Geoneutrino oscillations approach to discriminate distributions of HPE in the Earth's mantle

Daniel Forero-Sánchez[1,2]

[1]Geosciences Department
[2]Physics Department

Universidad de los Andes
Bogotá, Colombia

November 28, 2017

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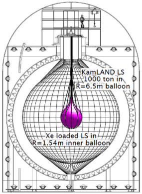
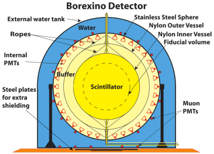
Introduction I

- Mantle structure is widely debated.
  - Uniform HPE: Mantle is chemically homogeneous.
  - Two-layer HPE: Chemically Enriched Layer (EL) near the nucleus. + Depleted Layer (DL).
- (Anti)Neutrinos, almost, do not interact with the Earth.
- Antineutrino detectors are being built.



(a) KamLAND.



(b) Borexino

Figure: Taken from [5].

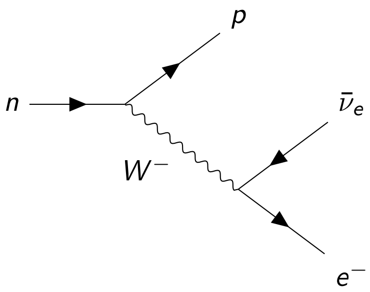
Introduction II

- There is an exact, relatively simple, way to include an exact solution to the evolution of a neutrino through matter.
- Another way to indirectly probe the deep Earth is then possible.
- This probing can **bring information on the distribution of HPE** distribution as well as heat flux.

Radioactive Decay & Geoneutrinos

$$n \rightarrow p + e^{-} + \bar{\nu}_e \tag{1}$$

- Alpha ($^4He^{+2}$)
- Gamma ($E_\gamma \sim 100keV$)
- Beta
  - β^+ (e^+, ν_e)
  - β^- (e^-, $\bar{\nu}_e$ ←**geoneutrino**)
  - Electron capture



Geoneutrinos are **naturally produced** electron antineutrinos (from β^- decay.).

Figure: Feynman diagram for β^- process. This is the process that **generates geoneutrinos**.

Decay Chains

The isotopes that dominate the geoneutrino production are

$$^{238}U \rightarrow ^{206} Pb + 8\alpha + 8e^- + 6\bar{\nu}_e + 51.7 MeV, \quad (2)$$

$$^{235}U \rightarrow ^{207} Pb + 7\alpha + 4e^- + 4\bar{\nu}_e + 46.4 MeV, \quad (3)$$

$$^{232}Th \rightarrow ^{208} Pb + 6\alpha + 4e^- + 4\bar{\nu}_e + 42.7 MeV, \quad (4)$$

$$^{40}K \rightarrow ^{40} Ca + e^- + \bar{\nu}_e + 1.31 MeV, \quad (5)$$

$$^{40}K + e^- \rightarrow ^{40} Ar + \bar{\nu}_e + 1.505 MeV. \quad (6)$$

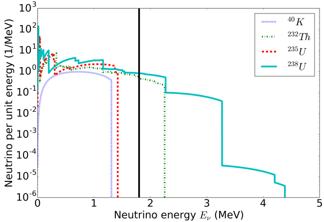


Figure: Geoneutrino energy spectrum from [2].

Bulk Silicate Earth

Bulk Silicate Earth (BSE) = Mantle + Crust $\leftarrow$ Silicate Earth.

There are three **main models**.

BSE Model	$A_{Th}^{BSE}(ppb)$	$A_{U}^{BSE}(ppb)$
Geochemical	79.5	20.3
Geodynamical	140 ± 14	35 ± 4
Cosmochemical	43 ± 4	12 ± 2

Table: Compositional BSE models. Taken from [6, 8].

Crustal Abundances

		ρ (g/cm^3)	d (km)	M (10^{21} kg)	Mass U (10^{15} kg)	Mass Th (10^{15} kg)	Mass K, (10^{19} kg)
CC	Sed	2.25[a]	1.5 ± 0.3	0.7 ± 0.1	$1.2^{+0.2}_{-0.2}$	$5.8^{+1.1}_{-1.1}$	$1.3^{+0.2}_{-0.2}$
	UC	2.76	11.6 ± 1.3	6.7 ± 0.8	$18.2^{+4.8}_{-4.3}$	$70.7^{+10.7}_{-10.2}$	$15.6^{+2.3}_{-2.1}$
	MC	2.88	11.4 ± 1.3	6.9 ± 0.9	$6.6^{+4.1}_{-2.5}$	$33.3^{+30.0}_{-15.5}$	$10.4^{+5.7}_{-3.7}$
	LC	3.05	10.0 ± 1.2	6.3 ± 0.7	$1.0^{+0.9}_{-0.4}$	$6.0^{+7.7}_{-3.3}$	$4.1^{+2.2}_{-1.4}$
	LM	3.37	140 ± 71	97 ± 47	$2.9^{+5.4}_{-2.0}$	$14.5^{+29.4}_{-9.4}$	$3.1^{+4.7}_{-1.8}$
OC	Sed	2.03	0.6 ± 0.2	0.3 ± 0.1	$0.6^{+0.2}_{-0.2}$	$2.8^{+0.9}_{-0.9}$	$0.6^{+0.2}_{-0.2}$
	C	2.88	7.4 ± 2.6	6.3 ± 2.2	$0.4^{+0.2}_{-0.2}$	$1.3^{+0.7}_{-0.5}$	$0.4^{+0.2}_{-0.2}$
DM[b]		4.66	2090	3207	25.7	70.6	48.7
EM[c]		5.39	710	704	24.0	113.7	28.7
BSE[d]		4.42	2891	4035	80.7	318.8	113.0

Figure: Values used for calculating the average abundance of U, Th, and K in the Earth's crust. Modified from [4].

The obtained values are $A_U^C = 453.19\ ppb$ and $A_{Th}^C = 1940.64\ ppb$.

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Mantle Abundances

These are calculated according to the mass balance relation

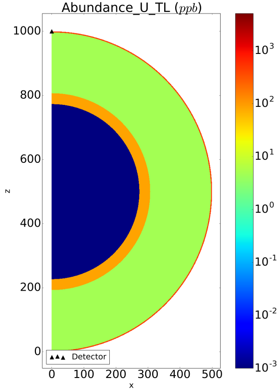
$$m_{BSE} A_X^{BSE} = m_C A_X^C + m_M A_X^M, \tag{7}$$

And the following values are obtained:

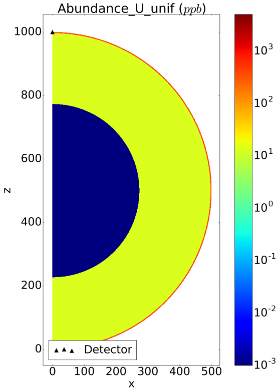
	Cosmochemical	Geochemical	Geodynamical
A_U^M (ppb)	5.14	13.26	28.49
A_{Th}^M (ppb)	13.48	51.06	111.99

Distribution of HPE

I tested two distributions:



(a) Two-layer model. Enriched layer is 10% of mantle's mass.



(b) Uniform model.

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Neutrino Oscillation

- Neutrinos are produced in flavor states $|\nu_\alpha\rangle$: electron ($\alpha = e$), tau ($\alpha = \tau$) or muon ($\alpha = \mu$).
- From QM: Time evolution of a state is given by the hamiltonian $\widehat{H}$.
- Flavor states are **not** energy (mass) eigenstates.
- Flavor states will change while traveling, they will **oscillate**.
- They travel in a mass eigenstate $|\nu_a\rangle$, $a = 1,\ 2,\ 3$.
- The flavor eigenstates can be expressed as a superposition of the mass eigenstates as:

$$|\nu_\alpha\rangle = \sum_{a=1}^{3} U^*_{\alpha a}|\nu_a\rangle. \tag{8}$$

- **Only the electron flavor** is affected by a potential

$$\mathcal{A} \approx \sqrt{2}G_F\frac{\rho(r)}{m_N}. \tag{9}$$

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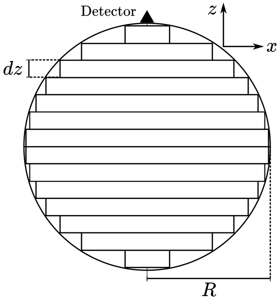
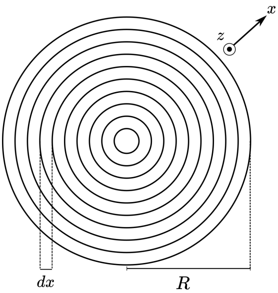
Modeling The Earth I



(c) Cross section of the model-Earth parallel to z-axis.



(d) Cross section of the model-Earth parallel to x-axis (map view).

Figure: The model consists of a 500×1000 matrix in which every element represents a **ring**. For each $z \in [-R, R]$, there are $N(z)$ rings.

Modeling The Earth II

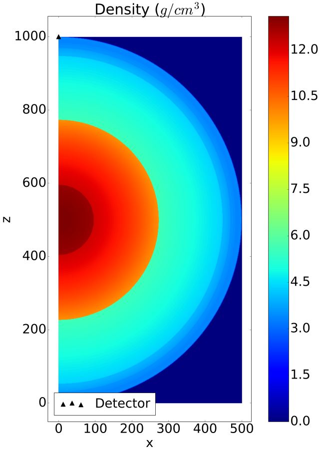


Figure: PREM [1] Model implemented.

Modeling is done in C++.
Each ring has various attributes:

- Coordinates: x, z, r.
- Volume ΔV and Attenuation factor $|\mathbf{r} - \mathbf{r'}|^{-2}$
- Mass density (according to reference [1]), Isotopic abundance.
- Flux contribution.
- A path from each one to the detector.

The Flux Integral I

The flux at a given position **r**, due to HPE at position **r'** is, for isotope X:

$$\Phi_X(\mathbf{r}) = \frac{n_X \lambda_X}{4\pi} \int_\Omega \int_\oplus \frac{a_X(\mathbf{r'})\rho(\mathbf{r'})P_{ee}(\mathbf{r}-\mathbf{r'}, E_\nu)}{|\mathbf{r}-\mathbf{r'}|^2} d^3 r dE_\nu \qquad (10)$$

	^{238}U	^{232}Th
$\mathcal{X}$	0.9927	1
M (u)	238.051	232.038
$\tau_{1/2}$	4.468	14.05
λ $(10^{-18}/s)$	4.916	1.563
n	6	4

Table: Values of the different constants in equation 10. Taken from reference [8].

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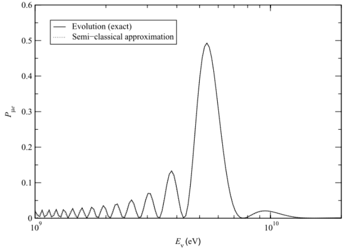
Survival Probability I

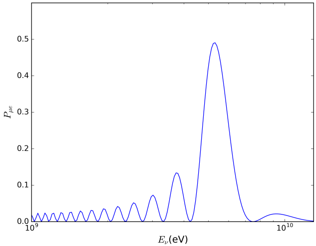
Probability that a geoneutrino is detected by an efficiency 1 detector.

Software: **UANdINO** $\leftarrow$ **path (density) + (anti)neutrino energy** (E_ν)

The results are comparable with reference those from [7]:



(a) Transition Probability (TP) from [7]



(b) TP from UANdINO

Survival Probability II

In the case of antineutrinos, we get

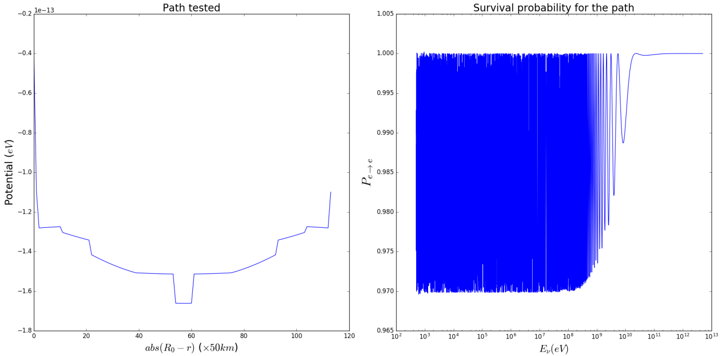
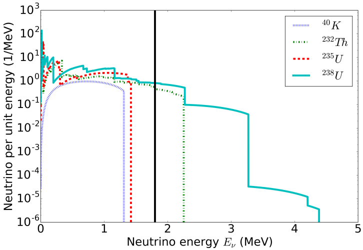


Figure: Antineutrino survival probability for the path shown, which is across the Earth.

Survival Probability III



- The survival probability is heavily oscillating in the relevant energy domain.
- The probability depends on the spectra, the average should reflect this. **How?**

- Calculate the probability $P(E_\nu)$ for K energy samples within the relevant domain $\Rightarrow$ **Average**.
- Sample spectrum in the same energies to obtain K f_i's.
- Compute

$$w_i = \frac{f_i}{\sum_i f_i}$$

the weights for each probability.
- Weight the probabilities to obtain an energy-averaged probability $P(\mathbf{r} - \mathbf{r'})$.
- Do this for each ring.

Survival Probability IV

This results in an average value of

$$\langle P_{ee} \rangle \approx 0.99.$$

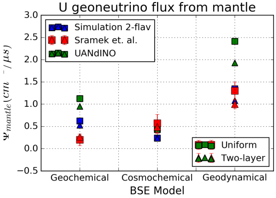
According to other authors (references [4, 5]):

$$\langle P_{ee} \rangle \approx \cos^4 \theta_{13} \left(1 - \frac{1}{2} \sin^2 2\theta_{12}\right) + \sin^4 \theta_{13} \approx 0.54. \tag{11}$$

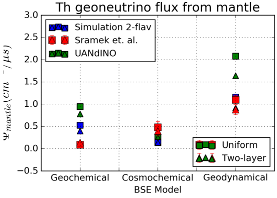
I simulated the flux with **both probabilities**.

Mantle's Flux I

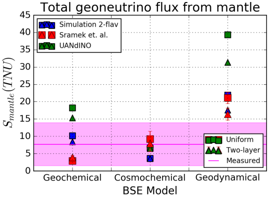
For the three BSE models and both HPE distribution models, the flux was calculated.



(a) *U* flux



(b) *Th* flux



(c) Total geoneutrino mantle signal.

Figure: Simulated geoneutrino signal from the mantle compared to reference [8].

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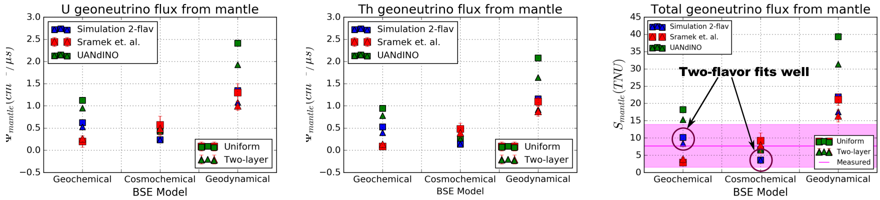
Mantle's Flux II

For the three BSE models and both HPE distribution models, the flux was calculated.



(a) *U* flux

(b) *Th* flux

(c) Total geoneutrino mantle signal.

Figure: Simulated geoneutrino signal from the mantle compared to reference [8].

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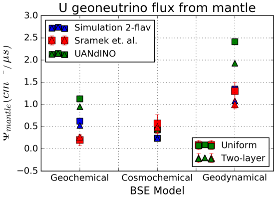
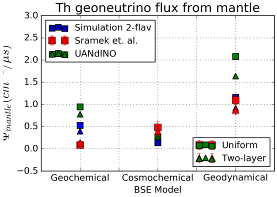
Mantle's Flux III

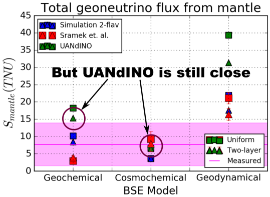
For the three BSE models and both HPE distribution models, the flux was calculated.



(a) *U* flux.



(b) *Th* flux.



(c) Total geoneutrino mantle signal.

Figure: Simulated geoneutrino signal from the mantle compared to reference [8].

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Earth's Flux I

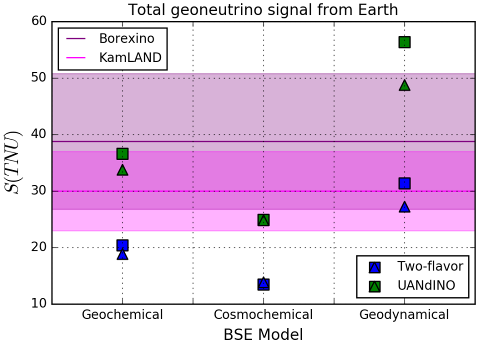


Figure: Total geoneutrino flux expected from the different models. Colored bands correspond to signals measured by the detectors according to reference [5].

Earth's Flux II

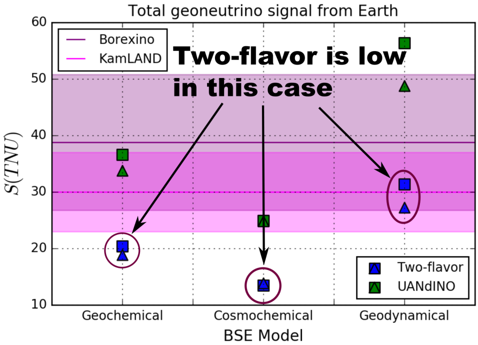


Figure: Total geoneutrino flux expected from the different models. Colored bands correspond to signals measured by the detectors according to reference [5].

Earth's Flux III

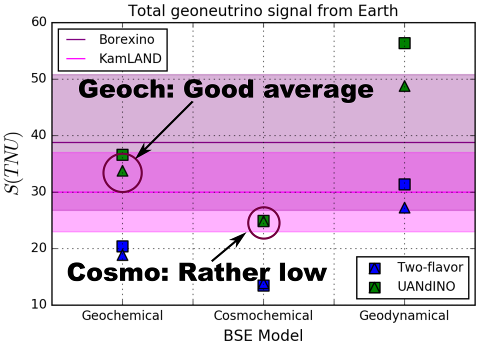
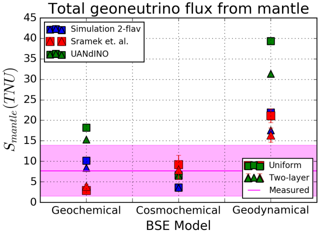
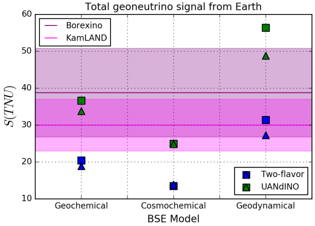


Figure: Total geoneutrino flux expected from the different models. Colored bands correspond to signals measured by the detectors according to reference [5].

Analysis





- UANdINO overestimates mantle flux.
- Nevertheless it gives “good” results for total flux.
- Cosmochemical BSE fits in both situations but does not allow to discriminate HPE distributions
- Geochemical BSE shows fits into total flux and is rather close in mantle flux.
- Two-layer model may be more accurate.

Conclusions I

- I successfully modeled the Earth and some of its physical properties.
- The survival probability of the geoneutrinos was also modeled and tested.
- Geochemical BSE model seems to be the more accurate.
- The two-layer mantle model gives results closer to the measurements in both, mantle and total, flux.

Conclusions II

- The uncertainties in the measurements of neutrino flux, as well as this simulation, are relevant limitations on the study.
- Further work is needed, especially regarding the difference between the survival probability given by the software and the average one given in the literature, and why, despite these quite different probabilities, the results in terms of geoneutrino flux are still reasonable. This involves additional testing to UANdINO to certify the accuracy of the results.

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Acknowledgements

I would like to acknowledge, in the first place, my parents and family, whose hard work and teaching have allowed me to get here.
Then, I acknowledge my advisors, who guided me through this project.
Also, thanks to my friends, who supported me even though they don't remember what this project is about.
Finally, thanks to the people at Stackoverflow, who taught me how to code.

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