

**Example 1.13** In answering a question on a multiple choice test a student either knows the answer or he guesses. Let  $p$  be the probability that she knows the answer and  $1 - p$  the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability  $1/m$ , where  $m$  is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question given that she answered it correctly?

**Solution:** Let  $C$  and  $K$  denote respectively the event that the student answers the question correctly and the event that she actually knows the answer. Now

$$\begin{aligned} P(K|C) &= \frac{P(KC)}{P(C)} = \frac{P(C|K)P(K)}{P(C|K)P(K) + P(C|K^c)P(K^c)} \\ &= \frac{p}{p + (1/m)(1 - p)} \\ &= \frac{mp}{1 + (m - 1)p} \end{aligned}$$

Thus, for example, if  $m = 5$ ,  $p = \frac{1}{2}$ , then the probability that a student knew the answer to a question she correctly answered is  $\frac{5}{6}$ . ♦