Probability 1

Lecture 09 : Illustration of the Central Limit Theorem

Dr. Daniel Flores-Agreda,

(based on the notes of Prof. Davide La Vecchia)

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Outline

Central Limit Theorem

Theorem

Let $X_1, X_2, ..., X_n, ...$ be a sequence of i.i.d. random variables and let $Y = h(X)$ be such that

$$\begin{aligned} E[Y] = E\left[h(X)\right] &= \mu_Y \\ Var(Y) = Var\left(h(X)\right) &= \sigma_Y^2 < \infty\,. \end{aligned}$$

Set

$$\overline{Y}_n = \frac{1}{n}\sum_{s=1}^{n} Y_s \quad \textit{where} \quad Y_s = h(X_s)\,, \quad s = 1, \dots, n\,.$$

Then (under quite general regularity conditions)

$$\frac{\sqrt{n}\left(\overline{Y}_n - \mu_Y\right)}{\sigma_Y} \overset{D}{\to} N\left(0,1\right) \Leftrightarrow P\left(\frac{\sqrt{n}\left(\overline{Y}_n - \mu_Y\right)}{\sigma_Y} \leq x\right) \underset{n\to\infty}{\longrightarrow} \Phi(x)$$

Central Limit Theorem

Implications

Example (Ross, Example 3e)

An instructor has 50 exams that will be graded in sequence.

The times required to grade the 50 exams are independent, with a common distribution that has mean 20 minutes and standard deviation of 4 minutes.

Approximate the probability that the instructor will grade at least 25 of the exams in the first 450 minutes of work.

Central Limit Theorem

Example (Ross, Example 3e)