Problem Set 2

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Question 1 Background:

In a parliamentary system, newly elected members of parliament choose a prime minister and a cabinet ("the government"). Typically the government stays in power for a fixed term; however, a no-confidence vote from the parliament or a decision by the prime minister can lead to early elections. Suppose we want to understand the duration of governments in a parliamentary democracy with five-year terms – that is, elections must be held at least every five years, but might be held earlier (in the case of a no-confidence vote or a prime ministerial decision to dissolve the government).

1a) If we can measure the government duration in infinitely small units of time (ie, continuously), what is the sample space for this experiment?

The sample space, if continuous, must be greater than 0 (more than no time in office) and less than or equal to five (it can technically last five years). So it would look like: $\Omega = (0, 5]$.

1b) The random variable $X(\omega)$ is the amount of time in years or fractions of years) between the last election and the calling of the next election. Suppose we know that $X(\omega)$ has the probability density function:

$$f(x) = \begin{cases} kx^3 & 0 < x < 5 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant. What is k (hint: remember the rules of pdfs)?

In this case, k = $\frac{1}{125}$. When the highest x-value gets plugged into the equation, the result should be 1. Since 5 is the highest value, we would end up with $k * (5)^3 = 1$. Thus, $k = \frac{1}{5^3}$ or $\frac{1}{125}$.

1c) What is the CDF of X?

We know that the CDF can be defined as $F(x) = \int_{-\infty}^{\infty} f(x)\, dx$. Thus, here we can say that for the first interval we have:

$\int_0^y \frac{3}{125}x^2\, dx$. This is simply the derivative of our initial equation, $(\frac{1}{125}x^3)$.

For the remainder, we can say the following:

$F(x) = \int_{-\infty}^{y} 0\, dx$

Also, we already know that F(5) = 1, so F(x) = 1 for x > 5.