

CS 4720/5720 Design and Analysis of Algorithms

Homework #5b

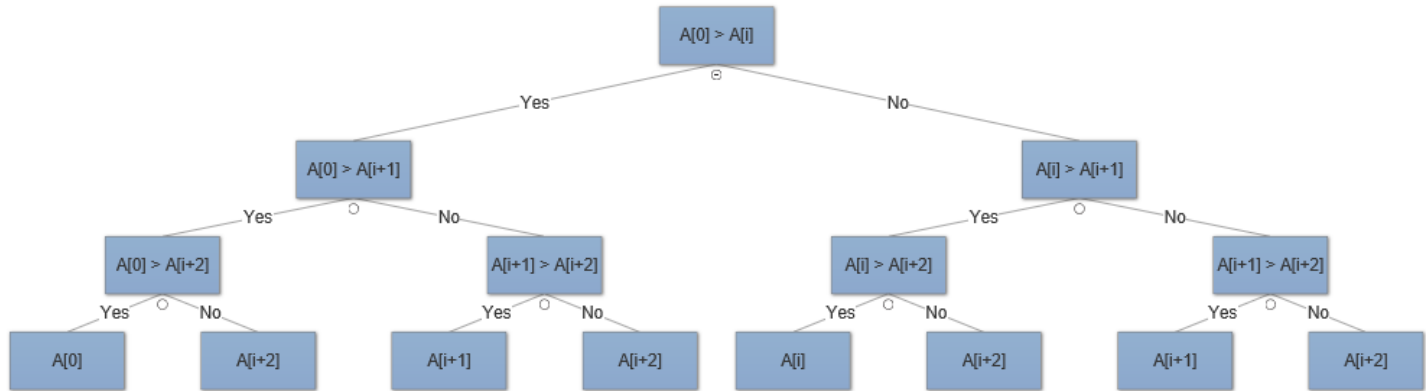
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Answers to homework problems:

1. *Lower bounding on the complexity of array search*

Trivially, all elements must be checked. A number is only the maximum if every other element "lost" at least once (every other element is less than the maximum), which would require $n - 1$ comparisons.

Consider the decision tree of an array size 3 $A[0, 2]$, it makes $n - 1$ comparisons to arrive at an maximum:

2. *Tractability*

- (a)
- $\Theta(n \log \log n)$
- running time**

Tractable since this is bounded by a polynomial.

- (b)
- $\Theta(5^n)$
- running time**

Intractable since this is exponential.

- (c)
- $\Theta(n!)$
- running time**

Intractable since this is factorial.

3. *Greedy Knapsack*

Consider a knapsack with capacity $W = 10$, and items with values $v_i : \{6, 6, 10\}$ and weights $w_i : \{3, 7, 8\}$

GREEDY: value = 10, weight = 8

OPT: value = 12, weight = 10

$$r = \frac{12}{10} = 1.2$$

Consider the values $v_i : \{\varepsilon, \varepsilon, 10 - \varepsilon\}$

$r = \frac{10 - \varepsilon}{\varepsilon} = \frac{10}{\varepsilon} - 1 \Rightarrow \varepsilon \rightarrow 0 = \infty \therefore$ Greedy Knapsack has an arbitrarily poor performance ratio.