Problem Set 3

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Question 1 Background:

Suppose that X and Y are identical and independently distributed (i.i.d.) random variables distributed $\mathcal{N}(\mu_X, \sigma_X^2)$ and $\mathcal{N}(\mu_Y, \sigma_Y^2)$, respectively. Note that σ_X represents the standard deviation of X, and σ_X^2 the variance. Find the following, expressed in terms of μ and σ:

1a) $E(7X - 6Y + 12)$

Additivity and homogeneity are two properties of expectations. Thus, we can write this expression as $7E[X] - 6E[Y] + 12$, or $7\mu_X - 6\mu_Y + 12$.

1b) Var$(X + 5Y)$

We know that $Var(aX + bY + c) = a^2Var(X) + b^2Var(Y) + 2abCov(X, Y)$. Inserting a 1 and 5 as a and b, we get $\sigma_X^2 + 25\sigma_Y^2 + 10Cov(X, Y)$. $Cov(X, Y) = E[XY] - E[X]E[Y] = E[(X - \mu_X)(Y - \mu_Y)]$. So then we get: $Var(X + 5Y) = \sigma_X^2 + 25\sigma_Y^2 + 10E[(X - \mu_X)(Y - \mu_Y)]$.

Because we know that the two variables are so-called "i.i.d." variables, we know that the co-variance is 0. Thus, we can remove the last term entirely and end up with $Var(X + 5Y) = \sigma_X^2 + 25\sigma_Y^2$.

1c) $E(5X^2 - 12XY + 16Y^2)$

First, we can split these up according to properties shown in 1a. So we get $5E[X^2] - 12E[XY] + 16E[Y^2]$.

Now, we can already deal with the first and last bits of this expression. We know that $\sigma_X^2 = E[X^2] - \mu_X^2$ and $\sigma_Y^2 = E[Y^2] - \mu_Y^2$. Thus, $E[X^2] = \sigma_X^2 + \mu_X^2$ and $E[Y^2] = \sigma_Y^2 + \mu_Y^2$. So we can set this as $5\sigma_X^2 + 5\mu_X^2 + 12E[XY] + 16\sigma_Y^2 + 16\mu_Y^2$.

Finally, the question tells us that X and Y are independent random variables. So we know therefore that $E[XY] = E[X]E[Y]$. We can write this as $\mu_X\mu_Y$.

Our final expression: $5\sigma_X^2 + 5\mu_X^2 + 12\mu_X\mu_Y + 16\sigma_Y^2 + 16\mu_Y^2$.

Question 2 Background:

Papers like this one have found that married couples tend to sort along ideological lines ("assortative mating"). Say we know the true distribution of ideology among heterosexual married couples along a scale of 1 (very conservative) to 4 (very liberal). The joint distribution for X (the woman's ideology) and Y (the man's ideology) is:

Y	X 1	2	3	4
1	.12	.05	.01	.01
2	.08	.17	.05	.02
3	.02	.01	.23	.05
4	.02	.02	.04	.10

2a) What is the expected value of X and the expected value of Y?

To find the expected value of X, we can run the formula: $E[X] = \Sigma(x)P(x)$. So we multiply the value x by each corresponding probability and add them up. So $E[X] = ((1 * .12) + (1 * .08) + ... + (4 * .10))$. The final answer is **2.57**.

To find the expected value of Y, we can run the same formula, but for Y instead of X. So we multiply the value y by each corresponding probability and add them up. So $E[Y] = ((1 * .12) + (1 * .05) + ... + (4 * .10))$. The final answer is **2.48**.

2b) What are the variances of X and of Y?