Geoneutrino oscillations approach to discriminate distributions of HPE in the Earth's mantle using the Monte Carlo Technique

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References

Facts

- Earth's structure and composition is not absolutely understood.
- It is impossible to directly probe the deep Earth.
- Seismic methods can be useful to indirectly probe for rheological information and possibly relate it with composition..
- Neutrinos do not strongly interact with matter.
- They respond to the electron density of the matter they cross and thus, can bring us information about it.

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Then...

Introduction

Neutrino geoscience is seen as the door to deep Earth studies due to the weak interaction of these particles with matter. Not even photons nor phonons, may be able to map the Earth at such depths, compositionally-wise.

This study aims to test the different hypothesis on the distribution of Heat Producing Elements (HPE) in the Earth's mantle through a simulation, using the Monte Carlo technique.

I will test the **uniform** and **two-layer** distributions. These will be explained later.

Relevance of the Study

- Mantle structure is widely debated.
- Antineutrino detectors are being built.
- There is an exact, relatively simple, way to include an exact solution to the evolution of a neutrino through matter.
- Another way to indirectly probe the deep Earth is then possible.
- This probing can bring information on distribution of HPE distribution as well as heat flux.

Classification of Elements

Goldschmidt [3] did provide a classification for the chemical elements, based on their "affinity".

- **Siderophiles** or iron-loving elements, tend to be close to the nucleus (iron).
- **Chalcophiles** or sulfur-loving do not like the deep Earth, as there is no sulfur there.
- **Lithophiles** or oxygen-loving elements tend to accumulate in the "uppermost" part of the planet.
- **Atmophiles** like to be in the atmosphere.

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Classification of Elements

Siderophile	Chalcophile	Lithophile	Atmophile
Fe*, Co*, Ni*	(Cu), Ag	Li, Na, K, Rb, Cs	(H), N, (O)
Ru, Rh, Pd	Zn, Cd, Hg	Be, Mg, Ca, Sr, Ba	He, Ne, Ar, Kr, Xe
Os, Ir, Pt	Ga, In, Tl	B, Al, Sc, Y, REE	
Au, Re†, Mo†	(Ge), (Sn), Pb	Si, Ti, Zr, Hf, Th	
Ge*, Sn*, W‡	(As), (Sb), Bi	P, V, Nb, Ta	
C‡, Cu*, Ga*	S, Se, Te	O, Cr, U	
Ge*, As†, Sb†	(Fe), Mo, (Os)	H, F, Cl, Br, I	
	(Ru), (Rh), (Pd)	(Fe), Mn, (Zn), (Ga)	

* Chalcophile and lithophile in the Earth's crust.
† Chalcophile in the Earth's crust.
‡ Lithophile in the Earth's crust.

Figure: Goldschmidt's classification for chemical elements. Taken from [10].

Most of the radioactive elements are lithophiles. In this study we concentrate on ^{40}K, ^{235}U, ^{238}U and ^{232}Th, since they dominate the **geoneutrino** production.

Classification of elements

Elements are also discriminated based on their condensation temperature:

- **Refractory** elements condense at high temperatures.
- **Volatile** elements condense al low temperatures.

Uranium and Thorium are refractory, so the current abundances are thought to be the same as the ones 4.6 *Gya*. Potassium, on the other hand is relatively volatile so some of it has since escaped.

Bulk Silicate Earth

The Bulk Silicate Earth (BSE) is the combination of silicate parts of the Earth, this is, **crust and mantle**. It's composition has been widely debated [5]. Three main currents have arisen in this matter...

Composition of BSE

- **Geochemical** model is based on geochemical data from peridotites [6]. Reference [6] reports values of:
$\boxed{A_{Th}^{BSE} = 79.5\ ppb,\ A_{K}^{BSE} = 240\ ppm,\ A_{U}^{BSE} = 20.3\ ppb}$.
- **Geodynamical** model is based on heat flow measurements in the surface. In reference [8], the reported values of the abundances are:
$\boxed{A_{Th}^{BSE} = 140 \pm 14\ ppb,\ A_{K}^{BSE} = 350 \pm 35\ ppm,\ A_{U}^{BSE} = 35 \pm 4\ ppb}$.
- **Cosmochemical** model is based on the geochemical analysis of enstatite chondrites, the abundance of iron could easily explain the Earth's core [5]. Reference [8] reports the following values relevant to this project:
$\boxed{A_{Th}^{BSE} = 43 \pm 4\ ppb,\ A_{K}^{BSE} = 146 \pm 29\ ppm,\ A_{U}^{BSE} = 12 \pm 2\ ppb}$.

Crust

- "Uppermost" part of the Earth.
- **Oceanic crust** is basalt with average thickness of 6 *km*.
- **Continental crust** is granite with average thickness of 35 *km*.

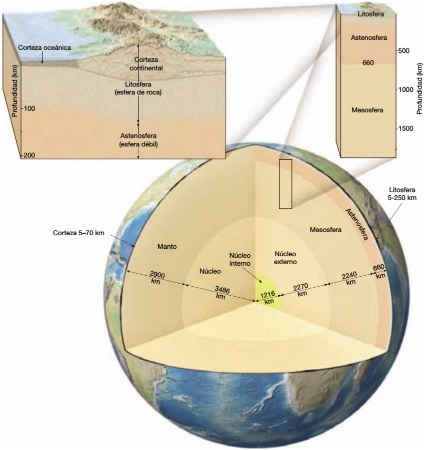

Figure: The internal structure of the Earth. Taken from [9].

Crust Abundances

		ρ (g/cm³)	d (km)	M (10^{21} kg)	Mass		
					U (10^{15} kg)	Th (10^{15} kg)	K, (10^{19} kg)
CC	Sed	2.25[a]	1.5 ± 0.3	0.7 ± 0.1	$1.2^{+0.2}_{-0.2}$	$5.8^{+1.1}_{-1.1}$	$1.3^{+0.2}_{-0.2}$
	UC	2.76	11.6 ± 1.3	6.7 ± 0.8	$18.2^{+4.8}_{-4.3}$	$70.7^{+10.7}_{-10.2}$	$15.6^{+2.3}_{-2.1}$
	MC	2.88	11.4 ± 1.3	6.9 ± 0.9	$6.6^{+4.1}_{-2.5}$	$33.3^{+30.0}_{-15.5}$	$10.4^{+5.7}_{-3.7}$
	LC	3.05	10.0 ± 1.2	6.3 ± 0.7	$1.0^{+0.9}_{-0.4}$	$6.0^{+7.7}_{-3.3}$	$4.1^{+2.2}_{-1.4}$
	LM	3.37	140 ± 71	97 ± 47	$2.9^{+5.4}_{-2.0}$	$14.5^{+29.4}_{-9.4}$	$3.1^{+4.7}_{-1.8}$
OC	Sed	2.03	0.6 ± 0.2	0.3 ± 0.1	$0.6^{+0.2}_{-0.2}$	$2.8^{+0.9}_{-0.9}$	$0.6^{+0.2}_{-0.2}$
	C	2.88	7.4 ± 2.6	6.3 ± 2.2	$0.4^{+0.2}_{-0.2}$	$1.3^{+0.7}_{-0.5}$	$0.4^{+0.2}_{-0.2}$
DM[b]		4.66	2090	3207	25.7	70.6	48.7
EM[c]		5.39	710	704	24.0	113.7	28.7
BSE[d]		4.42	2891	4035	80.7	318.8	113.0

Figure: Values used for calculating the average abundance of U, Th, and K in the Earth's crust. Modified from [4].

The obtained values are $A_U^C = 453.19\ ppb$ and $A_{Th}^C = 1940.64\ ppb$.

Mantle

- Intermediate part of the Earth.
- Ranges from $r = 3480\ km$ to $r = 6346\ km$.
- Makes up most of the Earth's volume.
- Partially molten rock.

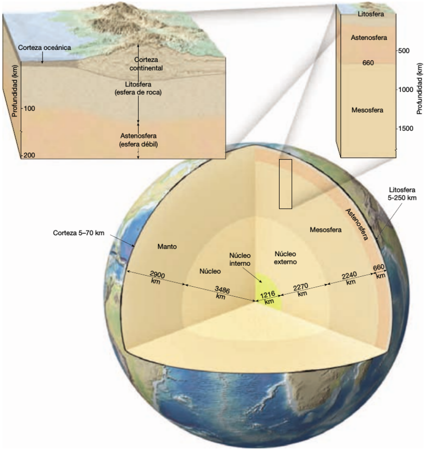

Figure: The internal structure of the Earth. Taken from [9].

Mantle Abundances

These are calculated according to the mass balance relation

$$m_{BSE} A_X^{BSE} = m_C A_X^C + m_M A_X^M, \tag{1}$$

And the following values are obtained:

	Cosmochemical	Geochemical	Geodynamical
A_U^M (ppb)	5.14	13.26	28.49
A_{Th}^M (ppb)	13.48	51.06	111.99

Radioactive Decay

There are different types of radioactive decay, all working through different mechanisms.

- **Alpha decay** is described by

$$ {}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}X'_{N-2} + \alpha \tag{2} $$

- **Gamma decay** is the electromagnetic emission that occurs when an unstable isotope decays into a more stable one.
- **Beta decay** is due to weak interaction and includes three processes: **beta plus (β^+), beta minus (β^-)** and **electron capture**.

Beta Minus Decay

We shall concentrate in β^- decay, in which

$$n \to p + e^- + \bar{\nu}_e \tag{3}$$

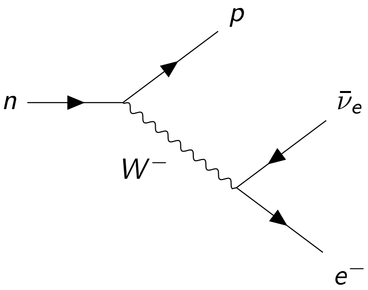

Figure: Feynman diagram for β^- process.

The isotopes that dominate the geoneutrino production are

$$^{238}U \rightarrow ^{206}Pb + 8\alpha + 8e^{-} + 6\bar{\nu}_e + 51.7 MeV, \quad (4)$$

$$^{235}U \rightarrow ^{207}Pb + 7\alpha + 4e^{-} + 4\bar{\nu}_e + 46.4 MeV, \quad (5)$$

$$^{232}Th \rightarrow ^{208}Pb + 6\alpha + 4e^{-} + 4\bar{\nu}_e + 42.7 MeV, \quad (6)$$

$$^{40}K \rightarrow ^{40}Ca + e^{-} + \bar{\nu}_e + 1.31 MeV, \quad (7)$$

$$^{40}K + e^{-} \rightarrow ^{40}Ar + \bar{\nu}_e + 1.505 MeV. \quad (8)$$

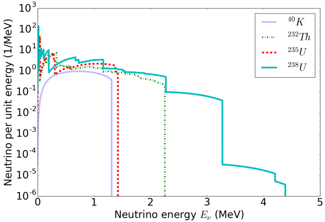

Figure: Geoneutrino energy spectrum from [2].

Spectrum and Sampling

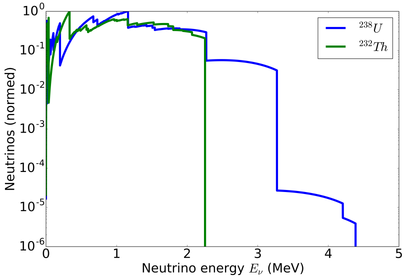

Figure: Energy spectra for the beta minus decay, for the relevant isotopes ^{238}U and ^{232}Th according to [2].

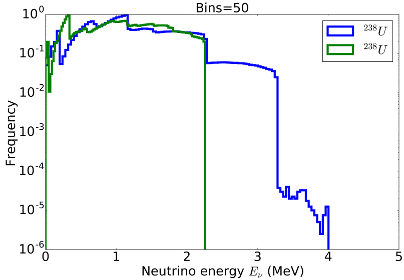

Figure: Metropolis-Hastings sampling for the energy spectra of the relevant isotopes.

Neutrino Oscillation

- Neutrinos are produced in flavor states $|\nu_\alpha\rangle$: electron ($\alpha = e$), tau ($\alpha = \tau$) or muon ($\alpha = \mu$).
- From QM: Time evolution of a state is given by the hamiltonian H.
- Flavor states are **not** energy (mass) eigenstates.
- Flavor states will change while travelling, they will oscillate.

- They travel in a mass eigenstate $|\nu_a\rangle$, $a = 1,\ 2,\ 3$.
- The flavor eigenstates can be expressed as a superposition of the mass eigenstates as:

$$|\nu_\alpha\rangle = \sum_{a=1}^{3} U^*_{\alpha a} |\nu_a\rangle \tag{9}$$

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The transformation matrix is called the PMNS matrix, and is defined as follows [7]:

$$U = \begin{bmatrix} C_2C_3 & S_3C_2 & S_2 \\ -S_3C_1 - S_1S_2C_3 & C_1C_3 - S_1S_2S_3 & S_1C_2 \\ S_1S_3 - S_2C_1C_3 & -S_1C_3 - S_2S_3C_1 & C_1C_2 \end{bmatrix} \tag{10}$$

Where $S_i \equiv \sin(\theta_i)$ and $C_i \equiv \cos(\theta_i)$. Parameters θ_i are the vacuum mixing angles. Here we have assumed there is no charge-parity (CP) phase, thus, $U = U^*$.

Time Evolution

The hamiltonian rules how the state evolves in time, for neutrinos in mass state we have:

$$|\nu_a(t)\rangle = \exp(-i\hat{H}_m(t-t_0))|\nu_a(t_0)\rangle \quad (11)$$

Where

$$\hat{H}_{m,unperturbed} = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix} \quad (12)$$

Inside matter, the Hamiltonian of the system is perturbed in the following way

$$\hat{V}_f = \mathcal{A}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad (13)$$

Where $\mathcal{A}$ depends on the density of the matter the neutrino is going through, and is defined as

$$\mathcal{A}(r) \approx \pm\sqrt{2}G_F\frac{\rho(r)}{m_N} \quad (14)$$

Time Evolution

Then, in mass basis we will have a hamiltonian

$$\hat{H}_m = \begin{bmatrix} E_1 & 0 & 0 \\ 0 & E_2 & 0 \\ 0 & 0 & E_3 \end{bmatrix} + U^{-1} V_f U \quad (15)$$

When introducing this hamiltonian into time evolution equation, after a rather complex algebra, we will be able to calculate the transition/survival probability

$$P_{\alpha \to \beta} = |\langle \beta | \hat{U}_f | \alpha \rangle|^2 \quad (16)$$

Time evolution: Some details I

Let us go back to

$$|\nu_a(t)\rangle = \exp(-i\hat{H}_m(t - t_0))|\nu_a(t_0)\rangle$$

. The time evolution operator is

$$U(t) = \exp(-i\hat{H}_m(t - t_0))$$

.

The exponential of a matrix M is calculated as

$$\exp(M) = \sum_{n=0}^{\infty} \frac{1}{n!} M^n.$$

Time evolution: Some details II

Cayley-Hamilton theorem assures that $p(M) = 0$ where p is the characteristic polynomial of the matrix M. This means that

$$0 = \sum_{n=0}^{N} a_n M^n,$$

which means that

$$M^N = -\sum_{n=0}^{N-1} c_n M^n$$

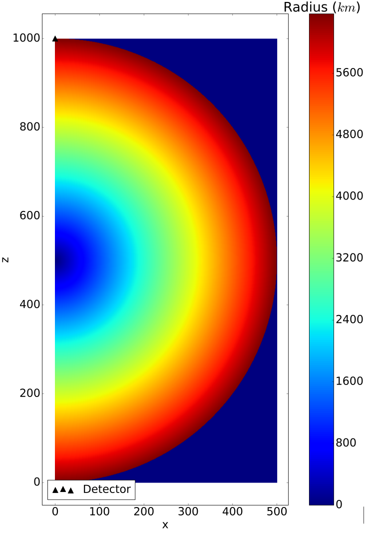
is an exact way for calculating M^p, given that we calculate each $c_n^{(p)}$ correctly for any exponent $p \geq N$, thus, the infinite series turns into

$$\exp(M) = \sum_{n=0}^{N-1} a_n M^n,$$

which may also be calculated, **exactly**.

Modelling The Earth I

Modeling is being developed in C++, the model consists of a 500×1000 matrix in which every element represents a **ring**.
For each $z \in [-R, R]$, there are $N(z)$ rings.



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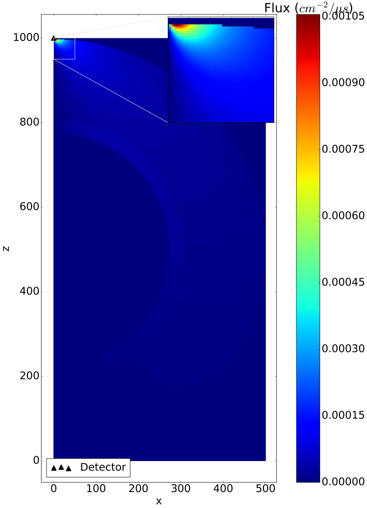
Modelling The Earth II

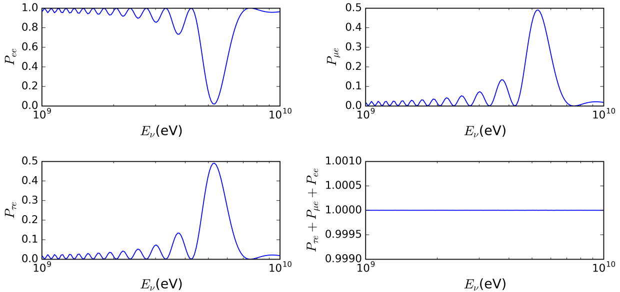


Each ring has various attributes:

- Coordinates: x, z, r.
- Volume ΔV and Attenuation factor $|\vec{r} - \vec{r'}|^{-2}$
- Mass density (according to reference [1]), Isotopic abundance.
- Flux contribution.
- A path from each one to the detector.

Survival Probability I

Following reference [7], a program that calculates the survival probability given a **path** and an **energy** was written. As a test the following plots were made with a density profile with $\rho = 7.8\ g/cm^3$ ($A \approx 3.0 \times 10^{13}\ eV,\ L = 12742\ km$) and with parameter values: $\theta_1 = 45^\circ,\ \theta_2 = 5^\circ,\ \theta_3 = 45^\circ,\ \Delta m^2_{21} = 0$, and $\Delta m^2_{32} = 3.2 \times 10^3\ e^2$:



If we compare with reference [7]:

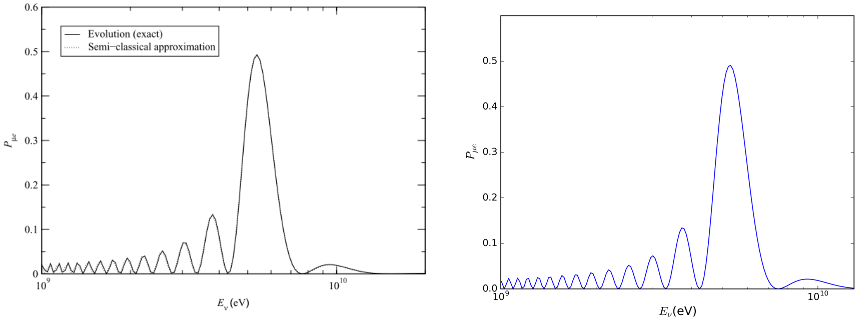


Figure: Left: Transition probability from reference [7]. Right: Plot produced.

Survival Probability III

Then, the survival probability for different paths and antineutrinos is

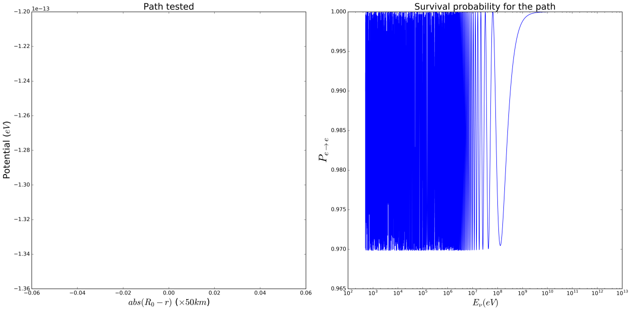


Figure: Antineutrino survival probability for the path shown. In this case, path is only two points long.

Survival Probability IV

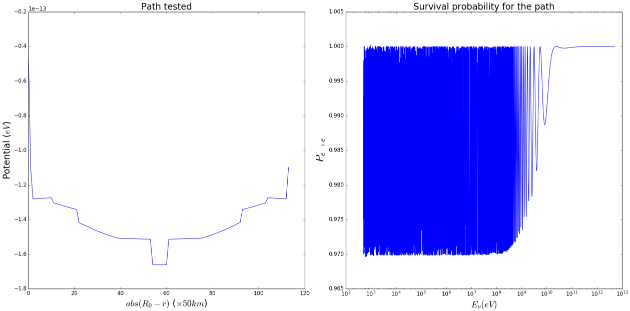
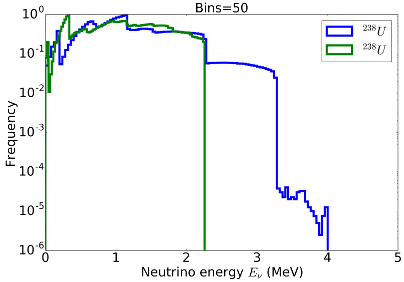


Figure: Antineutrino survival probability for the path shown, which is across the Earth.

Survival Probability V



- The survival probability is heavily oscillating in the relevant energy domain.
- The "wisest" choice may be to average the probability for each path.
- Idea: Generate M geoneutrinos in each ring and assign an energy to each one, following the sampling in the left. Calculate the survival probability for each one and take an average in each ring.
- This way the average probability will respond to these spectra.
- Recalculate the flux taking into account the oscillation probability.

Survival Probability VI

There is an **issue**: Different authors (references [4, 5]) say that the survival probability is well averaged by two-flavor oscillation formula and that matter effect represents a little contribution to it.

$$\langle P_{ee} \rangle \approx \cos^4 \theta_{13} \left(1 - \frac{1}{2} \sin^2 2\theta_{12}\right) + \sin^4 \theta_{13} \approx 0.54. \tag{17}$$

On the other hand, the program developed gives an average value of

$$\langle P_{ee} \rangle \approx 0.97.$$

What happens?

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Conclusions

- Inner structure and processes of the Earth are complex and direct probing seems, until now, impossible, thus, the development of new geophysical methods is important.
- Discrepancies in the oscillation probabilities were not expected and must be explained.
- Until now, the formalism used for the oscillation phenomena is an "overkill", given the behavior of the curve in the relevant energy domain.
- Different mantle HPE distributions can be evaluated using this method, as long as they have an azimuthal symmetry.

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