

**Example.** Let  $X$  and  $Y$  be two discrete random variables with joint probability function

	$Y = 0$	$Y = 1$	$Y = 2$	$Y = 3$	marginal of $X$
$X = 0$	$h$	$2h$	$3h$	$4h$	$10h$
$X = 1$	$4h$	$6h$	$8h$	$2h$	$20h$
$X = 2$	$9h$	$12h$	$3h$	$6h$	$30h$
marginal of $Y$	$14h$	$20h$	$14h$	$12h$	$\sum_{(x,y)} = 60h$

Hence,  $h = 1/60$ . We compute all moments up to order 2:

$$E[X] = \sum_x xp_X(x) = 0 \cdot 10h + 1 \cdot 20h + 2 \cdot 30h = 80h = 4/3;$$

$$E[Y] = \sum_y yp_Y(y) = 0 \cdot 14h + 1 \cdot 20h + 2 \cdot 14h + 3 \cdot 12h = 84h = 7/5;$$

$$E[X^2] = \sum_x x^2 p_X(x) = 0^2 \cdot 10h + 1^2 \cdot 20h + 2^2 \cdot 30h = 140h = 7/3;$$

$$E[Y^2] = \sum_y y^2 p_Y(y) = 0^2 \cdot 14h + 1^2 \cdot 20h + 2^2 \cdot 14h + 3^2 \cdot 12h = 184h = 46/15;$$

$$E[XY] = \sum_{(x,y)} xyp_{(X,Y)}(x,y) = 5/3.$$

Thus  $E[X] = 4/3$ ,  $E[Y] = 7/5$ ,  $Var(X) = 7/3 - (4/3)^2 = 5/9$ ,  $Var(Y) = 46/15 - (7/5)^2 = 83/75$  and  $Cov(X, Y) = 5/3 - 4/3 \cdot 7/5 = -1/5$ ,  $\rho(X, Y) = \frac{-1/5}{\sqrt{(5/9)(83/75)}} = -0.255$ .