Scalable Gaussian Processes with Quasiseparable Matrices[1]

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Abstract: An abstract. (Foreman-Mackey et al., 2017; Foreman-Mackey, 2018)

[1]This manuscript was prepared using the show your work! workflow (Luger, 2021). The source code used to generate this version can be found at dfm/quasisep-gps on GitHub.

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1 Quasiseparable matrices

There exist a range of definitions for *quasiseparable matrices* in the literature, so to be explicit, let's select the one that we will consider in all that follows. The most suitable definition for our purposes is nearly identical to the one used by Eidelman & Gohberg (1999), with some small modifications that will become clear as we go.

Let's start by considering an $N \times N$ *square quasiseparable matrix* $\boldsymbol{M}$ with lower quasiseparable order m_l and upper quasiseparable order m_u. In this note, we represent this matrix $\boldsymbol{M}$ as:[3]

$$M_{ij} = \begin{cases} d_i \quad , & \text{if } i = j \\ \boldsymbol{p}_i^\mathrm{T} \left(\prod_{k=i-1}^{j+1} \boldsymbol{A}_k\right) \boldsymbol{q}_j \quad , & \text{if } i > j \\ \boldsymbol{h}_i^\mathrm{T} \left(\prod_{k=i+1}^{j-1} \boldsymbol{B}_k^\mathrm{T}\right) \boldsymbol{g}_j \quad , & \text{if } i < j \end{cases} \tag{1}$$

[3]Comparing this definition to the one from Eidelman & Gohberg (1999), you may notice that we have swapped the labels of $\boldsymbol{g}_j$ and $\boldsymbol{h}_i$, and that we've added an explicit transpose to $\boldsymbol{B}_k^\mathrm{T}$. These changes simplify the notation and implementation for symmetric matrices where, with our definition, $\boldsymbol{g} = \boldsymbol{p}$, $\boldsymbol{h} = \boldsymbol{q}$, and $\boldsymbol{B} = \boldsymbol{A}$.

where

- i and j both range from 1 to N,
- d_i is a scalar,
- $\boldsymbol{p}_i$ and $\boldsymbol{q}_j$ are both vectors with m_l elements,
- $\boldsymbol{A}_k$ is an $m_l \times m_l$ matrix,
- $\boldsymbol{g}_j$ and $\boldsymbol{h}_i$ are both vectors with m_u elements, and
- $\boldsymbol{B}_k$ is an $m_u \times m_u$ matrix.

In Equation 1, the product notation is a little sloppy so, to be more explicit, this is how the products expand:

$$\begin{aligned} \prod_{k=i-1}^{j+1} \boldsymbol{A}_k &\equiv \boldsymbol{A}_{i-1}\,\boldsymbol{A}_{i-2}\cdots\boldsymbol{A}_{j+2}\,\boldsymbol{A}_{j+1} \quad \text{, and} \\ \prod_{k=i+1}^{j-1} \boldsymbol{B}_k^\mathrm{T} &\equiv \boldsymbol{B}_{i+1}^\mathrm{T}\,\boldsymbol{B}_{i+2}^\mathrm{T}\cdots\boldsymbol{B}_{j-2}^\mathrm{T}\,\boldsymbol{B}_{j-1}^\mathrm{T} \quad . \end{aligned} \tag{2}$$

$$\boldsymbol{M} = \begin{pmatrix} d_1 & \boldsymbol{h}_1^\mathrm{T}\boldsymbol{g}_2 & \boldsymbol{h}_1^\mathrm{T}\boldsymbol{B}_2^\mathrm{T}\boldsymbol{g}_3 & \boldsymbol{h}_1^\mathrm{T}\boldsymbol{B}_2^\mathrm{T}\boldsymbol{B}_3^\mathrm{T}\boldsymbol{g}_4 \\ \boldsymbol{p}_2^\mathrm{T}\boldsymbol{q}_1 & d_2 & \boldsymbol{h}_2^\mathrm{T}\boldsymbol{g}_3 & \boldsymbol{h}_2^\mathrm{T}\boldsymbol{B}_3^\mathrm{T}\boldsymbol{g}_4 \\ \boldsymbol{p}_3^\mathrm{T}\boldsymbol{A}_2\boldsymbol{q}_1 & \boldsymbol{p}_3^\mathrm{T}\boldsymbol{q}_2 & d_3 & \boldsymbol{h}_3^\mathrm{T}\boldsymbol{g}_4 \\ \boldsymbol{p}_4^\mathrm{T}\boldsymbol{A}_3\boldsymbol{A}_2\boldsymbol{q}_1 & \boldsymbol{p}_4^\mathrm{T}\boldsymbol{A}_3\boldsymbol{q}_2 & \boldsymbol{p}_4^\mathrm{T}\boldsymbol{q}_3 & d_4 \end{pmatrix} \tag{3}$$

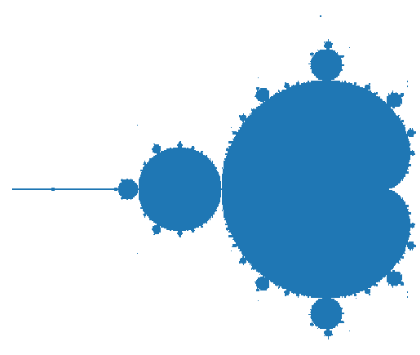

Figure 1: Face.

References

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