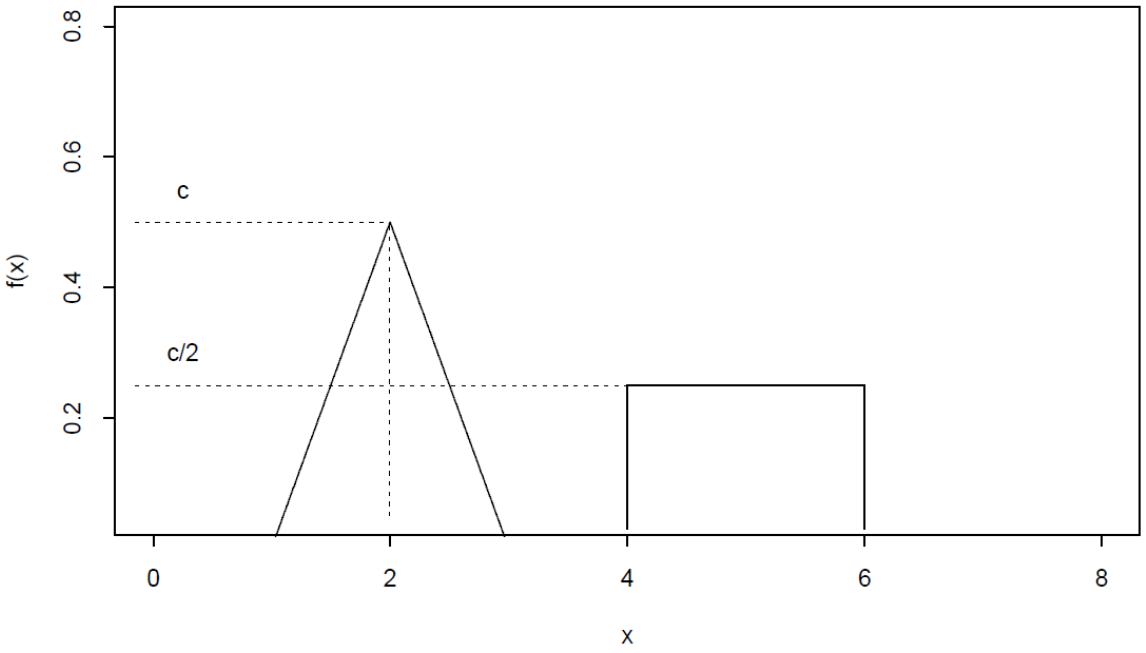
University of Geneva
Probability 1
Professor Davide La Vecchia
ASSIGNMENT 08

Bachelor in Economics and Management
Bachelor in International Relations
Spring 2018
April 23th-27th

Exercise 1



1. Determine the value of c so that the function $f(x)$ in the figure above is a density function.
2. Determine the density function $f(x)$.
3. Calculate $\mathrm{E}(X)$.
4. Determine $P(X \geq 2)$, $P(X \leq 4.5)$ and $P(2.5 \leq X \leq 5)$.

Exercise 2

You arrive at a bus stop knowing that the waiting time is a random variable uniformly distributed between 0 and 30 minutes.

The density function of this variable is written as:

$$f(t) = \begin{cases} 0, & t \leq 0; \\ \frac{1}{30}, & 0 < t < 30; \\ 0, & t \geq 30. \end{cases}$$

1. Determine the distribution function $F(t)$ of this variable.
2. What is the probability that you have to wait more than 10 minutes.
3. If after 15 minutes' waiting the bus has not arrived yet, what is the probability that you have to wait at least 10 more minutes?

Exercise 3

Let Z be a random variable with the following distribution function:

$$F(z) = kz^3 \left(\frac{1}{3} - \frac{z}{2} + \frac{z^2}{5} \right), \quad 0 \leq z \leq 1,$$

with k a positive constant.

1. Find the density function of Z as well as the value of k.
2. Calculate the expectation and variance of Z.
3. Calculate $P(0.75 < Z < 1.5)$ and $P(Z \geq 0.15)$.
4. Suppose we know in a particular case that Z is not less than 0.25. Find the probability that Z is greater than 0.5 in this case.

Exercise 4

The lifetime X in years of a television follows an exponential law with density:

$$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0.$$

1. Compute the cumulative distribution function $F(x)$.
2. Compute the α-quantile $q_\alpha = F^{-1}(\alpha)$.
3. The expected life of your television is 8 years. What is the probability that the lifetime of your television is more than 8 years ? Evaluate the median.
4. Compute the variance of X for any λ.

Exercise 5 (Optional)

Everyday, Sveta goes to the university by bike and follows the same 15km path. Her speed is a random variable V which depends on the climate and traffic conditions. Its density has the following form:

$$f_V(v) = \begin{cases} Cv\exp(-\lambda v) & v \geq 0 \\ 0 & otherwise. \end{cases}$$

Sveta is really athletic and rides at a speed of 30 km/h in average.

1. Determine the values of C and λ.
2. The duration of the ride is described by a the variable $T = \frac{15}{V}$. Compute the density and the expectation of T.