Sol_HW_8

April 25, 2018

```
In [1]: from sympy import *
        init_printing()
        from IPython.display import display
        from sympy.functions import Abs
```

0.1 Problema 3

```
In [2]: p,x,k = symbols('p x k',\
                            real=True)
        s, h, m, t, k0, p1 = symbols('sigma hbar m t k_0 p_1', real=True, \
                    positive=True)
        mom_repr = nsimplify(exp(-Abs(p)/(h*k0)))
        Nsq=integrate(mom_repr*mom_repr, (p,-oo,oo), conds='none')
        mom_repr/=sqrt(Nsq)

        mom_repr_t = exp(-I*(p**2/(2*m))*t/h)*mom_repr
        display(simplify(mom_repr_t))
        abs(mom_repr)**2==abs(mom_repr_t)**2
```

$$\frac{1}{\sqrt{\hbar}\sqrt{k_0}}e^{\frac{1}{\hbar}\left(-\frac{ip^2t}{2m}-\frac{|p|}{k_0}\right)}$$

```
Out[2]: True
```

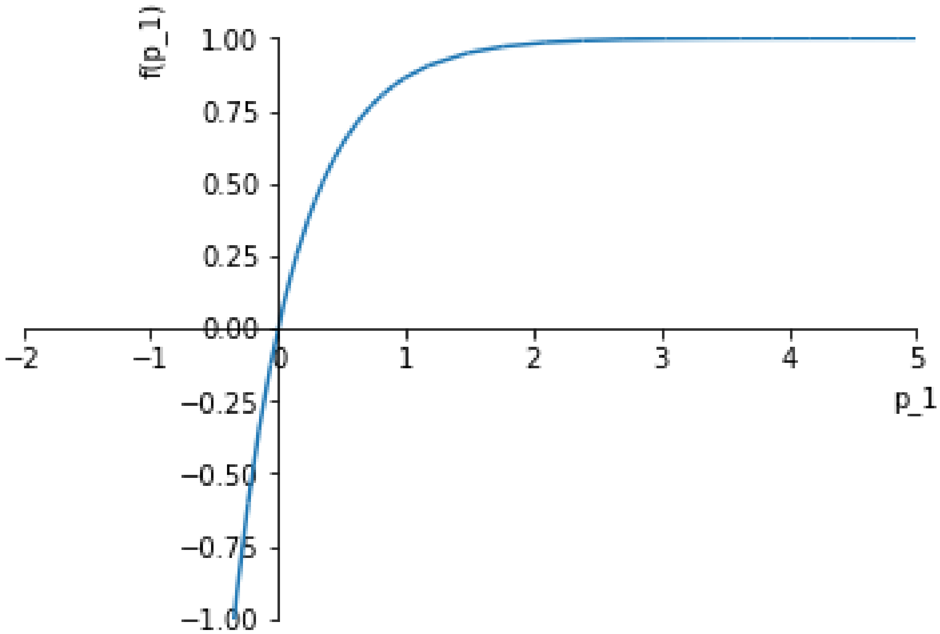
```
In [3]: P_p1 = 2*integrate(exp(-2*p/(h*k0))/Nsq, (p, 0, p1), conds='none')
        pl=plot(P_p1.subs(h,1).subs(k0,1), xlim=(-2,5), ylim=(-1,1))
        display(P_p1)
```

```
<matplotlib.figure.Figure at 0x7f7f6deeb490>
```

$$1-e^{-\frac{2p_1}{\hbar k_0}}$$

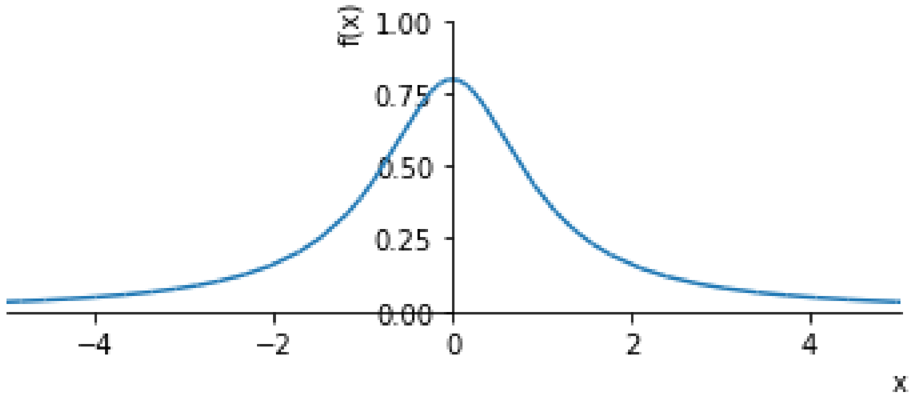
```
<matplotlib.figure.Figure at 0x7f7f691f1f90>
```

```
In [ ]: pl.show()
```



In [4]:
```
pos_repr = (1/sqrt(2*pi*h))*integrate(simplify(mom_repr*exp(I*p*x/h)), (p,-oo,oo), conds
display(simplify(cancel(pos_repr)))
Nsq = integrate(abs(pos_repr)**2, (x,-oo,oo), conds='none')
pos_repr/=sqrt(Nsq)
pt=plot(simplify(pos_repr.subs(k0,1)), xlim=(-5,5), ylim=(-1,1))
```

$$\frac{\sqrt{2}\sqrt{k_0}}{\sqrt{\pi}\left(k_0^2x^2+1\right)}$$



In [5]:
```
delta_p_sq = integrate(mom_repr*mom_repr*p**2, (p,-oo,oo), conds='none')
avg_p = integrate(mom_repr*mom_repr*p, (p,-oo,oo), conds='none')
display(avg_p)
delta_x_sq = integrate(pos_repr*pos_repr*x**2, (x,-oo,oo), conds='none')
display(sqrt(delta_x_sq), sqrt(delta_p_sq))
sqrt(delta_x_sq*delta_p_sq)>=h/2
```

$$0$$

$$\frac{1}{k_0}$$

$$\frac{\hbar k_0}{2}\sqrt{2}$$

Out[5]:

$$\frac{\sqrt{2}\hbar}{2} \geq \frac{\hbar}{2}$$

In [6]:
```
delta_p_t_sq = integrate(Abs(mom_repr_t)**2*p**2, (p,-oo,oo), conds='none')
display(delta_p_t_sq)
```

$$\frac{\hbar^2 k_0^2}{2}$$

1 Punto 14

In [7]:
```
from sympy.physics.matrices import msigma
a,b, w = symbols('a b omega_0', real=True)
H = h*w*diag(1, 2, 2)
A = a*diag(1,msigma(1))
B = b*diag(msigma(1), 1)
```

In [8]:
```
display(H.eigenvects())
Psi_0 = nsimplify(1/sqrt(2))*H.eigenvects()[0][2][0]+nsimplify(0.5)*H.eigenvects()[1][2]
display(Psi_0)
display('avg E', nsimplify(0.5*H.eigenvects()[0][0] + 2*0.25*H.eigenvects()[1][0]))

display('delta E', sqrt(simplify(nsimplify(0.5*H.eigenvects()[0][0]**2 + 2*0.25*H.eigenv
      nsimplify(0.5*H.eigenvects()[0][0] + 2*0.25*H.eigenvects()[1][0])**2))
```

$$\left[\left(\hbar\omega_0,\quad 1,\quad \left[\begin{bmatrix}1\\0\\0\end{bmatrix}\right]\right),\quad \left(2\hbar\omega_0,\quad 2,\quad \left[\begin{bmatrix}0\\1\\0\end{bmatrix},\quad \begin{bmatrix}0\\0\\1\end{bmatrix}\right]\right)\right]$$

$$\begin{bmatrix}\frac{\sqrt{2}}{2}\\ \frac{1}{2}\\ \frac{1}{2}\end{bmatrix}$$

'avg E'

$$\frac{3\omega_0}{2}\hbar$$

'delta E'

$$\frac{\hbar\left|\omega_0\right|}{2}$$

In [9]:
```
A.eigenvects()
#Note que los vectores propios de A son u1 (vp a), N*(u2+u3) (vp a) y N*(-u2+u3) (vp -a)
```

Out[9]:

$$\left[\left(-a,\quad 1,\quad \left[\begin{bmatrix}0\\-1\\1\end{bmatrix}\right]\right),\quad \left(a,\quad 2,\quad \left[\begin{bmatrix}1\\0\\0\end{bmatrix},\quad \begin{bmatrix}0\\1\\1\end{bmatrix}\right]\right)\right]$$

In [10]:
```
Psi_t = Matrix([[exp(-I*t*H.eigenvects()[0][0])*Psi_0[0]],\
                [exp(-I*t*H.eigenvects()[1][0])*Psi_0[1]],[exp(-I*t*H.eigenvects()[1][0
display(Psi_t)
Psi_t_T = adjoint(Psi_t)
display(Psi_t_T)
```

$$\begin{bmatrix}\frac{\sqrt{2}}{2}e^{-i\hbar\omega_0 t}\\ \frac{1}{2}e^{-2i\hbar\omega_0 t}\\ \frac{1}{2}e^{-2i\hbar\omega_0 t}\end{bmatrix}$$

$$\begin{bmatrix}\frac{\sqrt{2}}{2}e^{i\hbar\omega_0 t} & \frac{1}{2}e^{2i\hbar\omega_0 t} & \frac{1}{2}e^{2i\hbar\omega_0 t}\end{bmatrix}$$

```
In [11]: display(Psi_t_T*A*Psi_t)
         display(Psi_t_T*B*Psi_t)
```

$$\begin{bmatrix}a\end{bmatrix}$$

$$\begin{bmatrix}\frac{\sqrt{2}b}{4}e^{i\hbar\omega_0 t} + \frac{b}{4} + \frac{\sqrt{2}b}{4}e^{-i\hbar\omega_0 t}\end{bmatrix}$$

```
In [12]: simplify((A*Psi_t)) #Se mide a, note que A*Psi_t = a* Psi_t
```

Out[12]:

$$\begin{bmatrix}\frac{\sqrt{2}a}{2}e^{-i\hbar\omega_0 t}\\ \frac{a}{2}e^{-2i\hbar\omega_0 t}\\ \frac{a}{2}e^{-2i\hbar\omega_0 t}\end{bmatrix}$$

```
In [13]: B.eigenvects()
```

Out[13]:

$$\left[\left(-b,\quad 1,\quad \left[\begin{bmatrix}-1\\1\\0\end{bmatrix}\right]\right),\quad \left(b,\quad 2,\quad \left[\begin{bmatrix}1\\1\\0\end{bmatrix},\quad \begin{bmatrix}0\\0\\1\end{bmatrix}\right]\right)\right]$$

```
In [14]: #Medir -b
         display(simplify((abs((nsimplify(1/sqrt(2))*adjoint(B.eigenvects()[0][2][0])*Psi_t)[0])
         #Medir b
         display(simplify((abs(nsimplify(1/sqrt(2))*adjoint(B.eigenvects()[1][2][0])\
                          *Psi_t)[0]**2+abs(adjoint(B.eigenvects()[1][2][1])*Psi_t)[0]**2).r
```

$$-\frac{\sqrt{2}}{4}\cos\left(\hbar\omega_0 t\right) + \frac{3}{8}$$

$$\frac{\sqrt{2}}{4}\cos\left(\hbar\omega_0 t\right) + \frac{5}{8}$$

2 Problema 9

Tenemos

$$\psi(r) = \sqrt{\rho(r)}\exp(i\zeta(r))$$

(a)

$$J_i = \frac{\hbar}{2mi}(\psi^*\partial_i\psi - \psi\partial_i\psi^*)$$

$$J_i = \frac{-i\hbar}{2m}(i\rho\partial_i\zeta + i\rho\partial_i\zeta) = \frac{\hbar}{m}\rho\partial_i\zeta$$