Geodynamics - Homework 4

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Pueden escribir $\operatorname{erf}\lambda, \operatorname{erfc}\lambda$ All problems extracted from (Turcotte, Schubert and Turcotte, 2002).

Problem 4.7

Table 4.4 gives a series of surface heat flow and heat production measurements in the Sierra Nevada Mountains in California. Determine the reduced heat flow q_m and the scale depth h_r.

Table 4.4 *Surface Heat Flow and Heat Production Data for the Sierra Nevada Mountains*

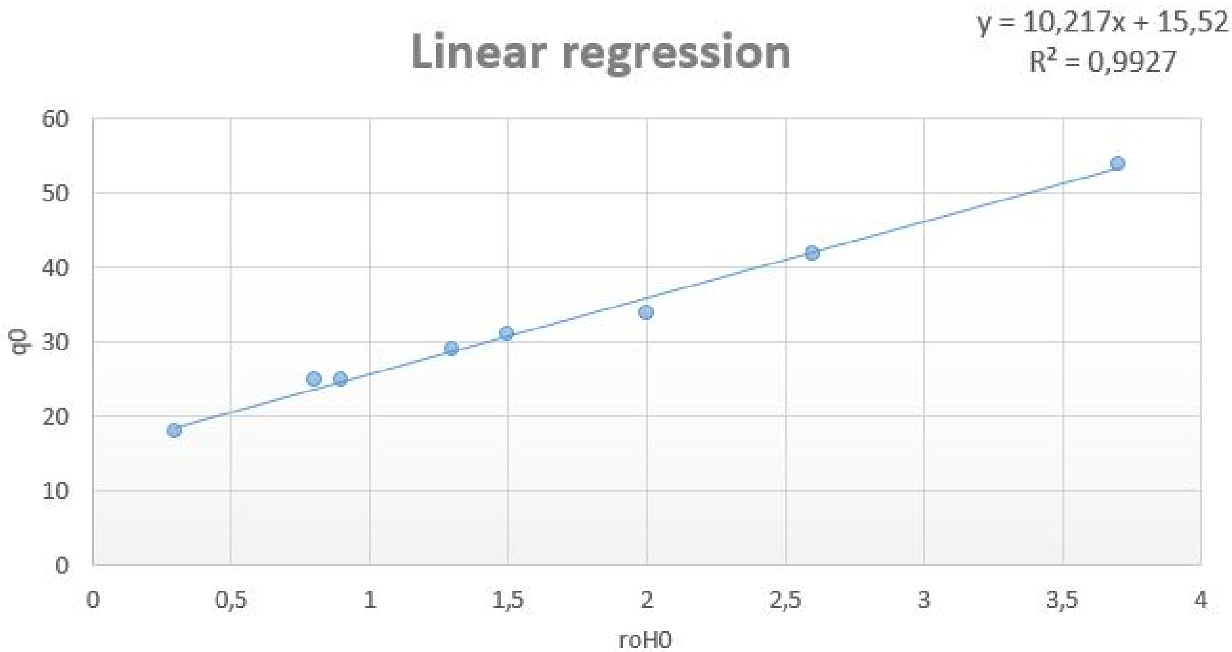
q_0 (mW m^{-2})	ρH_0 (μW m^{-3})	q_0 (mW m^{-2})	ρH_0 (μW m^{-3})
18	0.3	31	1.5
25	0.8	34	2.0
25	0.9	42	2.6
29	1.3	54	3.7

Solution:

In order to find these values, we have to use the following equation:

$$q_0 = q_m + \rho h_r H_0 \tag{1}$$

Because we have the respective values of q_0 and ρH_0, we can associate this equation with the equation of a linear function $y = mx + b$, where based on a linear regression of these data we can associate: $m = h_r$ and $b = q_m$.



Based on the above graph, we can see that the formula of the regression is:

$$y = 10.217x + 15.52 \tag{2}$$

Therefore:

$$q_m = 15.52\ mW/m \tag{3}$$
$$h_r = 10.217\ Km \tag{4}$$

Problem 4.11

The exponential depth dependence of heat production is preferred because it is self preserving upon erosion. However, many alternative models can be prescribed. Consider a two layer model with $H = H1$ and $k = k1$ for $0 \leq y \leq h1$, and $H = H2$ and $k = k2$ for $h1 \leq y2$. For $y > h2, H = 0$ and the upward heat flux is q_m. Determine the surface heat flow and temperature at $y = h2$ for $\rho_1 = 2600\ kgm^{-3}, \rho_2 = 3000\ kgm^{-3}, k1 = k2 = 2.4\ Wm^{-1}K^{-1}, h1 = 8\ km, h2 = 40\ km, \rho_1 H_1 = 2\ \mu Wm^{-3}, \rho_2 H_2 = 0.36\ \mu Wm^{-3}, T_0 = 0^o\ C$, and $qm = 28\ mWm^{-2}$

Solution:

Using the surface heat flow equation and modify for two layer model:

$$q_o = q_m + \rho h_r H_o \tag{5}$$

$$q_o = q_m + \rho_1 h_1 H_1 + \rho_2 h_2 H_2 \tag{6}$$
$$q_o = 28mW/m^2 + (2 * 10^{-3} mWm^3 * 8 * 10^3 m) + (0,36 * 10^{-3} mWm^3 32 * 10^3 m) \tag{7}$$
$$q_o = 28mW/m^2 + 16mW/m^2 + 11,52mW/m^2 = 55,52mW/m^2 \tag{8}$$

For temperature at y=h2 we use the equation:

$$T = T_o + \frac{q_o}{k} y - \frac{\rho H}{2k} y^2 \tag{9}$$

modify for two layer model:

$$T = T_o + \frac{q_o}{k}(y_1 + y_2) - \frac{\rho_1 H_1}{2k_1}(y_1)^2 - \frac{\rho_2 H_2}{2k_2} y_2^2 \tag{10}$$

$$T = 273k + \frac{55,5 * 10^{-3} mw/m^2}{2,4w/mk}(40 * 10^3 m) - \frac{2 * 10^{-6} mw/m^3}{2 * 2,4w/mk}(8 * 10^3 m)^2 - \frac{0,36 * 10^{-6} mw/m^3}{2 * 2,4w/mk}(32 * 10^3 m)^2 \tag{11}$$

$$T = 273K + 925K - 26,66K - 76,8K = 1094,9K = 820^o C \tag{12}$$

Problem 4.23

Using the relation $\tau = l^2/\kappa$ and taking $\kappa = 1\ mm^2/s$, determine the characteristic times for the conductive cooling of the Earth, Moon, Mars, Venus, and Mercury. What are the implications of these estimates?

Solution:

Taking that $\kappa = 1\ mm^2/s$:

$$\tau = l^2/\kappa \tag{13}$$

$$\tau = l^2 \tag{14}$$

Then, we can find that:

	$l(mm)$	$l^2(mm^2)$	$\tau(s)$	$\tau(years)$
Earth	$6371 * 10^6$	$4.06 * 10^{19}$	$4.06 * 10^{19}$	$1.29 * 10^{12}$
Moon	$1737 * 10^6$	$3.01 * 10^{18}$	$3.01 * 10^{18}$	$9.56 * 10^{10}$
Mars	$3390 * 10^6$	$1.15 * 10^{19}$	$1.15 * 10^{19}$	$3.64 * 10^{11}$
Venus	$6052 * 10^6$	$3.66 * 10^{19}$	$3.66 * 10^{19}$	$1.16 * 10^{12}$
Mercury	$2440 * 10^6$	$5.95 * 10^{18}$	$5.95 * 10^{18}$	$1.89 * 10^{11}$

Based on this values we can see that for this κ the time of conductive cooling is proportional to the radius of the planet. This because if we have the same value of κ for all the planets we are taking that the heat could propagate at the same velocity for all the planets independently of the own characteristics of each one.

Problem 4.44

Use the results of the sudden half-space heating problem, Equation (4–117), to estimate the time required for dike solidification by setting $Q = \rho Lb$. How does this time compare with the 10.9 days computed in the example?

Solution:

Using the equation 4-117 and modify to estimate the time required for dike solidification we obtain:

$$\rho Lb = \frac{2k(T_0 - T_1)}{\sqrt{\kappa\pi}} \sqrt(t) \tag{15}$$

By plug in the values used on the problem we get

$$L = 320\ kJ/Kg$$

$$T_m - T_o = 1000\ K$$

$$c = 1,2kJ/Kgk$$

$$b = 1\ m$$
$$\kappa = 0,5mm^2/s$$
$$\left(\frac{\rho}{k}\frac{Lb\sqrt{\kappa\pi}}{2(T_m - T_o)}\right)^2 = t \tag{16}$$

Using the definition of the thermal diffusivity we get the unknown terms K and ρ:

$$\kappa = \frac{K}{\rho c} \tag{17}$$

$$\frac{\rho}{K} = \frac{1}{\kappa c} = 0,6\frac{mm^2 kJ}{sKgK} \tag{18}$$

$$\left(0,6\frac{mm^2 kJ}{sKgK}\frac{320kJ/Kg1 * 10^3 mm\sqrt{0,5mm^2/s\pi}}{2(1000k)}\right)^2 = t \tag{19}$$

$$\left(266,67s/mm\sqrt{0,5mm^2/s\pi}\right)^2 = t \tag{20}$$

$$t = 111701,672s * \frac{1\ day}{86400\ s} = 1,29\ days. \tag{21}$$

If we compare this result with the 10.9 days computed in the example, we notice that the time required to solidify this intrusion is lower if we use the equation of the sudden half-space heating problem, Equation (4–117).

Problem 4.49

Determine the surface stress after 10 km of erosion. Take $E = 60\ GPa$, $v = 0.25$, $\alpha_l = 10^{-5}\ K^{-1}$, $\rho = 2700\ kgm^{-3}$, and $\beta = 20\ K.km^{-1}$.

Solution:

We have to take the equation:

$$\sigma_1 = \sigma_2 = \frac{h}{1-\nu}[(1-2\nu)\rho g - E\alpha_l\beta] \tag{22}$$

Then:

$$\sigma_1 = \sigma_2 = \frac{10\ km}{1-0.25}[(1-2(0.25))2700\ kg/m^3\ 9.8m/s - 60\ GPa10^{-5}\ K^{-1}20\ K.km^{-1}] \tag{23}$$

$$\sigma_1 = \sigma_2 = 176.4\ MPa - 160\ MPa = 16.4\ MPa \tag{24}$$

Problem 4.52

The ocean ridges are made up of a series of parallel segments connected by transform faults, as shown in Figure $1-12$. Because of the difference of age there is a vertical offset on the fracture zones. Assuming the theory just derived is applicable, what is the vertical offset (a) at the ridge crest and (b) 100 km from the ridge crest in Figure $4-46$ ($\rho m = 3300\ kgm^{-3}$, $\kappa = 1mm^2s^{-1}$,$\alpha_v = 3 * 10^{-5}K^{-1}, T1 - T0 = 1300K, u = 50mmyr^{-1}$).

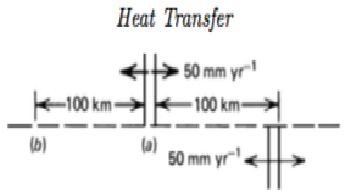


Figure 1: Problem 4.52

Solution:

Using the equation

$$w = \frac{2\rho_m \alpha_v (T_1 - T_0)}{(\rho_m - \rho_w)} \sqrt{\frac{\kappa x}{\pi u_0}} \tag{25}$$

$$w = \frac{2 * 3300Kg/m^3 3 * 10^{-5} K^{-1} 1300K}{(2300Kg/m^3)} \sqrt{\frac{1mm^2/s * x}{\pi 50 * (315336000)^- 1mm/s}} \tag{26}$$

$$w = 0,11191\sqrt{200764,411mm * x} \tag{27}$$

for x=100km

$$w = 0,11191\sqrt{200764,411mm * 100 * 10^6 mm} = 501432,24mm * 10^{-3} m/mm = 500m \tag{28}$$

for x=200

$$w = 0,11191\sqrt{200764,411mm * 200 * 10^6 mm} = 501432,24mm * 10^{-3} m/mm = 700m \tag{29}$$

know consider the that the ocean ridges are made up of a series of parallel segments connected by transform faults we have that the vertical offset 100km from the ridgw crest is:

$$x_2 - x_1 = 200km - 100km = 100km \tag{30}$$

$$w_2 - w_1 = 700m - 500m = 200m \tag{31}$$

Problem 4.53

Because of its cooling, the seafloor subsides relative to a continent at a passive continental margin. Determine the velocity of subsidence if $\rho_m = 3300\ kgm^{-3}$, $\kappa = 1\ mm^2 s^{-1}$, $T_1 - T_0 = 1300\ K$, $\alpha_v = 3 * 10^{-5}\ K^{-1}$, and the age is 20 Ma.

Solution:

Taking that:

$$\omega = \frac{2\rho_m \alpha_v (T_1 - T_0)}{\rho_m - \rho_w} (\frac{\kappa x}{\pi u_0})^{1/2} \tag{32}$$

and,

$$t = \frac{x}{u_0} \tag{33}$$

Then:

$$\omega = \frac{2\rho_m \alpha_v (T_1 - T_0)}{\rho_m - \rho_w} (\frac{\kappa t}{\pi})^{1/2} \tag{34}$$

Differentiating:

$$\frac{d\omega}{dt} = \frac{\rho_m \alpha_v (T_1 - T_0)}{\rho_m - \rho_w} \left(\frac{kt}{\pi}\right)^{-1/2} \frac{k}{\pi} \tag{35}$$

so:

$$\frac{d\omega}{dt} = v = \frac{3300\ kg/m^3\ 3 * 10^{-5}\ K^{-1}(1300\ K)}{2300\ kg/m^3} \left(\frac{1\ mm^2/s(6.31 * 10^{14}\ s)}{\pi}\right)^{-1/2} \frac{1\ mm^2/s}{\pi} \tag{36}$$

$$v = 1.304 * 10^{-9}\ mm/s \tag{37}$$

$$v = 0.04\ mm/year \tag{38}$$

Problem 4.55

What would be the decrease in sea level due to a 10 % reduction in the area of the continents? Assume the depth of deep ocean basins to be 5 km.

Solution

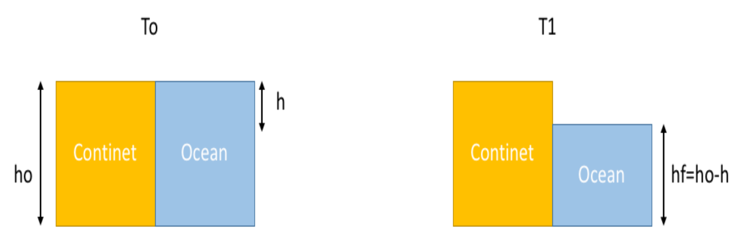


Figure 2: Problem 4.55

We propose 2 different moments where the volumes of the water are:

$$Vol_1 = (A_{total} - A_{continent})h \tag{39}$$

$$Vol_2 = (A_{total} - 0,9A_{continent})hf \tag{40}$$

assume that the volume of water is conserved

$$(A_{total} - A_{continent})h = (A_{total} - 0,9A_{continent})hf \tag{41}$$

$$\frac{(A_{total} - A_{continent})}{(A_{total} - 0,9A_{continent})}h = hf \tag{42}$$

we use $A_{total} = 5,1 * 10^{14}m^2, A_{continent} = 2 * 10^{14}m^2 and h_o = 5km$ and we get:

$$\frac{(5,1 * 10^{14}m^2 - 2 * 10^{14}m^2)}{(5,1 * 10^{14}m^2 - 0,9 * 2 * 10^{14}m^2)}5 * 10^3m = hf \tag{43}$$

$$4969,9m = hf \tag{44}$$

finally we get

$$\Delta h = ho - hf = 303,03m \tag{45}$$

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
        from scipy.special import erfinv, erf, erfc, erfcinv
```

Problem 4.2

```
In [2]: #Turcotte Problem 4.2
        #Import data from the table.
        data42=np.loadtxt('tab4-1.csv', delimiter=';', dtype='float')
        print data42
```

```
[[ 380.      18.362    3.2  ]
 [ 402.      18.871    1.7  ]
 [ 412.      19.33     5.3  ]
 [ 465.      20.446    6.1  ]
 [ 475.      20.58     3.4  ]
 [ 510.      21.331    1.9  ]
 [ 515.      21.51     0.   ]]
```

```
In [3]: def heatFlux(k, DT, thick):
            '''Returns the average heat flux q(W/m**2) from a single layer
            with k (W/m*K) thermal conductivity constant, thickess 'thick (m)'
            and with a temperature difference of DT (C, K)'''
            return k*DT/thick
        #Extract data from initial matrix
        depths=data42[:,0] #m
        Ts=data42[:,1]+273 #K
        #Calculate temperature differences
        DeltaT=np.roll(Ts, -1)-Ts
        DeltaT=DeltaT[:-1]
        theks=data42[:,2][:-1] #W/m*K
        #Calculate thicknesses
        thickness=np.roll(depths, -1)-depths
        thickness=thickness[:-1]
        resultq=heatFlux(theks, DeltaT, thickness)*1e3 #mW m^-2
        for i in range(len(resultq)):
            print 'The obtained value for the heat flux in the %d layer \n \
            is %.2f mW/m^2 \n '%(i+1, resultq[i])
        print 'Then, the mean value for q across the section \
        is %.2f mW/m^2'%np.mean(resultq)
```

```
The obtained value for the heat flux in the 1 layer
     is 74.04 mW/m^2

The obtained value for the heat flux in the 2 layer
     is 78.03 mW/m^2

The obtained value for the heat flux in the 3 layer
     is 111.60 mW/m^2

The obtained value for the heat flux in the 4 layer
     is 81.74 mW/m^2
```

```
The obtained value for the heat flux in the 5 layer
     is 72.95 mW/m^2

The obtained value for the heat flux in the 6 layer
     is 68.02 mW/m^2

Then, the mean value for q across the section is 81.06 mW/m^2
```

Problem 4.32

$$\frac{T - T_1}{T_0 - T_1} = erfc\frac{y}{2\sqrt{\kappa t}}$$

$$T|_{y=0} = T_0$$

$$T|_{t=0} = T_1$$

Thus, $T_0 - T_1 = 10K$ or assume $T_0 = 10K,\ T_1 = 0K$
$y = 1m$; $T = 2K$

$$\eta = \frac{y}{2\sqrt{\kappa t}}$$

$$\sqrt{t} = \frac{y}{2\sqrt{\kappa}\eta}$$

$$t = \left(\frac{y}{2\sqrt{\kappa}\eta}\right)^2$$

In [4]:
```
# Turcotte Problem 4.32
#Given values
kappa=1. #mm**2/s
y=1e3 #mm
Tf=2. #K
T1=0
T0=T1+10
#Value of erfc with given conditions.
erfval=(Tf-T1)/(T0-T1)
#Value of the argument eta of erfc(eta).
eta=erfcinv(erfval)
#Time calculated from eta.
t=(3600.*24)**-1 * (y/(2*np.sqrt(kappa)*eta))**2 #days

print 'The time it takes to increase temperature in 2K at 1m depth \
is %.2f days'%t
```

```
The time it takes to increase temperature in 2K at 1m depth is 3.52 days
```

Additional Problem

In [5]:
```
# Additional problem
```

Assume a sill of basaltic magma (1200K) 5 meters thick is intruded into 200K crust (5km depth). A) How long does it take to solidify assuming ($L = 320kJ/kg, c = 1.2kJ/kgK, kappa = 0.5mm^2/s$)?. B) What is the temperature of the sill - country rock contact at that time? C) Plot the approximate temperature distribution from the sill assuming a full-space solution after 1 year and 10 years.

The equation for the solidification of a sill is:

$$\frac{L\sqrt{\pi}}{c(T_m - T_0)} = \frac{\exp{(-\lambda_2^2)}}{\lambda_2(1 + erf\lambda_2)}$$

where

$$\lambda_2 = -\frac{y_m}{2\sqrt{\kappa t}} \Rightarrow \sqrt{t} = -\frac{y_m}{2\sqrt{\kappa}\lambda_2}$$

and y_m represents the depth from the surface of the sill to which it has solidified, so for knowing when it has solidified completely we shall say

$$y_m = b$$

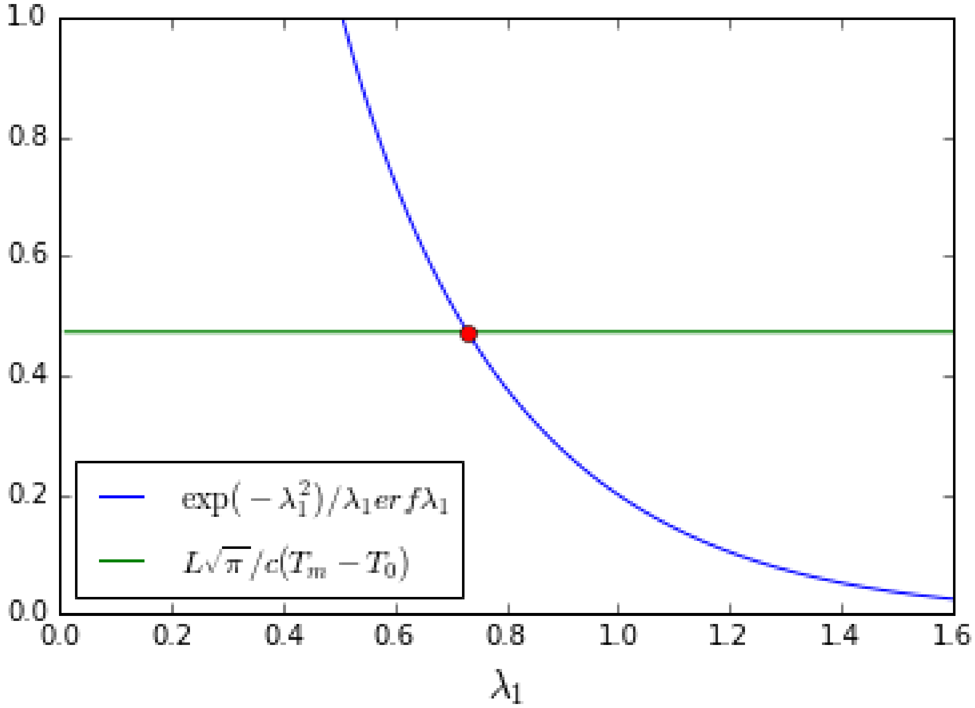
where $2b$ is the thickness of the sill.

In [9]:
```
#Array of possible lambda2s
lambda2=np.linspace(0.01, 1.6, 10000)

def theRightSide(lambda2):
    '''Returns the (adimensional) right side of the equation 4-149
    from Turcotte for a given value of lambda2'''
    return np.exp(-lambda2**2)/(lambda2*(1+erf(lambda2)))

#Array for possible values of the right side of the equation.
rightSide=theRightSide(lambda2)
#Given values.
L=320e3 #J/kg
c=1.2e3 #J/kgK
kappa=0.55*(1e-3)**2 #m**2/s
th=5. #m =2*b
b=th/2 #m
Tm=1200. #K
T0=200. #K
#Left side of the equation 4-149 from Turcotte.
leftSide=L*np.sqrt(np.pi)/(c*(Tm-T0))
#Array with leftside constant values.
horiz=np.ones(len(lambda2))*leftSide
```

In [10]:
```
#Plot of both sides of the equation 4-149.
plt.plot(lambda2, rightSide, \
         label='$\exp(-\lambda_1^2)/\lambda_1 erf\lambda_1$')
plt.plot(lambda2, horiz, label='$L\sqrt{\pi}/c(T_m-T_0)$')
plt.xlim(0, 1.6)
plt.ylim(0, 1)
#Graphical solution for the equation.
idx = np.argwhere(np.isclose(rightSide, horiz, atol=1e-4)).reshape(-1)
plt.plot(lambda2[idx], rightSide[idx], 'ro')
plt.xlabel('$\lambda_1$', fontsize=15)
plt.legend(loc=0)
plt.show()
print 'The value for lambda that solves the equation is %.4f'%lambda2[idx]
```




```
The value for lambda that solves the equation is 0.7305
```

In [16]:
```
theLambda=lambda2[idx]
#Time needed for the sill to solidify is calculated.
t=b/(2*np.sqrt(kappa)*theLambda)
t=(3600.*24)**-1 *t**2 #days
print 'The time necessary for the sill \
to be completely solidified is t=%.3f days'%t
```

```
The time necessary for the sill to be completely solidified is t=61.617 days
```

$$T - T_0 = \frac{erfc(\eta)}{erfc(-\lambda_2)}(T_m - T_0)$$

$$\eta = \frac{y}{2\sqrt{\kappa t}}$$

The sill - country rock boundary is defined to be $y = 0$, so $\eta = 0\ \forall t > 0$ at the interface. Therefore

$$T|_{y=0} = \frac{1}{erfc(-\lambda_2)}(T_m - T_0) + T_0$$

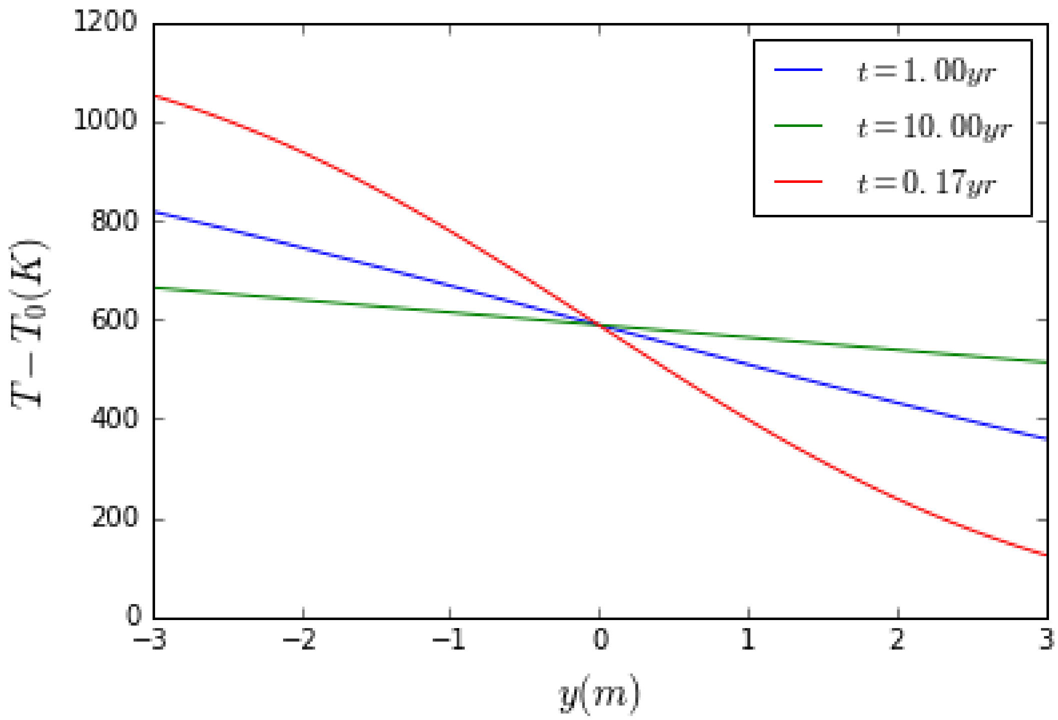
In [17]:
```
interfaceT=(erfc(-theLambda))**-1* (Tm-T0) + T0
print 'The temperature at the sill - country rock interface \
is %.3f K'%interfaceT
```

```
The temperature at the sill - country rock interface is 788.777 K
```

```
In [18]: def TmenT0(eta):
             return (Tm-T0)*erfc(eta)/(1.+erf(theLambda))
         def etafromy(y, t):
             return y/(2*np.sqrt(kappa*t))

In [25]: ys=np.linspace(-3, 3, 100)
         theEta=etafromy(ys, 1.)
         ayear=24*3600.*365.25#s/yr
         possibleTimes=ayear*np.array([1., 10, 1./6])#s
         for i in possibleTimes:
             plt.plot(ys, TmenT0(etafromy(ys, i)), label='$t=%.2f yr$'%(i/ayear))
         plt.legend(loc=0)
         plt.ylim(0, 1200)
         plt.xlabel('$y(m)$', fontsize=15)
         plt.ylabel('$T-T_0 (K)$', fontsize=15)

Out[25]: <matplotlib.text.Text at 0x81e04a8>
```



Since the time it takes for the sill to solidify is less than a year, it is expected that after 10 years the sill has not just solidified but reached thermal equilibrium with the country rock. This accounts for the almost-horizontal line at 10 years time.