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$$K = h^\alpha \left| \frac{\partial(h+h')}{\partial x} \right|^\beta$$

(1)

$$\frac{\partial h}{\partial t} = A \frac{\partial}{\partial x} \left[h^\alpha \left| \frac{\partial(h+h')}{\partial x} \right|^\beta \frac{\partial(h+h')}{\partial x} \right]$$

$$\frac{\partial h}{\partial t} = A \frac{\partial}{\partial x} \left[K \frac{\partial(h+h')}{\partial x} \right]$$

$$\frac{\partial h}{\partial t} = A \frac{\partial}{\partial x} \left[K \left(\frac{\partial h}{\partial x} + \frac{\partial h'}{\partial x} \right) \right]$$

$$\frac{\partial h}{\partial t} = A \frac{\partial}{\partial x} \left[K \frac{\partial h}{\partial x} + K \frac{\partial h'}{\partial x} \right]$$

$$\frac{\partial h}{\partial t} = A \frac{\partial}{\partial x} \left[K \frac{\partial h}{\partial x} \right] + A \frac{\partial}{\partial x} \left[K \frac{\partial h'}{\partial x} \right]$$

$$\frac{\partial h}{\partial t} = A \left(\frac{\partial K}{\partial x} \frac{\partial h}{\partial x} + K \frac{\partial^2 h}{\partial x^2} \right) + A \left(\frac{\partial K}{\partial x} \frac{\partial h'}{\partial x} + K \frac{\partial^2 h'}{\partial x^2} \right)$$

$$\frac{h_i^{k+1} - h_i^k}{\Delta t} = A \left(\frac{K_{i+1}^k - K_{i-1}^k}{2\Delta x} \cdot \frac{h_{i+1}^k - h_{i-1}^k}{2\Delta x} + K_i^k \frac{h_{i+1}^k - 2h_i^k + h_{i-1}^k}{\Delta x^2} \right) + A \left(\frac{K_{i+1}^k - K_{i-1}^k}{2\Delta x} \cdot \frac{h_{i+1}'^k - h_{i-1}'^k}{2\Delta x} + K_i^k \frac{h_{i+1}'^k - 2h_i'^k + h_{i-1}'^k}{\Delta x^2} \right)$$

$$h_i^{k+1} = \frac{A\Delta t}{4\Delta x^2} \left(K_{i+1}^k h_{i+1}^k - K_{i-1}^k h_{i-1}^k - K_{i+1}^k h_{i-1}^k + K_{i-1}^k h_{i+1}^k \right. \\ \left. + 4K_i^k h_{i+1}^k - 8K_i^k h_i^k + 4K_i^k h_{i-1}^k \right. \\ \left. + K_{i+1}^k h_{i+1}'^k - K_{i-1}^k h_{i-1}'^k - K_{i+1}^k h_{i-1}'^k + K_{i-1}^k h_{i+1}'^k \right. \\ \left. + 4K_i^k h_{i+1}'^k - 8K_i^k h_i'^k + 4K_i^k h_{i-1}'^k \right) + h_i^k$$

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$$C_A = \frac{A \Delta t}{4 \Delta x^2}$$

(2)

$$\begin{aligned} h_i^{k+1} = & (C_A K_{i-1}^k + 4C_A K_i^k - C_A K_{i+1}^k) h_{i-1}^k \\ & + (1 - 8C_A K_i^k) h_i^k \\ & + (-C_A K_{i-1}^k + 4C_A K_i^k + C_A K_{i+1}^k) h_{i+1}^k \\ & + (C_A K_{i-1}^k + 4C_A K_i^k - C_A K_{i+1}^k) h_{i-1}^{1k} \\ & + (-8C_A K_i^k) h_i^{1k} \\ & + (-C_A K_{i-1}^k + 4C_A K_i^k + C_A K_{i+1}^k) h_{i+1}^{1k} \end{aligned}$$

$$K = h^\alpha \left| \frac{\partial(h+h^1)}{\partial x} \right|^B = h^\alpha \left| \frac{\partial h}{\partial x} + \frac{\partial h^1}{\partial x} \right|^B$$

$$K_i^k = h_i^k \left| \frac{h_{i+1}^k - h_{i-1}^k}{2\Delta x} + \frac{h_{i+1}^{1k} - h_{i-1}^{1k}}{2\Delta x} \right|^B$$

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$$\begin{bmatrix} h_i^{K+1} \\ h_i^K \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ c_{AK_{i-1}}^K + 4c_{AK_i}^K - c_{AK_{i+1}}^K & -c_{AK_{i-1}}^K + 4c_{AK_i}^K + c_{AK_{i+1}}^K & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} h_i^K \\ h_i^K \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ c_{AK_{i-1}}^K + 4c_{AK_i}^K - c_{AK_{i+1}}^K & -c_{AK_{i-1}}^K + 4c_{AK_i}^K + c_{AK_{i+1}}^K & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} h_i^K \\ h_i^K \end{bmatrix}$$

Boundary Conditions:

$$h|_{x=740N} = 0$$

$$h|_{x=300N} = 0$$