Rigid and Variable Embodiment – Theory and Applications in Formal Ontology

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Introduction

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A "flat" view has been the standard in philosophy and ontology

- The whole is a mereological sum of its parts.
- This part-whole structure is governed by Classical Extensional Mereology (CEM). So, there is **no nested structure**, e.g.:

$$(a + b) + (c + d) = a + b + c + d$$

- The whole exists iff each of its parts exists. Thus, there is no requirement of the parts having any particular "form" or arrangement.

E.g., BFO, where 'table' is a phase-sortal (Otte et al., 2022).

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	sets	$H_2 0$ molecule	a polymer
nested structure	✓	✗	✓
hylomorphic structure	✗	✓	✓

Structure via “embodiments”

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This provides a **general and flexible framework** that enables us to “construct” a great variety of structured wholes on demand. We get:

- **fine-grained wholes** with nested and/or hylomorphic structure, which have
- **richer properties**, and
- a built-in account of **identity conditions** (incl. across time and possible worlds)

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- Ham sandwich: $b_1, h, b_2/R$
- Some components being assembled together to make up a car
- *Qua* objects, such as Biden *qua* president and Biden *qua* father: b/P and b/F

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Terminology

- the objects $a, b, \ldots$ are *the objectual parts* of the embodiment
- the relation R is *the principle of embodiment* $\rightsquigarrow$ the type of object

Rigid embodiments as objects

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This is important when a complex object is **assembled in steps** (more shortly).

However, taking rigid embodiments to be objects introduces a **threat of paradox** (more later).

Postulates concerning existence and location

(R1) The rigid embodiment $a, b, c/R$ **exists at a time** t iff R holds of a, b, c at time t.

(R2) If the rigid embodiment $e = a, b, c, \ldots/R$ exists at time t, then e is **located at the point** p at t iff at least one of $a, b, c, \ldots$ is located at p.

The identity of rigid embodiments

A natural first attempt:

(R3) The rigid embodiment $a, b, c/R$ and the rigid embodiment $a', b', c'/R'$ are **identical** iff $a = a'$ and $b = b'$ and $c = c'$ and $R = R'$.

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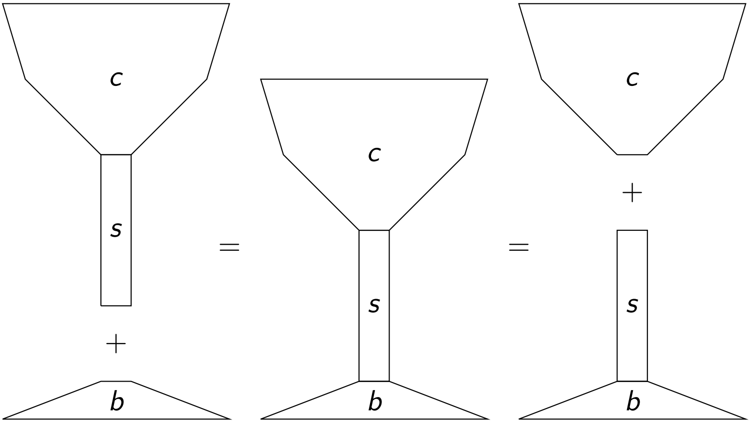
The identity postulate R3 is too strong, for two reasons:

- **converse relations** plausibly yield the same rigid embodiment, e.g.:

$$\text{Mary, John}/\textsc{Loves} \;=\; \text{John, Mary}/\textsc{Loved-by}$$

- the **order of assembly** typically does not matter.

Invariance under order of assembly (Thanks to Giovanni Codognato)



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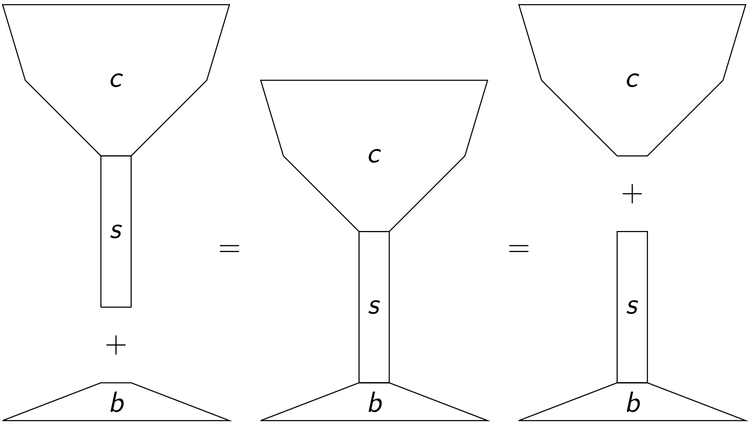

Let R be the relation of two objects being connected. We want:

$$(c, s/R), b/R = c, (s, b/R)/R$$

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(where $r[a/o]$ is the result of replacing o with a in r).

- then "divide out" by this equivalence, i.e., use the well-known, general technique to identify equivalent objects.

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Thus, we propose:

(R3*) The rigid embodiment $a, b, c/R$ and the rigid embodiment $a', b', c'/R'$ are identical iff the embodiments are equivalent in the sense describe above.

Parts of a rigid embodiment: sufficient conditions

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Thus, the parts of Glass are: c, s, b, and R but also $c, s/R$, $s, b/R$, and possibly Glass itself.

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Consider a three-piece suit. Its immediate parts are the three garments, not *their* proper parts.

Necessary conditions for *mediate* parthood: two options

To obtain the notion of **mediate parthood**, we can adopt Fine's principle:

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Although a rigid embodiment has its parts timelessly, these parts may *themselves* have temporary parts.

E.g., the slices of bread that make up a ham sandwich (understood as a rigid embodiment) may lose some crumbs.

We formally define **general mediate part** by additionally closing under *temporary* immediate part.

Properties of rigid embodiments

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For this to work, φ_F must be **invariant** under the equivalence (associated with the relation) $\sim$ that we use to individuate rigid embodiments:

$$\vec{a}, R \sim \vec{b}, S \;\rightarrow\; (\varphi_F(\vec{a}, R) \leftrightarrow \varphi_F(\vec{b}, S)) \tag{Invar}$$

Introduction to variable embodiments

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The notation: $/R/$

Again, the relation R is called *the principle of embodiment*.

A metaphor for variable embodiments

A container of water as a metaphor

- the container corresponds to the principle of embodiment
- the container with a particular portion of water in it corresponds to the manifestation
- the container with the ever-changing water corresponds to the variable embodiment

Some principles concerning variable embodiments

(V1) The variable embodiment $f = /F/$ **exists at time** t iff it has a manifestation at t.

(V2) If the variable embodiment $f = /F/$ exists at t, then its **location** is that of its manifestation f_t (assuming that f_t has a location).

(V3) The variable embodiments $/F/$ and $/G/$ are **identical** iff their principles F and G are the same.

How to understand the principle of embodiment

Is the principle of variable embodiment specific to the given individuals?

- If *no*, the account cannot handle two qualitatively identical variable embodiments.
- If *yes*, the principle is trope-like—which is problematic.
- Might the principle be *sui generis*? If so, this is left under-specified by (V1)–(V3). We will now propose a way to flesh it out.

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We begin with a triadic relation R of individual objects: $R(b_1, h, b_2)$.

The fact $R(b_1, h, b_2)$ is more perspicuously represented as $R'(bb, h)$, where R' is a two-place relation whose first argument place takes a **plurality of objects**, and $R'(bb, h)$ represents that bb flank h.

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The **roles** that objects play in R correspond to the **argument places** of R'.

Further, we might consider all relations R'' obtained by **permuting the argument places** of R'. Then we can identity roles across the various relations R' and R'' as well.

Multigrade relations? A digression

The relation R of individual objects must be **multigrade**, i.e., flexible as to how many arguments it takes.

The move to the plural relation R' removes one reason for going multigrade: instead of $a_1, a_2, \ldots$, we use a single plural argument aa.

Are there reasons to allow the plural relation R' too to be multigrade?

The identity of variable embodiments across time (I)

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Three easy types of **non-disruptive change**

1. **replacement** (of an object with another in the same role); e.g., getting a heart transplant
2. **reassignment** (reassigning an occupant of one role in the variable embodiment to another role); e.g., president and vice-president swapping roles
3. **gain or loss of occupants**: e.g., an organization gains or loses a member.

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(iii) **gain or loss of occupants**: e.g., an organization gains or loses a member.

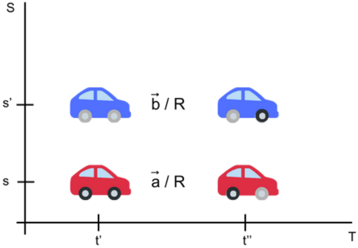
Two less obvious types of **non-disruptive change**

(iv) **formal change**: the principle R changes. An organization decides to have two vice-presidents rather than just one (Guarino and Guizzardi, 2024,); metamorphosis in biological development

(v) a variable embodiment can continue to exist even when one of its **roles is temporarily unoccupied**.

The identity of variable embodiments across time (II)

Identity across time is the result of **stringing together chains of NDCs**.



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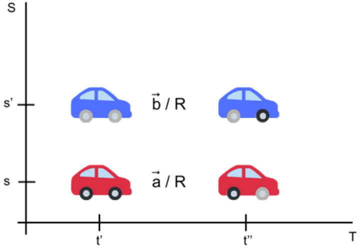


More precisely:

- NDC is a two-place reflexive, symmetric and not-transitive relation of non-disruptive change relating rigid embodiments.
- Take the transitive closure of this relation.
- The resulting relation is an equivalence relation by construction.
- Then, divide out to identify equivalent objects.

Empty argument places? A digression

How to model a car (as a variable embodiment) that temporarily loses its steering wheel? Or a company that is temporarily without a treasurer?

Two options

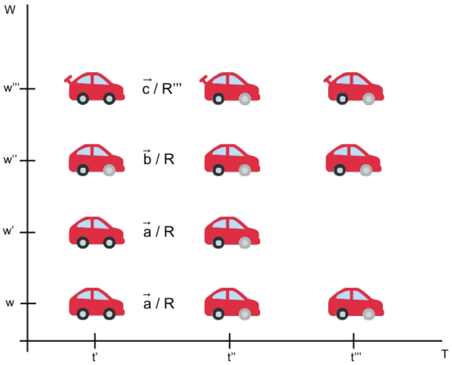
- invoke a **formal change**, i.e., the principle R (which *requires* a steering wheel) is replaced by R' (that does *not*).
- avoid formal change by allowing a relation to obtain even when *some* of its **argument places are empty**.

We can do the latter (without "empty objects") by using **co-partial functions**, i.e. functions that can be defined even where some arguments are missing.

The identity of var. embodiments across possible worlds

We use NDCs to track them across possible worlds as well. Three cases:

- *identical* manifestation: w'
- *material counterpart* of the manifestation: w''
- *formal counterpart* thereof: w'''



Fleshing out variable embodiments

A variable embodiment is represented by the set $\langle W_1, E \rangle$, where W_1 is a subset of the set of possible worlds W, E is a set of equivalence classes obtained from NDCs according to the previous three cases and the procedure to identify equivalent objects, s. t. each possible world w_j in W_1 is coupled with the corresponding equivalence class built from NDCs in w_j.

The associated function f from worlds and times to rigid embodiments represents the principle of embodiment.

I. It is a function by construction: given each of the three cases, for each world, w_i, f selects at most one equivalence class.
II. f is a partial function: our car may not exist in every world - this is why W_1 is a subset of the set of possible worlds, W.
III. The mapping from time-indices to rigid embodiments is ensured by how NDCs have been introduced.

Thus, we can still use the notation $/R/$—in a sense that is now defined.

Parthood of variable embodiments

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This suggests the following view of temporary parthood:

$$a \leq_t v \;\leftrightarrow\; \exists r(a \leq r \wedge r = v_t) \qquad (\leq_t/\leq\text{-Link})$$

This implies that any temporary part at t of $/v/$ is an atemporal part of its manifestation at t.

Constant parthood of variable embodiments

Next, we can define a notion of *constant* parthood of a variable embodiment. Consider two variable embodiments $/F/$ and $/G/$. Suppose:

(i) whenever $/F/$ exists, $/G/$ exists too, and
(ii) for every moment of time t at which $/F/$ exists, we have $/F/ \leq_t /G/$.

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Further notions of parthood can be defined as well. E.g., we might want to allow any kind of object to be a constant part of a variable embodiment.

Properties of variable embodiments

(V7) A variable embodiment inherits its *pro tem* properties from its manifestations:

$$P_t(v) \;\leftrightarrow\; \exists r(v_t = r \wedge P(r))$$

E.g., being built from Chinese-made parts or being all red.

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Yet other properties derive from the principle of embodiment: (necessarily) being a car, (necessarily) being a company.

Paradoxes of rigid embodiments

A **set-like version of Russell's paradox**: the mapping

$$xx \quad \mapsto \quad \begin{cases} xx/R, & \text{if } Rxx; \\ xx/\neg R, & \text{if } \neg Rxx. \end{cases}$$

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$$xx \quad \mapsto \quad \begin{cases} xx/R, & \text{if } Rxx; \\ xx/\neg R, & \text{if } \neg Rxx. \end{cases}$$

is one-to-one from pluralities to objects (at least for suitable principles R). This threatens a violation of (a generalization of) Cantor's theorem.

Constructionalism (Florio and Linnebo, 2024)—*from FOIS 2024*—as a solution:

- objects are successively defined, stage by stage
- each of xx has to be available **before** we can "form" or define xx/R

Analogy with the iterative conception of sets (Boolos, 1971).

Paradoxes of variable embodiments

A **property-like version of Russell's paradox**: $F \mapsto s/F$ or to $/F/$ is a one-to-one map of properties into objects—and so an apparent violation of (another generalization of) Cantor's theorem.

This is different from the set-like version and calls for a different response.

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According to **the Vicious Circle Principle**, when we define F, both entities that figure directly in the definition *and entities over which this definition quantifies* must be available. There are also various ways to liberalize VCP (Linnebo and Shapiro, 2023).

Concluding summary

We have presented Kit Fine's theory of embodiments and clarified several aspects of the theory:

- **the identity of rigid embodiments**: order of assembly, converse relations
- **the roles** in a relation
- the various forms of **non-disruptive change** (NDC)
- **the principle of (variable) embodiment**, using NDCs
- how this principle accounts for the identity of variable embodiments **across time and possible worlds**
- how a **constructionalist approach** to embodiments blocks the paradoxes

These clarifications will facilitate applications of the theory.

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