University of Geneva
Probability 1
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ASSIGNMENT 02

Bachelor in Economics and Management
Bachelor in International Relations
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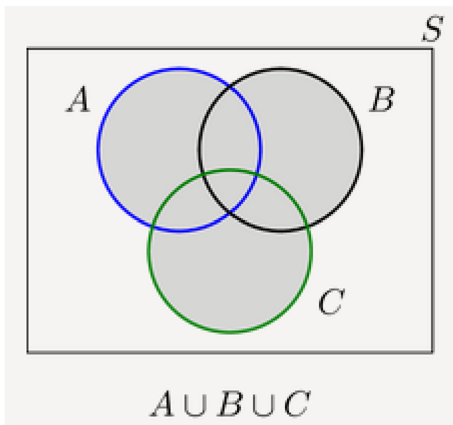
Exercise 1

1. Let A, B and C be three sets as shown in the following Venn diagram.



For each of the following sets, draw a Venn diagram and shade the area representing the given set.

a) $A \cup B \cup C$

b) $A \cap B \cap C$

c) $A \cup (B \cap C)$

d) $A - (B \cap C)$

e) $A \cup (B \cap C)^c$

2. Using the Venn Diagrams, verify the following identities.

a) Transitive Property: If $A \subset B$ and $B \subset C$, then $A \subset C$.

b) Distributive Property: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

c) De Morgan's Laws: $(A \cap B)^c = A^c \cup B^c$, $(A \cup B)^c = A^c \cap B^c$

d) $A = (A \cap B) \cup (A - B)$

e) If A and B are finite sets, we have $|A \cup B| = |A| + |B| - |A \cap B|$

Note that the symbol $|.|$ indicates the cardinality, i.e. the measure of the "number of elements of the set".

3. Determine whether each of the following sets is countable or uncountable:

a) $A = \{(x, y) | x \in N, y \in Z\}$

b) $B = (0, 0.1]$

c) $C = \{\frac{1}{n} | n \in N\}$

d) $D = \mathbb{Q}$

Exercise 2

We consider an urn with N marbles containing r reds and $N - r$ blacks. We draw n marbles randomly without repetition.

1. How many possible cases (more properly, let us call them "outcomes") do we have?

2. How many of these outcomes would include k red marbles.

3. Denoting by P(choosing k red marbles) the probability of choosing k red marbles, use the formula:

$$P(\text{choosing k red marbles}) = \frac{\text{number of outcomes including k red marbles}}{\text{number of possible outcomes}},$$

to deduce P(choosing k red marbles).

Exercise 3

There are 3 pairs of shoes of different color in a drawer. We randomly draw 2 shoes without repetition; determine the probability[1] associated with each of the following events:

A : 'they belong to the same pair';
B : 'there is a right shoe and a left shoe'.

[1] To compute the probability for an event E, use the formula

$$P(E) = \frac{\text{number of cases in E}}{\text{number of possible cases}}.$$

Exercise 4

Let A be a die whose faces display the values 2, 2, 4, 4, 9, 9. Let B be a die whose faces display the values 1, 1, 6, 6, 8, 8. We roll the two dice.

1. Write all the possible cases—more properly, let us call them "outcomes".
2. What is the probability[2] that the result of A is greater than the result of B?
3. What is the probability[3] that the sum of the two dice equals 10?

Exercise 5

A cafeteria offers a three-course menu. We choose a main course, a starch and a dessert. The possible choices are given below:

- 'Main course': Chicken (C) or roast beef (B);
- 'Starch': Pasta (P) or rice (R) or potatoes (T);
- 'Dessert': Ice cream(I) or jelly (J) or apple pie (A) or peach pie (P).

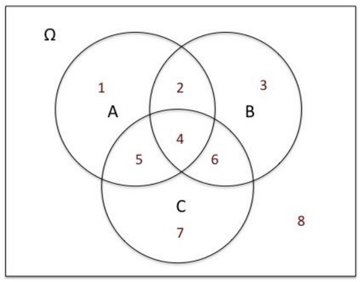
A person chooses a dish from each category.

1. How many possible menus are there in the basic set?
2. Let A be the event: 'we choose ice cream'. How many menus are there in A?
3. Let B be the event: 'we choose the chicken'. How many menus are there in B?
4. Give all the possible menus of the event $A \cap B$.
5. Let C be the event: 'we choose rice'. How many menus are there in C?
6. It is assumed that a person randomly selects his menu by associating an equal probability with all the options for each category. What is the probability that the chosen menu belongs to event A? Answer the same questions for event B and for event C.

[2]Use the same formula as in footnote 1.
[3]Use the same formula as in footnote 1.

Exercise 6 (Optional)

In this problem, you are given descriptions in words of certain events (e.g., at least one of the events A,B,C occurs). For each one of these descriptions, identify the correct symbolic description in terms of A, B, C from Events E1-E7 below. Also identify the correct description in terms of regions (i.e., subsets of the sample space Ω) as depicted in the Venn diagram below. (For example, Region 1 is the part of A outside of B and C.)



Symbolic descriptions:
Event E1: $A \cap B \cap C$
Event E2: $(A \cap B \cap C)^c$
Event E3: $A \cap B \cap C^c$
Event E4: $B \cup (B^c \cap C^c)$
Event E5: $A^c \cap B^c \cap C^c$
Event E6: $(A \cap B) \cup (A \cap C) \cup (B \cap C)$
Event E7: $(A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$
Which Event or Regions satisfy the following conditions.
1. At least two of the events A, B, C occur.
2. At most two of the events A, B, C occur.
3. None of the events A, B, C occurs.
4. All three events A, B, C occur.
5. Exactly one of the events A, B, C occurs.
6. Events A and B occur, but C does not occur.
7. Either event B occurs or, if not, then C also does not occur.

Exercise 7 (Optional)

A,B and C take turns flipping a coin, that is: A flips first, then B, then C, then A and so on. The first one get a head wins.

1. Define the sample space S of all the possible outcomes.

2. Define the following events in terms of S:

   (a) The event A: A wins.

   (b) The event B: B wins.

   (c) $(A \cup B)^c$