the inner-most planet in a low-mass CBP system typically results in little or no change in the orbits and hence transitability of the surviving

exterior planets. Although rarely the inner-most planet’s ejection can destabilize the entire system, the dominant result that the rest of the system remains stable and effectively unchanged. If the Kepler-47 system did previously have a planet interior to Kepler-47b that was ejected after falling into the region of dynamical instability, it could have been ejected from the system without any noticeable observational impact on the surviving planetary system. With three planets on nearly co-planar and low-eccentricity orbits, the Kepler-47 planetary system seems relatively dynamically cold, supporting this picture and making its current state compatible with the STEEP process.

If Kepler-47 did not previously have an additional close-in planet, we examine Kepler-47b, the innermost planet in the Kepler-47 system, which resides at a semi-major axis that is about $1.46a_{crit}$ for the Kepler-47 binary. As shown in § 3.5.6 and § 3.5.9, it is plausible for a binary to attain a_{crit} ratios near 1.4 depending on the initial stellar P_{rot} and stellar tidal Qs. With Kepler-47’s relatively longer orbital period, there was likely an initially larger angular momentum reservoir in the orbit relative to that in the stellar rotations so any transfer would probably not increase the orbital period significantly. We display such a case of Kepler-47’s potential past a_{crit} evolution in Fig. 3.16. For this simulation, we used the observed stellar masses (Orosz et al., 2012), initial stellar tidal Qs of 5×10^5 , $r_g = 0.45$, an initial $e = 0.22$, and an initial orbital period of 7.5 days. Note that this case does not represent the precise past evolution of Kepler-47, but instead shows one possible past evolution that is consistent with observations. In this simulation, given Kepler-47’s longer initial orbital period, the binary orbital period and hence a_{crit} grows less via tidal transfer making it less likely to potentially destabilize Kepler-47b. Under this scenario, Kepler-47b likely formed or migrated to a location far enough away from the central binary to insulate it from the central binary’s coupled stellar-tidal evolution, preventing a destabilizing event and making this system compatible with the STEEP process. Note that accurately modeling Kepler-47’s past evolution to assess how the STEEP process could have impacted the circumbinary planetary system requires running a large number of simulations of coupled stellar-tidal evolution and comparing their results with observations. The results of such a simulation suite could potentially constrain parameters, such as tidal Qs, but that analysis is beyond the scope of this work.

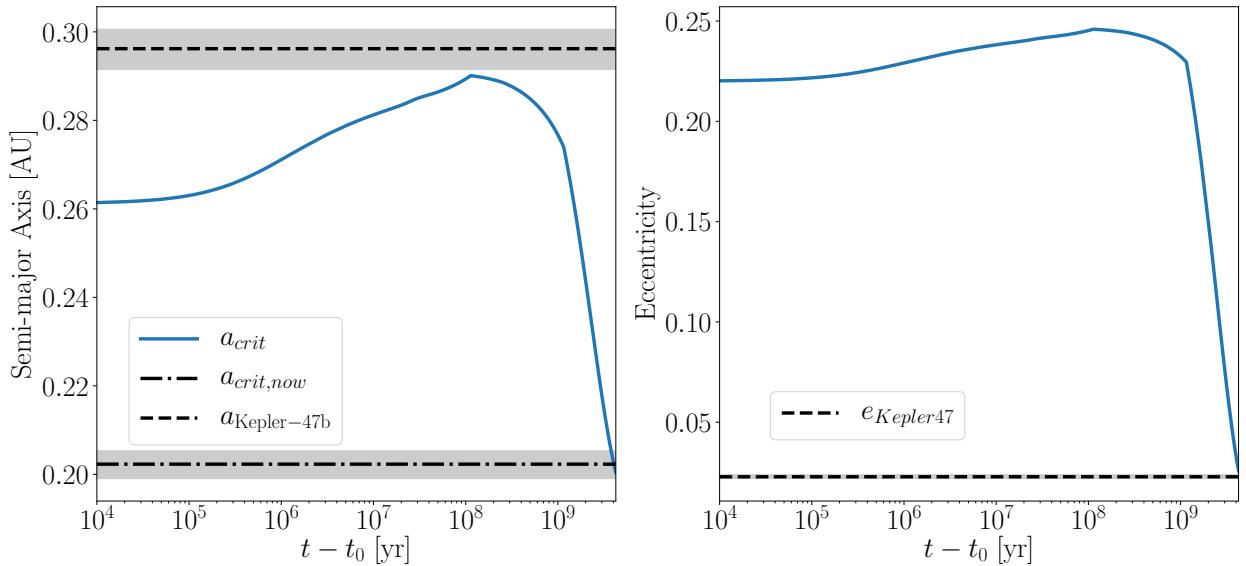


Figure 3.16: One potential coupled stellar-tidal evolutionary history of Kepler-47. *Left:* a_{crit} evolution. The blue curve shows a potential past evolution of Kepler-47's a_{crit} . The dashed line represents Kepler-47b's current semi-major axis while the dot-dashed line indicates Kepler-47's observed a_{crit} . The grey regions display the uncertainties on Kepler-47b's semi-major axis and on Kepler-47's a_{crit} from Orosz et al. (2012). *Right:* Eccentricity evolution. The blue curve displays a potential past evolution of Kepler-47's e . The dashed line and grey regions denote Kepler-47's observed e and the associated uncertainties, respectively. This potential past evolution of the Kepler-47 achieves an a_{crit} nearly large enough to potentially eject Kepler-47b while also reaching an a_{crit} and e consistent with the observed values (Orosz et al., 2012). A more rigorous statistical examination of this system is required to constrain its actual past coupled stellar-tidal evolution.

3.8 Discussion

In this work, we proposed an explanation for the apparent lack of CBPs around isolated binary stars that arises as a natural consequence of short-period binary star evolution. We showed that binary stars that start with P_{rot} faster than the orbital period transfer angular momentum into the orbit as tides drive the system to a tidally locked state. The addition of angular momentum to the orbit increases a , expanding a_{crit} . Since CBPs tend to preferentially exist near a_{crit} , they are then enveloped by the expanding dynamical stability limit and expelled from the system, explaining their observed lack. We referred to this Stellar Tidal Evolution Ejection of Planets as the STEEP process. Binary star systems are a product of complex coupled stellar-tidal evolution and their current observed state likely differs appreciably from its past. Any CBPs that exist around short-period isolated binaries must orbit at large a_{CBP} relative to the binary's a as the past larger region of dynamical stability precludes stable orbits closer to the binary and since the binary's a decays with time post tidal locking due to magnetic braking. If any future surveys discover new CBPs around short-period isolated binaries, their current state must be understood in the context of the binary's previous evolution.

We examined the dynamical stability of circumbinary planetary systems in which the innermost planet falls within the region of dynamical instability by running a series of N-body simulations and found that in most cases, at least one planet is ejected. We performed mock transit observations of such systems after the dynamical stability integration to examine how a potential planetary ejection can impact the detectability any remaining planets. Most surviving planets' orbits exterior to the ejected planet did not appreciably change after a planet ejection, allowing them to remain detectable via the transit method, although more massive planets tended to scatter away from transiting configurations. Given the sizable population of short-period binary systems (Kirk et al., 2016) around which there are no discovered CBPs, destabilization and subsequent ejection via the STEEP process could have ejected many CBPs contributing to the population of free-floating planets (Veras and Raymond, 2012). This population could be examined by microlensing surveys (e.g. Sumi et al., 2011) and compared with expected free-floating planet population pro-

duced by other mechanisms such as planet-planet scattering in single-star systems to gauge its significance.

Our treatment of stellar evolution has its limitations as well, even though we used modern stellar evolution models (Baraffe et al., 2015) and magnetic braking laws (Reiners and Mohanty, 2012; Repetto and Nelemans, 2014). Future examinations of coupled stellar-tidal evolution should model realistic evolving stellar radii of gyration, stellar metallicity effects (e.g. Bolmont et al., 2017), differential rotation (e.g. Lanza and Mathis, 2016), and the effect of binarity on stellar-tidal evolution in order to produce more quantitatively accurate models. Additionally, future studies should consider directly coupling an N-body code with a coupled stellar-tidal evolution model to robustly model the STEEP process, but we note that such simulations would be quite computationally expensive given the \sim Gyr timescales of coupled stellar-tidal evolution.

Future examinations of the STEEP process should focus on how coupled stellar-tidal evolution proceeds at large e . If a binary star system tidally locks at $e \gtrsim 0.2$, it can get captured into a pseudo-synchronous rotation state or a higher order spin-orbit resonance. We examined the case of an eccentric binary tidally locking into a 3:2 spin-orbit resonance in § 3.5.7. We found that at larger e , the orbit has less angular momentum so, for given initial stellar P_{rot} , tidal transfer of angular momentum to the orbit leads to larger increases in a_{crit} than the synchronous rotation case. Our model tends to break down at $e \gtrsim 0.3$ as the CPL model is derived to second order in e and does not resolve tidal locking into pseudo-synchronous rotation like the CTL model does, so future work could examine coupled stellar-tidal evolution using the CTL model. The precise details of tidal evolution at large e , however, is speculative and likely poorly constrained by linear equilibrium tidal models like the CPL and CTL models (e.g. Ferraz-Mello et al., 2008; Greenberg, 2009).

One effect not modeled by the STEEP process is the impact of mean motion resonances (MMRs) between the CBP and inner binary on the stability of CBP orbits. Holman and Wiegert (1999) found that the inner-most $n : 1$ MMR exterior to a_{crit} produced “islands” of instability such that CBPs orbiting exterior to a_{crit} near the MMR could still go unstable and be ejected from the system, a finding confirmed by the recent study of Lam and Kipping (2018). Similar to how a_{crit}

expands as the binary orbital period increases, the location of the inner-most $n : 1$ MMR will extend outward, potentially destabilizing CBPs still residing exterior to a_{crit} . This phenomenon can potentially make the STEEP process more effective at ejecting close-in CBPs and should be examined in future studies.

3.8.1 Future Prospects

The prospect for detecting additional CBPs appears bright as future surveys and algorithmic improvements can potentially increase the known population of CBPs. Newly discovered CBPs will help characterize the true underlying distribution of a_{CBP} relative to a_{crit} and provide a direct observational test of the destabilization of CBPs via coupled stellar-tidal evolution. Refined detection algorithms could uncover previously undetected CBPs in both *Kepler* and *K2* observations. Future *TESS* observations are expected to find approximately 1,100 eclipsing binaries (Sullivan et al., 2015) that could host additional CBPs, improving population statistics and helping to settle this remaining issue. Coupled with the previously-discovered *Kepler* eclipsing binaries (Kirk et al., 2016), the eclipsing binaries *TESS* will discover will provide a rich dataset that could be used to constrain how coupled stellar-tidal evolution proceeds and could potentially allow for constraints on parameters such as stellar tidal Qs. Launching between 2022-2024, the *ESA*-led *PLATO* mission will monitor nearly 1,000,000 stars searching for transits with a focus on low-mass terrestrial planets, potentially discovering new CBPs (Rauer et al., 2014). Additionally, the *Gaia* mission can potentially probe the population of gaseous CBPs and the CBP-binary mutual inclination distribution allowing for comparison with the *Kepler* circumbinary population (Sahlmann et al., 2015).

A key component required to understand the observed CBP population is CBP planet formation. Although numerous studies have examined planet formation in circumbinary systems (e.g. Alexander, 2012; Paardekooper et al., 2012; Meschiari, 2012a,b; Pelupessy and Portegies Zwart, 2013; Bromley and Kenyon, 2015; Vartanyan et al., 2016), no previous study has examined the impact of coupled stellar-tidal evolution on young binary stars embedded in a circumbinary disk. Given that complex disk-binary interactions can lead to significant changes in both the orbit of the binary and the structure of the circumbinary disk (e.g. Fleming and Quinn, 2017), coupled

stellar-tidal evolution would necessarily play an important role in that feedback especially since appreciable tidal orbital evolution can occur over the ~ 1 Myr disk lifetime (Haisch et al., 2001). Although likely computationally non-trivial, accounting for coupled stellar-tidal evolution in simulations of binaries embedded in protoplanetary circumbinary disks could yield new insights into how CBPs form and migrate in circumbinary disks.

In recent years, numerous theoretical modeling efforts have sought to characterize how the presence of two stars impacts the potential habitability of CBPs (e.g. Kane and Hinkel, 2013; Forgan, 2014; Popp and Eggl, 2017). As future studies look to characterize and detect potentially habitable CBPs, we suggest that such efforts should focus on longer-period binaries, those with $P_{orb} \gtrsim 7.5$ days, as in this regime, the STEEP process is less likely to result in the ejection of close-in CBPs.

As future observations discover and characterize new CBPs, probing the true underlying CBP population, the evolution of the binary stars that host these planets must be well known. Understanding coupled stellar-tidal evolution in young, short-period binary star systems can provide critical insights into how binaries form and host circumbinary planetary systems and hence provide insights into the observed CBP population. In this work, we outlined a theoretical framework, the STEEP process, for the long-term evolution of short-period binary stars that provides an explanation for the lack of CBPs around such binary systems. Future detections or non-detections of CBPs around short-period binaries will provide the best indirect observational test of the STEEP process.

3.9 acknowledgments

The authors wish to thank the anonymous referee for their careful reading of the manuscript and their insightful suggestions that improved the quality of the manuscript. We also would like to thank Russell Deitrick and Jacob Lustig-Yaeger for constructive conversations that improved the quality of this work. This work was facilitated through the use of advanced computational, storage, and networking infrastructure provided by the Hyak supercomputer system and funded by the STF at the University of Washington. DPF is supported by an NSF IGERT DGE-1258485 fellowship. This work was supported by NASA Headquarters under the NASA Earth and Space

Science Fellowship Program - Grant 80NSSC17K0482. DPF, RB, RL, and TRQ acknowledge that this work was supported by the NASA Astrobiology Institute's Virtual Planetary Laboratory under Cooperative Agreement number NNA13AA93A. R.L. acknowledges support from NASA grant NNX14AK26G. Software: matplotlib: Hunter (2007), numpy: van der Walt et al. (2011), pandas: McKinney (2010)

Chapter 4

STELLAR ROTATION RATES IN BINARY STARS: THE COMPETITION BETWEEN MAGNETIC BRAKING AND TIDAL TORQUES

Portions of this chapter were originally published in collaboration with Rory Barnes, James R. A. Davenport, and Rodrigo Luger in the August 2019 edition of the Astrophysical Journal (Fleming et al., 2019, ApJ, Vol. 881, 2; 2019 ©American Astronomical Society), and are reproduced below with permission of the American Astronomical Society.

4.1 abstract

We examine how tides, stellar evolution, and magnetic braking shape the rotation period (P_{rot}) evolution of low-mass stellar binaries up to orbital periods (P_{orb}) of 100 d across a wide range tidal dissipation parameters using two common equilibrium tidal models. We find that many binaries with $P_{orb} \lesssim 20$ d tidally lock, and most with $P_{orb} \lesssim 4$ d tidally lock into synchronous rotation on circularized orbits. At short P_{orb} , tidal torques produce a population of fast rotators that single-star only models of magnetic braking fail to produce. In many cases, we show that the competition between magnetic braking and tides produces a population of subsynchronous rotators that persists for Gyrs, even in short P_{orb} binaries, qualitatively reproducing the subsynchronous eclipsing binaries (EBs) discovered in the *Kepler* field by Lurie et al. (2017). Both equilibrium tidal models predict that binaries can tidally-interact out to $P_{orb} \approx 80$ d, while the Constant Phase Lag tidal model predicts that binaries can tidally lock out to $P_{orb} \approx 100$ d. Tidal torques often force the P_{rot} evolution of stellar binaries to depart from the long-term magnetic braking-driven spin down experienced by single stars, revealing that P_{rot} is not be a valid proxy for age in all cases, i.e. gyrochronology can underpredict ages by up to 300% unless one accounts for binarity. We suggest that accurate determinations of orbital eccentricities and P_{rot} can be used to discriminate between

which equilibrium tidal models best describes tidal interactions in low-mass binary stars.

4.2 Introduction

The long-term angular momentum evolution of low-mass ($M \lesssim 1 M_{\odot}$) stars is controlled by magnetic braking, the torque exerted on stars due to the coupling of stellar winds to the surface magnetic field (Mestel, 1968). Early in stellar lifetimes, stars spin-up as they contract along the pre-main sequence. Once stars reach the main sequence, stellar radii remain mostly constant while magnetic braking removes angular momentum from the stars, gradually spinning them down over time (Skumanich, 1972). Although the precise details of how magnetic braking operates are not fully known, models of magnetic braking have been used to successfully model the bulk trends of P_{rot} distributions in clusters (e.g. Praesepe, Reiners and Mohanty, 2012; Matt et al., 2015; Douglas et al., 2017) and field stars (e.g. the *Kepler* field, Matt et al., 2015; van Saders et al., 2018). Furthermore, the magnetic braking-driven long-term spin-down of stars has been used to estimate stellar ages, a method known as gyrochronology (Skumanich, 1972; Barnes, 2003, 2007; Mamajek and Hillenbrand, 2008; Barnes, 2010), with older stars assumed to have lost more angular momentum due to magnetic braking and therefore rotate more slowly.

In contrast, the angular momentum evolution in low-mass short-period ($P_{orb} \lesssim 10$ d) stellar binaries is dominated by tides. Tidal torques drive secular changes in the binary orbit and stellar spins, eventually circularizing the orbit and synchronizing the stellar spins in the long-term (Counselman, 1973). Orbital circularization is ubiquitous for short-period binaries, owing to the tidal torque's strong radius and semi-major axis dependence, with both theoretical (e.g. Zahn and Bouchet, 1989; Claret et al., 1995) and observational (e.g. Meibom and Mathieu, 2005; Mazeh, 2008; Lurie et al., 2017) studies finding that most binaries with $P_{orb} \lesssim 10$ d are circularized. For short-period binaries, tidal torques work quickly on ~ 100 Myr timescales, as Zahn and Bouchet (1989) found that the orbit of solar twin binaries circularize during the stellar pre-main sequence. Observations by Meibom and Mathieu (2005) support this picture as they find short-period binaries in the ~ 150 Myr old cluster M35 tend to have circular orbits.

Tides impart a significant signature in the long-term angular momentum evolution for binary

stars, especially for stellar spins. Tidal torques drive binaries towards the tidally locked state in which the stellar P_{rot} is equal to the equilibrium rotation period (P_{eq}) predicted by tidal models, with a familiar example of this effect being spin-orbit synchronization where $P_{rot} = P_{eq} = P_{orb}$. Tidal-locking occurs much earlier than orbital circularization with the tidal-locking timescale estimated to be $2 - 3$ orders of magnitude less than the circularization timescale (Zahn and Bouchet, 1989; Witte and Savonije, 2002; Mazeh, 2008) as there is typically much less angular momentum in stellar spins than the binary orbit. As a result, tidal-locking is expected for binaries with $P_{orb} \lesssim 20$ d (e.g. Levato, 1974; Meibom et al., 2006; Mazeh, 2008; Zahn, 2008; Meibom et al., 2015).

In low-mass binaries, both magnetic braking and tidal torques compete to shape the stellar P_{rot} evolution. When tides dominate, in particular at close orbital separations, tides can fix $P_{rot} = P_{orb}$, or more generally $P_{rot} = P_{eq}$ for eccentric orbits. In such situations, magnetic braking still operates, removing angular momentum from each star, forcing tides to compensate for each star's loss of angular momentum by spinning up the stars to maintain the tidally locked equilibrium, removing angular momentum from the orbit, hardening the binary (Verbunt and Zwaan, 1981; Repetto and Nelemans, 2014; Fleming et al., 2018). Tides do not win out over magnetic braking in general, however, as magnetic braking can spin-down the stars past the tidally locked state into subsynchronous rotation ($P_{rot} > P_{eq}$, Habets and Zwaan, 1989; Zahn, 1994; Keppens, 1997). This behavior seems to be borne out in nature, as Lurie et al. (2017) discovered a substantial population of subsynchronous short-period binaries in the *Kepler* field, clustered near $P_{orb}/P_{rot} \approx 0.9$, in defiance of the expectation of tidal locking at such short orbital separations. The competition between magnetic braking and tidal torques can lead to complex angular momentum evolution in low-mass stellar binaries, and no previous work has conducted a systematic study to examine how this evolution proceeds across a wide range of tidal dissipation parameters and P_{orb} .

Understanding the interaction between tidal torques and magnetic braking is of paramount importance as P_{rot} distributions measured in clusters (e.g. Praeseppe, Agüeros et al., 2011; Douglas et al., 2017) and field stars (e.g. *Kepler*, Reinhold et al., 2013; McQuillan et al., 2014) are likely contaminated by unresolved binaries given that roughly half of Sun-like stars are in stellar binaries (Raghavan et al., 2010; Duchêne and Kraus, 2013), and that binaries are difficult to resolve in pho-

tometric surveys. In the *Kepler* field, for example, Simonian et al. (2018) recently found that most rapid rotators ($P_{rot} \lesssim 7.5$ d) are likely non-eclipsing, tidally-synchronized short-period photometric binaries, indicating that tidal torques in binaries can significantly impact observed P_{rot} distributions. Tidally-interacting binaries impart a contaminating signal that is not currently accounted for by models. Moreover, any ages inferred from rotation periods of stars in unresolved binaries using gyrochronology could be incorrect owing to the influence of tidal torques. No previous study has quantified this effect.

There is currently a large number of *Kepler* binaries with known P_{rot} and P_{orb} (e.g. Lurie et al., 2017). Both the extended *Kepler* mission (K2, Howell et al., 2014) and the Transiting Exoplanet Survey Satellite (TESS, Ricker et al., 2014; Sullivan et al., 2015) are expected to detect additional low-mass eclipsing binaries, with Gaia parallaxes (Gaia Collaboration et al., 2016) poised to help refine these stellar parameters, potentially creating a rich dataset of the angular momentum budgets of low-mass binaries. Developing a framework for the angular momentum evolution of low-mass binaries can enable the characterization of the nature of tidal torques in binaries by conditioning on datasets of the spin and orbital states of stellar binaries.

Here, we present a model for the angular momentum evolution of low-mass stellar binaries over their full premain and main sequence lifetimes using a realistic treatment of stellar evolution, magnetic braking, and tidal torques. We investigate under what conditions tidal-locking occurs, and how tidal torques influence rotation in stellar binaries as a function of binary P_{orb} and tidal dissipation parameters for two widely-used equilibrium tidal models. We show how tidal torques can impact stellar rotation in binaries out to $P_{orb} = 100$ d, causing stellar rotation periods to not strongly correlate with age, making the predictions of gyrochronology models fail in such systems. We describe our model in § 4.3 and our simulation procedure in § 4.4. We discuss our results in § 4.5, apply our model to the *Kepler* field in § 4.5.5, and discuss our results’ implications in § 4.6.

4.3 Methods

We simulate coupled stellar-tidal evolution for low-mass binaries using an improved version of the model presented in Fleming et al. (2018). We implement our model in the open-source code

VPLanet¹ (Barnes et al., 2019). We integrate all model equations (see § 4.3.1 and § 4.3.2) using the 4th order Runge-Kutta scheme with adaptive timestepping described in Fleming et al. (2018).

4.3.1 Stellar Evolution

We improve upon the interpolation of the Baraffe et al. (2015) stellar evolution models employed by Fleming et al. (2018), STELLAR, by additionally performing a bicubic interpolation of the stellar radius of gyration, r_g , over mass and time of the Baraffe et al. (2015) models. This updated version of STELLAR now tracks the full moment of inertia evolution of low-mass stars according to the Baraffe et al. (2015) stellar evolution models, a critical requirement for modeling the angular momentum evolution of low-mass stars.

We simulate magnetic braking using the model derived by Matt et al. (2015) as this formalism has been shown to successfully model the spin-down of low-mass stars across many ages in both the Praesepe cluster and in the *Kepler* field. This model depends on the stellar Rossby number, $Ro = P_{rot}/\tau_{cz}$, the ratio of the stellar P_{rot} to the stellar convective turnover timescale, τ_{cz} . The Matt et al. (2015) model predicts that below a certain Ro for rapidly-rotating stars, stellar magnetic activity saturates at a constant value, producing a magnetic braking torque that is directly proportional to the stellar rotation rate. The angular momentum loss for rapidly-rotating saturated stars is given by

$$\frac{dJ}{dt} = -\left. \frac{dJ}{dt} \right|_0 \chi^2 \left(\frac{\omega}{\omega_\odot} \right) \quad (4.1)$$

while for more slowly-rotating unsaturated stars,

$$\frac{dJ}{dt} = -\left. \frac{dJ}{dt} \right|_0 \left(\frac{\tau_{cz}}{\tau_{cz\odot}} \right)^2 \left(\frac{\omega}{\omega_\odot} \right)^3 \quad (4.2)$$

where

$$\left. \frac{dJ}{dt} \right|_0 = 6.3 \times 10^{30} \text{ erg} \left(\frac{R}{R_\odot} \right)^{3.1} \left(\frac{M}{M_\odot} \right)^{0.5}. \quad (4.3)$$

¹VPLanet is publicly available at <https://github.com/VirtualPlanetaryLaboratory/vplanet>.

Saturated magnetic braking occurs for $Ro \leq Ro_{\odot}/\chi$ for $\chi = 10$ where Matt et al. (2015) defines $\chi = Ro_{\odot}/Ro_{sat}$. We adopt all model parameters given in Table 1 from Matt et al. (2015), with the correction from Matt et al. (2019), and compute τ_{cz} using Eqn. (36) from Cranmer and Saar (2011).

We model the net change in the stellar rotation rate due to stellar evolution and magnetic braking via the following equation

$$\dot{\omega} = \frac{J_{mb}}{I} - \frac{2\dot{R}\omega}{R} - \frac{2\dot{r}_g\omega}{r_g} \quad (4.4)$$

where the moment of inertia $I = Mr_g^2R^2$, J_{mb} is the angular momentum loss due to magnetic braking, and the time derivatives of the stellar R and r_g are computed numerically using our interpolation of the Baraffe et al. (2015) stellar evolution grids.

Core-Envelope Coupling

Our simplified model assumes that stars follow solid body rotation, whereas in real low-mass stars, coupling between the radiative core and convective envelope can impact the surface rotation period evolution (MacGregor and Brenner, 1991; Allain, 1998; Bouvier, 2008; Irwin and Bouvier, 2009). Recent work by Gallet and Bouvier (2013) and Gallet and Bouvier (2015) find that the rapidly-rotating stellar core acts as an angular momentum reservoir for the convective envelope, potentially transferring angular momentum within the stellar interior and into the envelope for up to 1 Gyr, depending on the adopted magnetic braking model, the initial rotation rate, and the stellar mass. We anticipate that internal angular momentum transport would work against the spin-down caused by tidal torques, increasing tidal locking timescales. Internal angular momentum transport torques could potentially balance both tidal and magnetic braking torques near the tidally locked state, producing slight supersynchronous rotation, analogous to the subsynchronous case examined in § 4.5.1. Modeling core-envelope coupling is beyond the scope of this work, however.

Example Stellar Evolution

In Fig. 4.1, we plot the evolution of R , r_g , and P_{rot} for $0.2 M_\odot$, $0.7 M_\odot$, and $1 M_\odot$ mass stars, representing an M, K, and G dwarf, respectively, computed according to STELLAR and the Matt et al. (2015) magnetic braking model. We assume all stars have an initial $P_{rot} = 1$ d and have an initial age of 5 Myr. All stars' radii contract along the pre-main sequence, spinning the stars up (right panel). Once the stars reach the main sequence, their structure changes slowly, allowing magnetic braking to dominate the stellar angular momentum evolution, significantly spinning-down the stars over long timescales. The r_g evolution noticeably differs between the stars as the late M dwarf's (green) r_g varies little as it remains fully convective, while the K and G dwarf grow a radiative core while on the pre-main sequence, decreasing r_g until both reach the main sequence.

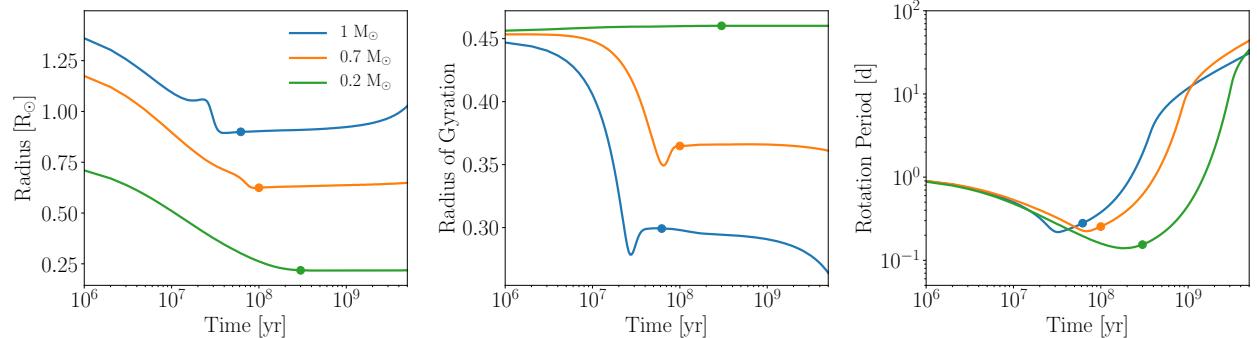


Figure 4.1: Stellar R (left), r_g (middle), and P_{rot} (right) evolution for $0.2 M_\odot$ (M, green), $0.7 M_\odot$ (K, orange), and $1 M_\odot$ (G, blue) mass stars computed according to STELLAR, our interpolation of the Baraffe et al. (2015) stellar evolution models (§ 4.3.1) combined with the Matt et al. (2015) magnetic braking model. Each dot denotes the approximate time when each star reaches the main sequence.

4.3.2 Tidal Evolution

Equilibrium tidal models, first introduced by Darwin (1880), track the secular evolution of an orbiter's semi-major axis, a , eccentricity, e , and the rotation rates, ω_i , and obliquities ψ_i , of both gravitating bodies due to tidal torques. Equilibrium tidal models assume that tidally interacting bodies raise tidal bulges on their companions that remain offset from the line connecting the bod-

ies’ centers of mass due to friction within each body. This assumption is typically referred to as the “weak friction approximation” (Zahn, 2008). The tidal bulges cause torques that permit the exchange of angular momentum between the orbit and both bodies’ spins. Equilibrium tidal models are linear since they assume that the tidal waves that comprise the tidal bulge raised on a body are uncoupled. Under these assumptions, the tidal evolution is analogous to a driven, damped harmonic oscillator (Greenberg, 2009). For low-mass stars, equilibrium tidal models assume that tidal forces primarily dissipate energy in the outer-convective regions via viscous turbulence (see Zahn, 2008). Although simple, equilibrium tidal models have been used to model the secular orbital and rotation evolution of both Solar System bodies and exoplanets (e.g. Goldreich and Soter, 1966; Jackson et al., 2009; Leconte et al., 2010; Heller et al., 2011; Barnes et al., 2013; Barnes, 2017) and stellar binaries (e.g. Zahn and Bouchet, 1989; Zahn, 2008; Khaliullin and Khaliullina, 2011; Repetto and Nelemans, 2014; Fleming et al., 2018). We refer the reader to Barnes (2017) for an in-depth discussion of the assumptions and limitations of equilibrium tidal models. Here, we consider two common equilibrium tidal models to study the secular spin-orbital evolution of low-mass stellar binaries.

Constant Phase Lag Model

The “Constant Phase Lag” (CPL) (Ferraz-Mello et al., 2008; Heller et al., 2011) equilibrium tidal model assumes that the tidal torque on one body due to its companion arises from a linear combination of several discrete, uncoupled tidal bulges, each with its own associated frequency, that maintain a fixed phase offset with respect to the line connecting the two stars’ centers of mass. We use the EQTIDE implementation of the CPL model in VPLanet following the derivation of Ferraz-Mello et al. (2008). The equations that govern the secular change in e and a are as follows:

$$\frac{de}{dt} = -\frac{ae}{8Gm_1m_2} \sum_{i=1}^2 Z_{i,\text{CPL}} \left(2\epsilon_{0,i} - \frac{49}{2}\epsilon_{1,i} + \frac{1}{2}\epsilon_{2,i} + 3\epsilon_{5,i} \right) \quad (4.5)$$

$$\frac{da}{dt} = \sum_{i=1}^2 \frac{da_i}{dt} \quad (4.6)$$

where if the i^{th} body is tidally locked in a synchronous orbit,

$$\frac{da_{i,\text{sync}}}{dt} = -\frac{a^2}{Gm_1m_2} Z_{i,\text{CPL}} (7e^2 + \sin^2(\psi_i)) \varepsilon_{2,i}, \quad (4.7)$$

otherwise

$$\begin{aligned} \frac{da_i}{dt} = & \frac{a^2}{4Gm_1m_2} Z_{i,\text{CPL}} \left(4\varepsilon_{0,i} + e^2 \left[-20\varepsilon_{0,i} + \frac{147}{2}\varepsilon_{1,i} \right. \right. \\ & \left. \left. + \frac{1}{2}\varepsilon_{2,i} - 3\varepsilon_{5,i} \right] - 4\sin^2(\psi_i) [\varepsilon_{0,i} - \varepsilon_{8,i}] \right). \end{aligned} \quad (4.8)$$

The CPL equations for ψ and ω evolution are

$$\frac{d\psi_i}{dt} = \frac{Z_{i,\text{CPL}} \sin(\psi_i)}{4m_i r_{g,i}^2 R_i^2 n \omega_i} ([1 - \xi_i] \varepsilon_{0,i} + [1 + \xi_i] (\varepsilon_{8,i} - \varepsilon_{9,i})) \quad (4.9)$$

$$\begin{aligned} \frac{d\omega_i}{dt} = & -\frac{Z_{i,\text{CPL}}}{8m_i r_{g,i}^2 R_i^2 n} (4\varepsilon_{0,i} + e^2 [-20\varepsilon_{0,i} + 49\varepsilon_{1,i} + \varepsilon_{2,i}] \\ & + 2\sin^2(\psi_i) [-2\varepsilon_{0,i} + \varepsilon_{8,i} + \varepsilon_{9,i}]) \end{aligned} \quad (4.10)$$

where G is Newton's gravitational constant, n is the binary's mean motion, and the index i denotes that i^{th} body. The tidal phase lags signs, ε , for the i^{th} body are given by

$$\begin{aligned} \varepsilon_{0,i} &= \Sigma(2\omega_i - 2n) \\ \varepsilon_{1,i} &= \Sigma(2\omega_i - 3n) \\ \varepsilon_{2,i} &= \Sigma(2\omega_i - n) \\ \varepsilon_{5,i} &= \Sigma(n) \\ \varepsilon_{8,i} &= \Sigma(\omega_i - 2n) \\ \varepsilon_{9,i} &= \Sigma(\omega_i) \end{aligned} \quad (4.11)$$

where the function $\Sigma(x)$ returns 1 for positive x , -1 for negative x , and 0 otherwise.

The intermediate variable $Z_{\text{CPL},i}$ is given by

$$Z_{i,\text{CPL}} = 3G^2 k_{2,i} M_j^2 (M_i + M_j) \frac{R_i^5}{a^9} \frac{1}{nQ_i} \quad (4.12)$$

where the j^{th} body is the i^{th} body's companion, k_2 is the body's Love number of degree 2, and Q is the tidal quality factor ("tidal Q"). The tidal Q parameterizes the energy dissipation due to tidal evolution, with lower tidal Qs, i.e. larger phase differences between the tidal bulges, driving more rapid tidal evolution.

The other intermediate variable, ξ_i , is defined as

$$\xi_i = \frac{r_{g,i}^2 R_i^2 \omega_i a n}{GM_j}. \quad (4.13)$$

Constant Time Lag Model

The "Constant Time Lag" (CTL) (Hut, 1981; Leconte et al., 2010) equilibrium tidal model assumes a constant time interval between the body's tidal bulge and the passage of the tidally interacting companion. In this formalism, unlike the CPL model, the CTL model is continuous over a range of tidal wave frequencies and applicable for large e . However, if the assumption of linearity is relaxed, i.e. frequencies associated with tidal bulges are allowed to depend on a spin or orbital forcing frequency, then this model is only valid over a small range of frequencies (Greenberg, 2009). We use the EQTIDE implementation of the CTL model in VPLanet following the derivation of Leconte et al. (2010). The equations that govern the secular changes in e , a , ω , and ψ are as follows:

$$\frac{de}{dt} = \frac{11ae}{2GM_1M_2} \sum_{i=1}^2 Z_{\text{CTL},i} \left(\cos(\psi_i) \frac{f_4(e)}{\beta^{10}(e)} \frac{\omega_i}{n} - \frac{18}{11} \frac{f_3(e)}{\beta^{13}(e)} \right), \quad (4.14)$$

$$\frac{da}{dt} = \frac{2a^2}{GM_1M_2} \sum_{i=1}^2 Z_{\text{CTL},i} \left(\cos(\psi_i) \frac{f_2(e)}{\beta^{12}(e)} \frac{\omega_i}{n} - \frac{f_1(e)}{\beta^{15}(e)} \right), \quad (4.15)$$

$$\frac{d\omega_i}{dt} = \frac{Z_{\text{CTL},i}}{2M_i r_{g,i}^2 R_i^2 n} \left(2 \cos(\psi_i) \frac{f_2(e)}{\beta^{12}(e)} - [1 + \cos^2(\psi)] \frac{f_5(e)}{\beta^9(e)} \frac{\omega_i}{n} \right), \quad (4.16)$$

and

$$\frac{d\psi_i}{dt} = \frac{Z_{\text{CTL},i} \sin(\psi_i)}{2M_i r_{g,i}^2 R_i^2 n \omega_i} \left(\left[\cos(\psi_i) - \frac{\xi_i}{\beta} \right] \frac{f_5(e)}{\beta^9(e)} \frac{\omega_i}{n} - 2 \frac{f_2(e)}{\beta^{12}(e)} \right). \quad (4.17)$$

where the intermediate variables are given by

$$Z_{i,\text{CTL}} = 3G^2 k_{2,i} M_j^2 (M_i + M_j) \frac{R_i^5}{a^9} \tau_i, \quad (4.18)$$

and

$$\begin{aligned} \beta(e) &= \sqrt{1 - e^2}, \\ f_1(e) &= 1 + \frac{31}{2}e^2 + \frac{255}{8}e^4 + \frac{185}{16}e^6 + \frac{25}{64}e^8, \\ f_2(e) &= 1 + \frac{15}{2}e^2 + \frac{45}{8}e^4 + \frac{5}{16}e^6, \\ f_3(e) &= 1 + \frac{15}{4}e^2 + \frac{15}{8}e^4 + \frac{5}{64}e^6, \\ f_4(e) &= 1 + \frac{3}{2}e^2 + \frac{1}{8}e^4, \\ f_5(e) &= 1 + 3e^2 + \frac{3}{8}e^4. \end{aligned} \quad (4.19)$$

In both the CPL and CTL model, We assume $k_2 = 0.5$. This choice of k_2 does not impact our results as k_2 is degenerate with Q in the CPL model, e.g. the k_2/Q scaling in Eq. (4.12), and with τ in the CTL model, e.g. $k_2\tau$ scaling in Eq. (4.18), so we instead examine how our results scale with Q and τ . Any constraints we derive as a function Q or τ can trivially be scaled to other values of k_2 . For example, a common re-parameterization of Q is the reduced tidal quality factor, $Q' = 3Q/2k_2$ (e.g. Leconte et al., 2010). Given our choice of $k_2 = 0.5$, this reduces to $Q' = 3Q$.

Tidal Locking

Tidal torques drive a body's rotation rate towards the tidally locked state. When a body tidally locks, tidal torques fix P_{rot} to the equilibrium P_{rot} , P_{eq} . Typically, tidal locking is understood in the context of a synchronized rotator, e.g. when $P_{rot} = P_{eq} = P_{orb}$. Although spin-orbit synchronization is an expected outcome of tidal evolution (Counselman, 1973), in general for tidally locked

bodies on non-circular orbits, both the CPL and CTL model predict pseudosynchronous, or supersynchronous rotation, e.g. Mercury's 3:2 spin-orbit resonance ($P_{rot} = 2/3 P_{orb}$, Goldreich and Peale, 1966).

The CPL model, owing to its assumption of a finite number of discrete tidal lags, only permits a 1:1 and 3:2 spin-orbit state where, following Barnes (2017), the CPL P_{eq} is given by

$$P_{eq}^{\text{CPL}} = \begin{cases} P_{orb} & \text{if } e < \sqrt{1/19} \\ \frac{2}{3}P_{orb} & \text{if } e \geq \sqrt{1/19}. \end{cases} \quad (4.20)$$

Therefore, the CPL model predicts synchronous rotation for $e \lesssim 0.23$, and a supersynchronous 3:2 spin-orbit state otherwise for tidally locked rotators.

We note that two discrete rotation states are not the only permitted ones for tidally locked systems under the CPL formalism. For example, an alternate derivation of P_{eq} for orbiters with rotation axes perpendicular to the orbital plane under the CPL model predicts

$$P_{eq} = \frac{P_{orb}}{1 + 9.5e^2}, \quad (4.21)$$

a continuous function of e (Goldreich, 1966; Murray and Dermott, 1999). Here, we follow the suggestions of both Barnes et al. (2013) and Barnes (2017) and use the discrete P_{eq} version of the CPL model for self-consistency.

The CTL model is continuous over a range of tidal frequencies and therefore predicts a P_{eq} that is a continuous function of both e and ψ . Following Barnes (2017), we define the CTL P_{eq} by

$$P_{eq}^{\text{CTL}} = P_{orb} \frac{\beta^3 f_5(e)(1 + \cos^2(\psi))}{2f_2(e)\cos(\psi)}. \quad (4.22)$$

The CTL model predicts that bodies on eccentric orbits tidally lock into supersynchronous rotation, and only bodies with aligned spins on circular orbits are synchronous rotators.

In general, a continuous P_{eq} and the discrete 1:1 and 3:2 spin-orbit commensurabilities are not the only equilibrium rotation states for tidally locked rotators predicted by equilibrium tidal

models. For example, Rodríguez et al. (2012) show that tidally interacting bodies can get captured into many spin-orbit resonances states, e.g. 2:1, 5:2, 4:3, etc, and below, we search for evidence of them in data of the spin-orbital states of *Kepler* EBs. Note that our model does not resolve capture into such states as the CPL model, owing to its inclusion of only 4 discrete tidal lags, only allows a body to enter into 3:2 and 1:1 spin - orbit commensurabilities. The CTL model predicts a continuous equilibrium period as a function of the P_{orb} , e , and obliquity, for tidally locked bodies, only resolving capture into 1:1 synchronous rotation.

Numerical Details of Tidal Locking

Due to the discontinuities in the equilibrium tidal model equations, for example in Eq. (4.11) when $\omega \approx n$, and due to the inherent discreteness of numerical integrations, numerical solutions for the CPL and CTL models can produce unphysical evolution. We follow Barnes et al. (2013) and Fleming et al. (2018) and fix $P_{rot} = P_{eq}$ according to Eq. (4.20) or Eq. (4.22) for the CPL and CTL models, respectively, when P_{rot} is within 1% of P_{eq} . To ensure that tidal torques dominate over torques due to magnetic braking and stellar evolution when forcing tidal-locking, we additionally require that the P_{rot} derivative points towards P_{eq} on both sides of P_{eq} , i.e. when the gradient of P_{rot} points towards the tidally locked state, before fixing $P_{rot} = P_{eq}$. We find that this scheme produces physically and numerically accurate results.

The Dynamical Tide

An additional mechanism for tidal dissipation in low-mass stellar binaries is the dynamical tide. This effect arises from the turbulent viscous damping of inertial waves that are excited in the stellar convective envelope by a tidal perturber, with Coriolis acceleration serving as the restoring force (Zahn, 1975; Ogilvie and Lin, 2007). Under the dynamical tide formalism, the stellar mass, evolving stellar structure, rotation rate, and tidal forcing frequency can all strongly impact the strength of tidal dissipation, which can span many orders of magnitude (Ogilvie and Lin, 2007; Ogilvie, 2013; Mathis, 2015; Gallet et al., 2017). For example, adopting the tidal frequency-averaged model for

tidal dissipation of Ogilvie (2013), both Mathis (2015) and Gallet et al. (2017) show that dynamical tidal dissipation is enhanced during the pre-main sequence due to the expansion of the stellar radiative core and rapid rotation, whereas the magnetic braking-driven spin-down on the main sequence decreases the tidal dissipation. Dissipation due to the dynamical tide could be important for some of the systems considered in this work since, for binary stars on circular orbits, inertial waves are excited in the stellar convective envelopes for $P_{orb} > P_{rot}/2$ and can drive significant spin and orbital evolution (e.g. Witte and Savonije, 2002; Ogilvie and Lin, 2007; Bolmont and Mathis, 2016). Although semi-analytic models for dynamical tidal dissipation that account for the evolving stellar structure and rotation exist (e.g. Mathis, 2015; Bolmont and Mathis, 2016; Gallet et al., 2017), we do not consider them here as they are currently limited to circular orbits. We instead focus on exploring the impact of the equilibrium tide across a wide range of parameter space and leave an examination of how the combination of the dynamical and equilibrium tide impacts the rotation period evolution of low-mass binary stars for future work.

Example Tidal Evolution

We plot the tidal evolution for a , e , and P_{rot} , ignoring stellar evolution, for a solar-twin binary with an initial $P_{orb} = 10$ d, $P_{rot} = 1$ d, $e = 0.2$ for the CPL model and CTL model, assuming $Q = 10^6$ and $\tau = 0.1$ seconds, respectively, in Fig. 4.2. The CPL and CTL model predict the same qualitative evolution: both the binary’s e and P_{orb} slightly increase as tides force the spins toward the tidally locked state, transferring rotational angular momentum into the orbit, increasing the orbital angular momentum by $\sim 1\%$ in the process. At late times, both the CPL and CTL drive the binaries towards orbital circularization, with tidal dissipation decreasing P_{orb} . The predictions of the CPL and CTL model, differ, however, when the binaries tidally lock. Under the CPL model, the binary tidally locks into a synchronous orbit when $e < \sqrt{1/19}$, e.g. Eq. (4.20), while the CTL model predicts supersynchronous rotation due to the CTL model’s equilibrium period eccentricity dependence, e.g. Eq. (4.22).

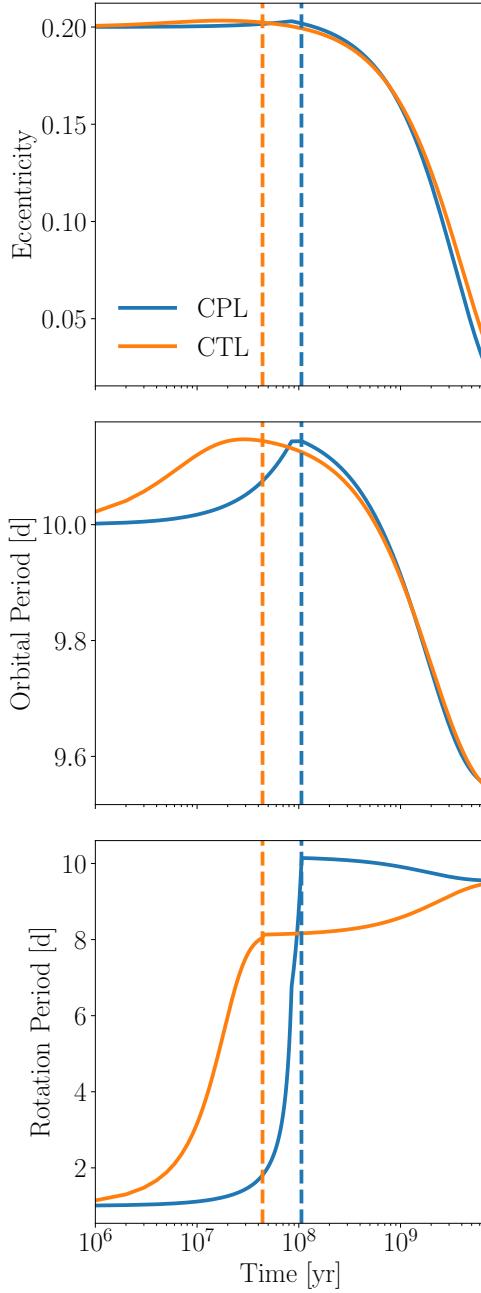


Figure 4.2: Tidal evolution of a $1 M_{\odot} - 1 M_{\odot}$ stellar binary's e (top), P_{orb} (middle), and P_{rot} (bottom) for the CPL (blue) and CTL (orange) model. The blue (CPL) and orange (CTL) vertical dashed lines denote when the stellar binary tidally locks. Both the CPL and CTL model predict the same qualitative evolution. The rotational evolution differs, however, as under the CPL model, the binary tidally locks into a synchronous orbit as $e < \sqrt{1/19}$, e.g. Eq. (4.20), while the CTL model predicts supersynchronous rotation due to the CTL model's equilibrium period eccentricity dependence (see Eq. (4.22)).

4.3.3 Coupled Stellar-Tidal Evolution For tidally locked Systems

Following Fleming et al. (2018), when one or both binary stars are tidally locked, tidal forces prevent magnetic braking from spinning down the tidally locked star(s), and any angular momentum lost comes at the expense of the binary orbit, decreasing a as a result (Verbunt and Zwaan, 1981). Below in Eq. (4.23) and Eq. (4.24), we modify the a decay equations due to stellar evolution and magnetic braking in tidally locked binaries from Fleming et al. (2018), their Eqs. (18) and (20), to additionally account for r_g evolution when one or both stars tidally lock, respectively, assuming conservation of angular momentum:

$$\dot{a}_{coupled}^{(1)} = \frac{-J_{mb} - 2\omega \left(m_1 r_{g,1}^2 R_1 \dot{R}_1 - m_1 r_{g,1} \dot{r}_{g,1} R_1^2 \right)}{\frac{\mu^2 GM(1-e^2)}{2J_{orb}} - \frac{3\omega}{2a} m_1 r_{g,1}^2 R_1^2} \quad (4.23)$$

and

$$\dot{a}_{coupled}^{(2)} = \frac{-J_{mb} - 2\omega \left(\sum_{i=1}^2 m_i r_{g,i}^2 R_i \dot{R}_i + m_i r_{g,i} \dot{r}_{g,i} R_i^2 \right)}{\frac{\mu^2 GM(1-e^2)}{2J_{orb}} - \frac{3\omega}{2a} \left(m_1 r_{g,1}^2 R_1^2 + m_2 r_{g,2}^2 R_2^2 \right)}, \quad (4.24)$$

where J_{orb} is the orbital angular momentum.

4.4 Simulations

We examine stellar angular momentum evolution in low-mass binaries by simulating two sets of 10,000 stellar binaries, one modeled using the CPL model and the other using the CTL formalism. We simulate both stars' spin evolution but mainly consider the P_{rot} evolution for the primary, i.e. more massive star in binaries, as it is observationally easier to measure a P_{rot} on the more massive, and hence brighter, star (e.g. Meibom et al., 2006; Lurie et al., 2017). For each simulation, we sample the primary's mass uniformly over $[0.1, 1] M_\odot$. Following Matt et al. (2015), we uniformly sample the \log_{10} of P_{rot} over $[0.8, 15]$ days, a distribution that approximates the P_{rot} distribution of young stars in the ~ 2 Myr old Orion Nebula Cluster (Stassun et al., 1999; Herbst et al., 2001, 2002; Rodríguez-Ledesma et al., 2009). We compute the secondary star's mass by uniformly sampling the mass ratio over $[0.1, 1]$ following observations of mass ratios in low-mass binaries (Raghavan

et al., 2010; Moe and Kratter, 2018). Given the inherent uncertainty in and complexity of the formation of short-period binaries (e.g. Bonnell and Bate, 1994; Bate, 2000; Bate et al., 2002; Moe and Kratter, 2018) and the potential for dynamical processing via tides or stellar close encounters (e.g. Mardling and Aarseth, 2001; Hurley et al., 2002; Ivanova et al., 2005; Meibom and Mathieu, 2005), we take an agnostic approach to the initial orbital configuration by uniformly randomly sampling the initial eccentricity (e) over $[0.0, 0.3]$, consistent with eccentricities of field binaries that likely have not been tidally-processed (Raghavan et al., 2010). Although the CTL model is applicable for $e \gtrsim 0.3$, the CPL model is not and can predict qualitatively incorrect evolution in that regime (see Section 4.1 in Leconte et al., 2010), so we restrict $e \leq 0.3$ to allow us to compare both models. We uniformly sample the initial P_{orb} over $[3, 100]$ d and do not consider $P_{orb} < 3$ d as these binaries are likely to have a tertiary companion (Tokovinin et al., 2006) which can significantly impact the inner binary’s dynamical evolution (e.g. Fabrycky and Tremaine, 2007; Muñoz and Lai, 2015; Martin et al., 2015; Hamers et al., 2016; Moe and Kratter, 2018).

Values for stellar tidal Q_s and τ_s for low-mass stars are highly uncertain due to complex viscous evolution within the stars (Ogilvie and Lin, 2007), and can differ for stars of the same spectral class (Barker and Ogilvie, 2009). These parameters can also vary as a function of stellar mass or age (Bolmont and Mathis, 2016; Van Eylen et al., 2016), likely due to low-mass stars’ evolving convective regions where the tidal dissipation predominantly occurs (Zahn, 2008). Typical values of Q and τ for Sun-like stars are estimated to be of order $Q \approx 10^6$ and $\tau \approx 0.1$ s, respectively (e.g. Meibom and Mathieu, 2005; Ogilvie and Lin, 2007; Jackson et al., 2008), however a range of values exist in the literature. Therefore, we consider a wide range of tidal parameters by sampling stellar tidal Q_s log-uniformly over $[10^4, 10^8]$ and τ log-uniformly over $[10^{-2}, 10]$ s. There is no general expression to compute Q as a function of τ , or vice versa, except in some special cases where approximations exist, e.g. Eqn. (2) from Heller et al. (2011). All stars have an initial age of 5 Myr unless stated otherwise as by this time, the gaseous protoplanetary circumbinary disk that can drive significant dynamical evolution in the binary (e.g. Fleming and Quinn, 2017) would likely have dissipated (Haisch et al., 2001). We also perform a smaller subset of simulations to illustrate the behaviour of our coupled model and describe their initial conditions as we introduce

them. All code used to run simulations and generate figures is available online.²

4.5 Results

4.5.1 Interaction Between Magnetic and Tidal Braking: Subsynchronous Rotation

Here we focus on binaries in the “weak tides” regime, i.e. long P_{orb} and large Q or small τ , to identify the boundary between evolution dominated by tides or magnetic braking via analytic calculations and simulations.

Analytic Torque Balance

In the weak tides regime, spin-down due to magnetic braking will drive the stellar P_{rot} past P_{eq} , resulting in subsynchronous rotation, $P_{rot} > P_{eq}$. For long P_{orb} , the stars will be slowly-rotating and in the unsaturated regime (Matt et al., 2015). Since magnetic braking scales as P_{rot}^{-3} for unsaturated rotators, e.g. Eqn. (4.2), magnetic braking torques weaken as the stellar rotation slows down, so at some P_{rot} , tidal torques will balance magnetic braking, producing a long-lasting state of subsynchronous rotation. We compute the P_{rot} at which this balance occurs as a function of P_{orb} , k_2 , and τ in § 4.8 by setting the sum of Eqn. (4.16) and Eqn. (4.2) equal to 0, considering tidal torques under the CTL formalism. For simplicity, we assume both stars are solar-mass with 0 obliquity, a circular binary orbit, and that the torque balance occurs while the stars are on the main sequence where stellar properties change slowly. Although solar mass stars are the most massive stars we consider in this work, and hence will have the strongest tidal torque for a given tidal dissipation parameter and P_{orb} , they can still exhibit subsynchronous rotation and serve as a useful end member case to examine here and in simulations below. We display the results of this calculation in Fig. 4.3, normalizing P_{rot} by P_{eq} , which for binary stars with 0 obliquity on circular orbits is simply P_{orb} .

Our calculations show that subsynchronous rotation occurs across a wide range of tidal parameters and P_{orb} . In general as tides weaken, i.e. increasing P_{orb} and/or decreasing $\log_{10}(k_2\tau)$,

²<https://github.com/dflemin3/sync>.

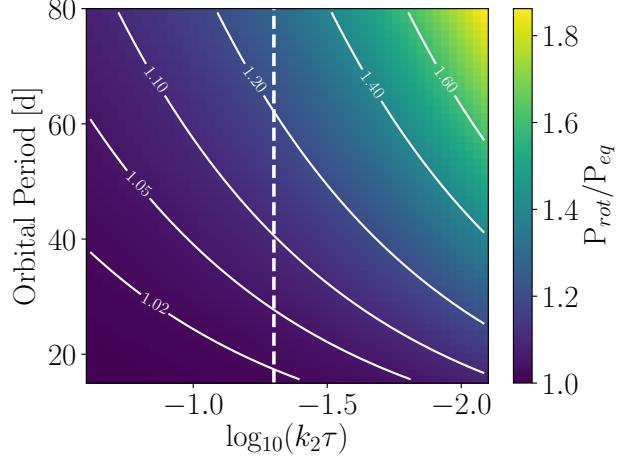
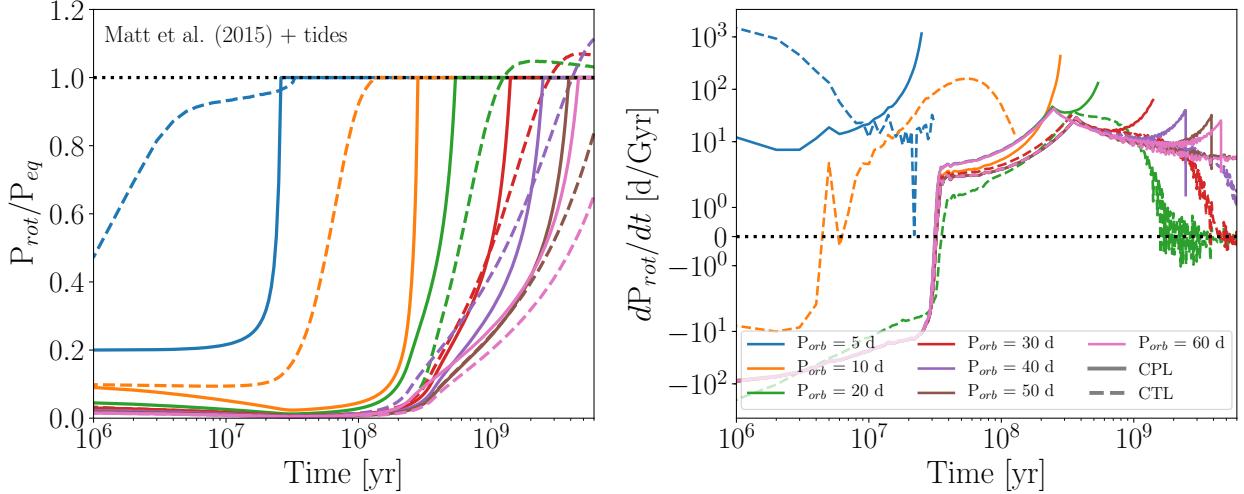


Figure 4.3: The stellar P_{rot} , normalized by P_{eq} , at which the torques due to magnetic braking and tides balance for a $1 M_\odot - 1 M_\odot$ binary on a circular orbit according to Eqn. 4.28. The white dashed line indicates our fiducial values for k_2 and τ , 0.5 and 0.1 s, respectively, that we adopt in the simulations in § 4.5.1.

tidal and magnetic braking torques balance at longer P_{rot} . For strong tides, $\log_{10}(k_2\tau) \gtrsim -1$, tidal torques overpower magnetic braking for $P_{orb} \lesssim 40$ d, tidally locking binaries into synchronous rotation. For our fiducial values of $k_2 = 0.5$ and $\tau = 0.1$ s (white dashed line in Fig. 4.3), solar-twin binaries will rotate subsynchronously for $P_{orb} \gtrsim 20$ d, with more severe subsynchronism at longer P_{orb} . This simple calculation, however, does not account for stellar evolution or secular tidal orbital evolution, e.g. tidal friction that will shrink the orbit, gradually strengthening tidal torques, so we turn to simulations to characterize this evolution.

Torque Balance

We simulate the full coupled stellar-tidal evolution of $1 M_\odot - 1 M_\odot$ binaries on initially circular orbits to examine how stellar binaries evolve towards subsynchronous rotation. In Fig. 4.4, we plot P_{rot} , normalized by P_{eq} , and its time derivative for $P_{orb} \in [5, 60]$ d modeled using both the CPL (solid line, $Q = 10^6$) and CTL (dashed line, $\tau = 0.1$ s) models. Both tidal models predict that binaries with $P_{orb} < 10$ d will tidally lock within 100 Myr, in agreement with observations (Meibom and Mathieu, 2005) and previous theoretical work (Zahn and Bouchet, 1989). The CPL



For $P_{orb} \leq 10$ d, both the CPL and CTL models predict that the binaries lock into synchronous rotation. For all P_{orb} , the CPL models tidally lock whereas the CTL model predicts subsynchronous rotation that persists for Gyrs.

Figure 4.4: Evolution of stellar P_{rot} , normalized by P_{eq} (see Eqn. (4.20) and Eqn. (4.22)), for initial circular binary orbits according to the CPL (solid) and CTL (dashed) models with $Q = 10^6$ and $\tau = 0.1$ s, respectively, using the Matt et al. (2015) magnetic braking model. Left: P_{rot}/P_{eq} for stars with P_{orb} ranging from 5 d to 60 d. The black dotted line indicates the tidally locked state. Right: Net P_{rot} derivative due to stellar evolution, tidal torques, and magnetic braking. We truncate each curve when the binary tidally locks. The legend denotes the initial binary orbital period and we note that the orbital periods do not vary by more than a few percent over the course of the simulations.

model predicts that all binaries tidally lock, even out to $P_{orb} = 60$ d, indicating that tidal locking is not necessarily restricted to short P_{orb} systems. As anticipated by our analytic calculations, the CTL model predicts subsynchronous rotation for $P_{orb} \geq 20$ d as magnetic braking overpowers tidal torques. For $P_{orb} = 20$ d, magnetic braking pushes $P_{rot}/P_{eq} \approx 1.05$, with the maximum value set by the torque balance. As shown in § 4.5.1, the peak P_{rot}/P_{eq} grows for longer P_{orb} since tides weaken with increasing binary separation, e.g. Eqn. (4.18), allowing magnetic braking to dominate the spin evolution.

Subsynchronous rotation does not persist indefinitely, however, as P_{rot} eventually decreases back towards the tidally locked state in the long-term due to a combination of three simultaneous physical effects. First, magnetic braking weakens at long P_{rot} as its torque scales as P_{rot}^{-3} for unsat-

urated rotators (Matt et al., 2015). Second, as P_{rot} increases further from the tidally locked state, tidal torques strengthen as they try to force P_{rot} back towards P_{eq} (see Eqn. (4.16)). Third, when $P_{rot} > P_{eq}$, tides transfer angular momentum from the orbit into stellar rotations, decreasing P_{orb} , gradually strengthening tidal torques that strongly depend on the binary separation as $a^{-6.5}$. These effects combine to shift the balance of power from magnetic braking-controlled stellar spin down to tidal torques spinning-up stars, shepherding them towards P_{eq} in the long-term.

We can see this process unfold in the right panel of Fig. 4.4 where we plot the total P_{rot} time derivative due to tidal torques, stellar evolution, and magnetic braking. Early on, $\dot{P}_{rot} < 0$ as stars contract along the pre-main sequence until about 60 Myr when the stars reach the zero age main sequence. Tides and magnetic braking then combine to spin down stars towards the tidally locked state. For the CTL models with $P_{orb} > 10$ d, $\dot{P}_{rot} > 0$ as magnetic braking dominates, driving the stars into subsynchronous rotation. In the long-term, $\ddot{P}_{rot} < 0$, however, as the three processes described above gradually strengthen tidal torques relative to magnetic braking. Tidal torques eventually overpower magnetic braking, seen as a slight negative P_{rot} derivative, slowly driving P_{rot} back towards P_{eq} , producing a population of subsynchronous rotators that can persist for Gyrs. We explore this point further in § 4.5.2.

4.5.2 Influence of P_{orb} , Q and τ

We next examine how P_{rot} evolution in stellar binaries depends on P_{orb} and the strength of tidal dissipation, parameterized by Q and τ for the CPL and CTL models, respectively. In Fig. 4.5, we bin our simulation results after the full 7 Gyr evolution by P_{orb} and Q or τ and compute the median P_{orb}/P_{rot} in each bin, marginalizing over all other parameters.

Spin-orbit synchronization is the typical outcome for binaries with $P_{orb} < 10$ d according to the CPL model for most values of Q . The strong tidal torques predicted by the CPL model can even tidally lock binaries out to $P_{orb} \gtrsim 80$ d for $Q < 10^5$, well beyond the expected limit of 20 d (Meibom et al., 2006). According to the CTL model, binaries with $P_{orb} < 10$ d typically tidally lock for $\tau \gtrsim 0.1$ s, and seldomly tidally lock for $P_{orb} > 20$ d, except for systems with strong tides, $\tau \gtrsim 3$ s. Both models predict a substantial population of subsynchronous rotators (red regions in Fig. 4.5,

$P_{rot} > P_{orb}$), consistent with magnetic braking dominating weak tidal torques. The population of supersynchronous rotators (blue regions in Fig. 4.5, $P_{rot} < P_{orb}$) with $P_{orb} > 60$ d does not in general correspond to binaries tidally locking into supersynchronous rotation, but rather, typically arises from the combination of weak tidal torques and magnetic braking not spinning down stars enough for P_{rot} to be close to the tidally locked state. At a given age, longer P_{orb} binaries will tend to rotate faster as they experience weaker tidal torques, and hence require longer to spin down towards the tidally locked state.

Both tidal models predict a population of nearly synchronous rotators near $P_{orb} \approx 60$ d. This population corresponds to the evolution described in § 4.5.1 in which magnetic braking initially spins down stars past the tidally locked state, but in the long-term, tidal torques spin up the stars, shepherding them towards the tidally locked state. This process can keep stellar $P_{rot} \gtrsim P_{eq}$ for several Gyrs or longer, depending on the P_{orb} and Q or τ (see Fig. 4.4, § 4.5.2).

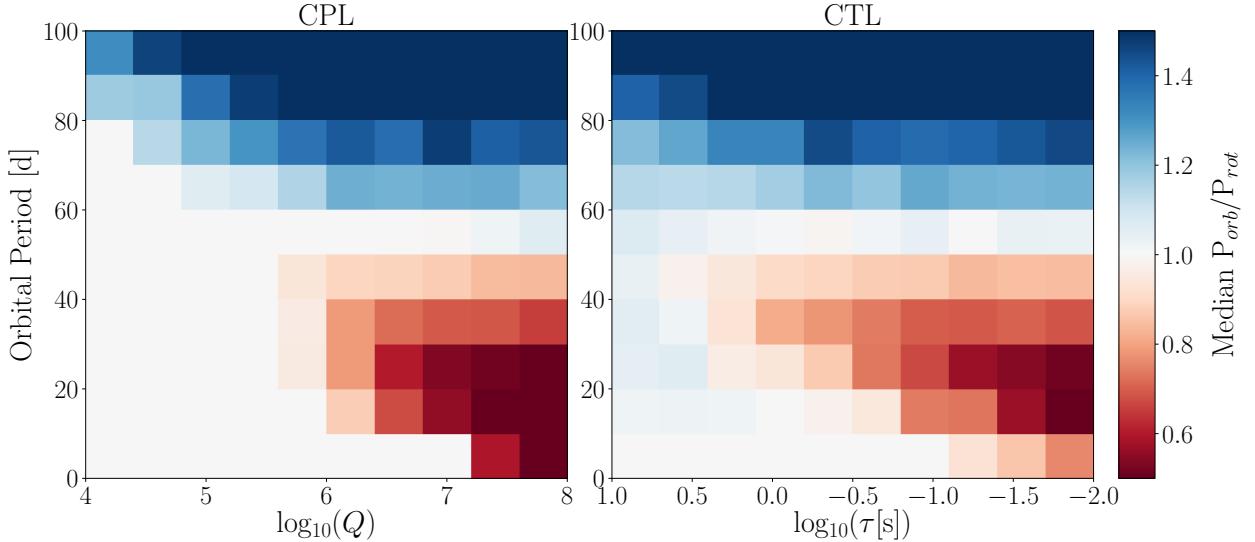


Figure 4.5: Median P_{orb}/P_{rot} at the end of the simulation according to the CPL (left) and CTL (right) models binned by $\log_{10}(Q)$ and $\log_{10}(\tau)$, respectively, and P_{orb} . For $P_{orb} < 10$ d, the CPL model predicts that most systems will tidally lock into synchronous rotation, whereas the CTL model requires $\tau \geq 0.1$ s to tidally lock. For large $Q (> 10^7)$ and small $\tau (\tau < 0.1$ s), weak tidal torques cannot prevent magnetic braking from spinning down stars past the tidally locked state, producing a population of subsynchronous rotators (red regions, $P_{rot} > P_{orb}$).

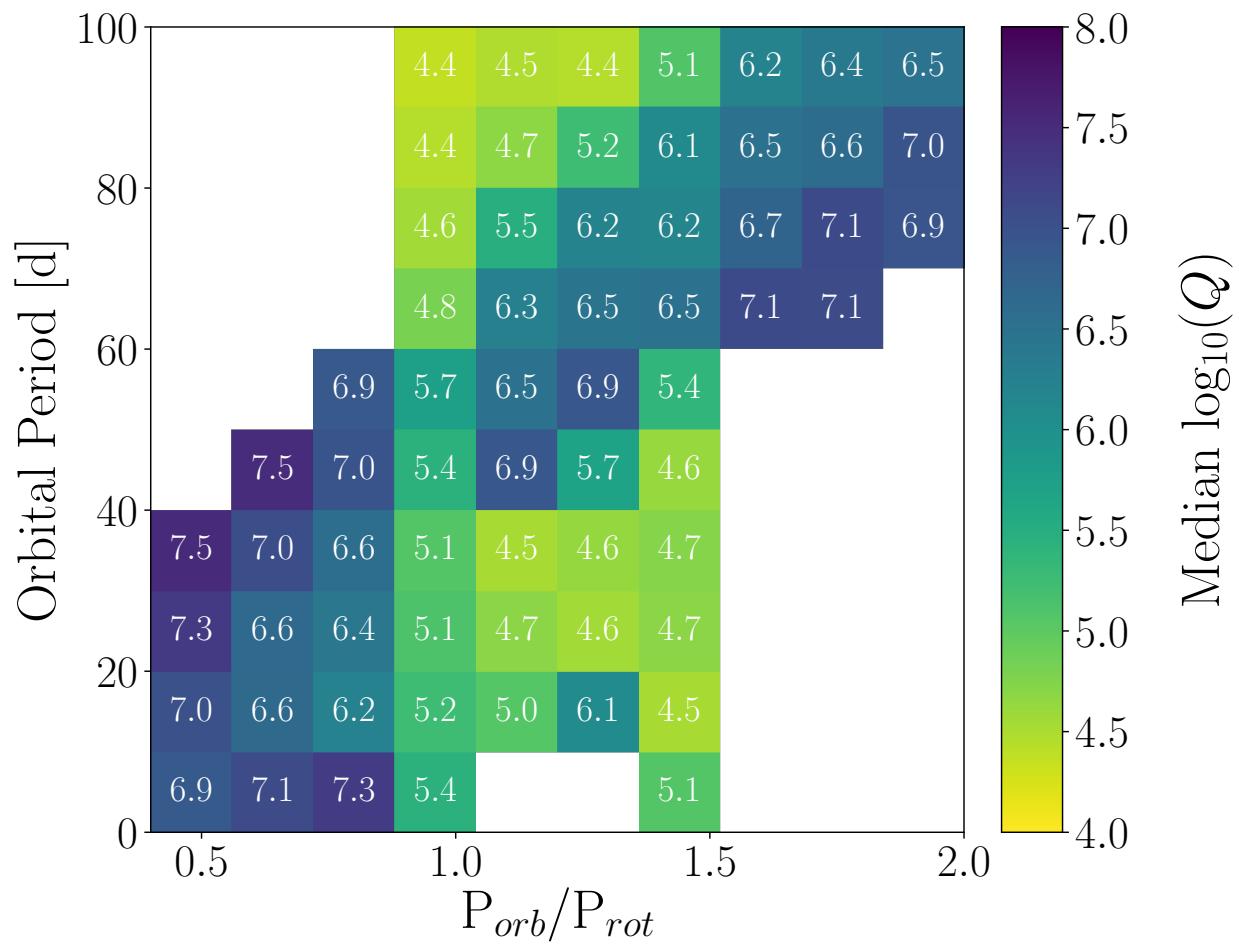


Figure 4.6: Median $\log_{10}(Q)$ of primary stars binned by P_{orb} and P_{orb}/P_{rot} evolved using the CPL model.

We isolate the impact of Q and τ on the spin-orbital state of tidally interacting stellar binaries by binning our CPL and CTL simulation results after 7 Gyr of evolution by P_{orb} and P_{orb}/P_{rot} in Figures 4.6 and 4.7, respectively. In these figures, we estimate the typical strength of tidal torques, using Q and τ as a proxy, that can produce various spin-orbital states. For the CPL simulations depicted in Fig. 4.6, synchronous and supersynchronous rotators have systematically low Q s, typically $Q < 10^6$, as strong tidal torques are required to tidally lock these binaries. For $P_{orb} < 10$ d, there are no rotators with $1.0 < P_{orb}/P_{rot} < 1.5$, nor do any stars have $P_{orb}/P_{rot} > 1.5$ for $P_{orb} < 60$ d, as in the CPL model, binaries with eccentric orbits can only tidally lock into a 1:1 or 3:2 spin-orbit commensurability, see Eqn. (4.20).

Subsynchronous rotators have systematically larger Q s, typically $Q > 10^6$, and hence experience weak tidal torques that are dominated by magnetic braking. Subsynchronous rotation can occur under the CPL model for binaries with $P_{orb} < 50$ d. In this regime, the median Q tends to increase with decreasing P_{orb}/P_{rot} , except near the tidally locked state, as magnetic braking dominates weaker tidal torques, yielding longer P_{rot} . This trend reverses at longer $P_{orb} > 60$ d where supersynchronous rotation arises from the inability of tidal torques and magnetic braking to spin-down stars enough to approach the tidally locked state by the end of the simulation. In this case, the more supersynchronous the rotation, the weaker the tidal torques must be, and hence the larger the Q must be.

According to the CTL model simulations, depicted in Fig. 4.7, many binaries tidally lock for $P_{orb} \lesssim 20$ d when $\tau \gtrsim 0.1$ s, with some tidally locking up to $P_{orb} \approx 50$ d when $\tau \gtrsim 1$ s. Subsynchronous rotation typically occurs for stars with $\tau < 0.1$ s. Similar to the behavior depicted in Fig. 4.6, longer P_{rot} are produced by binaries with weaker tidal interactions since P_{orb}/P_{rot} decreases monotonically with τ for $P_{orb} < 40$ d. For $P_{orb} > 50$ d, magnetic braking dominates the evolution seen in the diagonal sequence with a median $\tau \approx 0.1$ s, a value that is typically insufficient for tides to strongly influence the evolution given the wide orbital separations. The shape of this diagonal region arises from the combination of magnetic braking and our flat initial P_{orb} distribution. In this P_{orb} regime, most binaries rotate supersynchronously as tides and magnetic braking fail to sufficiently spin down the stars by the age of the system. At longer P_{orb} , some

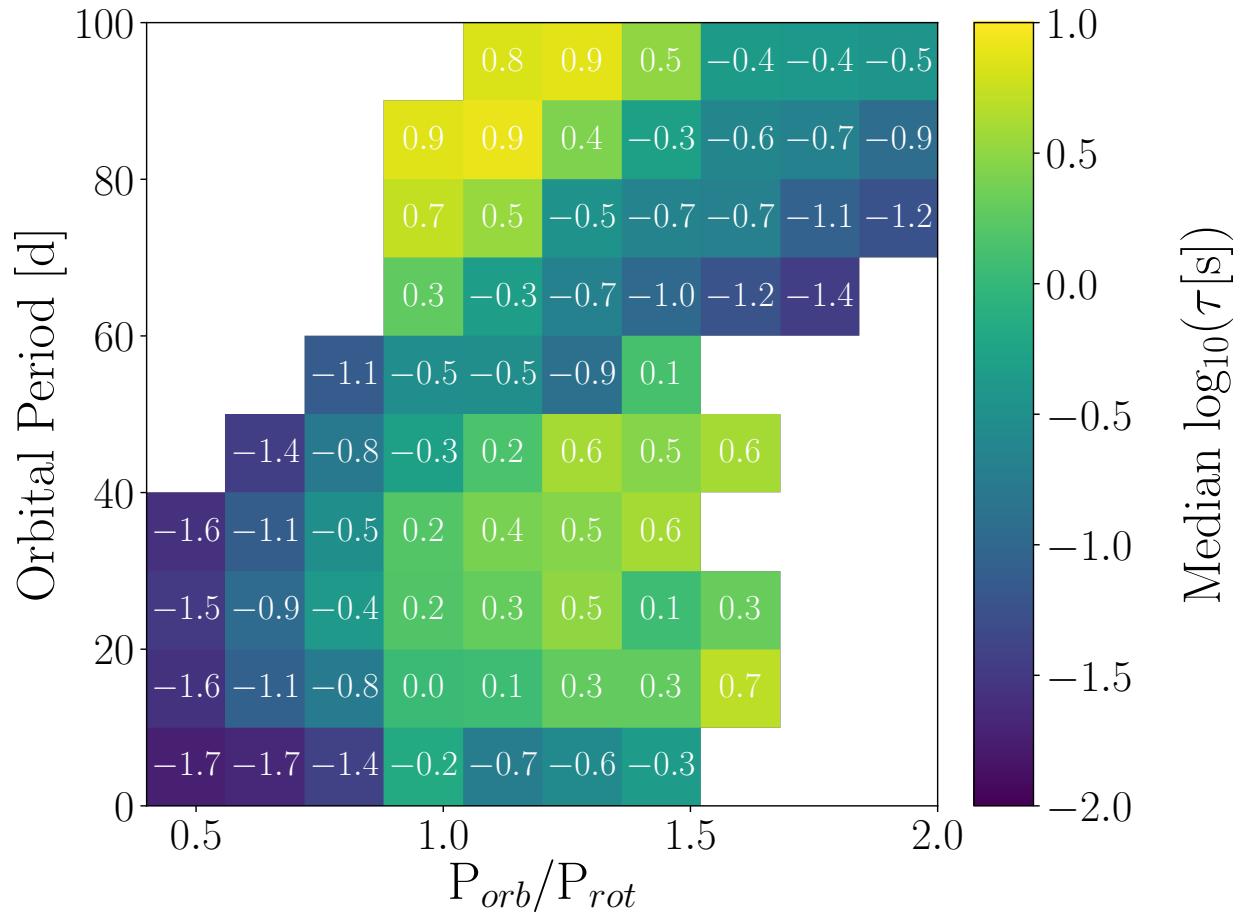


Figure 4.7: Same format as Fig. 4.6, but for $\log_{10}(\tau[\text{s}])$ under the CTL model.

binaries can strongly tidally-interact, but these systems require extreme tidal $\tau \gtrsim 10$ s. Unlike the CPL simulations depicted in Fig. 4.6, these binaries with $P_{orb} \approx 90$ d are not tidally locked as the tides are still not strong enough to lock the system. We explore this point further in § 4.5.3. We do not often observe $P_{orb}/P_{rot} \gtrsim 1.5$ as we only consider eccentricities up to $e = 0.3$, limiting how rapid supersynchronous systems can rotate according to Eqn. (4.22).

Subsynchronous Rotation at Short P_{orb}

As seen in Fig. 4.5, subsynchronous rotation can even occur for short P_{orb} binaries, where tidal-locking is the expectation, if the tidal torques are sufficiently weak. In Fig. 4.8, we examine subsynchronous rotation in short P_{orb} binaries by displaying the P_{rot} evolution for a $P_{orb} = 7.5$ d binary for various tidal dissipation parameters. Subsynchronous rotation occurs in general for weak tidal torques, $Q > 10^7$ or $\tau < 0.1$ s in these cases, and is not restricted to long P_{orb} binaries. Previous theoretical studies have also predicted subsynchronous rotation in short P_{orb} binaries arising from the balance between tidal torques and magnetic braking (e.g. Habets and Zwaan, 1989; Zahn, 1994; Keppens, 1997) suggesting that this behavior is not an artifact of our choice of tidal or magnetic braking models, but rather a general outcome of the competition between magnetic braking and tidal evolution in low-mass binaries. Short P_{orb} subsynchronous rotators can eventually tidally lock after several Gyrs, e.g. the $Q = 10^8$ case in Fig. 4.8, via the mechanism described above where tidal torques gradually strengthen relative to magnetic braking.

Short P_{orb} subsynchronous binaries exist in nature, such as many *Kepler* EBs (Lurie et al. (2017), see § 4.5.5 for further discussion), Kepler-47 (Orosz et al., 2012), EPIC 219394517 (Torres et al., 2018), and in “Binary 6211” observed by Meibom et al. (2006), suggesting that this theoretical observation is real and borne out in nature. Spin-orbit synchronization should therefore not be assumed for short P_{orb} binaries and sunsynchronous rotation should be expected in many tidally interacting binaries. We explore these effect further and compare our theory to observations of *Kepler* EBs in § 4.5.5.

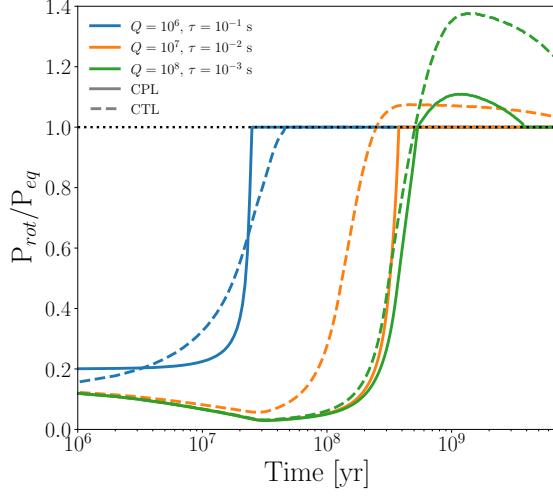


Figure 4.8: Evolution of stellar P_{rot} , normalized by P_{eq} (see Eqn. (4.20) and Eqn. (4.22)), for initial circular binary orbits with initial $P_{orb} = 7.5$ d according to the CPL (solid) and CTL (dashed) models for several values of Q and τ , respectively, using the Matt et al. (2015) magnetic braking model. Systems with strong tidal torques tidally lock, whereas in systems with weaker tidal torques (larger Q and smaller τ , respectively), magnetic braking initially overpowers tidal torques, spinning down the stars past the tidally locked state, resulting in subsynchronous rotation.

4.5.3 P_{rot} Distribution of a Synthetic Population of Stellar Binaries

Here we examine how the competition between tidal torques and magnetic braking shape the P_{rot} distribution of low-mass stellar binaries. We consider two cases where tidal torques dominate: ‘‘Locked’’, where $P_{rot} = P_{eq}$, and ‘‘Interacting’’, where P_{rot} is within 10% of P_{eq} as in this regime, tides are likely shepherding P_{rot} towards the tidally locked state as we demonstrated in § 4.5.1. We refer to the remaining binaries as ‘‘not locked’’ as magnetic braking and stellar evolution likely dominate their angular momentum evolution. In Fig. 4.9 and Fig. 4.10, we plot P_{rot} as a function of mass for the primary stars in stellar binaries for both the CPL and CTL model, respectively, integrated to system ages uniformly sampled over 1 – 7 Gyr, consistent with ages of stars in the *Kepler* field (Chaplin et al., 2014).

Both models predict a substantial population of tidally locked fast rotators with $P_{rot} \lesssim 20$ d, with tidally locked stars systematically rotating faster (median CPL, CTL $P_{rot} = 22.6$ d and 8.8 d) than not locked (median CPL, CTL both $P_{rot} = 32.4$ d) binaries. The CTL model predicts that the

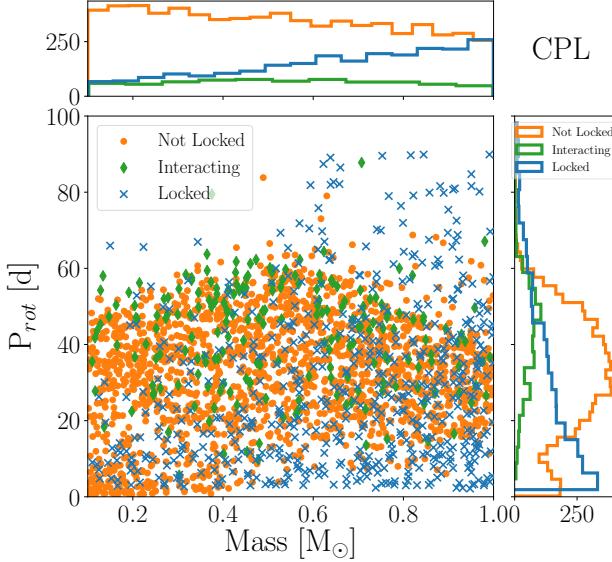


Figure 4.9: Rotation state for tidally locked (blue, $P_{rot} = P_{eq}$), interacting (green, P_{rot} within 10% of P_{eq} and not locked), and not locked (orange, remainder of binaries) stellar binaries. Left: P_{rot} as a function of stellar mass and age according to our CPL simulations integrated to system ages uniformly sampled over 1 – 7 Gyr. Right: Marginalized P_{rot} distribution for each case. Top: Marginalized mass distributions.

majority of tidally locked binaries, 83%, lock into rapid rotation with $P_{rot} \lesssim 20$ d, typically in short P_{orb} binaries where tidal torques are strongest. The CPL model, however, predicts that binaries can tidally lock into a wide range of rotation states as only 46% of locked binaries have $P_{rot} < 20$ d, while the rest can lock out to $P_{rot} \approx 100$ d in long P_{orb} binaries. More massive stars are more likely to tidally lock compared to less massive stars as tidal torques scale with the stellar masses and as R^5 , with R increasing with stellar mass. This feature is seen in the enhanced density of locked systems at larger masses for both tidal models, but in particular for the CPL model. We highlight this enhanced density of locked binaries in systems with more massive primaries in the marginalized mass distributions in the top panels of Fig. 4.9 and Fig. 4.10.

The interacting population tends to rotate more slowly than the not locked population as at short P_{orb} , and hence P_{rot} , binaries preferentially tidally lock due to stronger tidal torques. At longer P_{orb} , weaker tidal torques allow magnetic braking to spin down the stars past P_{eq} , with tidal

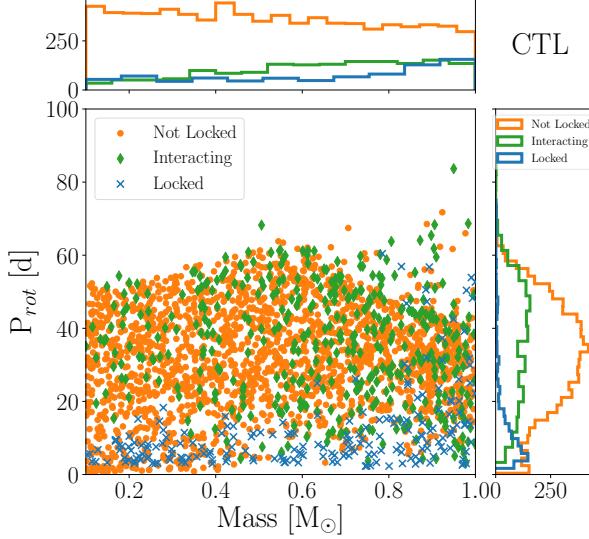


Figure 4.10: Same format as Fig. 4.9, but for the CTL simulations.

torques eventually strengthening enough to shepherd P_{rot} towards P_{eq} via the mechanism discussed in § 4.5.1. The CPL and CTL models predict that 31% and 24% of stars, respectively, are either tidally locked or interacting, demonstrating that tidal torques play a pivotal role in shaping the angular momentum evolution in stellar binaries across a wide range of parameters. The P_{rot} - mass distribution for not locked binaries resembles the single star sequence as magnetic braking and stellar evolution dictate their angular momentum evolution.

Tidal locking is not limited to $P_{orb} \lesssim 20$ d, however, as we find stellar binaries can tidally lock over a wide range of P_{rot} up to $P_{rot} = P_{orb} \approx 100$ d according to the CPL model, producing a slow-rotating population above the P_{rot} distribution envelop of solar-mass single stars. This behavior is consistent with observations of P_{rot} in *Kepler* eclipsing binaries by Lurie et al. (2017) who find tentative evidence that binaries can tidally lock up to their detection limit of $P_{orb} = P_{rot} = 45$ d. Under the CTL model, however, binaries predominantly tidally lock out to only $P_{orb} \approx 20$ d, although binaries with more massive primaries can occasionally lock, or at least tidally-interact, out to $P_{orb} \approx 80$ d. We highlight this behavior with a histogram of locked and interacting binaries over P_{rot} for both tidal models in Fig. 4.11. The CTL model predicts fewer tidally locked binaries at longer P_{rot} , concentrating most of the locked CTL distribution's density

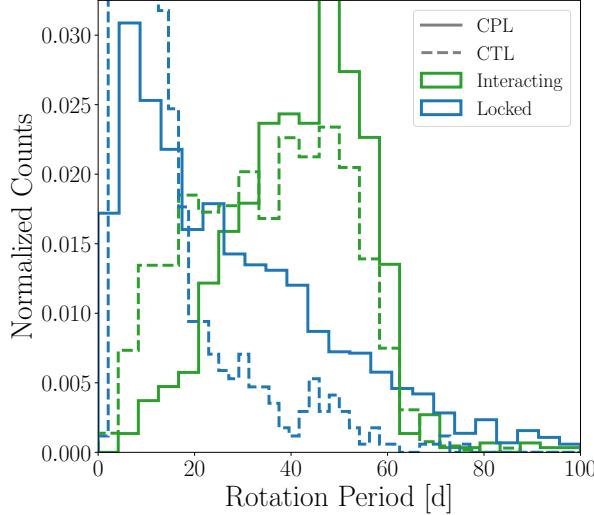


Figure 4.11: P_{rot} distribution for tidally locked (blue) and interacting (green, P_{rot} within 10% of P_{eq} and not locked) binaries according to the CPL (solid line) and CTL (dashed line) models.

at short P_{rot} , whereas the CPL distribution has a heavy tail extending towards longer P_{rot} . The CPL model, however, predicts larger tidal-locking rates than the CTL model as seen in the enhanced numbers of tidally locked binaries at low P_{rot} Fig. 4.9 compared with Fig. 4.10. The presence of $P_{orb} > 20$ d locked population, or lack there of, could be a powerful observational discriminant between which equilibrium tidal model acts in low-mass stellar binaries. We discuss this point further in § 4.5.6.

4.5.4 Deviations From Single Star P_{rot} Evolution: Implications for Gyrochronology

We compare the P_{rot} and age distributions of tidally interacting stellar binaries from our CPL and CTL simulations with that of single stars to gauge the impact of tidal torques on driving P_{rot} distributions away from that of single stars and what implications that may have for estimating stellar ages using gyrochronology. We simulate 10,000 single star systems according to the evolution described in § 4.3.1 with initial conditions sampled from the same mass and P_{rot} distributions used for the binary simulations described § 4.3. In Fig. 4.12, we display P_{rot} as a function of mass and age for binaries simulated using both the CPL and CTL model and for single stars.

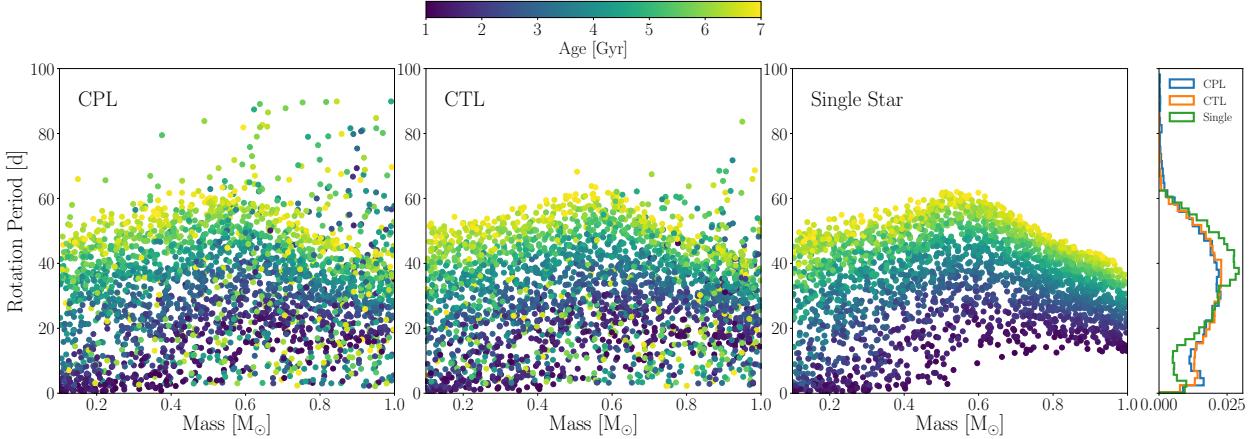


Figure 4.12: P_{rot} as a function of stellar mass and age according to our CPL (left), CTL (left center), and single star (right center) simulations integrated to system ages uniformly sampled over 1 – 7 Gyr using the Matt et al. (2015) magnetic braking model. For each case, we only plot 2,500 systems for clarity but account for all systems when computing the marginalized distributions. Right: The P_{rot} distribution for each case, marginalized over stellar mass.

In binaries, tidal torques tend to drive the P_{rot} evolution away from that of single stars and towards P_{eq} , either maintaining rapid rotation in tidally locked short P_{orb} systems, or working with magnetic braking to slow P_{rot} beyond that of single stars of the same age. The impact of tidal torques on the binary P_{rot} distribution is clear: strong tidal torques in short P_{orb} binaries produce a substantial population of rapid rotators with $P_{rot} \lesssim 20$ d. Except for stars with ages $\lesssim 1$ Gyr, or young late M-dwarfs who are either contracting along the pre-main sequence or have just reached the main sequence, our single star simulations fail to produce a population of rapid rotators. This theoretical result is consistent with Simonian et al. (2018) who find that the population of *Kepler* stars with $P_{rot} < 7.5$ d is likely dominated by tidally interacting binaries. The influence of tides extends to longer P_{orb} systems producing a slowly-rotating population above the upper envelope of the single star sequence for $M \gtrsim 0.6 M_{\odot}$, e.g. Fig. 4.9 and Fig. 4.10, and is a population that single-star models fail to produce.

In the single star population, there is a clear monotonic relation between P_{rot} and age, with older stars rotating more slowly, a trend that is borne out in nature and is the critical assumption of gyrochronology methods that link P_{rot} to stellar ages via the magnetic braking-driven long-term

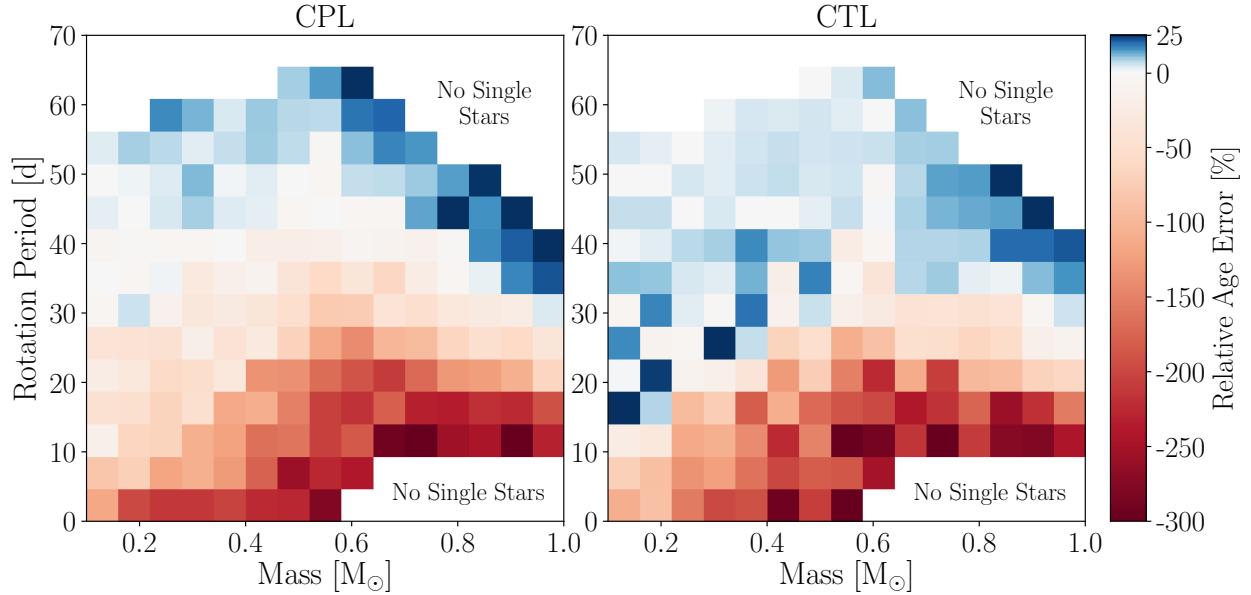


Figure 4.13: Relative age error between single and binary stars for both the CPL (left) and CTL (right) tidal models. The error is computed as the percent difference between the mean ages of single and tidally interacting binary stars in mass and P_{rot} bins.

spin down of low-mass stars (e.g. Skumanich, 1972; Barnes, 2003, 2007; Mamajek and Hillenbrand, 2008; Barnes, 2010; Meibom et al., 2015). This trend is a generic outcome of magnetic braking and is not specific to our choice of magnetic braking model. In stark contrast, both tidal models predict that age does not always strongly correlate with P_{rot} as tidally interacting binaries, at a given primary star mass and P_{rot} , can assume a wide range of ages, especially for $P_{rot} \lesssim 20$ d.

We quantify the impact of binarity on gyrochronology age estimates in Fig. 4.13 by computing the percent difference between the mean ages of single and binary stars, for both tidal models, in mass and P_{rot} bins for the populations depicted in Fig. 4.12. This quantity, referred to here as the “Relative Age Error”, represents the systematic error incurred by assigning a tidally interacting binary star the age expected for single stars at a given mass and P_{rot} . For this comparison, we select the subset of tidally locked and tidally interacting binaries as classified in Fig. 4.9 and Fig. 4.10.

For most values of P_{rot} , gyrochronology methods systematically underestimate the ages of tidally interacting binaries, with the relative age error increasing with decreasing P_{rot} and increas-

ing with primary star mass. For binaries with $P_{rot} \lesssim 30$ d, gyrochronology ages are underestimated by 50%, with this error growing to 300% for $P_{rot} \approx 10$ d. For slow rotators with $P_{rot} \gtrsim 40$ d, gyrochronology ages are slightly overestimated by up to 25%, with the largest errors occurring for near solar-mass primary stars where tides and magnetic braking combine to spin down binary stars. The relative age errors would be more pronounced for the most rapidly-rotating stars, e.g. tidally interacting near-solar mass binaries with $P_{rot} \lesssim 10$ d, however, our single star-only models fail to produce such rotation states.

The age distribution of binaries with $P_{rot} < 20$ d is inconsistent with that of single stars. In this range, the median ages and 68% interval are $2.3^{+2.9}_{-0.9}$ Gyr and $2.4^{+3.0}_{-1.1}$ Gyr according to the CPL and CTL models, respectively, compared to the much younger single stars with ages of $1.6^{+0.8}_{-0.4}$ Gyr. We highlight this dichotomy in Fig. 4.14 by plotting a histogram of system ages from Fig. 4.12 for single or primary stars in binaries with $P_{rot} < 20$ d.

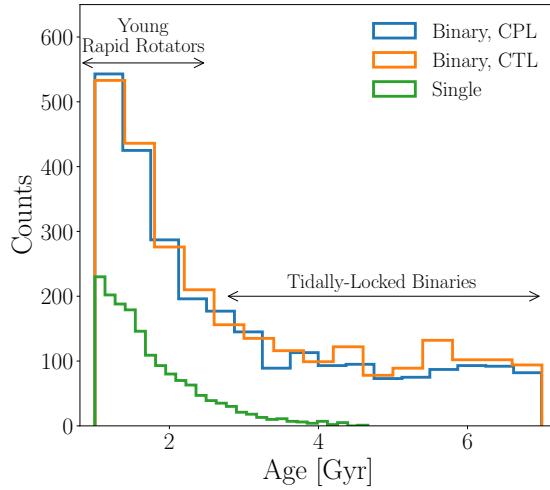


Figure 4.14: Histogram of rapidly-rotating ($P_{rot} < 20$ d) star ages for single and primary stars in binaries from Fig. 4.12. Rapidly-rotating single stars must be young (ages $\lesssim 2$ Gyr), while tidally locked rapidly-rotating binaries exhibit a wide range of ages.

Tidal torques pose a fundamental problem for inferring ages of stars via gyrochronology. Regardless of the choice of equilibrium tidal model or magnetic braking model, stellar binaries readily tidally lock, or at least strongly tidally-interact, across a wide range of P_{orb} and primary star masses,

decoupling P_{rot} from age. For example, if one observed a rapidly rotating star with $P_{rot} \lesssim 20$ d, gyrochronology models would predict ages $\lesssim 1.6$ Gyr. If the star is actually an unresolved binary, as could be the case for many *Kepler* rapid rotators (Simonian et al., 2018), it would likely be tidally locked, decoupling P_{rot} from age, causing the predictions of gyrochronology models to fail. This effect is most likely to manifest in rapid rotators ($P_{rot} < 20$ d), but persists across all P_{rot} up to 100 d, producing a contaminating signal, e.g. Fig. 4.12 and Fig. 4.13.

In general, it is difficult to accurately determine if a source is single star or a stellar binary via longterm photometric monitoring, e.g. via *Kepler* or *TESS*, as only a small fraction of stars in binaries will occult one another. Observations of the binarity of field stars by Raghavan et al. (2010) and Duchêne and Kraus (2013) indicate that roughly half of stars are in stellar binaries, with 10% of these binaries having $P_{orb} \lesssim 100$ d, suggesting that unless one accounts for binarity, stellar binaries will produce a contaminating signal in any study of stellar rotation periods and any ages inferred via gyrochronology are potentially subject to systematic errors. Moreover, this problem could be more significant as Simonian et al. (2018) found that most rapid rotators with $P_{rot} \leq 7.5$ d in the *Kepler* field are consistent with tidally-synchronized photometric binaries, suggesting that binary contamination in P_{rot} studies could be widespread. We caution that any application of gyrochronology methods to predict ages for stars, especially those with $P_{rot} \lesssim 20$ d, should rule out or account for stellar binarity, or otherwise risk deriving systematically incorrect ages. Tidal torques do not just produce spin-orbit synchronization at short P_{orb} , but can produce a rich variety of rotation states that deviate from the expected long-term spin-down experienced by single stars, e.g. Fig. 4.6 and Fig. 4.7. We recommend that the application, or calibration, magnetic braking models to a sample of stellar rotation periods control for binarity.

4.5.5 Comparison to *Kepler*

We compare our simulation results to P_{rot} measurements of primary stars in *Kepler* low-mass eclipsing binaries by Lurie et al. (2017) to gauge if our model predictions, which by design populate a wide, but physically-plausible, region of parameter space, can reproduce features observed in the data. Lurie et al. (2017) measured 816 rotation periods for primary stars in *Kepler* EBs with star

spot modulations and visually inspected each light curve to ensure their accuracy. The Lurie et al. (2017) dataset is the largest homogenous set of P_{rot} measurements available for low-mass stellar binaries and represents the state of the art benchmark for studies of the influence of tides on P_{rot} in stellar binaries. We compare our results to the $P_{1,min} P_{rot}$ values reported by Lurie et al. (2017) as the authors demonstrated that these values are likely to be close to the equatorial P_{rot} that we track in our simulations. In Fig. 4.15, we display P_{orb}/P_{rot} as a function of P_{orb} for both the CPL and CTL models where each simulation was integrated to an age uniformly sampled over 1 – 7 Gyr, consistent with ages of *Kepler* field stars (Chaplin et al., 2014).

Qualitatively, the CTL model appears to do a better job of reproducing features seen in the Lurie et al. (2017) data than the CPL model. The CPL model, for example, cannot produce the observed cluster of supersynchronous rotators with $P_{orb}/P_{rot} \lesssim 1.2$ for $P_{orb} < 10$ d whereas the CTL model can. Instead, owing to the its discrete P_{eq} , the CPL model predicts that all tidally locked supersynchronous rotators lie on the line $P_{orb}/P_{rot} = 1.5$. This prediction is inconsistent with the data as no obvious spin-orbit commensurability, aside from 1:1 synchronization, is present in the Lurie et al. (2017) data, likely because stellar convective envelopes lack a fixed shape, making resonant coupling difficult unless it occurs with internal gravity or pressure modes (Burkart et al., 2014; Lurie et al., 2017). Neither model reproduces the very supersynchronous, $P_{orb}/P_{rot} > 1.6$, binaries in the Lurie et al. (2017) data as they typically have orbital eccentricities in excess of 0.3 and are outside of the region of parameter space we consider. The CTL model, however, could in principle reproduce these points if they are tidally locked binaries as its P_{eq} is a continuous function of e and is applicable for large e , in contrast to the discrete P_{eq} predicted by the CPL model that is valid for smaller e . Both tidal models predict that nearly all binaries with $P_{orb} < 4$ d have circularized orbits and synchronized spins due to strong tidal torques at short stellar separations, in agreement with the Lurie et al. (2017) observations. At very short P_{orb} , in the absence of a perturbing tertiary companion, circularization and synchronization is the inevitable end state for low-mass binaries (Counselman, 1973).

For $P_{orb} \gtrsim 4$ d, our models produce a substantial number of subsynchronous rotators. Although Lurie et al. (2017) argues that differential rotation creates the subsynchronous population, we find

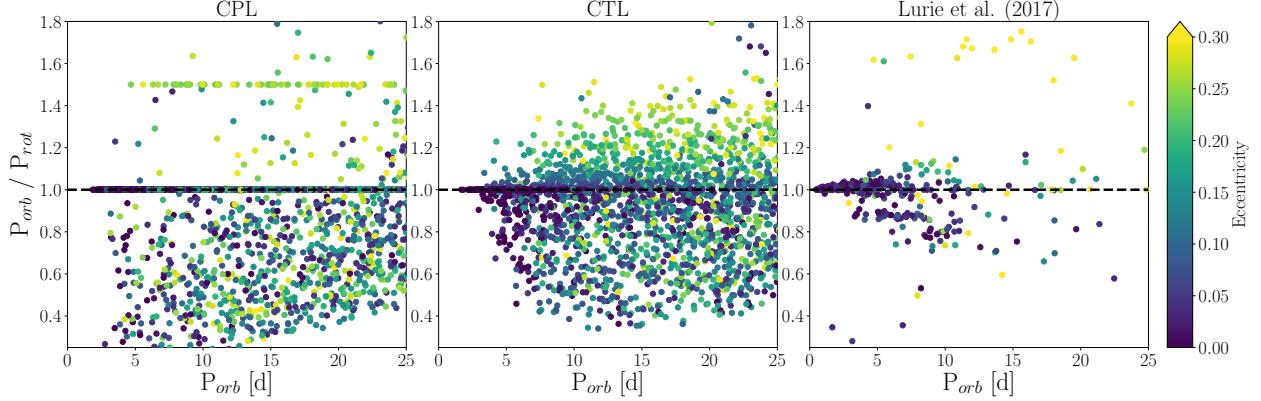


Figure 4.15: P_{orb}/P_{rot} as a function of P_{orb} according to the CPL model (left) and the CTL model (middle), and Lurie et al. (2017) *Kepler* EB observations (right). All points are colored by e . In the right panel, the *Kepler* EBs at low P_{orb} and low P_{orb}/P_{rot} are likely either brown dwarfs or exoplanets (Lurie et al., 2017), and hence are not modeled by our simulations, so we do not consider them, but we display them for completeness.

that the competition between weak tidal torques and magnetic braking described in § 4.5.1 naturally produces this population. The CPL model, however, struggles to populate the prominent cluster of subsynchronous rotators at $P_{orb}/P_{rot} \approx 0.9$ for $P_{orb} < 10$ d observed by Lurie et al. (2017). Lurie et al. (2017) find that 15% of their sample with $2 < P_{orb} < 10$ days has $P_{orb}/P_{rot} \in [0.84, 0.92]$, compared with 8% of our CTL population and only 2% of the CPL population.

Both models predict a large number of extremely subsynchronous rotators with $P_{orb}/P_{rot} < 0.7$ across all P_{orb} that is not present in the Lurie et al. (2017) data. Magnetic braking creates the lower limit of this subsynchronous population, a line of nearly constant $P_{rot} \approx 60$ d set by how much a star can spin down over 7 Gyr, the longest age considered in our simulations. Our choice of prior distributions for both Q and τ permit very weak tidal interactions that likely gives rise to this population and suggests that our prior does not reflect the underlying distribution of stellar tidal parameters in nature. Alternatively, the data could be incomplete where our models predict slowly-rotating subsynchronous rotators as the photometric amplitude of star spot modulations tends to decrease with increasing P_{rot} , making reliable rotation periods difficult to detect (McQuillan et al., 2014; Lurie et al., 2017; Reinhold et al., 2018).

Although the CTL model seems to better reproduce the Lurie et al. (2017) data, both tidal models can reproduce features observed in the *Kepler* EB distribution, e.g. the synchronized population and subsynchronous rotators, suggesting that our models reasonably approximate the dynamical interactions of tidally-evolving, low-mass stellar binaries. Our comparison between theory and observations is limited, however, because the Lurie et al. (2017) P_{rot} data lack uncertainties and Lurie et al. (2017) approximated the EB orbital e via transit durations and ingress/egress times, potentially leading to inaccurate e determinations. Unconstrained biases in the data, e.g. the lack of long P_{orb} binaries, further inhibit our ability to compare our predictions with the data. Moreover, our prior distributions were chosen to be plausible, but wide, in order to examine our model predictions over parameter space and are not suited for a robust statistical inference to select between which equilibrium tidal model best describes tidal interactions in low-mass binaries stars. Below, we offer observational tests that could discriminate between models.

4.5.6 CPL or CTL?

Accurate measurements of P_{rot} and e , especially out to long P_{orb} , can potentially discriminate between which equilibrium tidal model best describes tidal interactions in low-mass stellar binaries. Here, we outline three observational tests that can discriminate between the two models. The first test considers binaries with $P_{orb} < 10$ d that are likely tidally locked on eccentric orbits, but with $e < 0.23$. In this e regime, the CPL model predicts that the majority of systems are tidally locked into synchronous rotation and does not permit a supersynchronous rotation state, e.g. Eqn. (4.20). The CTL model, however, predicts a continuum of supersynchronous rotators on eccentric orbits, e.g. Eqn. (4.22). Supersynchronous rotation that is not due to tidal interactions can occur in extremely young, rapidly rotating systems that are still contracting along the pre-main sequence, or that have recently reached the main sequence. These young, supersynchronous rotators are unlikely to be tidally locked, usually have $P_{orb}/P_{rot} > 1.5$, and do not stay supersynchronous for long given that solar mass pre-main sequence lifetimes are $\lesssim 100$ Myr, distinguishing them from tidally locked binaries (see Fig. 4.15). If supersynchronous rotation is observed in binaries with $P_{orb} < 10$ d, $P_{orb}/P_{rot} < 1.5$, and $0 < e \lesssim 0.23$, it is evidence in favor of the CTL model over the CPL model.

Second, for tidally locked binaries with $e > 0.23$, the CPL model predicts supersynchronous rotation in the form of a 3:2 spin-orbit comensurability, e.g. the line at $P_{orb}/P_{rot} = 1.5$ seen in the left panel of Fig. 4.15, and no other spin state is permitted, compared to the continuum of supersynchronous rotation states in eccentric tidally locked rotators predicted by the CTL model. If a substantial clustering of stellar binaries with $P_{orb}/P_{rot} = 1.5$ is observed, it would be strong evidence in favor of the CPL model, but there is no obvious clustering of *Kepler* EBs near any spin-orbit resonance. These two tests can fail to discriminate between the CPL and CTL model, however, if the CPL model P_{eq} is a continuous function of e , e.g. Eqn. (4.21), as was argued by Goldreich (1966) and derived by Murray and Dermott (1999). In such a case, one would need a large number of accurate and precise measurements P_{orb} and e , with robust uncertainties, for tidally interacting binaries to discriminate between the CPL and CTL continuous P_{eq} , e.g. Eqn. (4.21) versus Eqn. (4.22). In practice, this is extremely observationally expensive as it requires extensive photometric and spectroscopic observations of many binaries.

A third test, the detection of tidally locked binaries with solar-mass primaries and $P_{rot} \gtrsim 60$ d, would provide strong evidence in favor of the CPL model as the CTL model cannot tidally lock stars beyond $P_{orb} \approx 60$ d, regardless of τ , e.g. Fig. 4.10. The CPL model, however, can tidally lock binaries out to $P_{orb} \gtrsim 90$ d. We recommend observers try to measure P_{rot} and e in binaries out to $P_{orb} = 100$ d to test this hypothesis, but we note that detecting P_{rot} for such slow rotators can be difficult due to small star spot modulation amplitudes (McQuillan et al., 2014; Lurie et al., 2017; Reinhold et al., 2018). Long term spectroscopic monitoring may be warranted in such cases.

4.6 Discussion

In this work, we probed the long-term angular momentum evolution of low-mass stellar binaries, with a focus on P_{rot} in short and intermediate P_{orb} binaries. We considered the impact of two common equilibrium tidal models, magnetic braking, and stellar evolution. We performed a large suite of simulations for binaries with physically-motivated initial conditions out to $P_{orb} = 100$ and across a wide range of tidal dissipation parameters to examine the competition between tidal torques and magnetic braking for controlling the stellar P_{rot} evolution.

In our simulations, nearly all binaries with $P_{orb} \lesssim 4$ d have tidally-synchronized spins and circularized orbits, in good agreement with observations of *Kepler* EBs and binaries in the field. We showed for $P_{orb} \gtrsim 4$ d, primary stars in stellar binaries can rotate subsynchronously for Gyrs due to the competition between tidal torques and magnetic braking, or supersynchronously if they tidally lock on eccentric orbits. Our predictions are not strongly dependant on the choice of magnetic braking model, but rather are generic outcomes of the interaction between magnetic braking and tidal torques. Both the CPL and CTL equilibrium tidal models predict that binaries tidally-interact at longer P_{orb} than have previously been considered, out to $P_{orb} \approx 60 - 100$ d. Many binaries with $P_{orb} \lesssim 20$ d tidally lock according to both models, in good agreement with previous results, but the CPL model predicts that binaries can readily tidally lock out to $P_{orb} \approx 100$ d. Tidal interactions can cause P_{rot} evolution in stellar binaries to differ from the long-term spin down due to magnetic braking experienced by single stars, decoupling P_{rot} from age. In tidally interacting binaries, gyrochronology, the technique of linking stellar P_{rot} to age, likely fails, potentially underestimating stellar ages by up to 300%. We caution that any application of gyrochronology methods to stars, especially those with $P_{rot} \lesssim 20$ d, should account for the possibility of stellar binarity to prevent deriving incorrect ages.

We compare the predictions of both the CPL and CTL models with observations of P_{rot} and P_{orb} of *Kepler* EBs by Lurie et al. (2017) and find that both can qualitatively reproduce many features seen in the data, validating our approach and suggesting that equilibrium tidal models can accurately model stellar-tidal evolution in low-mass stellar binaries. The lack of uncertainties on P_{rot} , the approximate orbital eccentricities derived by Lurie et al. (2017), and unconstrained completeness estimates prevent us from discriminating between which tidal model best describes tidal torques in low-mass binaries and from inferring tidal properties of low-mass stars given the *Kepler* EB data.

We described three observational tests that can distinguish between which equilibrium tidal model better describes tidal interactions in low-mass stellar binaries. We primarily suggest that observers measure stellar P_{rot} in binaries with solar-mass primaries for P_{orb} between $60 - 100$ d. If any tidally locked binaries are identified at long P_{orb} , this would be evidence in favor of the CPL

model as we found that only binaries tidally interacting via the CPL model could tidally lock at such long P_{orb} . At shorter P_{orb} , precise measurements of P_{rot} and binary e and P_{orb} could distinguish between the CPL and CTL model in tidally locked systems, e.g. identifying if P_{eq} follows Eq. (4.20) vs. Eq. (4.22), especially if the CPL P_{eq} is in fact a discrete function of e . The observations required by these tests, however, are non-trivial. Beyond these tests, our model could be used to infer the tidal properties of binary stars, perhaps in a Markov Chain Monte Carlo framework, by directly comparing simulation results with the observed stellar and orbital properties, given the observational uncertainties and reasonable prior probability distributions for parameters like the initial binary e . This analysis, however, is beyond the scope of this work and we leave it for future endeavors.

Our theoretical predictions outline a critical point: one cannot simply observe a short P_{orb} binary on a circular orbit and assume synchronization, nor can one observe a binary with $P_{orb} \gtrsim 20$ d and assume that tides have not impacted that system's angular momentum evolution. Stellar-tidal interactions can produce synchronous and subsynchronous rotation for short P_{orb} binaries on circular orbits, e.g. Fig. 4.4, depending on the age of the system, e.g. Fig. 4.8, and the strength of tidal dissipation, e.g. Fig. 4.6 and Fig. 4.7. Understanding the long-term angular momentum evolution of stellar binaries out to $P_{orb} = 100$ d requires detailed modeling of its coupled-stellar tidal evolution, and characterizing tidal dissipation parameters. Many new eclipsing stellar binaries will be discovered by TESS (e.g. Sullivan et al., 2015; Matson et al., 2018) and in analysis of K2 data. Obtaining precise orbital and rotational constraints for stellar binaries will permit detailed characterization of tidal interactions between low-mass stars and shed light into the long-term angular momentum evolution in stellar binaries.

4.7 acknowledgments

We thank the anonymous reviewer for helpful comments that improved the quality of this manuscript. This work was facilitated through the use of advanced computational, storage, and networking infrastructure provided by the Hyak supercomputer system and funded by the Student Technology Fund at the University of Washington. DPF was supported by NASA Headquarters under the

NASA Earth and Space Science Fellowship Program - Grant 80NSSC17K0482. RB acknowledges support from the NASA Astrobiology Institute's Virtual Planetary Laboratory under Cooperative Agreement number NNA13AA93A. JRAD acknowledges support from the DIRAC Institute in the Department of Astronomy at the University of Washington. The DIRAC Institute is supported through generous gifts from the Charles and Lisa Simonyi Fund for Arts and Sciences, and the Washington Research Foundation.

Software: matplotlib: Hunter (2007), numpy: van der Walt et al. (2011), pandas: McKinney (2010), VPLanet: Barnes et al. (2019)

4.8 Analytic Torque Balance

Here we derive the equation for the stellar P_{rot} at which tidal torques balance magnetic braking discussed in § 4.5.1. As in § 4.5.1, we assume that both stars have $M = 1M_\odot$, 0 obliquity, and we assume a circular binary orbit. We assume that the torque balance occurs while the stars are on the main sequence, where stellar properties change slowly, so the angular momentum evolution is controlled by the balance between tidal torques and magnetic braking, not stellar radius contraction. Under this assumption, we can set $R = 1R_\odot$ and assume constant moments of inertia. For simplicity, we assume that magnetic braking proceeds under the Matt et al. (2015) model and the CTL model describes tidal torques.

As discussed in § 4.5.1, both stars are in the unsaturated rotation regime, so the torque due to magnetic braking is given by Eqn. (4.2), which under the aforementioned assumptions, reduces to

$$\frac{dJ}{dt} \Bigg|_{MB} = -C_{MB} \left(\frac{P_{rot,\odot}}{P_{rot}} \right)^3 \quad (4.25)$$

where $P_{rot} = 2\pi/\omega$ and $C_{MB} = 6.3 \times 10^{30}$ ergs (Matt et al., 2015, 2019).

Under the CTL model and our assumptions, the change in rotation rate due to tidal torques, Eqn. 4.16, reduces to

$$\frac{d\omega}{dt} \Bigg|_{tides} = \frac{P_{orb}Z_{CTL}}{2\pi Mr_g^2 R^2} \left(1 - \frac{P_{orb}}{P_{rot}} \right) \quad (4.26)$$

where $P_{orb} = 2\pi/n$. For fixed moment of inertia, $dJ/dt = Id\omega/dt$, and after inserting Eqn. 4.18 for Z_{CTL} , the tidal torque on the stellar rotations becomes

$$\frac{dJ}{dt}\Big|_{tides} = \frac{C_{tides}k_2\tau}{P_{orb}^5} \left(1 - \frac{P_{orb}}{P_{rot}}\right). \quad (4.27)$$

where $C_{tides} = 24\pi^5 R_\odot^5/G$.

The torques due to tides and magnetic braking balance when $\frac{dJ}{dt}|_{tides} + \frac{dJ}{dt}|_{MB} = 0$,

$$\frac{C_{tides}k_2\tau}{P_{orb}^5} \left(1 - \frac{P_{orb}}{P_{rot}}\right) - C_{MB} \left(\frac{P_{rot,\odot}}{P_{rot}}\right)^3 = 0. \quad (4.28)$$

By specifying P_{orb} and $k_2\tau$, we can numerically solve Eqn. (4.28) for the P_{rot} at which torques due to magnetic braking and tides balance, often producing subsynchronous rotation as seen in Fig. 4.3 and our simulations in § 4.5.1.

Chapter 5

APPROXIMATE BAYESIAN INFERENCE WITH `approxposterior`

Chapter 6

INFERRING THE XUV HISTORY OF THE TRAPPIST-1 PLANETARY SYSTEM

Chapter 7

DISCUSSION

Chapter 8

CONCLUSIONS

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Appendix A

VITA

I am David P. Fleming. When I want to procrastinate on the much more important sections of my thesis, I will complete my vita.