Sol_HW_9

May 13, 2018

```
In [1]: from sympy import *
        init_printing()
        from IPython.display import display
        from sympy.physics.matrices import msigma
        from sympy.physics.quantum.dagger import Dagger
        from sympy.physics.quantum import Ket, Bra
        from sympy.physics.quantum.state import Wavefunction
        from sympy.physics.quantum import TensorProduct as TP
```

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```
In [2]: z = symbols('xi', real=True, positive=True)
        Ket_1 = Matrix([[1],[0]])
        Ket_2 = Matrix([[0],[1]])
        Ket_11 = TP(Ket_1,Ket_1)
        Ket_12 = TP(Ket_1,Ket_2)
        Ket_21 = TP(Ket_2,Ket_1)
        Ket_22 = TP(Ket_2,Ket_2)
        psi_0_B=1/sqrt(6) *Ket_11 + 1/sqrt(3)* Ket_12 + 1/sqrt(6) *Ket_21 + 1/sqrt(3) * Ket_22

        def E(n,q):
            return (n**2 + q**2)*z
        H1 = Matrix([[E(1,0),0],[0,E(2,0)]])
        H1_TP = TP(H1,eye(2))
        H2 = Matrix([[E(0,1),0],[0,E(0,2)]])
        H2_TP = TP(eye(2),H2)
        def do_HW(psi):
            avg_H1 = Dagger(psi)*H1_TP*psi
            display('avg_H1', avg_H1)
            avg_H2 = Dagger(psi)*H2_TP*psi
            display('avg_H2', avg_H2)
            display('avg_H1*avg_H2', avg_H1*avg_H2)
            avg_H1H2 = Dagger(psi)*TP(H1,H2)*psi
            display('avg_H1H2', avg_H1H2)
        do_HW(psi_0_B)
        #Como el estado psi_0 es un producto tensorial, los valores esperados son iguales.
```

```
'avg_H1'
```

$$\left[\frac{5\xi}{2}\right]$$

'avg_H2'

$$\left[3\xi\right]$$

'avg_H1*avg_H2'

$$\left[\frac{15\xi^2}{2}\right]$$

'avg_H1H2'

$$\left[\frac{15\xi^2}{2}\right]$$

In [3]:
```
t,h = symbols('t hbar', real=True, positive=True)
t_vec_B = Matrix([[exp(-I*E(1,1)*t/h)],[exp(-I*E(1,2)*t/h)],[exp(-I*E(2,1)*t/h)],[exp(-I
psi_t=t_vec_B.multiply_elementwise(psi_0_B)
do_HW(psi_t)
#Es claro que es igual a cuando el ket no depende del tiempo.
```

'avg_H1'

$$\left[\frac{5\xi}{2}\right]$$

'avg_H2'

$$\left[3\xi\right]$$

'avg_H1*avg_H2'

$$\left[\frac{15\xi^2}{2}\right]$$

'avg_H1H2'

$$\left[\frac{15\xi^2}{2}\right]$$

```
In [4]: psi_0_D=1/sqrt(5) *Ket_11 + sqrt(3)/sqrt(5)* Ket_12 + 1/sqrt(5) *Ket_21
        do_HW(psi_0_D)
        #Como en este caso no se trata de un estado producto tensorial, el producto de los valor
```

'avg_H1'

$$\left[\frac{8\xi}{5}\right]$$

'avg_H2'

$$\left[\frac{14\xi}{5}\right]$$

'avg_H1*avg_H2'

$$\left[\frac{112\xi^2}{25}\right]$$

'avg_H1H2'

$$\left[\frac{17\xi^2}{5}\right]$$

```
In [5]: psi_t_D = t_vec_B.multiply_elementwise(psi_0_D)
        do_HW(psi_t_D)
```

'avg_H1'

$$\left[\frac{8\xi}{5}\right]$$

'avg_H2'

$$\left[\frac{14\xi}{5}\right]$$

'avg_H1*avg_H2'

$$\left[\frac{112\xi^2}{25}\right]$$

'avg_H1H2'

$$\left[\frac{17\xi^2}{5}\right]$$

```
In [6]: def do_HW_2(psi,t_vec=t_vec_B):
            rho_0 = psi*Dagger(psi)
            display('rho_0',rho_0)
            psi_t = t_vec.multiply_elementwise(psi)
            rho_t = psi_t*Dagger(psi_t)
            display('rho_t',rho_t)
            one_kets = [Ket_1, Ket_2]
            two_kets = [Ket_1, Ket_2]
            rho_1 = MutableMatrix(zeros(2))
            for n in range(2):
                for np in range(2):
                    summ=0
                    for p in range(2):
                        summ += (Dagger(TP(one_kets[n],two_kets[p]))*rho_0*TP(one_kets[np],two_k
                    rho_1[n,np]=summ
            display('rho_1',rho_1)

            rho_2 = MutableMatrix(zeros(2))
            for p in range(2):
                for pp in range(2):
                    summ=0
                    for n in range(2):
                        summ += (Dagger(TP(one_kets[n],two_kets[p]))*rho_0*TP(one_kets[n],two_ke
                    rho_2[p,pp] = summ
            display('rho_2',rho_2)
            display('rho1 x rho_2',TP(rho_1,rho_2))
            display('rho1 x rho_2 == rho_0',TP(rho_1,rho_2)==rho_0)
```

```
In [7]: do_HW_2(psi_0_B)
```

'rho_0'

$$\begin{bmatrix} \frac{1}{6} & \frac{\sqrt{2}}{6} & \frac{1}{6} & \frac{\sqrt{2}}{6} \\ \frac{\sqrt{2}}{6} & \frac{1}{3} & \frac{\sqrt{2}}{6} & \frac{1}{3} \\ \frac{1}{6} & \frac{\sqrt{2}}{6} & \frac{1}{6} & \frac{\sqrt{2}}{6} \\ \frac{\sqrt{2}}{6} & \frac{1}{3} & \frac{\sqrt{2}}{6} & \frac{1}{3} \end{bmatrix}$$

'rho_t'

$$\left[\begin{matrix}\frac{1}{6} & \frac{\sqrt{2}}{6} e^{\frac{3 i}{\hbar} t \xi} & \frac{1}{6} e^{\frac{3 i}{\hbar} t \xi} & \frac{\sqrt{2}}{6} e^{\frac{6 i}{\hbar} t \xi}\\\frac{\sqrt{2}}{6} e^{- \frac{3 i}{\hbar} t \xi} & \frac{1}{3} & \frac{\sqrt{2}}{6} & \frac{1}{3} e^{\frac{3 i}{\hbar} t \xi}\\\frac{1}{6} e^{- \frac{3 i}{\hbar} t \xi} & \frac{\sqrt{2}}{6} & \frac{1}{6} & \frac{\sqrt{2}}{6} e^{\frac{3 i}{\hbar} t \xi}\\\frac{\sqrt{2}}{6} e^{- \frac{6 i}{\hbar} t \xi} & \frac{1}{3} e^{- \frac{3 i}{\hbar} t \xi} & \frac{\sqrt{2}}{6} e^{- \frac{3 i}{\hbar} t \xi} & \frac{1}{3}\end{matrix}\right]$$

'rho_1'

$$\left[\begin{matrix}\frac{1}{2} & \frac{1}{2}\\\frac{1}{2} & \frac{1}{2}\end{matrix}\right]$$

'rho_2'

$$\left[\begin{matrix}\frac{1}{3} & \frac{\sqrt{2}}{3}\\\frac{\sqrt{2}}{3} & \frac{2}{3}\end{matrix}\right]$$

'rho1 x rho_2'

$$\left[\begin{matrix}\frac{1}{6} & \frac{\sqrt{2}}{6} & \frac{1}{6} & \frac{\sqrt{2}}{6}\\\frac{\sqrt{2}}{6} & \frac{1}{3} & \frac{\sqrt{2}}{6} & \frac{1}{3}\\\frac{1}{6} & \frac{\sqrt{2}}{6} & \frac{1}{6} & \frac{\sqrt{2}}{6}\\\frac{\sqrt{2}}{6} & \frac{1}{3} & \frac{\sqrt{2}}{6} & \frac{1}{3}\end{matrix}\right]$$

'rho1 x rho_2 == rho_0'

True

```
In [8]: do_HW_2(psi_0_D)
```

'rho_0'

$$\left[\begin{matrix}\frac{1}{5} & \frac{\sqrt{3}}{5} & \frac{1}{5} & 0\\\frac{\sqrt{3}}{5} & \frac{3}{5} & \frac{\sqrt{3}}{5} & 0\\\frac{1}{5} & \frac{\sqrt{3}}{5} & \frac{1}{5} & 0\\0 & 0 & 0 & 0\end{matrix}\right]$$

'rho_t'

$$\begin{bmatrix} \frac{1}{5} & \frac{\sqrt{3}}{5}e^{\frac{3i}{\hbar}t\xi} & \frac{1}{5}e^{\frac{3i}{\hbar}t\xi} & 0 \\ \frac{\sqrt{3}}{5}e^{-\frac{3i}{\hbar}t\xi} & \frac{3}{5} & \frac{\sqrt{3}}{5} & 0 \\ \frac{1}{5}e^{-\frac{3i}{\hbar}t\xi} & \frac{\sqrt{3}}{5} & \frac{1}{5} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

'rho_1'

$$\begin{bmatrix} \frac{4}{5} & \frac{1}{5} \\ \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

'rho_2'

$$\begin{bmatrix} \frac{2}{5} & \frac{\sqrt{3}}{5} \\ \frac{\sqrt{3}}{5} & \frac{3}{5} \end{bmatrix}$$

'rho1 x rho_2'

$$\begin{bmatrix} \frac{8}{25} & \frac{4\sqrt{3}}{25} & \frac{2}{25} & \frac{\sqrt{3}}{25} \\ \frac{4\sqrt{3}}{25} & \frac{12}{25} & \frac{\sqrt{3}}{25} & \frac{3}{25} \\ \frac{2}{25} & \frac{\sqrt{3}}{25} & \frac{2}{25} & \frac{\sqrt{3}}{25} \\ \frac{\sqrt{3}}{25} & \frac{3}{25} & \frac{\sqrt{3}}{25} & \frac{3}{25} \end{bmatrix}$$

'rho1 x rho_2 == rho_0'

False

2 Problema 1

```
In [9]: th, phi, w = symbols('theta phi omega', real=True)
        chi_n_up = Matrix([[cos(th/2)],[sin(th/2)*exp(I*phi)]])
        chi_n_dn = Matrix([[sin(th/2)],[-cos(th/2)*exp(I*phi)]])
        sx_up = chi_n_up.subs(th,pi/2).subs(phi,0)
        sx_dn = chi_n_dn.subs(th,pi/2).subs(phi,0)
        sy_up = chi_n_up.subs(th,pi/2).subs(phi,pi/2)
        sy_dn = chi_n_dn.subs(th,pi/2).subs(phi,pi/2)
        sz_up = Matrix([[1],[0]])
        sz_dn =Matrix([[0],[1]])
        display('Px+', abs(Dagger(sx_up)*sz_up)**2)
        display('Px-', abs(Dagger(sx_dn)*sz_up)**2)
```

'Px+'

$$\left[\frac{1}{2}\right]$$

'Px-'

$$\left[\frac{1}{2}\right]$$

```
In [10]: psi_t = cos(w*t)*sz_up + sin(w*t)*sz_dn
         display('sx=+h/2',abs(Dagger(sx_up)*psi_t)**2)
         display('sx=-h/2',abs(Dagger(sx_dn)*psi_t)**2)
         display('sy=+h/2',abs(Dagger(sy_up)*psi_t)**2)
         display('sy=-h/2',abs(Dagger(sy_dn)*psi_t)**2)
         display('sz=+h/2',abs(Dagger(sz_up)*psi_t)**2)
         display('sz=-h/2',abs(Dagger(sz_dn)*psi_t)**2)
```

'sx=+h/2'

$$\left[\left(\frac{\sqrt{2}}{2}\sin\left(\omega t\right)+\frac{\sqrt{2}}{2}\cos\left(\omega t\right)\right)^2\right]$$

'sx=-h/2'

$$\left[\left(\frac{\sqrt{2}}{2}\sin\left(\omega t\right)-\frac{\sqrt{2}}{2}\cos\left(\omega t\right)\right)^2\right]$$

'sy=+h/2'

$$\left[\frac{1}{2}\sin^2\left(\omega t\right)+\frac{1}{2}\cos^2\left(\omega t\right)\right]$$

'sy=-h/2'

$$\left[\frac{1}{2}\sin^2\left(\omega t\right)+\frac{1}{2}\cos^2\left(\omega t\right)\right]$$

'sz=+h/2'

$$\left[\cos^2\left(\omega t\right)\right]$$

'sz=-h/2'

$$\left[\sin^2(\omega t)\right]$$

```
In [11]: T=2*pi/w
         n=4
         display('sx=+h/2',abs(Dagger(sx_up)*psi_t.subs(t,n*T))**2)
         display('sx=-h/2',abs(Dagger(sx_dn)*psi_t.subs(t,n*T))**2)
         display('sy=+h/2',abs(Dagger(sy_up)*psi_t.subs(t,n*T))**2)
         display('sy=-h/2',abs(Dagger(sy_dn)*psi_t.subs(t,n*T))**2)
         display('sz=+h/2',abs(Dagger(sz_up)*psi_t.subs(t,n*T))**2)
         display('sz=-h/2',abs(Dagger(sz_dn)*psi_t.subs(t,n*T))**2)
```

'sx=+h/2'

$$\left[\frac{1}{2}\right]$$

'sx=-h/2'

$$\left[\frac{1}{2}\right]$$

'sy=+h/2'

$$\left[\frac{1}{2}\right]$$

'sy=-h/2'

$$\left[\frac{1}{2}\right]$$

'sz=+h/2'

$$\left[1\right]$$

'sz=-h/2'

$$\left[0\right]$$