University of Geneva — Bachelor in Economics and Management
Probability 1 — Bachelor in International Relations
Dr. Daniel Flores Agreda — Spring 2021
ASSIGNMENT 01

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1 Integrals

Compute the following integrals:

$$\int 5x^3 - 10x^{-6} + 4dx$$
$$\int \pi^8 + \pi^{-8}dx$$
$$\int 3\sqrt[4]{x^3} + \frac{7}{x^5} + \frac{1}{6\sqrt{x}}dx$$
$$\int \frac{15}{x}dx$$
$$\int exp(x)dx$$
$$\int_1^2 y^2 + y^{-2}dy$$
$$\int_{-3}^1 6y^2 - 5y + 2dy$$
$$\int_0^u \lambda exp(\lambda y)dy$$
$$\int_{-\infty}^a \frac{1}{2b}exp\left(-\frac{|y-\mu|}{b}\right)dy, \quad \text{for} \quad \mu \in \mathbb{R} \quad \text{and} \quad b \in \mathbb{R}^+.$$

2 Combinatorics

Combinatorics is a branch of mathematics concerning the study of finite or countable discrete structures. We will enumerate the number of ways of selecting k items from a collection of n objects. In this class we will use several kinds of combinations:

1. Permutation with repetition: n^k
   we draw (with replacement) k elements from n objects.

2. Permutation without repetition: $P_n^n = n!$
   we draw (without replacement) n elements from n objects.

3. Partial permutations: $P_k^n = \frac{n!}{(n-k)!}$
   we draw (without replacement) k elements from n objects.

4. Combinations: $C\binom{n}{k} = \frac{n!}{k!(n-k)!}$
   Same as a partial permutation but the order of the items doesn't count. The formula comes from:

$$\text{Partial permutations} = \text{Permutations of the outcome} \times \text{Combinations}$$

$$\text{That is :} \quad P_k^n = P_k^k \times \binom{n}{k}$$

Theorem:

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r} \quad \text{for } 1 \le r \le n \tag{1}$$

1) Assume there are 6 women and 4 men taking an exam. Then they are ranked according to their grades.

- How many possible rankings can we get?
- If the women are ranked between them, and the men between them. How many possible global rankings do we have?

2) How many car number plates with 7 items does it exist:

- if the first two are letters and the five last are digits?
- same question but we assume that there is no repetition of the letters and the digits.

3) Suppose that a fair coin is tossed five times. The outcome of the experiment is registered with H if the result of the tossing is head and with T if the outcome is tail (the outcomes would look like $HHTHT, THTTH$ and so on).

- How many possible outcomes do we have?
- How many of these five dimensional outcomes would include two heads?

4) A committee of 3 is to be formed from a group of 20 people. How many different committees are possible?

3 Taylor Expansions

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k} \tag{2}$$

A Taylor series of a function $f(x)$ about x_0 is the power series

$$\begin{aligned} f(x) &= f(x_0) + f'(x_0)(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + ... + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + ... & (3) \\ &= \sum_{k=0}^{\infty} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k & (4) \end{aligned}$$

A Maclaurin series is a Taylor expansion of a function about $x_0 = 0$.

1) Using binomial theorem, compute the following expansions:

$$\begin{aligned} &(x+y)^2 \\ &(x-y)^2 \\ &x^2 - y^2 \\ &(x+y)^3 \\ &x^3 + y^3 \end{aligned}$$

2) Find the Maclaurin expansions of the following functions $f(x) = \frac{1}{1-x}$ and $g(x) = exp(x)$.

4 Sums

1) Compute the sum of the first 100 integers, namely find the value of

$$\sum_{i=1}^{100} i = 1 + 2 + 3 + ... + 100.$$

2) Find a general formula for: $\sum_{i=1}^{n} i$.