Problem Set 1

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First, let's define what we already know:

P(Man born in August|Gus) = P(A|G) = $\frac{3}{20}$ P(G) = $\frac{1}{2}$ P(A) = $\frac{1}{12}$

1a: What is the overall probability of being born in August?

$\frac{1}{12}$: We already know that all months are equally likely.

1b: What is P(Name=*Gus*|Man born in August)? That is, conditional on being born in August, what is the probability that a man is named Gus?

By Bayes' Rule, we know that P(A|B) = $\frac{P(B|A)*P(A)}{P(B)}$. So here, we can do P(G|A) = $\frac{P(A|G)*P(G)}{P(A)} = \frac{\frac{3}{20}*\frac{1}{2}}{\frac{1}{12}} = \frac{\frac{3}{40}}{\frac{1}{12}} = \frac{36}{40} = \frac{9}{10} = 90\%$.

1c: What is P(Name=*Gus*|Man born in July)?

If we assume that in each month other than August, "Gus"es are spread out evenly, then we can say the following: if the likelihood of a "Gus" being born in August is 15% ($\frac{3}{20}$), the likelihood for the rest of the year is ($\frac{17}{20}$). Then, divided by the remaining 11 months, we get ($\frac{17}{220}$). We can write this as P(J|G), or the probability that a man was born in July, given that his name is Gus. This gives us enough information to use Bayes' Rule again:

$$\text{P(G|J)} = \frac{P(J|G)*P(G)}{P(J)} = \frac{\frac{17}{220}*\frac{1}{2}}{\frac{1}{12}} = \frac{\frac{17}{440}}{\frac{1}{12}} = \frac{204}{440} = 46.36$$

1d: Imagine that there are 600 men born in 1982. Given the conditions above, what is the probability that at least two baby *Guses* are born on August 1, 1982?

1e: What is the probability that at least two baby *Hugos* are born on August 1, 1982?

1f: Given your answers to (d) and (e), what would you say to a pundit who pointed to two *Guses* with the same August 1, 1982 birthday as evidence of voter fraud in the form of double voting? What would you say to a pundit who pointed to two *Hugos* with the same August 1, 1982 birthday as evidence of voter fraud in the form of double voting?

I think that one cannot "conclude" anything either way, but there's a much higher chance that there are two Guses who were born on that day than that there are two Hugos.