Affine Toric Varieties and Torus Quotients

Honours Thesis Defence

Declan Fletcher

November 2024

Let V be a rational representation of an algebraic torus T.

Main result: the affine GIT quotient $V /\!\!/ T$ is a toric variety.

1. Algebraic sets
   - Algebraic sets, their ideals, the Nullstellensatz
   - Polynomial maps, direct products, open subsets
2. Affine varieties
   - Varieties, the maximal spectrum
   - Morphisms, tangent spaces
3. Convex geometry
   - Polyhedral, rational and strongly convex cones

4. Affine toric varieties
   - Semigroup algebras, toric varieties, their points
   - Faces and open affine subsets, the torus action, singularities
5. Torus quotients as toric varieties
   - Algebraic groups, the affine GIT quotient
   - The invariant ring $k[V]^T$, the cone of $V /\!\!/ T$, examples