Assignment 4

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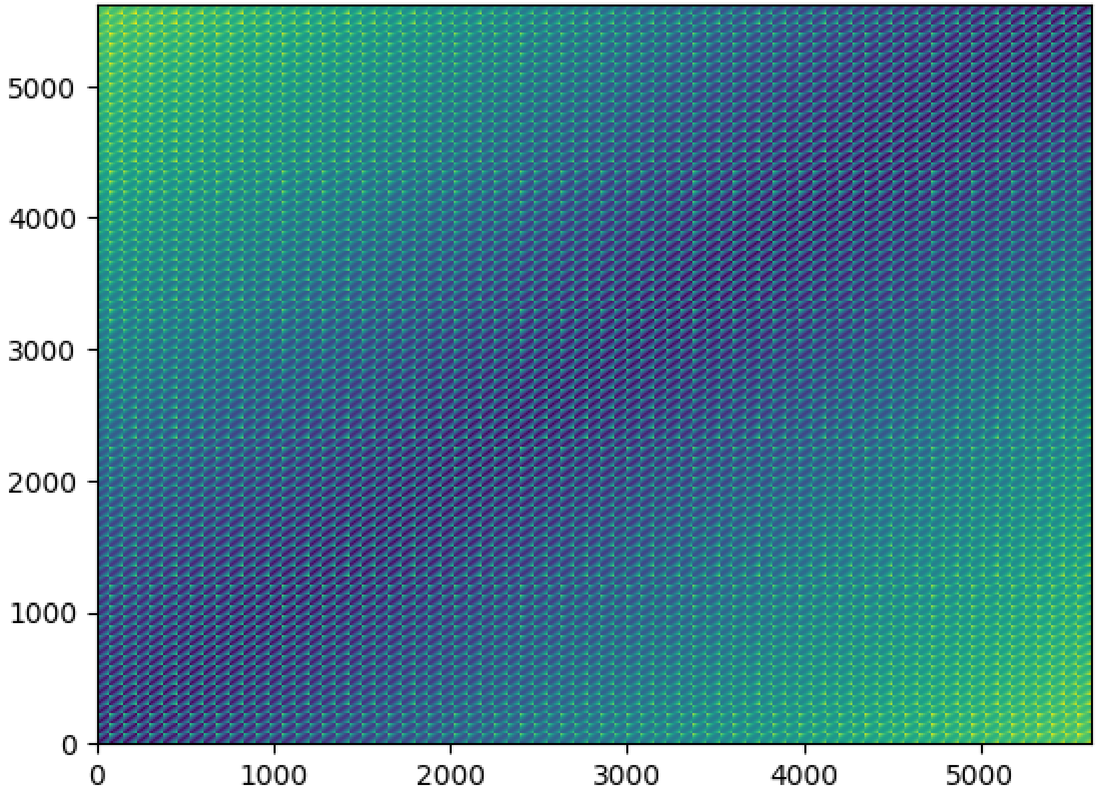
In [29]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as ss
import matplotlib.animation as animation
import matplotlib
%matplotlib inline
import pandas as pd
```

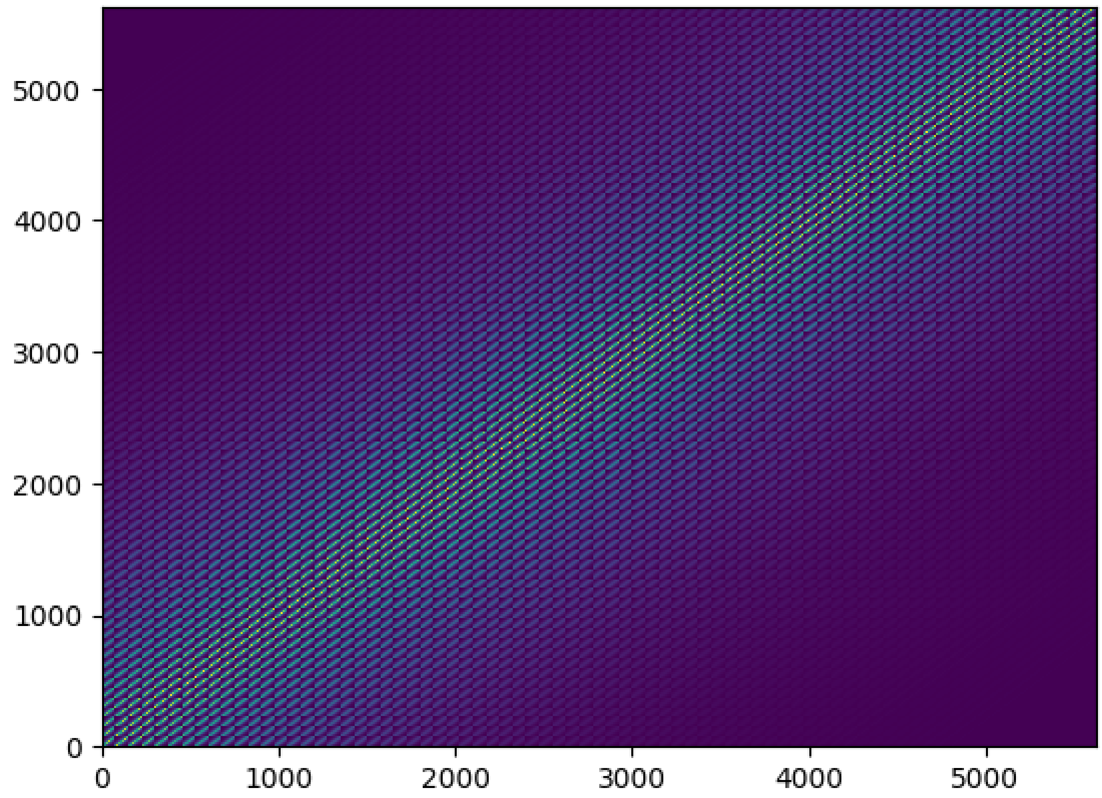
In [30]:
```
# Define number of grid cells
nx = 75
ny = 75

# Create mesh grid for x and y
x = np.arange(1,nx+1,1).reshape(-1,1)
y = np.arange(1,ny+1,1).reshape(-1,1)
x_grid, y_grid = np.meshgrid(x, y)

# Flatten each mesh grid
x_grid_flat = x_grid.reshape(1, nx*ny)
y_grid_flat = y_grid.reshape(1, nx*ny)

# Find distance from each grid point to every other grid point
x_dist = x_grid_flat - x_grid_flat.T
y_dist = y_grid_flat - y_grid_flat.T
dist = np.sqrt(np.square(x_dist) + np.square(y_dist))
plt.pcolormesh(dist)
```

Out[30]:
```
<matplotlib.collections.QuadMesh at 0x111328fcd50>
```



In [31]:
```
# Define input data as per assignment
mean_phi = 4
ra = 35
gamma = 1
sigma = 1
n_real = 200

# Calculate covariance matrix from distance matrix
Cov_phi = np.square(sigma)*np.exp(-3*np.power(dist/ra, gamma))
plt.pcolormesh(Cov_phi)
```

Out[31]: `<matplotlib.collections.QuadMesh at 0x11132126a90>`



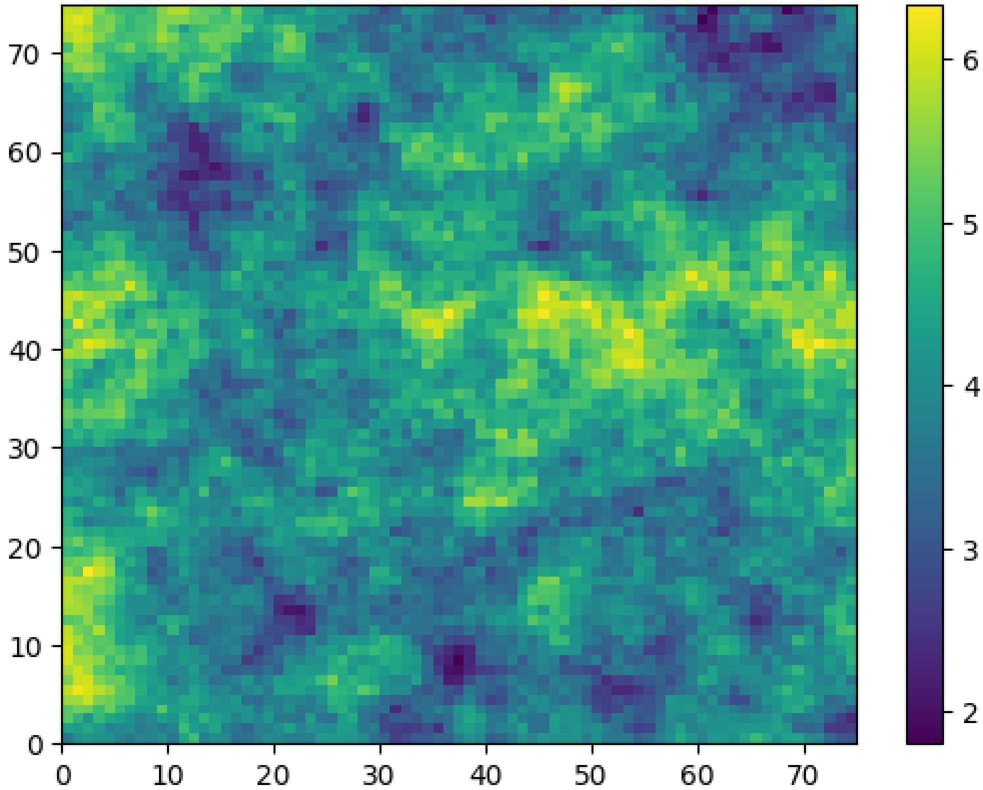
In [32]:
```
# Find square root of covaraince matrix / standard deviation
u, s, vh = np.linalg.svd(Cov_phi)
s = np.diag(s)
s_sqrt = np.sqrt(s)
Cov_phi_sqrt = np.linalg.multi_dot((u, s_sqrt, vh))
```

In [33]:
```
# Draw random numbers for each realization
z = ss.norm.rvs(loc=0, scale=1, size=(nx*ny, n_real))
```

In [34]:
```
# Calculate all realizations
phi_real_all = mean_phi + np.dot(Cov_phi_sqrt, z)
```

In [35]:
```
# Define function to fetch a given realization
def phi_real_i(i):
    return phi_real_all[:,i-1].reshape((nx, ny))

plt.pcolormesh(phi_real_i(1))
plt.colorbar()
```

Out[35]:
```
<matplotlib.colorbar.Colorbar at 0x11132755090>
```



In [56]:
```
# Switching matplotlib viewer to enable animations
matplotlib.use('WebAgg')

# Create the initial plot
fig, ax = plt.subplots()
im = ax.pcolormesh(x_grid, y_grid, phi_real_i(1), cmap='viridis')

# Define the update function for the animation
def animate(i):
    im.set_array(phi_real_i(i).ravel())
    ax.set_title(f"Frame {i}")
    return im,

# Create the animation
anim = animation.FuncAnimation(fig, animate, frames = np.arange(1,n_real+1), int
plt.show()
```

Task 2

In [37]:
```
%matplotlib inline
df_data = pd.read_excel (r'stock_price_data.xlsx')
df_data
```

Out[37]:

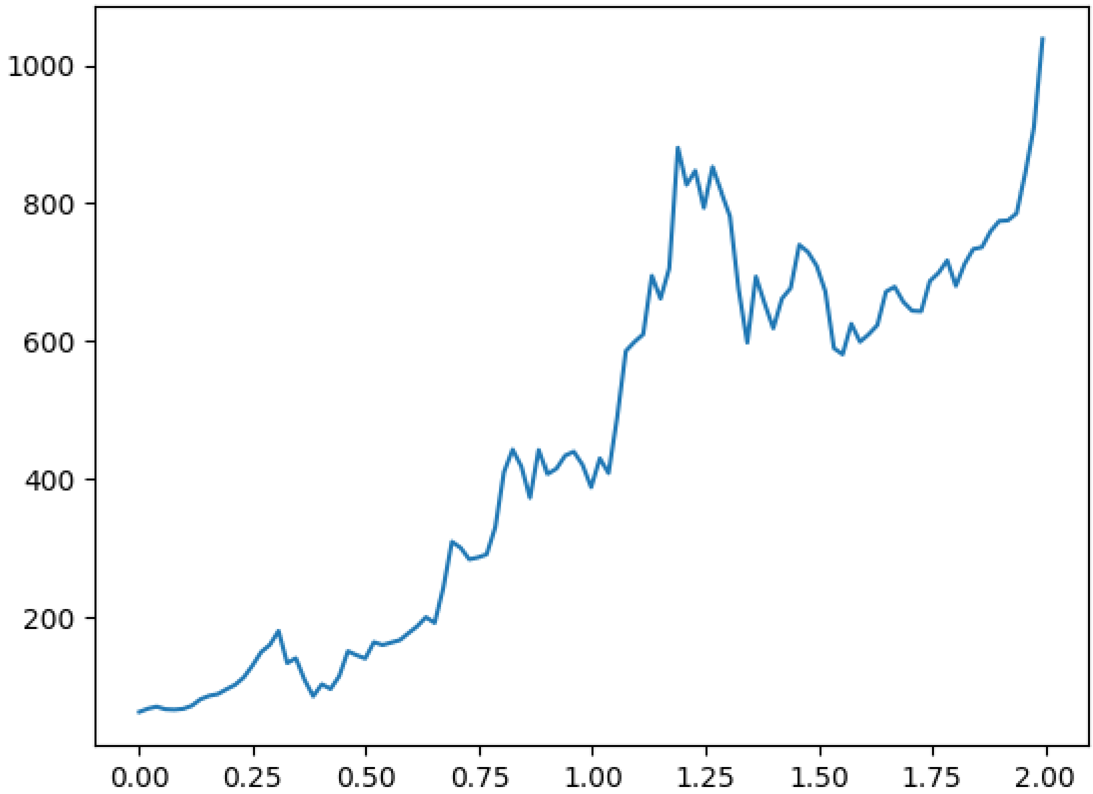
	Date	Relative Time (days)	Relative Time (years)	Price ($)
0	2019-10-28	0	0.000000	62.661999
1	2019-11-04	7	0.019178	67.428001
2	2019-11-11	14	0.038356	70.433998
3	2019-11-18	21	0.057534	66.608002
4	2019-11-25	28	0.076712	65.987999
...	...	...	...	...
100	2021-09-27	700	1.917808	775.219971
101	2021-10-04	707	1.936986	785.489990
102	2021-10-11	714	1.956164	843.030029
103	2021-10-18	721	1.975342	909.679993
104	2021-10-25	728	1.994521	1037.859985

105 rows × 4 columns

In [38]:

```
# Visalize data
plt.plot(df_data['Relative Time (years)'].values, df_data['Price ($)'].values)
```

Out[38]: [<matplotlib.lines.Line2D at 0x11133bde350>]



In [39]:

```
dt = 7/365 # years
```

a) GBM Calibration

Calculate $x_t=ln(S_{t+dt}/S_t)$

In [ ]:
```
# Calculate x_t according to formula
S_next = df_data['Price ($)'].values[1:]
S_pre = df_data['Price ($)'].values[0:-1]
x_t = np.log(S_next/S_pre)
```

In [41]:
```
# Calculate mean and standard devaition of x_t
x_mean = np.mean(x_t)
x_sd = np.std(x_t)

print(x_mean, x_sd)
```

```
0.026991932481463796 0.09876476949744876
```

In [42]:
```
# Calculate GBM parameters mu and sigma
mu_gbm = (2*x_mean+x_sd**2)/(2*dt)
sigma_gbm = x_sd/np.sqrt(dt)

print(mu_gbm, sigma_gbm)
```

```
1.661749699981164 0.7131805109375213
```

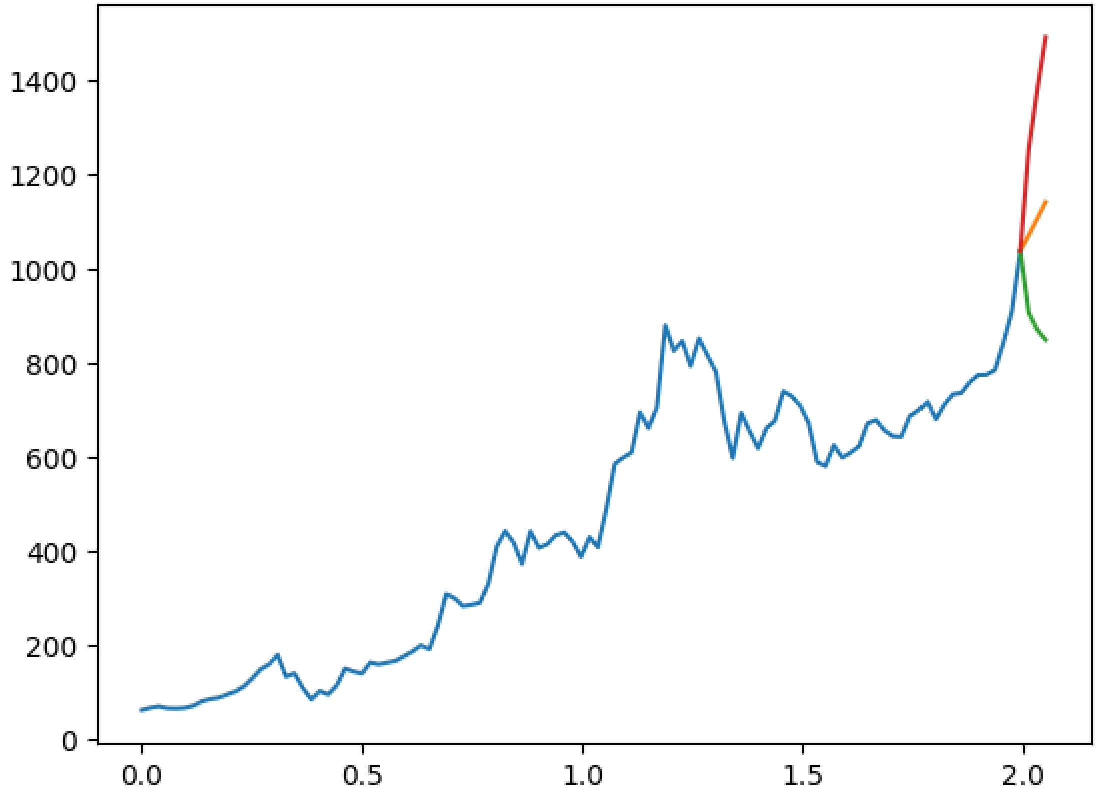
In [43]:
```
# Set S_0 to last value in time series
S_0 = df_data['Price ($)'].values[-1]

# Establish relative times for forecasting 3 weeks ahead
t_forecast = 7*np.arange(4)/365 #

# Generate forecast
S_forecast = S_0*np.exp(mu_gbm*t_forecast)
S_forecast
```

Out[43]:
```
array([1037.859985  , 1071.46841568, 1106.16516909, 1141.98548777])
```

In [ ]:
```
# Calculate P05 and P95 outcomes
lnS_t_mean_gbm = np.log(S_0)+(mu_gbm-sigma_gbm**2/2)*t_forecast
lnS_t_var_gbm = sigma_gbm**2*t_forecast
lnS_t_P05_gbm = ss.norm.ppf(q=0.05, loc=lnS_t_mean_gbm, scale=np.sqrt(lnS_t_var_
lnS_t_P05_gbm[0] = np.log(S_0)
lnS_t_P95_gbm = ss.norm.ppf(q=0.95, loc=lnS_t_mean_gbm, scale=np.sqrt(lnS_t_var_
lnS_t_P95_gbm[0] = np.log(S_0)
S_t_P05_gbm = np.exp(lnS_t_P05_gbm)
S_t_P95_gbm = np.exp(lnS_t_P95_gbm)
```

In [45]:
```
# Visalize forecast
plt.plot(df_data['Relative Time (years)'].values, df_data['Price ($)'].values)
plt.plot(t_forecast + df_data['Relative Time (days)'].values[-1]/365, S_forecast
plt.plot(t_forecast + df_data['Relative Time (days)'].values[-1]/365, S_t_P05_gb
plt.plot(t_forecast + df_data['Relative Time (days)'].values[-1]/365, S_t_P95_gb
```

Out[45]:
```
[<matplotlib.lines.Line2D at 0x11133c8fe50>]
```



In [46]:
```
print(f'P05-P95 interval after 3 weeks is {S_t_P05_gbm[-1]:.1f} to {S_t_P95_gbm[
```

```
P05-P95 interval after 3 weeks is 849.4 to 1491.1
```

b) GOU Calibration

Calculate $\pi_t=ln(S_t)$

In [47]:
```
pi_t = np.log(df_data['Price ($)'].values)
```

Let $x_t=\pi_t$ and $y_t=\pi_{t+dt}$

In [48]:
```
x_t = pi_t[:-1]
y_t = pi_t[1:]
```

In [49]:
```
regression = ss.linregress(x = x_t, y = y_t)
a = regression.slope
b = regression.intercept
print(a,b)
```

```
0.9817982221362007 0.13284985994263465
```

Calculate regression residuals and the SD of the residuals

In [50]:
```
y_reg = a*x_t + b
residuals = y_t - y_reg
residuals_sd = np.std(residuals, ddof=1)
residuals_sd
```

Out[50]:
```
0.09806476191560043
```

Calculate GOU parameters: μ, θ, and σ

In [51]:
```
mu_gou = b/(1-a)
theta_gou = -np.log(a)/dt
sigma_gou = residuals_sd*np.sqrt(-2*np.log(a)/(dt*(1-a**2)))
print(mu_gou, theta_gou, sigma_gou)
```

```
7.29872987884628 0.9578365551292606 0.7146395667743817
```

GOU Forecasting

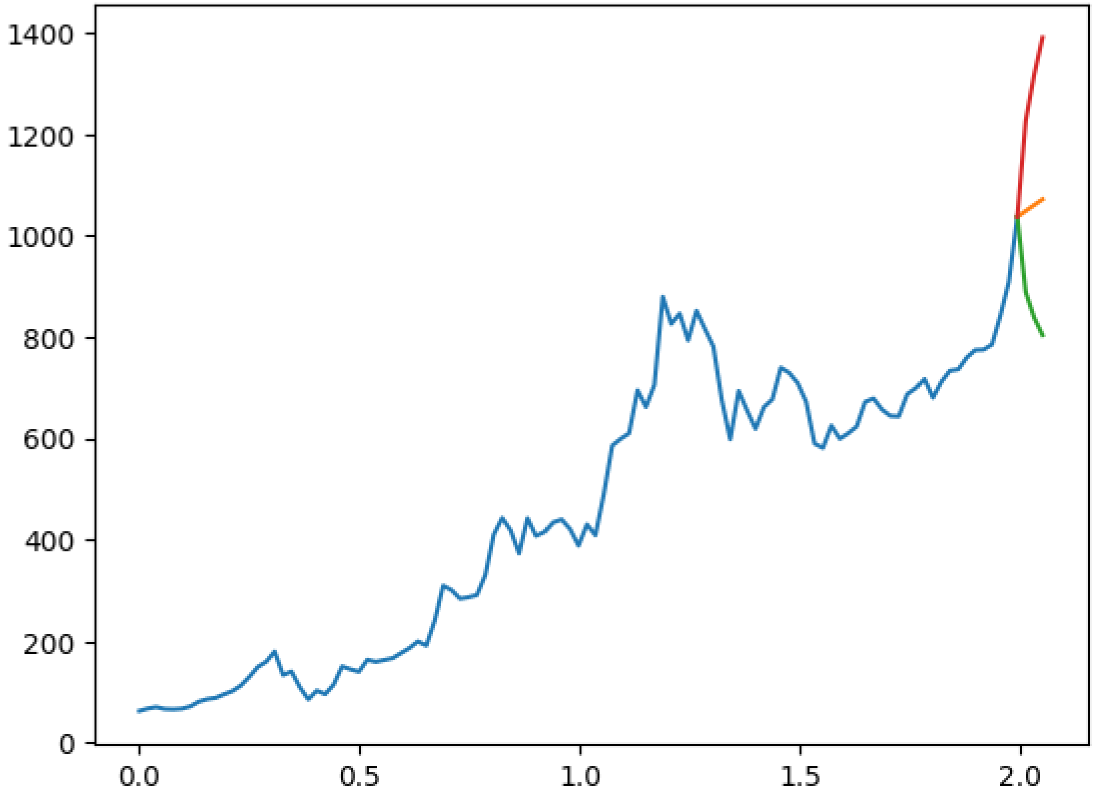
In [52]:
```
# Calculate forecasted mean

S_0 = df_data['Price ($)'].values[-1]
lnS_0 = np.log(S_0)

lnS_t_mean_gou = lnS_0*np.exp(-theta_gou*t_forecast)+mu_gou*(1-np.exp(-theta_gou
lnS_t_var_gou = sigma_gou**2/(2*theta_gou)*(1-np.exp(-2*theta_gou*t_forecast))
S_t_mean_gou = np.exp(lnS_t_mean_gou+lnS_t_var_gou/2)
S_t_mean_gou
```

Out[52]:
```
array([1037.859985  , 1049.60014129, 1061.16460936, 1072.55369358])
```

In [53]:
```
# Calculate forecasted P05 and P95

lnS_t_P05_gou = ss.norm.ppf(q=0.05, loc = lnS_t_mean_gou, scale = np.sqrt(lnS_t_
lnS_t_P05_gou[0] = np.log(S_0)

lnS_t_P95_gou = ss.norm.ppf(q=0.95, loc = lnS_t_mean_gou, scale = np.sqrt(lnS_t_
lnS_t_P95_gou[0] = np.log(S_0)

S_t_P05_gou = np.exp(lnS_t_P05_gou)
S_t_P95_gou = np.exp(lnS_t_P95_gou)
```

In [54]:
```
# Visalize GOU forecast
plt.plot(df_data['Relative Time (years)'].values, df_data['Price ($)'].values)
plt.plot(t_forecast + df_data['Relative Time (days)'].values[-1]/365, S_t_mean_g
plt.plot(t_forecast + df_data['Relative Time (days)'].values[-1]/365, S_t_P05_gc
plt.plot(t_forecast + df_data['Relative Time (days)'].values[-1]/365, S_t_P95_gc
```

Out[54]:
```
[<matplotlib.lines.Line2D at 0x11133e589d0>]
```



In [55]:
```
print(f'P05-P95 interval after 3 weeks is {S_t_P05_gou[-1]:.1f} to {S_t_P95_gou[
```

```
P05-P95 interval after 3 weeks is 803.9 to 1391.7
```

c) GBM vs GOU

We can see that the forecasts from the two different models are quite similar in this case and it would be tough to say which one predicts the uncertainty of the future stock price better.

In general I would not recommend using any of these models for predicting future stock prices. Stocks are influenced by other factors than its own history, such as human psychology. Mathematical models do not include such complex influences, which are critical to modern macroeconomic movements.