QUANTUM COMPUTATION

Analysing Qubit Decay with Bayesian Statistics

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The basic blocks of quantum computers are qubits. These can be any quantum object that can exist in a superposition of states and driven by a field. In practical applications of quantum computing, these qubits need to be entangled with each other and completely isolated from the environment. In practice the latter condition isn't possible and they interact with their surroundings, leaking information. When this happens the computation fails [1].

In order to build a quantum computer we first need to know the parameters that describe the qubits. Primarily, their decay constant Γ and their natural frequency of oscillation, their Rabi Frequency (RF), Ω. The ratio between these two values determines the how likely we are to finish the computation before decay occurs. Using Bayes Theorem we can easily obtain this by measuring the time intervals between decay events.

THE QUBIT WAVE FUNCTION

Qubits are ideally 2 state systems with a ground state, which represents the value 0 in classical computing, and an excited state, which represents 1. In practice this could be the spin of an atomic nucleus, aligned or unaligned to a magnetic field. Contrary to classical bits, qubits can be in a superposition of both 0 and 1.

$$\left|\Psi(t)\right\rangle = a(t)\left|0\right\rangle + b(t)\left|1\right\rangle$$

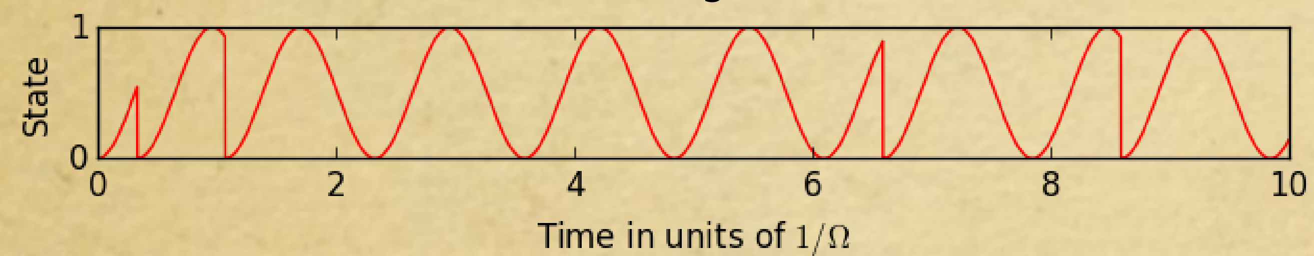
This system evolves with time, and in the case of the a nuclear spin qubit, we can drive evolution of the system harmonically using a laser field. To solve how the system evolves we can use the Optical Bloch Equations (OBEs):

$$i\hbar\frac{\partial\rho}{\partial t} = \left[\hat{H}, \rho\right]$$

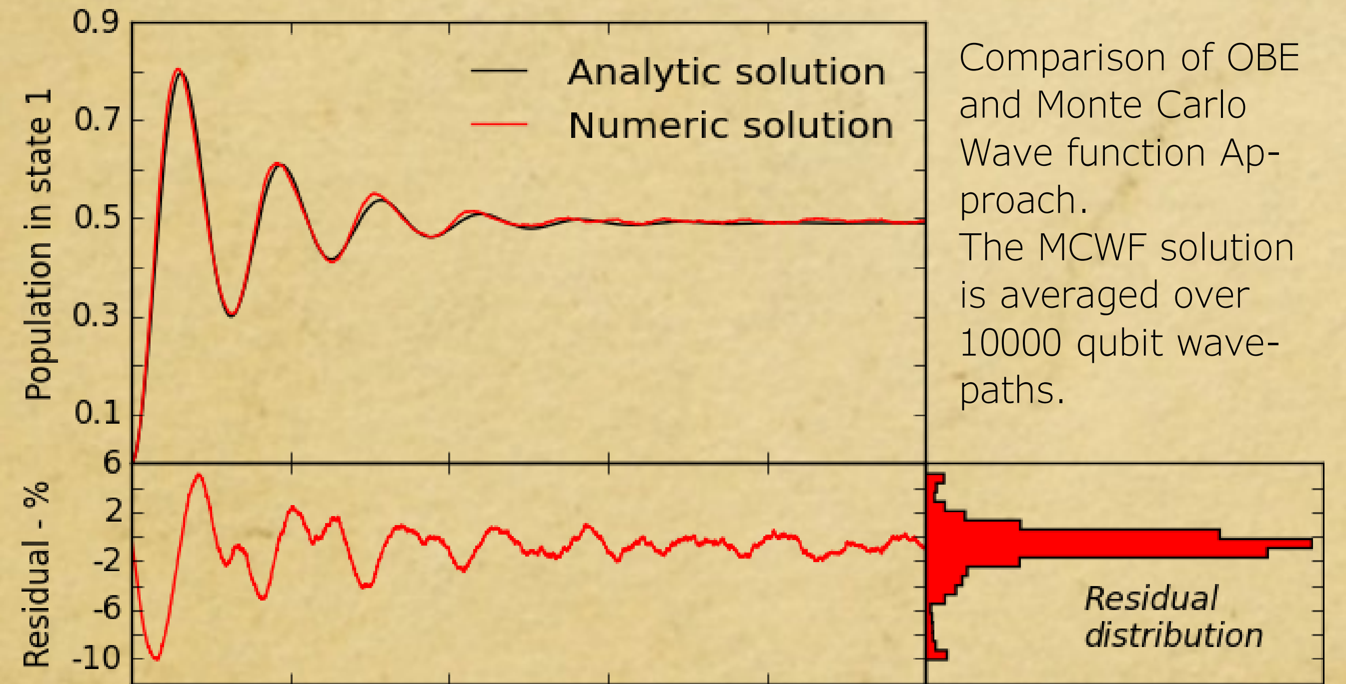
Where ρ is the outer product of the wave-vector Ψ with itself and $\hat{H}$ is the Hamiltonian of the system.

This can and has been solved analytically [2] but the solution is only observable in the real world in averages of large numbers of particles.

It's possible to approximate a single qubit system numerically using the differential form of the OBEs and Monte Carlo Methods to introduce random emission [3]. We used the fourth order Runge-Kutta integration method and the ratio of $\Omega = 5\Gamma$ in figure 1 below.



Whenever you sum up a great number of these individual wave paths it approaches the analytic solution to the OBE. See figure 2 below.



Comparison of OBE and Monte Carlo Wave function Approach.
The MCWF solution is averaged over 10000 qubit wave-paths.

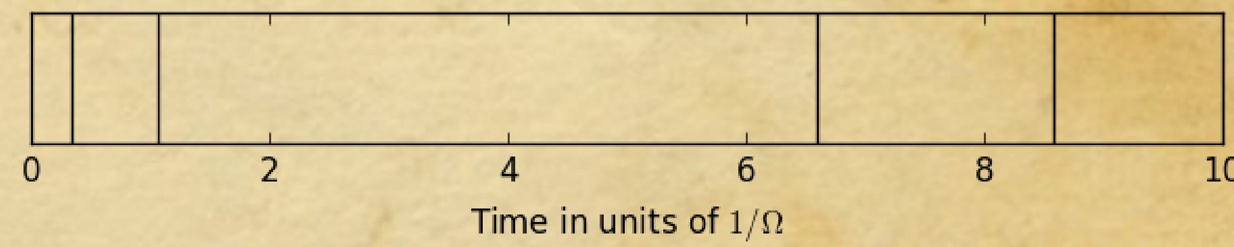
[1] Le Bellac, 2006, *Quantum Information and Quantum Information,* Cambridge University Press.

[2] H.-R. Noh and W. Jhe, Analytic Solutions of the Optical Bloch Equations, Opt. Commun. **283**, 2353 (2010).

[3] Molmer, Castin and Dalibard, *Monte Carlo Wave-Function Method in Quantum Optics*, J. Opt, Soc. Am. B **10**, 524 (1993).

THE BAYESIAN METHOD

Besides spontaneous decay, the qubit state evolves predictably in time and always decays into the same ground state. Therefore the entire wave form of fig1. can be represented without loss of information by the set of times at which it decays [4]. i.e. The times when a photon is emitted. Shown in figure 3 below.

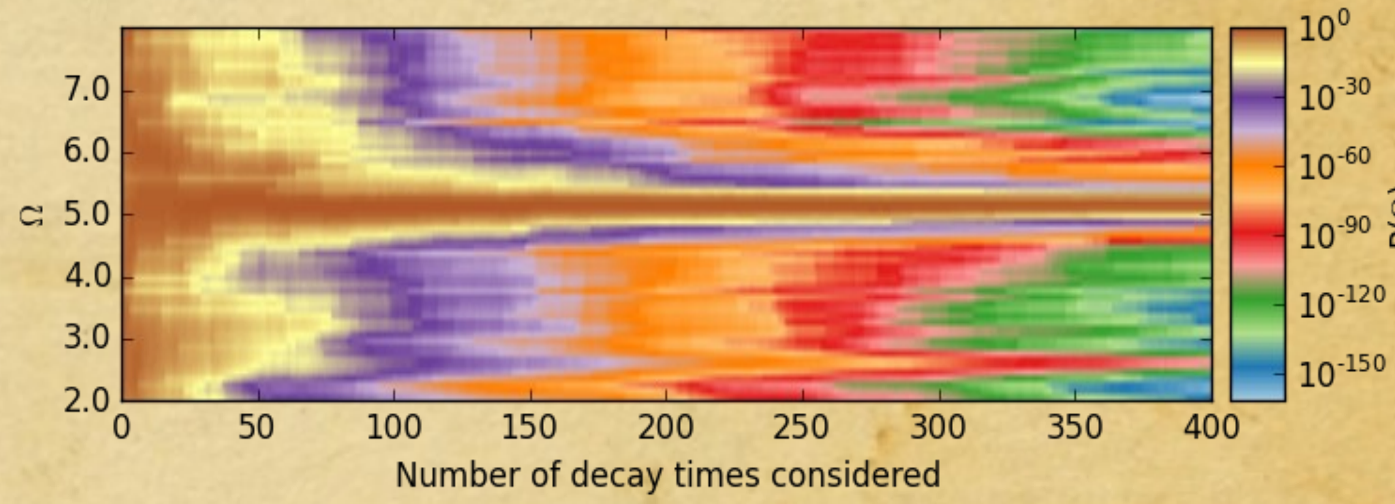


In this plot we appear to lose information about the qubits RF. However the decay times observed above are most probable for a qubit with RF equal to *fig1*. and from the wait time, τ, between emission events we can implement *Bayes Theorem* [5] and work backwards to find the RF. Bayes theorem states:

$$P(\Omega\,|\,\tau) = \frac{P(\tau\,|\,\Omega)P(\Omega)}{\int P(\tau\,|\,\Omega)P(\Omega)d\Omega}$$

Where $P(\Omega|\tau)$ is the probability that the RF is a value Ω given that we observe a wait time τ. $P(\Omega)$ is the prior probability (probability before considering the wait time τ) of the RF being a value Ω.

After we iterate this process over many photons the probabilities of all possible RFs evolve. The probability of the true RF tends to 1 whilst the probability of all others possible Ω values tend to zero. Figure 4 below.



CONCLUSION AND OUTLOOK:

We can use the Bayesian method to reconstruct the RF from a list of times when the qubit decays. This approach is easily extended to find the decay parameter. We hope in further work this system can be extended to analyse data collected in a lab setting where the efficiency of the detector is less than 1 and we can not initially drive the system without detuning.
This will allow different qubit systems to be evaluated as candidates for use in quantum computing.

[4] A. H. Kiilerich and K .Molmer, *Estimation of atomic interaction parameters by photon counting*, Phys. Rev. A 89, 052110 (2014).

[5] Eshky, *Bayesian Methods of Parameter Estimation,* Edinburgh, University of Edinburgh.

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