Geodynamics - Homework 3

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All problems extracted from (Turcotte, Schubert and Turcotte, 2002).

Problem 3.1

Determine the surface stress after the erosion of 10 km of granite. Assume that the initial state of stress is lithostatic and that $\rho = 2700\ kg/m^3$ and $\nu = 0.25$.

Solution:
Using the equation 3.30 from the book, the horizontal surface stresses are given by:

$$\bar{\sigma_2} = (\frac{1-2\nu}{1-\nu})\rho g h \tag{1}$$

If $h = 10\ km$, $\rho = 2700\ kg/m^3$ and $\nu = 0.25$, then:

$$\bar{\sigma_2} = (\frac{1-2(0.25)}{1-(0.25)})(2700\ kg/m^3)(9.8\ m/s^2)(10000\ m) \tag{2}$$

$$\bar{\sigma_2} = 176400000\ Pa = 176.4\ MPa \tag{3}$$

Problem 3.2

An unstressed surface is covered with sediments with a density of 2500 kg/m^3 to a depth of 5 km. If the surface is laterally constrained and has a Poisson's ratio of 0.25, what are the three components of stress at the original surface?

Solution:
For σ_1:
Taking $g = 9.8\ m/s^2$:

$$\bar{\sigma_1} = \sigma_1 + \Delta\sigma_1 \tag{4}$$

$$\Delta\sigma_1 = \bar{\sigma_1} - \sigma_1 \tag{5}$$

$$\Delta\sigma_1 = 0 - \rho g h \tag{6}$$

$$\Delta\sigma_1 = 122.5\ MPa \tag{7}$$

For σ_2 and σ_3:
Taking $g = 9.8\ m/s^2$:

$$\Delta\sigma_2 = \Delta\sigma_3 = (\frac{\nu}{1-\nu})\Delta\sigma_1 \tag{8}$$

$$\Delta\sigma_2 = \Delta\sigma_3 = 40.83\ MPa \tag{9}$$

Problem 3.14

A granite plate freely supported at its ends spans a gorge 20 m wide. How thick does the plate have to be if granite fails in tension at 20 MPa? Assume $\rho = 2700kgm^{-3}$ (Hint - use solution from 3-13 in the back of the book. If you'd like the additional challenge I encourage you to try to solve 3-13).

Solution:
Using equations from the solution 3-13

$$\sigma_{Maxbendingstress} = \frac{3L^2q}{4h^2} \tag{10}$$

Where $q = \rho g h_{thickness}$ and $\sigma_{Maxbendingstress} = 20Mpa$ so we obtain that:

$$\sigma = \frac{3L^2\rho g h}{4h^2} \tag{11}$$

$$\sigma_{=}\frac{3L^2\rho g}{4h} \tag{12}$$

$$h = \frac{3L^2\rho g}{4\sigma} \tag{13}$$

$$h = \frac{3 * 20^2m * 2700kg/m^3 * 10m/s^{"}}{4 * 20 * 10^6kg/ms^2} \tag{14}$$

$$h = 0,405m \tag{15}$$

Problem 3.22

The Amazon River basin in Brazil has a width of 400 km. Assuming that the basin is caused by a line load at its center and that the elastic lithosphere is not broken, determine the corresponding thickness of the elastic lithosphere. Assume $E = 70\ GPa$, $\nu = 0.25$, and $\rho_m - \rho_s = 700\ kg/m^3$.

(Note: I obtained a different answer from the back of the book, but you should be within $\pm 10\ km$)

Solution:

$$X_b = \pi * \alpha \tag{16}$$

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_s) * g}\right]^{\frac{1}{4}} \tag{17}$$

We use this to equations to get the variable D

$$\frac{X_b}{\pi} = \left[\frac{4D}{(\rho_m - \rho_s) * g}\right]^{\frac{1}{4}} \tag{18}$$

$$\left[\frac{X_b}{\pi}\right]^4 \left[\frac{(\rho_m - \rho_s) * g}{4}\right] = D \tag{19}$$

We know that

$$D = \frac{ET_e^3}{12(1 - \nu^2)} \tag{20}$$

Using this two equations we get

$$\left[\frac{X_b}{\pi}\right]^4 \left[\frac{(\rho_m - \rho_s) * g * 12 * (1-\nu^2)}{4E}\right] = T_e^3 \tag{21}$$

$$\left[\left[\frac{X_b}{\pi}\right]^4 \left[\frac{(\rho_m - \rho_s) * g * 12 * (1-\nu^2)}{4E}\right]\right]^{\frac{1}{3}} = T_e \tag{22}$$

Using the values of the problem we obtain that T_e is equal to:

$$\left[\left[\frac{200 * 10^3 m}{\pi}\right]^4 \left[\frac{(700 kg/m^3) * 10 m/s^2 * 12 * (1-0.25^2)}{4 * 70 * 10^9 kg/ms^2}\right]\right]^{\frac{1}{3}} = T_e \tag{23}$$

$$T_e = 16654.73m = 16.65km \tag{24}$$

Problem 5.2

Determine the ratio of the centrifugal acceleration to the gravitational acceleration at the Earth's equator.

Solution:
Using equation 5.48 of the book:

$$g_\omega = \omega^2 r \cos\phi \tag{25}$$

And taking $\omega = 7.292115.10^{-5}\ rad/s$, $r = 6371000\ m$ and $\phi = 0$ because we are ar the equator:

$$g_\omega = 0.033877\ m/s^2 \tag{26}$$

And the gravitational acceleration could be given by:

$$g(\phi) = 9.7803253359 \frac{1 + 0.00193185265241 \sin^2(\phi)}{\sqrt{1 - 0.00669437999014 \sin^2(\phi)}} \tag{27}$$

Then:

$$g(\phi) = 9.7803\ m/s^2 \tag{28}$$

Then, the ratio between the centrifugal acceleration to the gravitational acceleration at the Earth's equator is:

$$\frac{0.033877\ m/s^2}{9.7803\ m/s^2} = 3.4638.10^{-3} \tag{29}$$

Problem 5.5

In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

Problem 5-5 from Turcotte and Schubert: Determine the values of the acceleration of gravity at the equator and the poles using GRS 80 and the quadratic approximation given in Equation (5–72).

Equation 5-72:

$$g_0 = \frac{GM}{a^2}\left(1 + \frac{3}{2} J_2 \cos^2\phi\right) + a\omega^2(\sin^2\phi - \cos^2\phi). \tag{30}$$

GRS80 equation from T&S:

$$g0 = 9.7803267715(1 + 0.0052790414\sin^2\phi + 0.0000232718\sin^4\phi + 0.0000001262\sin^6\phi + 0.0000000007\sin^8\phi) \tag{31}$$

From Wikipedia's GRS80 page:
Defining geometrical constants :

- Semi-major axis = Equatorial Radius =

$a = 6378137m$
Defining physical constants:

- Geocentric gravitational constant, including mass of the atmosphere

$GM = 3986005 \times 10^8 m^3/s^2$

- Dynamical form factor

$J_2 = 108263 \times 10^{-8}$

- Angular velocity of rotation

$\omega = 7292115 \times 10^{-11} rad/s$

In [3]:
```
def GRS_g0(phi):
    #returns gravity acceleration according to GRS80 standard
    # in m/s^2
    return 9.7803267715*(1+0.0052790414*(np.sin(phi))**2+\
                         0.0000232718*(np.sin(phi))**4+\
                         0.0000001262*(np.sin(phi))**6+\
                         0.0000000007*(np.sin(phi))**8)

a=6378137. #m
GM=3986005e8 #m^3/s^2
J2=108263e-8 #dimensionless
omega=7292115e-11 #rad/s
def g0(phi):
    #returns gravity acceleration according to GRS80 constants and T&S' equation 5-72
    # in m/s^2
    global a
    global GM
    global J2
    global omega
    return (GM/(a**2))*(1+1.5*J2*(np.cos(phi))**2)\
             +(a*omega**2)*((np.sin(phi))**2-(np.cos(phi))**2)
```

In [17]:
```
#usingEquation=[northPole, southPole, equator]
usingGRS=np.array([GRS_g0(np.pi/2), GRS_g0(-np.pi/2), GRS_g0(0.)])
usingEq572=np.array([g0(np.pi/2), g0(-np.pi/2), g0(0.)])

print 'The value for gravity at the north and south poles \n \
from GRS80 equation are respectively %(north).6f \n and %(south)\
.6f, while for the equator it is %(equator).6f \n '\
%{'north':usingGRS[0],'south':usingGRS[1], 'equator':usingGRS[2]}
```

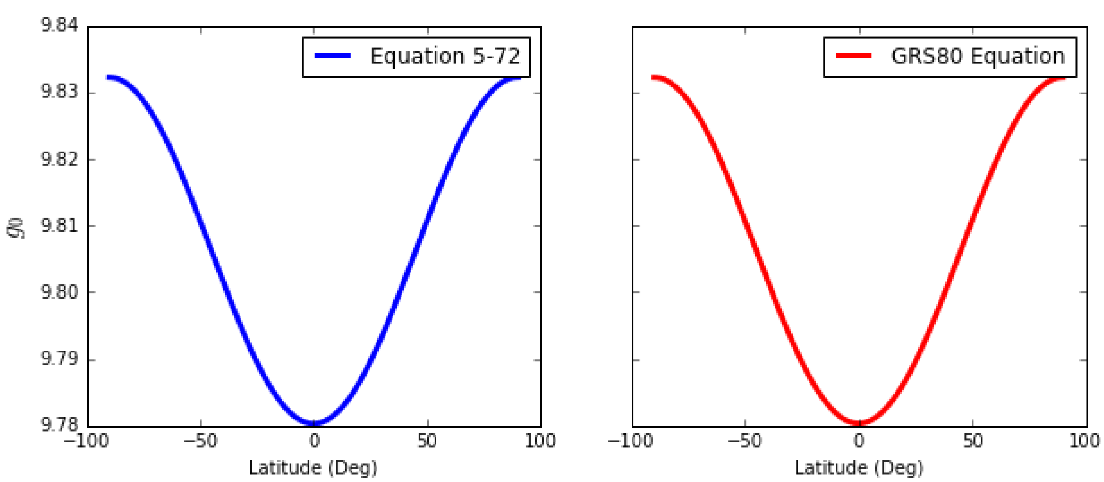
```
print 'The value for gravity at the north and south poles \n \
from equation 5-72 are respectively %(north).6f \n and %(south)\
.6f, while for the equator it is %(equator).6f \n '\
%{'north':usingEq572[0],'south':usingEq572[1], 'equator':usingEq572[2]}

errPerc=abs(100.*(usingEq572-usingGRS)/usingEq572)
print 'The percentage difference between methods is \n of the order of 4e-4%.',errPerc
```

```
The value for gravity at the north and south poles
 from GRS80 equation are respectively 9.832186
 and 9.832186, while for the equator it is 9.780327

The value for gravity at the north and south poles
 from equation 5-72 are respectively 9.832203
 and 9.832203, while for the equator it is 9.780283

The percentage difference between methods is
 of the order of 4e-4%. [ 0.00016525  0.00016525  0.0004467 ]
```

It is possible to see that there are slight differences between methods, but these are truly negligible as the percentual differences between values are of the order of 10^{-4}%.

In [5]:
```
latitude=np.linspace(-np.pi/2, np.pi/2, 1000)
gravities, ax=plt.subplots(1, 2, sharey=True, figsize=(10,4))

ax[0].plot(np.rad2deg(latitude), g0(latitude), label='Equation 5-72', lw=3)
ax[1].plot(np.rad2deg(latitude), GRS_g0(latitude), label='GRS80 Equation', c='r', lw=3)
for i in range(len(ax)):
    ax[i].set_xlabel('Latitude (Deg)')
    ax[i].legend(loc=0)
ax[0].set_ylabel('$g_0$', fontsize=15)
```

Out[5]: `<matplotlib.text.Text at 0x6fd23c8>`



In []:

Problem 5.12

A gravity profile across the Pyramid No. 1 ore body near Pine Point, Northwest Territories, Canada, is shown in Figure 5–10. A reasonable fit with Equation (5–102) is obtained taking b = 200 m and $4\pi GR^3\Delta\rho/3b^2 = 0.006mms-2$. Assume that the gravity anomaly is caused by lead–zinc ore with a density of 3650 kg m−3 and that the country rock has a density of 2650 kg m−3. Estimate the tonnage of lead–zinc ore, assuming a spherical body. The tonnage established by drilling in this ore body was 9.2 million tons.

Solution:
We know that

$$\frac{4\pi GR^3\Delta_\rho}{3b^2} = 0,006mms^{-2} \tag{32}$$

From this equation we can obtain the radios

$$\frac{3b^2 * 0,006mms^{-2}}{4\pi G\Delta_\rho} = R^3 \tag{33}$$

Where $b = 200m, G = 6,674 * 10^{-11}\frac{Nm^2}{Kg^2}$ and $\Delta_\rho = 3650Kg/m^3 - 2650Kg/m^3$

$$\frac{3(200m)^2 * 0.000006ms^{-2}}{4\pi G * 1000Kg/m^3} = R^3 \tag{34}$$

$$\frac{3(200m)^2 * 0.000006ms^{-2}}{4\pi 6,674 * 10^{-11}\frac{Nm^2}{Kg^2} * 1000Kg/m^3} = R^3 \tag{35}$$

$$R^3 = \frac{0,72m^3s^{-2}}{8.3868 * 10^{-7}s^{-2}} = 858492m^3 \tag{36}$$

Using the formula of the volume of the sphere we get:

$$V = \frac{4\pi R^3}{3} = \frac{4\pi(858492\ m^3)}{3} = 3,59604 * 10^6\ m^3 \tag{37}$$

If we use the density of the material we will get the total mass

$$m = \rho * vol. = 3650\ kg/m^3 * 3,59604 * 10^6\ m^3 = 1.31256 * 10^{10}\ kg \tag{38}$$

Finally we convert to tons and the final answer is that:

$$1.31256 * 10^{10}\ kg = 12918301,19\ tons \tag{39}$$

Problem 5.16

A seamount with a density of $2900kg/m^3$ rests on the seafloor at a depth of 5 km. What is the expected surface gravity anomaly if the seamount just reaches the sea surface? (Assume the width to height ratio of the seamount is large and that it does not deflect the seafloor on which it rests.)

Solution:

$\rho_{sm} = 2900\ kg/m^3$

$\rho_w = 1000\ kg/m^3$

h=5000 m

$G = 6,674 * 10^{-11} \frac{Nm^2}{Kg^2}$

Using the equation

$$\Delta_g = 2\pi G h \Delta_\rho \tag{40}$$

We obtain that the expected surface gravity anomaly is :

$$\Delta_g = 2\pi * 6,674 * 10^{-11} \frac{Nm^2}{Kg^2} * 5000m * (2900Kg/m^3 - 1000Kg/m^3) \tag{41}$$

$$\Delta_g = 0,003984m/s^2 = 3,98mm/s^2 \tag{42}$$

Problem 5.20

The surface gravity at a measuring site is $9.803243m/s^2$. The site has a latitude 43° 32' 16" N and an elevation of 542.3 m. Obtain the free-air and Bouguer gravity anomalies.

Solution:

To begin, we have lo calculate he value of g_0, using the formula 5.73 of the book:

$$g_0 = 9.7803267715(1+0.0052790414\sin^2(\phi)+0.0000232718\sin^4(\phi)+0.0000001262\sin^6(\phi)+0.0000000007\sin^8(\phi)) \tag{43}$$

Turning the coordinates to degrees, we obtain:

$$\phi = 43 + \frac{32}{60} + \frac{16}{3600} \tag{44}$$

$$\phi = 43.5348^\circ \tag{45}$$

Then, we obtain that:

$$g_0 = 9.8049\ m/s^2 \tag{46}$$

Now, the elevation correction is:

$$\Delta_{gh} = \frac{2hg_0}{r_0} \tag{47}$$

$$\Delta_{gh} = \frac{2(542.3\ m)(9.8049\ m/s^2)}{6378000\ m} \tag{48}$$

$$\Delta_{gh} = 0.0017\ m/s^2 \tag{49}$$

Then, the free air anomaly is:

$$\Delta g_{fa} = 9.803243\ m/s^2 - 9.8049\ m/s^2 + 0.0017\ m/s^2 \tag{50}$$

$$\Delta g_{fa} = 0.0339\ mm/s^2 \tag{51}$$

Now, the Bouguer gravitational anomaly is:

$$\Delta g_B = \Delta g_{fa} - 2\pi G \rho_c h \tag{52}$$

$$\Delta g_B = -0.5733\ mm/s^2 \tag{53}$$

References

Turcotte, D., Schubert, G. and Turcotte, D. (2002). Geodynamics. Cambridge: Cambridge University Press.