# Probability 1

## Lecture 03 : Probability Axioms

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(based on the notes of Prof. Davide La Vecchia)

Spring Semester 2021

# Objective

- Provide the Axioms upon which the Probability Theory is built.
- Explore some rules that result from the Axioms
- Illustrate how these rules can be used to compute probabilities "in real life"
- Introduce the concepts of Independence and Conditional Probability
- Present two important Theorems and illustrate their consequences.

# Outline

# Outline

# An Axiomatic Definition of Probability

## Definition

We define probability a set function with values in $[0, 1]$, which satisfies the following axioms:

(i) $P(A) \geq 0$, for every event $A$

(ii) $P(S) = 1$

(iii) If $A_1, A_2, \ldots$ is a sequence of mutually exclusive events (namely $A_i \cap A_j = \varnothing$, for $i \neq j$, and $i, j = 1, 2, \ldots$), and such that $A = \bigcup_{i=1}^{\infty} A_i$, then

$$P(A) = P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i). \quad (1)$$

# Outline

# Implications: Properties of $P(\cdot)$

We can use the three axioms to build more sophisticated statements, e.g.

- The Probability of the Empty Set
- The Addition Law of Probability
- The Complement Rule
- The Monotonicity Rule
- The Probability of the Union
- Boole's Inequality

# Implications: Properties of $P(\cdot)$

## Theorem (Probability of the Empty Set)

$$P(\varnothing) = 0$$

## Proof.

Take $A_1 = A_2 = A_3 = .... = \varnothing$. Then by (1) in Axiom (ii) we have

$$P(\varnothing) = P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{\infty} P(\varnothing)$$

which is true only if it is an infinite sum of zeros. Thus

$$P(\varnothing) = 0.$$

□

# Implications: Properties of $P(\cdot)$

## Theorem (The Addition Law of Probability)

*If $A_1, A_2, \ldots$ are mutually exclusive events, then*

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i). \tag{2}$$

## Proof.

Let $A_{n+1} = A_{n+2} = \ldots. = \varnothing$, then $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{\infty} A_i$, and, from (1) (see Axiom (iii)) it follows

$$P\left(\bigcup_{i=1}^{n} A_i\right) = P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i) = \sum_{i=1}^{n} P(A_i) + \underbrace{\sum_{i=n+1}^{\infty} P(A_i)}_{\equiv 0}.$$

$\square$

### Theorem (The Complement Rule)

*If $A$ is an event, then $P(A^c) = 1 - P(A)$.*

### Proof.

Let $A \cup A^c = S$ and $A \cap A^c = \varnothing$, so and

$$P(S) = P\left(A \cup A^c\right) = P(A) + P\left(A^c\right).$$

By Axiom (i) Axiom (ii) we have $P(S) = 1$, so the desired result follows from:

$$1 = P(A) + P\left(A^c\right).$$

$\square$

## Theorem (The Monotonicity Rule)

*For any two events $A$ and $B$, such that $B \subset A$, we have*

$$P(A) \geq P(B).$$

## Proof.

Let us write

$$A = B \cup (B^c \cap A)$$

and notice that $B \cap (B^c \cap A) = \phi$, so that

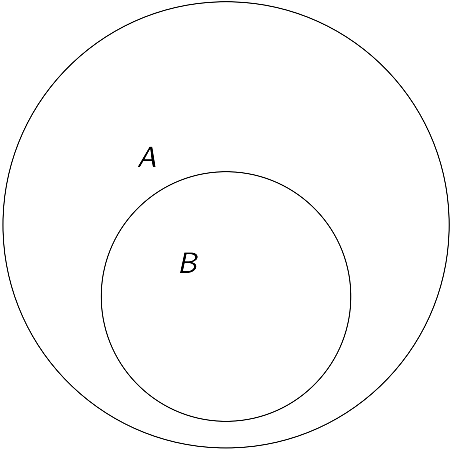
$$\begin{aligned} P(A) &= P\{B \cup (B^c \cap A)\} \\ &= P(B) + P(B^c \cap A) \end{aligned}$$

which implies (since $P(B^c \cap A) \geq 0$) that

$$P(A) \geq P(B).$$

$\square$

# Implications: Properties of $P(\cdot)$




$B \subset A$

# Implications: Properties of $P(\cdot)$

## Theorem (Probability of the Union of Two Events)

*For any two events $A$ and $B$ then*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

## Proof.

Consider that $A \cup B = A \cup (A^c \cap B)$, and $A \cap (A^c \cap B) = \phi$. Now remember[a] that $A^c \cap B = B - (A \cap B)$, so,

$$\begin{aligned} P(A \cup B) &= P(A) + P(A^c \cap B) \\ &= P(A) + P(B) - P(A \cap B). \end{aligned}$$

$\square$

[a] See Lecture 1 for the meaning of set difference.

# Implications: Properties of $P(\cdot)$

## Theorem (Boole's inequality)

*For the events $A_1, A_2, ...A_n$,*

$$P(A_1 \cup A_2 \cup .... \cup A_n) \leq \sum_{i=1}^{n} P(A_i).$$

For instance, let us consider $n = 2$. Then we have:

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq P(A_1) + P(A_2)$$

since $P(A_1 \cap A_2) \geq 0$ by definition.

## Remark

*It is worth notice that if $A_j \cap A_i = \varnothing$, for every $i$ and $j$, with $i \neq j$, then $P(A_1 \cup A_2 \cup .... \cup A_n) = \sum_{i=1}^{n} P(A_i)$, as stated in (2).*

# Outline

## Example (Flipping a coin twice)

**If we flip a balanced coin twice, what is the probability of getting at least one head?**

- The sample space is: $S = \{HH, HT, TH, TT\}$
- Balanced coin: outcomes are equally likely and $p_{HH} = p_{HT} = p_{TH} = p_{TT} = p = 1/4$
- Let $A$ denote the event **obtaining at least one Head**, i.e. $H = \{HH, HT, TH\}$

$$
\begin{aligned}
Pr(A) &= Pr(\{HH \cup HT \cup TH\}) \\
&= Pr(\{HH\}) + Pr(\{HT\}) + Pr(\{TH\}) \\
&= \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}
\end{aligned}
$$

## Example (Detecting Shoppers)

*Shopper TRK is an electronic device that counts the number of shoppers entering a shopping centre.*
*Two shoppers enter the shopping centre together, one walking in front of the other:*

1. *There is a 0.98 probability that the first shopper is detected*
2. *There is a 0.94 probability that the second shopper is detected.*
3. *There is a 0.93 probability that both shoppers are detected.*

**What is the probability that the device will detect at least one of the two shoppers entering?**

# Examples and Illustrations

## Example (Detecting Shoppers, continued)

Define the **events**:

- $D$ (shopper is detected) and
- $U$ (shopper is undetected).

Then, the Sample Space is $S = \{DD, DU, UD, UU\}$
Define the **probabilities**:

- $Pr(DD \cup DU) = 0.98$
- $Pr(DD \cup UD) = 0.94$
- $Pr(DD) = 0.93$

## Example (Detecting Shoppers, continued)

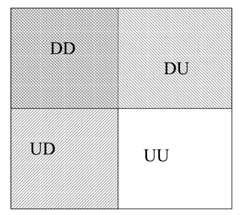
$$
\begin{aligned}
Pr(DD \cup UD \cup DU) &= Pr(\{DD \cup UD\} \cup \{DD \cup DU\}) \\
&= Pr(\{DD \cup UD\}) + Pr(\{DD \cup DU\}) \\
&\quad - Pr(\{DD \cup UD\} \cap \{DD \cup DU\})
\end{aligned}
$$

The only missing probability is : $P\{DD \cup UD\} \cap \{DD \cup DU\} =?$
To compute it, let's use the fact that **the union is distributive with respect to the intersection** (see Ch 2):

$$
(DD \cup UD) \cap (DD \cup DU) = DD \cup (UD \cap DU) = DD \cup \varnothing = DD
$$

## Example (Detecting Shoppers, continued)

Graphically:




So, the desired probability is:

$$\begin{aligned} Pr(DD \cup UD \cup DU) &= Pr(\{DD \cup UD\}) + Pr(\{DD \cup DU\}) - Pr(DD) \\ &= 0.98 + 0.94 - 0.93 \\ &= 0.99 \end{aligned}$$

## Example (De Morgan's law)

Given $P(A \cup B) = 0.7$ and $P(A \cup B^c) = 0.9$, find $P(A)$.

By De Morgan's law,

$$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - 0.7 = 0.3$$

and similarly

$$P(A^c \cap B) = 1 - P(A \cup B^c) = 1 - 0.9 = 0.1.$$

Thus,

$$P(A^c) = P(A^c \cap B^c) + P(A^c \cap B) = 0.3 + 0.1 = 0.4,$$

so

$$P(A) = 1 - 0.4 = 0.6.$$

# Examples and Illustrations

## Example (Probability, union, and complement)

*John is taking two books along on his holiday vacation. With probability 0.5, he will like the first book; with probability 0.4, he will like the second book; and with probability 0.3, he will like both books. What is the probability that he likes neither book?*

Let $A_i$ be the event that John likes book $i$, for $i = 1, 2$. Then the probability that he likes at least one book is:

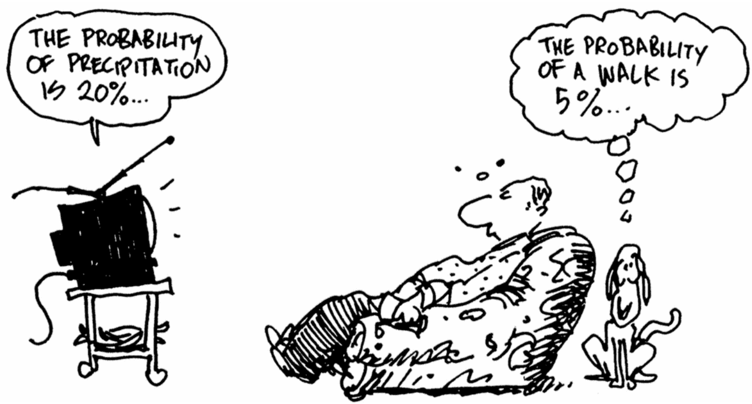
$$\begin{aligned} P(\bigcup_{i=1}^{2} A_i) &= P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2) \\ &= 0.5 + 0.4 - 0.3 = 0.6. \end{aligned}$$

Because the event the John likes neither books is the complement of the event that he likes at leas one of them (namely $A_1 \cup A_2$), we have

$$P(A_1^c \cap A_2^c) = P((A_1 \cup A_2)^c) = 1 - P(A_1 \cup A_2) = 0.4.$$

# Outline

# Conditional Probability



# Conditional Probability

Sometimes, probability depends on the information available.

## Example

Suppose you have two dice and throw them; the possible outcomes are:

| | | | | | |
|---|---|---|---|---|---|
| **(1,1)** | **(1,2)** | **(1,3)** | **(1,4)** | **(1,5)** | **(1,6)** |
| **(2,1)** | **(2,2)** | **(2,3)** | **(2,4)** | **(2,5)** | **(2,6)** |
| **(3,1)** | **(3,2)** | **(3,3)** | **(3,4)** | **(3,5)** | **(3,6)** |
| **(4,1)** | **(4,2)** | **(4,3)** | **(4,4)** | **(4,5)** | **(4,6)** |
| **(5,1)** | **(5,2)** | **(5,3)** | **(5,4)** | **(5,5)** | **(6,5)** |
| **(6,1)** | **(6,2)** | **(6,3)** | **(6,4)** | **(6,5)** | **(6,6)** |

## Example (cont'd)

Consider the event $A =$ getting 5, or equivalently $A = \{5\}$.
What is $P(A)$, namely, the probability of getting 5?

| | | | | | |
|---|---|---|---|---|---|
| **(1,1)** | **(1,2)** | **(1,3)** | **(1,4)** | **(1,5)** | **(1,6)** |
| **(2,1)** | **(2,2)** | **(2,3)** | **(2,4)** | **(2,5)** | **(2,6)** |
| **(3,1)** | **(3,2)** | **(3,3)** | **(3,4)** | **(3,5)** | **(3,6)** |
| **(4,1)** | **(4,2)** | **(4,3)** | **(4,4)** | **(4,5)** | **(4,6)** |
| **(5,1)** | **(5,2)** | **(5,3)** | **(5,4)** | **(5,5)** | **(6,5)** |
| **(6,1)** | **(6,2)** | **(6,3)** | **(6,4)** | **(6,5)** | **(6,6)** |

# Conditional Probability

## Example (cont'd)

The dice are fair so we can get 36 events with equal probability $1/36$. Namely:

$$Pr(i,j) = \frac{1}{36}, \quad \text{for} \quad i,j = 1,..,6$$

Thus, we can make use of the highlighted probabilities

$$\begin{aligned}
P(5) &= Pr\{(1,4) \cup (2,3) \cup (3,2) \cup (4,1)\} \\
&= Pr\{(1,4)\} + Pr\{(2,3)\} + Pr\{(3,2)\} + Pr\{(4,1)\} \\
&= 1/36 + 1/36 + 1/36 + 1/36 \\
&= 4/36 \\
&= 1/9.
\end{aligned}$$

## Example (cont'd)

Now, suppose that we throw the die first and we get 2.

**What is the probability of getting 5 given that we have observed 2 in the first throw?**

| | | | | | |
|---|---|---|---|---|---|
| **(1,1)** | **(1,2)** | **(1,3)** | **(1,4)** | **(1,5)** | **(1,6)** |
| **(2,1)** | **(2,2)** | **(2,3)** | **(2,4)** | **(2,5)** | **(2,6)** |
| **(3,1)** | **(3,2)** | **(3,3)** | **(3,4)** | **(3,5)** | **(3,6)** |
| **(4,1)** | **(4,2)** | **(4,3)** | **(4,4)** | **(4,5)** | **(4,6)** |
| **(5,1)** | **(5,2)** | **(5,3)** | **(5,4)** | **(5,5)** | **(6,5)** |
| **(6,1)** | **(6,2)** | **(6,3)** | **(6,4)** | **(6,5)** | **(6,6)** |

$\Pr\{\text{getting 5 given 2 in the first throw}\} = \Pr\{\text{getting 3 in the second throw}\} = 1/6$

# Conditional Probability

## Remark

- *Information changes the probability*
- *By knowing that we got 2 in the first throw, we have changed the sample space:*

| | | | | | |
|---|---|---|---|---|---|
| **(1,1)** | **(1,2)** | **(1,3)** | **(1,4)** | **(1,5)** | **(1,6)** |
| **(2,1)** | **(2,2)** | **(2,3)** | **(2,4)** | **(2,5)** | **(2,6)** |
| **(3,1)** | **(3,2)** | **(3,3)** | **(3,4)** | **(3,5)** | **(3,6)** |
| **(4,1)** | **(4,2)** | **(4,3)** | **(4,4)** | **(4,5)** | **(4,6)** |
| **(5,1)** | **(5,2)** | **(5,3)** | **(5,4)** | **(5,5)** | **(6,5)** |
| **(6,1)** | **(6,2)** | **(6,3)** | **(6,4)** | **(6,5)** | **(6,6)** |

- *Probability can change drastically — e.g., suppose that in our example we have 6 in the first throw $\Rightarrow$ the probability of observing 5 in two draws ... is zero !!!*

# Conditional Probability

## Definition

Let $A$ and $B$ be two events. The conditional probability of event $A$ given event $B$, denoted by $P\left(A|B\right)$, is defined by:

$$P\left(A|B\right) = \frac{P(A \cap B)}{P(B)}, \quad \text{if} \quad P(B) > 0,$$

and it is left undefined if $P(B) = 0$.

# Conditional Probability

## Example (A check)

Let us define the set $B$ as:

| | | | | | |
|---|---|---|---|---|---|
| (1,1) | (1,2) | (1,3) | (1,4) | (1,5) | (1,6) |
| (2,1) | (2,2) | (2,3) | (2,4) | (2,5) | (2,6) |
| (3,1) | (3,2) | (3,3) | (3,4) | (3,5) | (3,6) |
| (4,1) | (4,2) | (4,3) | (4,4) | (4,5) | (4,6) |
| (5,1) | (5,2) | (5,3) | (5,4) | (5,5) | (6,5) |
| (6,1) | (6,2) | (6,3) | (6,4) | (6,5) | (6,6) |

So we have

$$\begin{aligned} P(B) &= Pr\left\{(2,1) \cup (2,2) \cup (2,3) \cup (2,4) \cup (2,5) \cup (2,6)\right\} \\ &= Pr(2,1) + Pr(2,2) + Pr(2,3) + Pr(2,4) + Pr(2,5) + Pr(2,6) \\ &= 6/36 = 1/6 \end{aligned}$$

## Example (A check)

and consider $A \cap B$

| | | | | | |
|---|---|---|---|---|---|
| **(1,1)** | **(1,2)** | **(1,3)** | **(1,4)** | **(1,5)** | **(1,6)** |
| **(2,1)** | **(2,2)** | **(2,3)** | **(2,4)** | **(2,5)** | **(2,6)** |
| **(3,1)** | **(3,2)** | **(3,3)** | **(3,4)** | **(3,5)** | **(3,6)** |
| **(4,1)** | **(4,2)** | **(4,3)** | **(4,4)** | **(4,5)** | **(4,6)** |
| **(5,1)** | **(5,2)** | **(5,3)** | **(5,4)** | **(5,5)** | **(6,5)** |
| **(6,1)** | **(6,2)** | **(6,3)** | **(6,4)** | **(6,5)** | **(6,6)** |

So we have $P(A \cap B) = Pr(2, 3) = 1/36$ so,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/36}{1/6} = \frac{1}{6}.$$

# Outline

# Independence

## Definition

Two events $A$ and $B$ are independent if the occurrence of one event has no effect on the probability of occurrence of the other event. Thus,

$$P(A|B) = P(A)$$

or equivalently

$$P(B|A) = P(B)$$

Clearly, if $P(A|B) \neq P(A)$, then $A$ and $B$ are *dependent*.

Two events $A$ and $B$ are independent if

$$P(A|B) = P(A),$$

now by definition of conditional probability we know that

$$P(A|B) = \frac{P(A \cap B)}{P(B)},$$

so we have

$$P(A) = \frac{P(A \cap B)}{P(B)},$$

and rearranging the terms, we find that two events are independent iif

$$P(A \cap B) = P(A)P(B).$$

# Independence

## Example

A coin is toss three times and the eight possible outcomes

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

are assumed to be equally likely with probability 1/8

Define

$A$= an H occurs on each of the first two tosses

$B$= T occurs in the third toss

$D$= two Ts occur in the three tosses

**Q1:** Are $A$ and $B$ independent?

**Q2:** Are $B$ and $D$ independent?

# Independence

## Example (cont'd)

We have

$$A = \{HHH,\ HHT\} \qquad \Pr\{A\} = 2/8 = 1/4$$

$$B = \{HHT,\ HTT,\ THT,\ TTT\} \qquad \Pr\{B\} = 4/8 = 1/2$$

$$D = \{HTT,\ THT,\ TTH\} \qquad \Pr\{D\} = 3/8$$

$$A \cap B = \{HHT\} \qquad \Pr\{A \cap B\} = 1/8$$

$$B \cap D = \{HTT, THT\} \qquad \Pr\{B \cap D\} = 2/8 = 1/4$$

$$\Pr\{A\}\Pr\{B\} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \qquad \Pr\{A \cap B\} = \frac{1}{8} \Rightarrow$$ *A,B* independent

$$\Pr\{B\}\Pr\{D\} = \frac{1}{2} \times \frac{3}{8} = \frac{3}{16} \qquad Pr\{B \cap D\} = \frac{1}{4} \Rightarrow$$ *B* and *D* dependent

# Outline

# Theorem I: The Theorem of Total Probability

## Theorem (Total Probabilities)

*Let* $B_1, B_2, ..., B_k, ..., B_n$ *be mutually disjoint events, satisfying* $S = \cup_{i=1}^n B_i$, *and* $P(B_i) > 0$, *for every* $i = 1, 2, ..., n$ *then for every* $A$ *we have that:*

$$P(A) = \sum_{i=1}^{n} P(A|B_i)P(B_i). \tag{3}$$

## Proof.

Write $A = A \cap S = A \cap (\cup_{i=1}^n B_i) = \cup_{i=1}^n (A \cap B_i)$. Since the $\{B_i \cap A\}$ are mutually disjoint, we have

$$P(A) = P\left(\cup_{i=1}^n (A \cap B_i)\right) = \sum_{i=1}^{n} P\left(A \cap B_i\right) = \sum_{i=1}^{n} P(A|B_i)P(B_i).$$

□

**Remark**. The theorem remains valid even if $n = \infty$ in Eq. (3). (Double check, and re-do the proof using $n = \infty$.)

### Corollary

*Let $B$ satisfy $0 < P(B) < 1$; then for every event $A$:*

$$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

### Proof.

Exercise [Hint: $S = B \cup B^c$]. $\square$

# Outline

# Theorem II: Bayes' Theorem

Theorem I can be applied to derive the well-celebrated Bayes' Theorem.

## Theorem (Bayes' Theorem)

*Let $B_1, B_2, ..., B_k, ..., B_n$ be mutually disjoint events, satisfying*

$$S = \cup_{i=1}^{n} B_i,$$

*and $P(B_i) > 0$, for every $i = 1, 2, ..., n$. Then for every event $A$ for which $P(A) > 0$, we have that*

$$P(B_k|A) = \frac{P(A|B_k)P(B_k)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}. \tag{4}$$

# Theorem II: Bayes' Theorem

**Proof.**

Let us write

$$
\begin{aligned}
P(B_k|A) &= \frac{P(A \cap B_k)}{P(A)} \\
&= \frac{P(A \cap B_k)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)} \\
&= \frac{P(A|B_k)P(B_k)}{\sum_{i=1}^{n} P(A|B_i)P(B_i)}
\end{aligned}
$$

That concludes the proof. □

## Example

Let us consider a special case, where we have only two events $A$ and $B$.

From the definition of conditional probability

$$\Pr\{A|B\}=\frac{\Pr\{A\cap B\}}{\Pr\{B\}} \qquad \Pr\{B|A\}=\frac{\Pr\{A\cap B\}}{\Pr\{A\}}$$

This can be written as

$$\Pr\{A\cap B\}=\Pr\{A|B\}\Pr\{B\} \qquad \Pr\{A\cap B\}=\Pr\{B|A\}\Pr\{A\}$$

That is

$$\Pr\{A|B\}\Pr\{B\}=\Pr\{B|A\}\Pr\{A\}$$

Rearranging this we have

$$\Pr\{A|B\}=\frac{\Pr\{A\}}{\Pr\{B\}}\times\Pr\{B|A\} \quad \Leftarrow \text{ Bayes' Theorem}$$

... so thanks to Bayes' Theorem we can reverse the role of $A|B$ and $B|A$.

## Example (Guessing in a multiple choice exam)

**Example 1.13** In answering a question on a multiple choice test a student either knows the answer or he guesses. Let $p$ be the probability that she knows the answer and $1 - p$ the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where $m$ is the number of multiple-choice alternatives. What is the conditional probability that a student knew the answer to a question given that she answered it correctly?

**Solution:** Let $C$ and $K$ denote respectively the event that the student answers the question correctly and the event that she actually knows the answer. Now

$$P(K \mid C) = \frac{P(KC)}{P(C)} = \frac{P(C \mid K)P(K)}{P(C \mid K)P(K) + P(C \mid K^c)P(K^c)}$$

$$= \frac{p}{p + (1/m)(1 - p)}$$

$$= \frac{mp}{1 + (m - 1)p}$$

Thus, for example, if $m = 5$, $p = \frac{1}{2}$, then the probability that a student knew the answer to a question she correctly answered is $\frac{5}{6}$. ◆

## Example

Members of a consulting firm in Geneva rent cars from two rental agencies:

- 60% from AVIS
- 40% from Mobility

Now consider that

- 9% of the cars from AVIS need a tune-up
- 20% of the cars from Mobility need a tune-up

If a car delivered to the consulting firm needs a tune-up, what is the probability that the care came from AVIS?

## Aim

*$A := \{$car rented from AVIS$\}$ and $B := \{$car needs a tune-up$\}$. We know $P(B|A)$ and we look for $P(A|B) \Rightarrow$ Bayes' theorem!!*

## Example (cont'd)

$$\Pr\{A\}=0.6$$

$$\Pr\{B|A\}=0.09$$

$$\Pr\{B|A^C\}=0.2$$

$$\begin{aligned}\Pr\{B\} &= \Pr\left\{(B\cap A)\cup\left(B\cap A^C\right)\right\}\\ &= \Pr\{B\cap A\}+\Pr\left\{B\cap A^C\right\}\\ &= \Pr\{B|A\}\Pr\{A\}+\Pr\left\{B|A^C\right\}\Pr\left\{A^C\right\}\\ &= 0.09\times 0.6+0.2\times 0.4\\ &= 0.134\end{aligned}$$

$$\Pr\{A|B\}=\frac{\Pr\{A\}}{\Pr\{B\}}\times\Pr\{B|A\}=\frac{0.6}{0.134}0.09=0.402985$$

# Wrap-up

Take-home message

- The Probability Axioms are important, since they imply **computation rules**.
- These rules are key to compute probabilities "in real life".
- Probability of an event can be **conditional** on the realisation of another event.
- When an event has the **same probability** regardless of another it is called independent
- Theorem I (basic)

$$P(A) = P(A \cap B) + P(A \cap B^c) = P(A|B)P(B) + P(A|B^c)P(B^c)$$

- Theorem II (basic) If we have $P(A|B)$, we can find $P(B|A)$

$$P(B|A) = \frac{P(A|B)P(B)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

Thank you for your attention!
"See you" next week!