University of Geneva
Probability 1
Dr. Daniel Flores Agreda
ASSIGNMENT 03

Bachelor in Economics and Management
Bachelor in International Relations
Spring 2021
March 13th

Exercise 1

The mail order company 'CD-Bill' sends a questionnaire to households in its address file to know if they are interested in rap, classical or rock music, genres in which it is specialized. By searching the answers, it notes that 20% of households are not interested in any of these 3 genres of music, 35% of households are interested in rap, 20% in classical music and the number of households interested in rock is twice as many as in classical music. In addition, 5% households are interested in both rap and rock and no household is interested in both rap and classical music.

If one randomly chooses one of the households that responded to the survey:

1. What is the probability that he/she is interested in rap or classical music?
2. What is the probability that he/she is interested in rock, knowing that he/she is interested in rap?
3. What is the probability that he/she is interested in at least 2 kinds of music ?
4. What is the probability that he/she is interested in rap, knowing that he/she is interested in neither classical nor rock music?

Exercise 2

Roger Federer prepares for Wimbledon tennis. Since turf is the best surface, we know that regardless of the player he faces, the probability of winning a match is 0.85. To win the tournament a player has to win seven consecutive games.

1. What is the probability that Federer reaches at least the semi-finals?
2. Federer reached the quarter-finals (5th round). What is the probability that he wins the tournament?
3. A journalist says that if Novak Djokovic is in the final with Federer, the latter has a 60% chance of winning the tournament against 90% if Djokovic does not qualify. He says Djokovic has a 75% chance of reaching the final. Calculate the probability that Federer[1] wins the tournament according to the journalist.

[1] Assuming he is in the final.

Exercise 3

The probability that a new car battery works for over 30'000km is 0.8, the probability that it functions for over 60'000km is 0.4, and the probability that it works for over 90'000km is 0.1. If a new car battery is still working after 30'000km, what is the probability that

1. its total life will exceed 60'000km?
2. its additional life will exceed 60'000km?

Exercise 4

Consider the random experiment of tossing a fair coin twice. Let us define the following events:

- A: Observe a head (H) on the first toss
- B: Observe a head (H) on the second toss
- C: Observe the same outcome on both tosses

Are the events pairwise independent? Are the events jointly independent?

Exercise 5 (Optional)

Given the axioms of probability theory, show the following:

1. $P(\emptyset) = 0$
2. $P(A^c) = 1 - P(A)$
3. $P(A \cap B) \leq \min(P(A), P(B))$
4. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
5. If $A_1, A_2, ...$ are mutually exclusive events in $\mathcal{B}$, then

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i). \tag{1}$$

6. Let's A_n a collection of sets such that: $\lim_{n\to\infty} A_n = A$. Show that $\lim_{n\to\infty} P(A_n) = P(A)$.