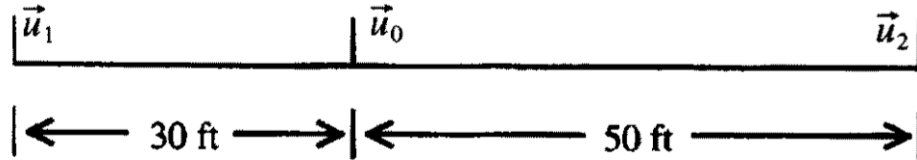# Assignment 3

Solve these problems using Python and hand in your assignment in the form of a Jupyter Notebook which includes both text (markdowns) and code.

1. **Simple problem – Simple Kriging:** (20 points)
Consider the data configuration, $x(\boldsymbol{u_1}) = x(\boldsymbol{u_2}) = 20, m = 18$ (see figure).

Further, assume that he isotopic variogram, representing the spatial relationship, is given as:

$$\gamma(L) = 100\left[1 - exp\left(\frac{-3h}{100}\right)\right] \ for \ h \geq 0$$

a) Estimate the value at location $\boldsymbol{u_0}$. Also, estimate the error variance.

b) If the spatial model is given as

$$\gamma(L) = 200\left[1 - exp\left(\frac{-3h}{100}\right)\right] \ for \ h \geq 0$$

How would the estimate and error variance change?

2. **Ordinary Kriging Problem with Spatial Variations:** (40 points)
    a) Estimate the value of a regionalized variable at point V from the four points in figure. Calculate the weights and value of the estimated variable, the local mean and the error variance. The values at the sample points are

| i	$z(u_i)$
1	20
2	50
3	30
4	100

Assume a spherical variogram with a sill of 1, range of 200 and no nugget effect.
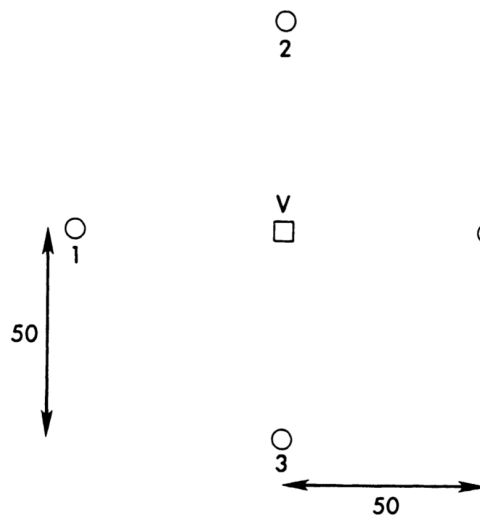
    b) Using the same point distribution, but in a spherical model with an overall sill of 1 and a nugget effect of 0.25, calculate the weights and value of the estimated variable, the local mean and the error variance.
    c) Consider now the configuration of wells in the figure below, in which two wells are very close to each other and therefore redundant, and two wells are roughly equidistant from each other and the cluster. Calculate the weights and value of the estimated variable, the local mean and the error variance using the variogram model from part (a).
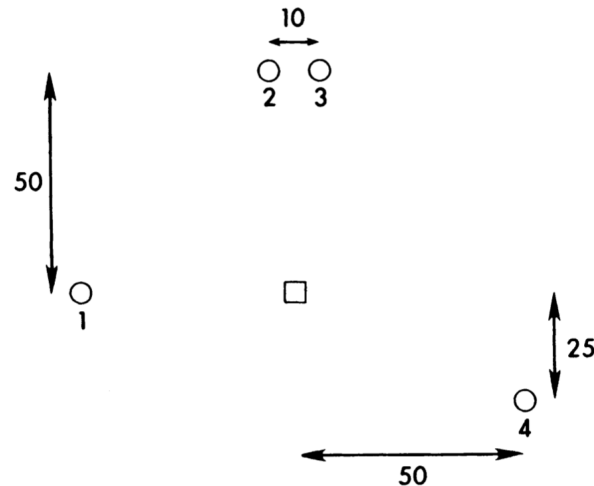
d) Going back to the configuration and variogram used in (a), evaluate the influence of anisotropy by using an anisotropy ratio of 1.5 in the east-west direction.

e) Using the previous isotropic variogram model, nugget effect of zero, sill of 1, and range equal to 200, solve the kriging equations and estimate the value at point V for the configuration below.
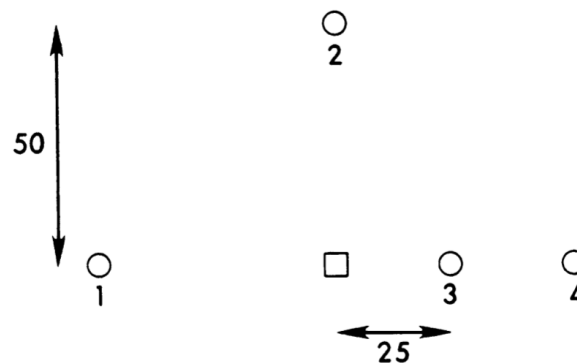
f) Compare and comment on the resulting weights and variable estimates in the four cases discussed in (a) through (e)

4. Simple Kriging and Ordinary Kriging problem: (10 points) Figure 3 shows three log permeability data points and an estimation location in a given reservoir. The following pieces of information are available to us:

1.   The distance between the estimation point and Point 1 is 1.2m.
2.   The distance between the estimation point and Point 2 is 2m.
3.   The distance between the estimation point and Point 3 is 0.9m.

Other distances are as indicated in the figure. The spatial correlation between the data is adequately represented by a spherical variogram model with a nugget of 1.2, a sill of 6.8 and a range of 10m.

Using (a) Simple kriging, (b) Ordinary kriging
(i)   Calculate the value of log permeability at the estimation point.
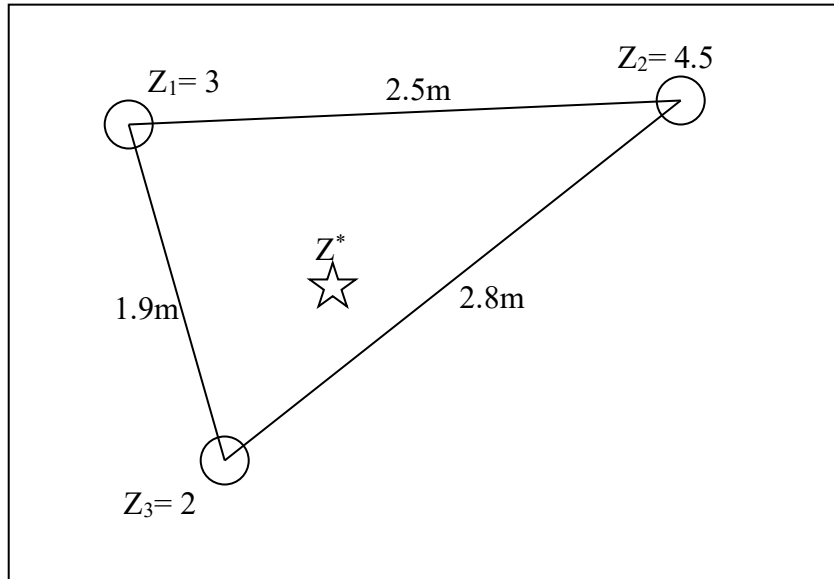(ii)   Calculate the error variance at the estimation point.

Fig. 3 - Data and Estimation Points in a reservoir

4. Variogram fitting and Simple kriging problem: (30 points) Table 1 shows the porosities of samples provided to you. You have to construct an experimental variogram from these data. Fit (a) a spherical variogram (b) an exponential variogram and (c) a Gaussian variogram model to your data. Remember to add the nugget effect if there is any. You should write a python code to do this fitting. Using the exponential variogram, compute the values of porosity and the error variances at 15m and 25m from the origin. Do these calculations using only the three closest data points to the estimation location.

Table 1: Porosities of samples taken at equidistant locations

x, m	z (%)
0	11
10	12
20	15
30	17
40	16
50	19
60	17