Exercise 1

You look at the following discrete probability distributions for two random variables X and Y respectively

X values	-1	1	3	5	7
Probability	0.1	0.2	0.4	0.2	0.1
Y values	-1	1	3	5	7
Probability	0.2	0.15	0.3	0.15	0.2

Compute the mean and the variance of both variables.

Exercise 2

In a TV game, a candidate faces 5 doors, one of which hides a gift. Viewers can make bets on the number of doors that the candidate will push until he finds the gift. Jules and Gaston are candidates for the game. Jules has a good memory and is not likely to push twice the same door. As far as Gaston is concerned, he has absolutely no memory.

- 1. Construct the probability function of the number of doors pushed by Jules. Calculate the expectation of this event.
- 2. Construct the probability function of the number of doors pushed by Gaston. Calculate the expectation of this event.

 $\underline{\text{Indication:}}$ The following mathematical property can be used (generic geometric series):

$$|x| < 1 \Rightarrow \sum_{k=1}^{\infty} kx^{k-1} = \frac{1}{(1-x)^2}.$$

3. Generalize the previous results to n doors.

Exercise 3

Two cards are randomly chosen from a box containing the following five cards: 1, 1, 2, 2, 3. X represents the average and Y is the maximum of the two numbers drawn.

- 1. Calculate the probability distribution, the cumulative distribution function, the mean and the variance of X, Y and Z = Y X.
- 2. If W is the sum of the two numbers, what are its expectation and its variance?

Exercise 4

We consider a die whose faces are numbered from 1 to 6 and we define X the random variable given by the number of the upper face. It is assumed that the die is rigged so that the probability of getting a face is proportional to the number on that face.

- 1. Determine the probability function of X, then calculate its expectation and variance.
- 2. We define Y = 3X. Calculate the expectation and variance of Y. Is it necessary to determine the probability function of Y for calculating its expectation and variance?
- 3. We define $Z = \frac{1}{X}$. Calculate the expectation and variance of Z. Is it necessary to determine the probability function of Z for calculating its expectation and variance?

Exercise 5

- 1. The following game is considered: The player rolls once a fair six sided die. If he gets 1, 2 or 3, he wins the equivalent in francs. Otherwise, he loses 2 francs. Let X be the random variable corresponding to the player's win (a negative value indicating a loss).
 - (a) Give the probability function of X and its distribution function F_X .
 - (b) Calculate the expectation and variance of X.
- 2. We modify the game as follows: The winnings remain the same for the results 1, 2 or 3, but if the player gets something else, he rolls the die again. If he then gets 3 or less, he wins 3 francs, otherwise he loses 5 francs. We define Y as the random variable corresponding to the gain of the player in this new game.
 - (a) Give the probability function of Y and its distribution function F_Y .
 - (b) Calculate the expectation and variance of Y.
- 3. Which game is the most advantageous for the player? To justify.

Exercise 6 (Optional)

The random variable X is Bernoulli(p) distribution if its probability mass function is given by:

$$P(X = x) = p^{x}(1 - p)^{1-x}$$
 for $x = 0, 1$

where 0 . Compute the Mean and the Variance of the Bernoulli distribution.

Exercise 7 (Optional)

Let X a discrete random variable following a Poisson distribution with parameter λ . Its probability function is given by:

$$P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!} \quad \text{for} \quad k = 0, 1, \dots$$

- 1. Check that P(X = k) is a probability function.
- 2. Prove that its expectation is equal to λ .
- 3. Find the variance of X.
- 4. Compare the expectation and the variance of X with the expectation and the variance of S, for a very large n. Here $S \sim Binomial(n, p)$.