Geoneutrino oscillations approach to discriminate distributions of Heat Producing Elements in the Earth's mantle

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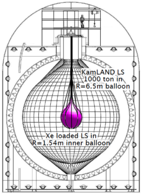
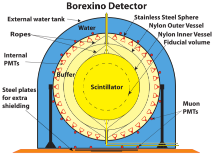
Outline I

Introduction I

- Mantle structure is widely debated.
  - Uniform HPE: Mantle is chemically homogeneous.
  - Two-layer HPE: Chemically Enriched Layer (EL) near the nucleus. + Depleted Layer (DL).
- (Anti)Neutrinos, almost, do not interact with the Earth.
- Different antineutrino detectors were built.



(a) KamLAND.



(b) Borexino

Figure: Taken from [5].

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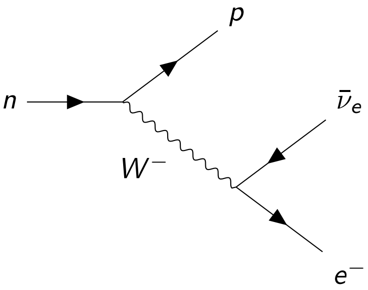
Introduction II

- An exact solution to the evolution of a neutrino through matter can be obtained.
- Another way to indirectly probe the deep Earth is then possible.
- Geoneutrinos can bring to the surface **information on the distribution of HPE** as well as heat flux.

Radioactive Decay & Geoneutrinos

$$n \to p + e^- + \bar{\nu}_e \qquad (1)$$

- Alpha ($^4He^{+2}$)
- Gamma ($E_\gamma \sim 100 keV$)
- Beta
  - β^+ (e^+, ν_e)
  - β^- (e^-, $\bar{\nu}_e$ ←**geoneutrino**)
  - Electron capture



Geoneutrinos are **naturally produced** electron antineutrinos (from β^- decay.).

Figure: Feynman diagram for β^- process. This is the process that **generates geoneutrinos**.

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Decay Chains

The isotopes that dominate the geoneutrino production are

$$^{238}U \rightarrow ^{206} Pb + 8\alpha + 8e^- + 6\bar{\nu}_e + 51.7 MeV, \quad (2)$$

$$^{235}U \rightarrow ^{207} Pb + 7\alpha + 4e^- + 4\bar{\nu}_e + 46.4 MeV, \quad (3)$$

$$^{232}Th \rightarrow ^{208} Pb + 6\alpha + 4e^- + 4\bar{\nu}_e + 42.7 MeV, \quad (4)$$

$$^{40}K \rightarrow ^{40} Ca + e^- + \bar{\nu}_e + 1.31 MeV, \quad (5)$$

$$^{40}K + e^- \rightarrow ^{40} Ar + \bar{\nu}_e + 1.505 MeV. \quad (6)$$

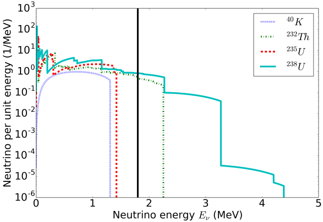

Figure: Geoneutrino energy spectrum from [3].

Bulk Silicate Earth

Bulk Silicate Earth (BSE) = Mantle + Crust ← Silicate Earth.

There are three **main models**.

BSE Model	$A_{Th}^{BSE}(ppb)$	$A_{U}^{BSE}(ppb)$
Geochemical	79.5	20.3
Geodynamical	140 ± 14	35 ± 4
Cosmochemical	43 ± 4	12 ± 2

Table: Compositional BSE models. Taken from [6, 9].

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Crustal Abundances

		ρ (g/cm^3)	d (km)	M (10^{21} kg)	Mass		
					U (10^{15} kg)	Th (10^{15} kg)	K, (10^{19} kg)
CC	Sed	2.25[a]	1.5 ± 0.3	0.7 ± 0.1	$1.2^{+0.2}_{-0.2}$	$5.8^{+1.1}_{-1.1}$	$1.3^{+0.2}_{-0.2}$
	UC	2.76	11.6 ± 1.3	6.7 ± 0.8	$18.2^{+4.8}_{-4.3}$	$70.7^{+10.7}_{-10.2}$	$15.6^{+2.3}_{-2.1}$
	MC	2.88	11.4 ± 1.3	6.9 ± 0.9	$6.6^{+4.1}_{-2.5}$	$33.3^{+30.0}_{-15.5}$	$10.4^{+5.7}_{-3.7}$
	LC	3.05	10.0 ± 1.2	6.3 ± 0.7	$1.0^{+0.9}_{-0.4}$	$6.0^{+7.7}_{-3.3}$	$4.1^{+2.2}_{-1.4}$
	LM	3.37	140 ± 71	97 ± 47	$2.9^{+5.4}_{-2.0}$	$14.5^{+29.4}_{-9.4}$	$3.1^{+4.7}_{-1.8}$
OC	Sed	2.03	0.6 ± 0.2	0.3 ± 0.1	$0.6^{+0.2}_{-0.2}$	$2.8^{+0.9}_{-0.9}$	$0.6^{+0.2}_{-0.2}$
	C	2.88	7.4 ± 2.6	6.3 ± 2.2	$0.4^{+0.2}_{-0.2}$	$1.3^{+0.7}_{-0.5}$	$0.4^{+0.2}_{-0.2}$
DM[b]		4.66	2090	3207	25.7	70.6	48.7
EM[c]		5.39	710	704	24.0	113.7	28.7
BSE[d]		4.42	2891	4035	80.7	318.8	113.0

Figure: Values used for calculating the average abundance of U, Th, and K in the Earth's crust. Modified from [4].

The obtained values are $A_U^C = 453.19\ ppb$ and $A_{Th}^C = 1940.64\ ppb$.

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Mantle Abundances

These are calculated according to the mass balance relation

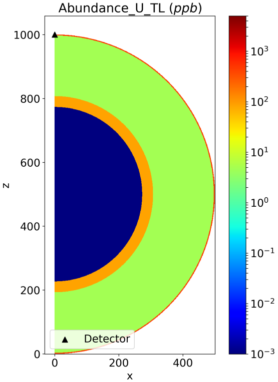
$$m_{BSE} A_X^{BSE} = m_C A_X^C + m_M A_X^M, \tag{7}$$

And the following values are obtained:

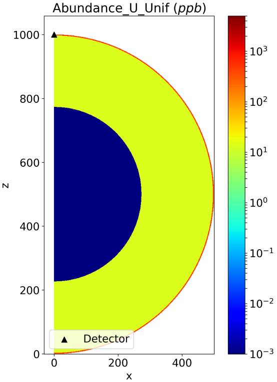
	Cosmochemical	Geochemical	Geodynamical
A_U^M (ppb)	5.14	13.26	28.49
A_{Th}^M (ppb)	13.48	51.06	111.99

Distribution of HPE

I tested two distributions:



(a) Two-layer model. Enriched layer is 10% of mantle's mass.



(b) Uniform model.

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Neutrino Oscillation I

- Neutrinos are produced in flavor states $|\nu_\alpha\rangle$: electron ($\alpha = e$), tau ($\alpha = \tau$) or muon ($\alpha = \mu$).
- From QM: Time evolution of a state is given by the hamiltonian $\widehat{H}$.
- Flavor states are **not** energy (mass) eigenstates.
- Flavor states will change while traveling, they will **oscillate**.
- $|\nu_a\rangle$, $a = 1,\ 2,\ 3$ are the (stationary) mass eigenstates.

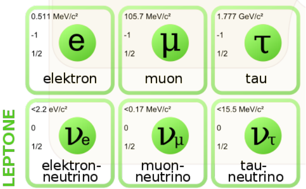

Figure: Lepton flavors. Modified from [7]

Neutrino Oscillation II

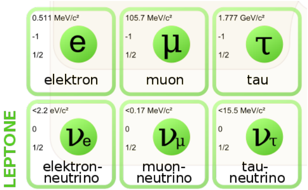

Figure: Lepton flavors. Modified from [7].

- The flavor eigenstates can be expressed as a superposition of the mass eigenstates as:

$$|\nu_\alpha\rangle = \sum_{a=1}^{3} U^*_{\alpha a}|\nu_a\rangle. \quad (8)$$

- **Only the electron flavor** is affected by a potential

$$\mathcal{A} \approx \pm\sqrt{2}G_F\frac{\rho(r)}{m_N}. \quad (9)$$

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MSW Effect I

In a **two flavor approximation** we have

$$P_{ee} = 1 - \sin^2 2\theta' \sin^2 \left(\frac{\Delta m'^2 L}{4E_\nu} \right). \tag{10}$$

If we calculate

$$\tan 2\theta' = \frac{\sin^2 2\theta}{-2\mathcal{A}E_\nu/\Delta m^2 + \cos 2\theta}, \tag{11}$$

it is evident that there is maximal mixing when

$$\mathcal{A} = \frac{\Delta m^2 \cos 2\theta}{2E_\nu} \equiv \mathcal{A}_R \sim \frac{\Delta m^2}{E_\nu}. \tag{12}$$

WHAT DOES THIS MEAN?

MSW Effect II

An example: The Sun

$$\mathcal{A}_{Sun} \sim 10^{-11} eV \Rightarrow E_\nu \sim 1 MeV \quad \text{or} \quad E_\nu \sim 10^2 MeV. \tag{13}$$

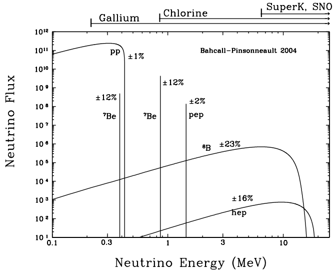


Figure: Solar neutrino spectrum and detection limits for different experiments. Taken from reference [1].

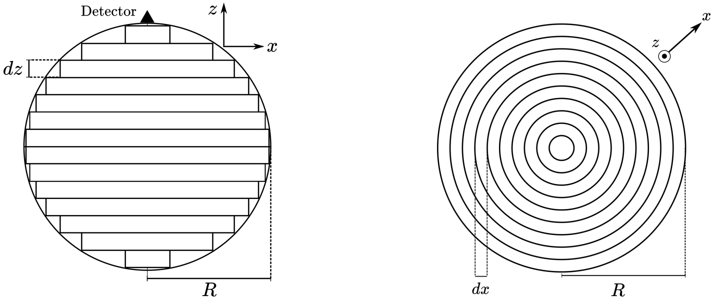
So we should expect **large transitions** from solar neutrinos

$\Downarrow$

SOLAR NEUTRINO PROBLEM

Missing measured flux from solar ν_e

Modeling The Earth I



(a) Cross section of the model-Earth parallel to *z*-axis.

(b) Cross section of the model-Earth parallel to *x*-axis (map view).

Figure: The model consists of a 500×1000 matrix in which every element represents a **ring**. For each $z \in [-R, R]$, there are $N(z)$ rings.

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Modeling The Earth II

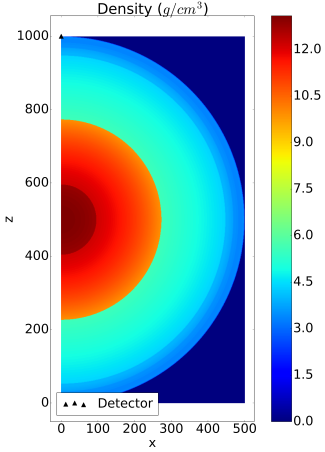

Figure: PREM [2] Model implemented.

Modeling is done in C++.
Each ring has various attributes:

- Coordinates: x, z, r.
- Volume ΔV and Attenuation factor $|\mathbf{r} - \mathbf{r'}|^{-2}$
- Mass density (according to reference [2]), Isotopic abundance.
- Flux contribution.
- A path from each one to the detector.

The Flux Integral I

The flux at a given position **r**, due to HPE at position **r'** is, for isotope X:

$$\Phi_X(\mathbf{r}) = \frac{n_X \lambda_X}{4\pi} \int_\Omega \int_\oplus \frac{a_X(\mathbf{r'})\rho(\mathbf{r'})P_{ee}(\mathbf{r}-\mathbf{r'}, E_\nu)}{|\mathbf{r}-\mathbf{r'}|^2} d^3 r dE_\nu \tag{14}$$

	^{238}U	^{232}Th
$\mathcal{X}$	0.9927	1
M (u)	238.051	232.038
$\tau_{1/2}$	4.468	14.05
λ $(10^{-18}/s)$	4.916	1.563
n	6	4

Table: Values of the different constants in equation 14. Taken from reference [9].

UANdINO I

$$\rho(\mathbf{r}) + E_\nu \rightarrow \textbf{UANdINO} \rightarrow P_{\alpha\beta}(\mathbf{r}, E_\nu)$$

$$\Downarrow$$

$$P_{\alpha\beta} = |\langle\beta| \, U_f(t \approx L) \, |\alpha\rangle|^2$$

$$U_f(t \approx L) \equiv \exp(-iH_f(\rho(\mathbf{r}))L) = \prod_{k=1}^{N} \exp(-iH_f(\rho_k)L_k), \qquad L = NL_k$$

This can be done with **neutrinos** ($\mathcal{A} > 0$) and **antineutrinos** ($\mathcal{A} < 0$)

- Works best at **high energies**.
- At **low energies**: $L_k \sim 10^{-2} m \Rightarrow N = \frac{L}{L_k} \approx 10^6$.
- **Computationally expensive!**

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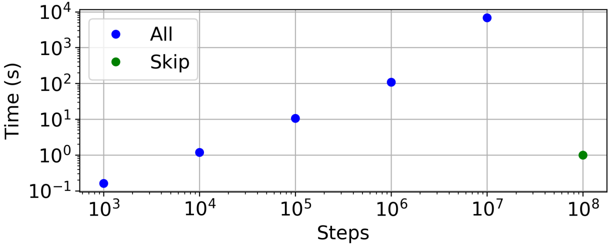
UANdINO II

- ρ won't change *that* much in this L_k scale.
- We could **skip** a lot of steps!
  If we want to skip M steps, we should do

$$U_f(ML_k, \rho_k) = U_f(L_k, \rho_k)^M$$

instead of calculating it M times.



Accuracy tests I

Reproducing plots in reference [8], in which the method of calculating U_f is explained.

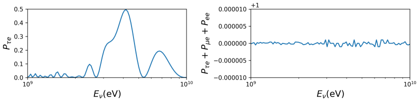


Figure: UANdINO's result

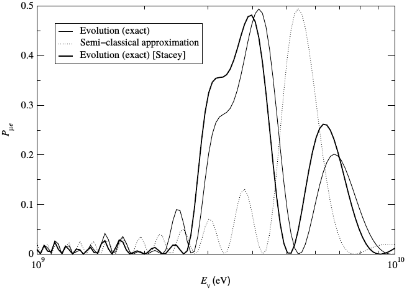


Figure: Taken from reference [8].

Accuracy tests II

Solar neutrino survival probability.

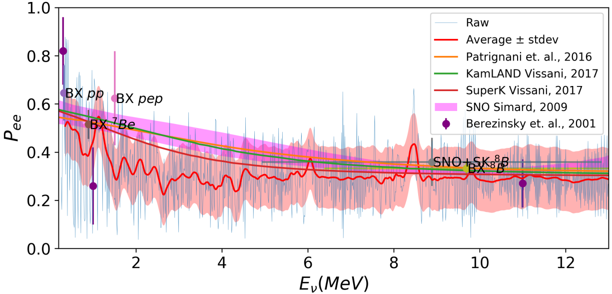
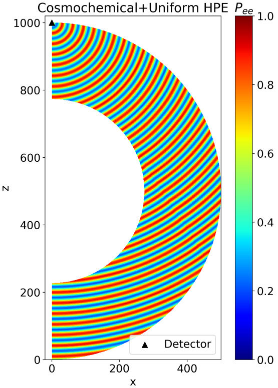


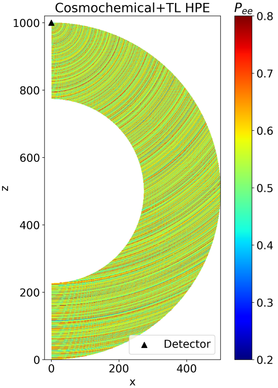
Figure: Survival probability for solar ν_e. Simulation vs. experiments.

Survival Probability: The Earth I

Some weeks later...



(a) P_{ee} for the uniform HPE distribution.



(b) P_{ee} for the two-layer (TL) HPE distribution.

Survival Probability: The Earth II

There's an average value of

$$\langle P_{ee} \rangle \approx 0.56.$$

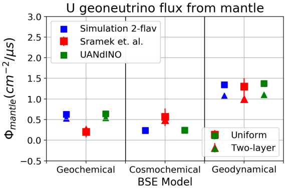
According to other authors (references [4, 5]):

$$\langle P_{ee} \rangle \approx \cos^4 \theta_{13} \left(1 - \frac{1}{2} \sin^2 2\theta_{12}\right) + \sin^4 \theta_{13} \approx 0.55. \tag{15}$$

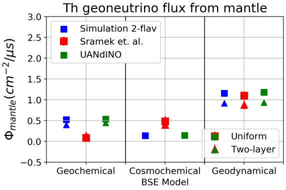
I simulated the flux with **both probabilities**.

Mantle's Flux I

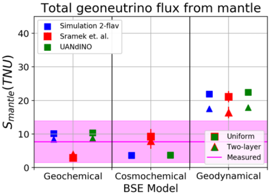
For the three BSE models and both HPE distribution models, the flux was calculated.



(c) U flux



(d) Th flux

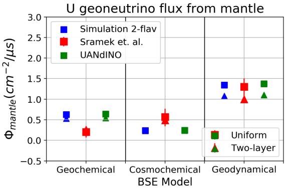


(e) Total geoneutrino mantle signal.

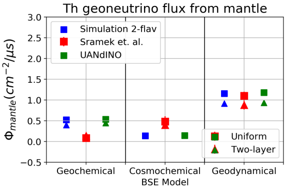
Figure: Simulated geoneutrino signal from the mantle compared to reference [9].

Mantle's Flux II

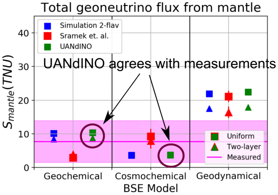
For the three BSE models and both HPE distribution models, the flux was calculated.



(a) *U* flux



(b) *Th* flux



(c) Total geoneutrino mantle signal.

Figure: Simulated geoneutrino signal from the mantle compared to reference [9].

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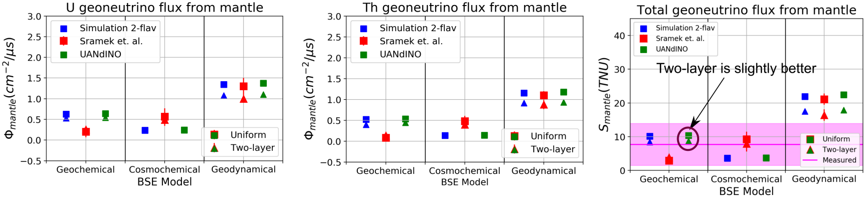
Mantle's Flux III

For the three BSE models and both HPE distribution models, the flux was calculated.



(a) *U* flux.

(b) *Th* flux.

(c) Total geoneutrino mantle signal.

Figure: Simulated geoneutrino signal from the mantle compared to reference [9].

Earth's Flux I

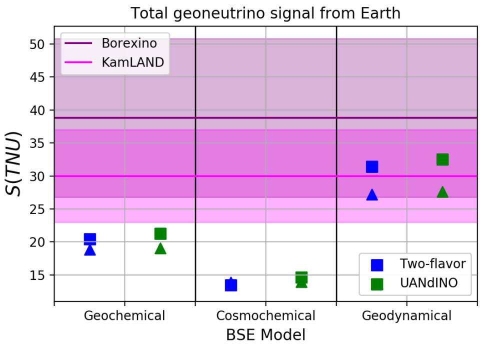


Figure: Total geoneutrino flux expected from the different models. Colored bands correspond to signals measured by the detectors according to reference [5].

Earth's Flux II

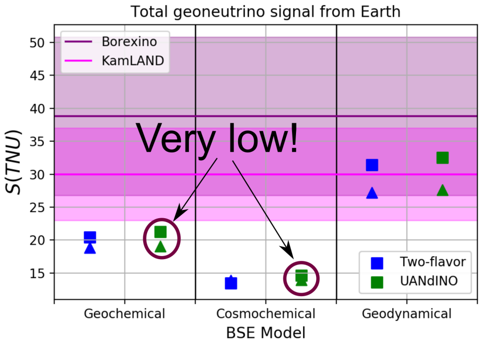


Figure: Total geoneutrino flux expected from the different models. Colored bands correspond to signals measured by the detectors according to reference [5].

Earth's Flux III

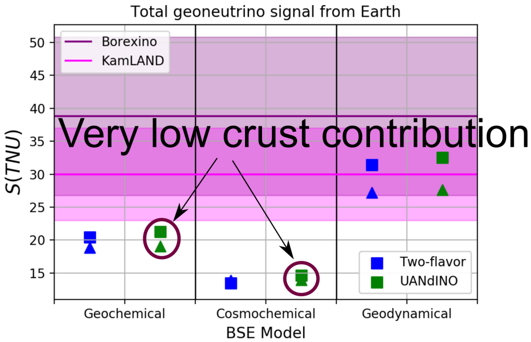
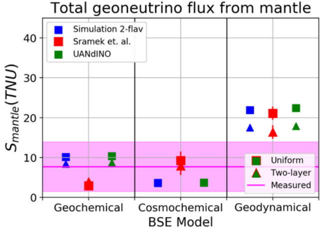
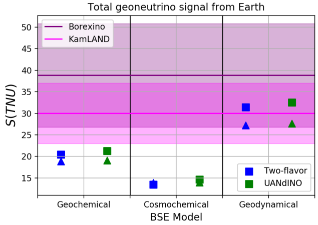


Figure: Total geoneutrino flux expected from the different models. Colored bands correspond to signals measured by the detectors according to reference [5].

Analysis





- Geodynamical model overestimates the mantle flux but fits in the crustal.
- For the other BSE, crustal contribution is underestimated while the mantle's is quite accurate.
- Cosmochemical BSE does not allow to discriminate HPE distributions.
- Geochemical BSE shows the best fit in mantle flux.
- Two-layer model may be more accurate.

Conclusions I

- The Earth's Model leads to accurate mantle flux but the total flux depends on the crustal model, which is rather simple at the moment.
- Geochemical BSE model seems to be the most accurate.
- The two-layer mantle model gives results closer to the measurements.
- Regarding HPE distributions, for the two-layer model, the possibilities are still many.
- The uncertainties in the measurements of neutrino flux, as well as in this simulation, are relevant limitations on the study.

Conclusions II

- UANdINO's results are in agreement with other results, as shown by the solar neutrino test.
- The survival probability of the geoneutrinos was also modeled and tested, showing an agreement with the fact that there is no MSW effect for antineutrinos.
- The software would be very useful, perhaps, in studies involving **neutrinos**, given it's accuracy.
- This work, in particular, could be improved by the use of more accurate crustal models available in order to correctly estimate it's contribution to the total flux.

Conclusions III

- The main conclusion is the need for more sensitive antineutrino detectors, with lower energy thresholds, in order to better discriminate between geological models.

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Then, I acknowledge my advisors, who guided me through this project.
Also, thanks to my friends, who supported me even though they don't remember what this project is about.
Finally, thanks to the people at Stackoverflow, who taught me how to code.

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