# Métodos de gradiente de política (continuación)

Fernando Lozano

Universidad de los Andes

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- Estimar $a_\pi(S_t, A_t)$

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**end for**

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donde

$$
\rho_\pi(s) = \mathbf{P}\left\{S_0 = s\right\} + \gamma \mathbf{P}\left\{S_1 = s\right\} + \gamma^2 \mathbf{P}\left\{S_2 = s\right\} + \dots
$$

con $a \sim \pi$

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- Con aproximación $\exists s$ para los que $\sum_a \tilde{\pi}(a \mid s) a_\pi(s, a) < 0$

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- Si $\pi_{\boldsymbol{\theta}} = \pi(a \mid s, \boldsymbol{\theta})$
  - $L_{\pi_{\boldsymbol{\theta}_0}}(\pi_{\boldsymbol{\theta}_0}) = \eta(\pi_{\boldsymbol{\theta}_0})$
  - $\nabla_{\boldsymbol{\theta}} L_{\pi_{\boldsymbol{\theta}}}(\pi_{\boldsymbol{\theta}})|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0} = \nabla_{\boldsymbol{\theta}} \eta(\pi_{\boldsymbol{\theta}})|_{\boldsymbol{\theta}=\boldsymbol{\theta}_0}$

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- Aproximación válida si $\tilde{\pi}$ es cercana a $\pi$

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- Teoría:

$$\eta(\tilde{\pi}) \geq L_\pi(\tilde{\pi}) - 4\frac{\epsilon\gamma}{(1-\gamma^2)}\alpha^2$$

  - $\alpha = \text{máx}\, D_{TV}(\tilde{\pi}, \pi)$
  - $\epsilon = \text{máx}_{s,a} |a_\pi(s,a)|$

- Sugiere:

$$\begin{aligned} \max \quad & L_{\boldsymbol{\theta}_{\text{old}}}(\boldsymbol{\theta}) \\ \text{sujeto a} \quad & \mathbb{E}_{\pi}\left[KL(\pi_{\boldsymbol{\theta}_{\text{old}}}(. \mid s_t)), \pi_{\boldsymbol{\theta}}(. \mid s_t))\right] \leq \delta \end{aligned}$$

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- Donde $KL(\mathbf{p}, \mathbf{q}) = \sum_i p_i \log \frac{pi}{q_i}$.
- $\boldsymbol{\theta}$ no es muy lejana a $\boldsymbol{\theta}_{\text{old}}$

# En la práctica

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\text{máx} \quad & \hat{\mathbb{E}}_t \left[ \frac{\pi_{\boldsymbol{\theta}}(a_t \mid s_t)}{\pi_{\boldsymbol{\theta}_{\text{old}}}(a_t \mid s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[ r_t(\boldsymbol{\theta}) \hat{A}_t \right] \\
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- Alternar muestreo y optimización.

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  - Algoritmos Off-Policy, On-policy, Policy Gradient.

# A3C: Asynchronous advantage actor critic

**Algorithm S3** Asynchronous advantage actor-critic - pseudocode for each actor-learner thread.

*// Assume global shared parameter vectors $\theta$ and $\theta_v$ and global shared counter $T = 0$*
*// Assume thread-specific parameter vectors $\theta'$ and $\theta'_v$*
Initialize thread step counter $t \leftarrow 1$
**repeat**
  Reset gradients: $d\theta \leftarrow 0$ and $d\theta_v \leftarrow 0$.
  Synchronize thread-specific parameters $\theta' = \theta$ and $\theta'_v = \theta_v$
  $t_{start} = t$
  Get state $s_t$
  **repeat**
    Perform $a_t$ according to policy $\pi(a_t|s_t;\theta')$
    Receive reward $r_t$ and new state $s_{t+1}$
    $t \leftarrow t+1$
    $T \leftarrow T+1$
  **until** terminal $s_t$ **or** $t - t_{start} == t_{max}$
  $R = \begin{cases} 0 & \text{for terminal } s_t \\ V(s_t, \theta'_v) & \text{for non-terminal } s_t \text{// Bootstrap from last state} \end{cases}$
  **for** $i \in \{t-1, \ldots, t_{start}\}$ **do**
    $R \leftarrow r_i + \gamma R$
    Accumulate gradients wrt $\theta'$: $d\theta \leftarrow d\theta + \nabla_{\theta'} \log \pi(a_i|s_i;\theta')(R - V(s_i;\theta'_v))$
    Accumulate gradients wrt $\theta'_v$: $d\theta_v \leftarrow d\theta_v + \partial\left(R - V(s_i;\theta'_v)\right)^2 / \partial\theta'_v$
  **end for**
  Perform asynchronous update of $\theta$ using $d\theta$ and of $\theta_v$ using $d\theta_v$.
**until** $T > T_{max}$

- NN compartida para $\pi$ y $\hat{v}$.

# A3C: Asynchronous advantage actor critic

**Algorithm S3** Asynchronous advantage actor-critic - pseudocode for each actor-learner thread.

```
// Assume global shared parameter vectors θ and θ_v and global shared counter T = 0
// Assume thread-specific parameter vectors θ' and θ'_v
Initialize thread step counter t ← 1
repeat
    Reset gradients: dθ ← 0 and dθ_v ← 0.
    Synchronize thread-specific parameters θ' = θ and θ'_v = θ_v
    t_start = t
    Get state s_t
    repeat
        Perform a_t according to policy π(a_t|s_t; θ')
        Receive reward r_t and new state s_{t+1}
        t ← t + 1
        T ← T + 1
    until terminal s_t or t − t_start == t_max
    R = { 0            for terminal s_t
        { V(s_t, θ'_v)  for non-terminal s_t // Bootstrap from last state
    for i ∈ {t − 1, ..., t_start} do
        R ← r_i + γR
        Accumulate gradients wrt θ': dθ ← dθ + ∇_θ' log π(a_i|s_i; θ')(R − V(s_i; θ'_v))
        Accumulate gradients wrt θ'_v: dθ_v ← dθ_v + ∂(R − V(s_i; θ'_v))^2/∂θ'_v
    end for
    Perform asynchronous update of θ using dθ and of θ_v using dθ_v.
until T > T_max
```

- NN compartida para $\pi$ y $\hat{v}$.
- Añade entropía de $\pi$ a la función objetivo $\rightarrow$ hiperparámetro $\beta$.

# Proximal Policy Optimization (PPO)

- TRPO:

$$\begin{aligned} \max_{\boldsymbol{\theta}} \quad & \hat{\mathbb{E}}_t \left[ \frac{\pi_{\boldsymbol{\theta}}(a_t \mid s_t)}{\pi_{\boldsymbol{\theta}_{\text{old}}}(a_t \mid s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[ r_t(\boldsymbol{\theta}) \hat{A}_t \right] \\ \text{sujeto a} \quad & \hat{\mathbb{E}}_t \left[ KL(\pi_{\boldsymbol{\theta}_{\text{old}}}(. \mid s_t)), \pi_{\boldsymbol{\theta}}(. \mid s_t)) \right] \leq \delta \end{aligned}$$

# Proximal Policy Optimization (PPO)

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$$\begin{aligned} \max_{\boldsymbol{\theta}} \quad & \hat{\mathbb{E}}_t \left[ \frac{\pi_{\boldsymbol{\theta}}(a_t \mid s_t)}{\pi_{\boldsymbol{\theta}_{\text{old}}}(a_t \mid s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[ r_t(\boldsymbol{\theta}) \hat{A}_t \right] \\ \text{sujeto a} \quad & \hat{\mathbb{E}}_t \left[ KL(\pi_{\boldsymbol{\theta}_{\text{old}}}(. \mid s_t)), \pi_{\boldsymbol{\theta}}(. \mid s_t)) \right] \leq \delta \end{aligned}$$

- Versión con penalización:

$$\max_{\boldsymbol{\theta}} \hat{\mathbb{E}}_t \left[ r_t(\boldsymbol{\theta}) \hat{A}_t - \beta KL(\pi_{\boldsymbol{\theta}_{\text{old}}}(. \mid s_t)), \pi_{\boldsymbol{\theta}}(. \mid s_t)) \right]$$

# Proximal Policy Optimization (PPO)

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$$\begin{aligned} \max_{\boldsymbol{\theta}} \quad & \hat{\mathbb{E}}_t \left[ \frac{\pi_{\boldsymbol{\theta}}(a_t \mid s_t)}{\pi_{\boldsymbol{\theta}_{\text{old}}}(a_t \mid s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[ r_t(\boldsymbol{\theta}) \hat{A}_t \right] \\ \text{sujeto a} \quad & \hat{\mathbb{E}}_t \left[ KL(\pi_{\boldsymbol{\theta}_{\text{old}}}(. \mid s_t)), \pi_{\boldsymbol{\theta}}(. \mid s_t)) \right] \leq \delta \end{aligned}$$

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  - Cota inferior en mejora de la política.

# Proximal Policy Optimization (PPO)

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  - En la práctica es difícil ajustar $\beta$

# Proximal Policy Optimization (PPO)

- TRPO:

$$\max_{\boldsymbol{\theta}} \quad \hat{\mathbb{E}}_t \left[ \frac{\pi_{\boldsymbol{\theta}}(a_t \mid s_t)}{\pi_{\boldsymbol{\theta}_{\text{old}}}(a_t \mid s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[ r_t(\boldsymbol{\theta}) \hat{A}_t \right]$$

$$\text{sujeto a} \quad \hat{\mathbb{E}}_t \left[ KL(\pi_{\boldsymbol{\theta}_{\text{old}}}(. \mid s_t)), \pi_{\boldsymbol{\theta}}(. \mid s_t)) \right] \leq \delta$$

- Versión con penalización:

$$\max_{\boldsymbol{\theta}} \hat{\mathbb{E}}_t \left[ r_t(\boldsymbol{\theta}) \hat{A}_t - \beta KL(\pi_{\boldsymbol{\theta}_{\text{old}}}(. \mid s_t)), \pi_{\boldsymbol{\theta}}(. \mid s_t)) \right]$$

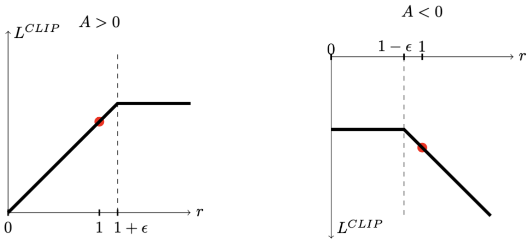
  - Cota inferior en mejora de la política.
  - En la práctica es difícil ajustar $\beta \longrightarrow \beta(t)$

- Objetivo recortado:

$$L^{\mathrm{CLIP}}(\boldsymbol{\theta}) = \hat{\mathbb{E}}_t \left[ \min(r_t(\boldsymbol{\theta})\hat{A}_t, \mathrm{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon)\hat{A}_t) \right]$$

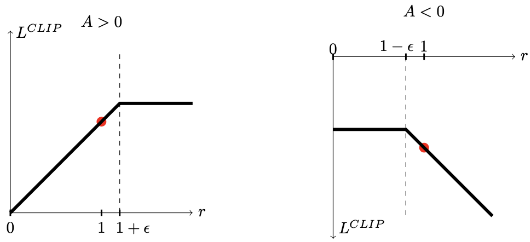
- Objetivo recortado:

$$L^{\text{CLIP}}(\boldsymbol{\theta}) = \hat{\mathbb{E}}_t \left[ \text{mín}(r_t(\boldsymbol{\theta})\hat{A}_t, \text{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon)\hat{A}_t) \right]$$



- Objetivo recortado:

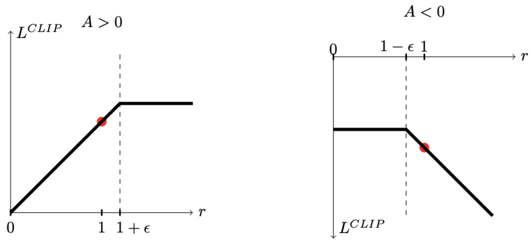
$$L^{\text{CLIP}}(\boldsymbol{\theta}) = \hat{\mathbb{E}}_t \left[ \text{mín}(r_t(\boldsymbol{\theta})\hat{A}_t, \text{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon)\hat{A}_t) \right]$$




- Mínimo entre objetivo y objetivo recortado.

- Objetivo recortado:

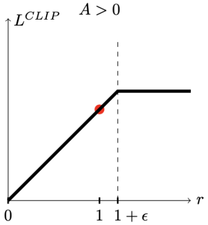
$$L^{\text{CLIP}}(\boldsymbol{\theta}) = \hat{\mathbb{E}}_t \left[ \text{mín}(r_t(\boldsymbol{\theta})\hat{A}_t, \text{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon)\hat{A}_t) \right]$$

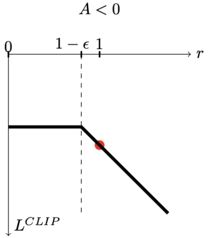



  - Mínimo entre objetivo y objetivo recortado.
  - Hiper parámetro $\epsilon$.

- Objetivo recortado:

$$L^{\text{CLIP}}(\boldsymbol{\theta}) = \hat{\mathbb{E}}_t \left[ \text{mín}(r_t(\boldsymbol{\theta})\hat{A}_t, \text{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon)\hat{A}_t) \right]$$

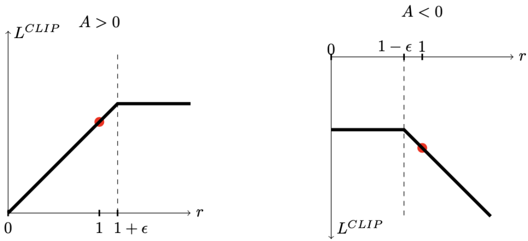



  - Mínimo entre objetivo y objetivo recortado.
  - Hiper parámetro $\epsilon$.
  - Cota inferior (pesimista) en el objetivo de TRPO.

- Objetivo recortado:

$$L^{\text{CLIP}}(\boldsymbol{\theta}) = \hat{\mathbb{E}}_t \left[ \text{mín}(r_t(\boldsymbol{\theta})\hat{A}_t, \text{clip}(r_t(\boldsymbol{\theta}), 1-\epsilon, 1+\epsilon)\hat{A}_t) \right]$$




  - Mínimo entre objetivo y objetivo recortado.
  - Hiper parámetro $\epsilon$.
  - Cota inferior (pesimista) en el objetivo de TRPO.

- NN compartida:

$$L(\boldsymbol{\theta}) = \hat{\mathbb{E}}_t \left[ L^{\text{CLIP}}(\boldsymbol{\theta}) - c_1 (v_{\boldsymbol{\theta}}(S_t) - V_t^{\text{targ}})^2 + c_2 H(\pi_{\boldsymbol{\theta}})(S_t) \right]$$

# PPO

**for** iteración=1,2,. . . , **do**

# PPO

**for** iteración=1,2,. . . , **do**

  **for** actor=1,2,. . . ,N **do**

# PPO

**for** iteración=1,2,. . . , **do**

  **for** actor=1,2,. . . ,N **do**

    Corra política $\pi_{\boldsymbol{\theta}_{\text{old}}}$ por $T$ pasos

# PPO

**for** iteración=1,2,. . . , **do**

**for** actor=1,2,. . . ,N **do**

Corra política $\pi_{\boldsymbol{\theta}_{\text{old}}}$ por $T$ pasos

Calcule estimativos $\hat{A}_t$, $t = 1, \ldots, T$

# PPO

**for** iteración=1,2,. . . , **do**

  **for** actor=1,2,. . . ,N **do**

    Corra política $\pi_{\boldsymbol{\theta}_{\text{old}}}$ por $T$ pasos

    Calcule estimativos $\hat{A}_t$, $t = 1, \ldots, T$

  **end for**

  Optimice $L(\boldsymbol{\theta})$ con $K$ épocas y minibatch de tamaño $M \leq NT$

# PPO

**for** iteración=1,2,. . . , **do**
    **for** actor=1,2,. . . ,N **do**

        Corra política $\pi_{\boldsymbol{\theta}_{\text{old}}}$ por $T$ pasos
        Calcule estimativos $\hat{A}_t$, $t = 1, \ldots, T$
    **end for**
    Optimice $L(\boldsymbol{\theta})$ con $K$ épocas y minibatch de tamaño $M \leq NT$
    $\boldsymbol{\theta}_{\text{old}} = \boldsymbol{\theta}$
**end for**