MOD600 Project A

Task 1

Explain how to obtain $S_t + f(S)_x = 0 \quad x \in [0,1]$

i. Assumptions:
- 1D reservoir with length L and cross section A
- Immiscible flow of two incompressible fluids w & o
- Homogenious reservoir i.e. const ϕ & k
- Capillary pressure is zero
- Reservoir initially oil filled
- Constant injection rate of water at one end and production at the other
- a unique physical solution exists.

Mass balance for both fluids:

$(\lambda_w P_x)_x = \phi S_{w,t}$

$(\lambda_o P_x)_x = \phi S_{o,t}$

λ_i - mobility $= \frac{k k_i}{\mu_i}$
- k = absolute permeability
- k_i = relative permeability of fluid i
- μ_i = viscosity of fluid i

P - pressure

ϕ - porosity

S_i - Saturation of fluid i

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Task 1

i cont..

Adding mass balance equations:

$$[(\lambda_w + \lambda_o) P_x]_x = \phi (S_w + S_o)_t$$

Applying $S_w + S_o = 1$ and $\lambda_w + \lambda_o = \lambda_T$:

$$(\lambda_T P_x)_x = 0$$

Expressing total Darcy velocity:

$$u_T = u_w + u_o = -\lambda_T P_x \Rightarrow P_x = -\frac{u_T}{\lambda_T}$$

Substituting pressure-expression into mass-balance equation:

$$-\left(\lambda_w \frac{u_T}{\lambda_T}\right)_x = \phi S_{w,t}$$

Setting $f = \frac{\lambda_w}{\lambda_T}$ and simplifying:

$$\phi S_{w,t} + u_T f_x = 0$$

Setting dimentionless parameters $x_D = \frac{x}{L}$, $t_D = \frac{u_T t}{\phi L}$

$$\phi \frac{\partial S_w}{\partial t} + u_T \frac{\partial f}{\partial x} = 0 \qquad x = x_D L, \; t = \frac{t_D \phi L}{u_T}$$

$$\phi \frac{\partial S_w u_T}{t_D \phi L} + u_T \frac{\partial f}{\partial x_D L} = 0 \quad \Big| \text{multiply by } \frac{L}{u_T}$$

$$\frac{\partial S_w}{\partial t_D} + \frac{\partial f}{\partial x_D} = 0$$

Leaving out subscript to obtain:

$$S_t + f(S)_x = 0$$

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Task 1

ii

$$f(S) = \frac{\lambda_w}{\lambda_T} = \frac{k\,k_{rw}/\mu_w}{k\,k_{rw}/\mu_w + k\,k_{ro}/\mu_o} = \Big|\ \text{expand by } \mu_w$$

$$= \frac{k_{rw}}{k_{rw} + k_{ro}\frac{\mu_w}{\mu_o}} = \frac{k_{rw}}{k_{rw} + k_{ro} M}$$

M - viscosity ratio $= \frac{\mu_w}{\mu_o}$

$k_{rw}(S) = S_w^{nw}$

$k_{ro}(S) = (1-S_w)^{no}$

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Task 3

(i) "What can be said about the total velocity $u_T = u_w + u_o$?"

- Not sure what is meant by this, could have been more specific.
- Maybe since the core is closed at the bottom this means that $U_T = 0$ as if one fluid goes up, the other must move down. with same but oposite velocity.

(ii) Find expression for P_w:

$$u_w = -k \lambda_w [P_{wx} + g \rho_w]$$

$$u_o = -k \lambda_o [P_{ox} + g \rho_o]$$

Using $U_T = U_w + U_o$ and $P_o = P_w + P_c$

$$U_T = -k \lambda_w [P_{wx} + g \rho_w] - k \lambda_o [P_{wx} + P_{cx} + g \rho_o] = 0$$

$$\lambda_w P_{wx} + \lambda_o P_{wx} = -\lambda_w g \rho_w - \lambda_o P_{cx} - \lambda_o g \rho_o$$

$$P_{wx} = \frac{-g(\lambda_w \rho_w + \lambda_o \rho_o) - \lambda_o P_{cx}}{\lambda_w + \lambda_o} \quad *$$

$$P_w = \int * \, dx$$

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Task 3 (ii) cont...

Find expression for u_w:

- inserting expression for P_{wx} in base expression for u_w

$$u_w = -k\lambda_w\left[\frac{-g(\lambda_w\rho_w + \lambda_o\rho_o) - P_{cx}\lambda_o}{\lambda_w + \lambda_o} + g\rho_w\right]$$

$$= -k\lambda_w\left[\frac{-\cancel{g\lambda_w\rho_w} - g\lambda_o\rho_o - P_{cx}\lambda_o + \cancel{\lambda_w g\rho_w} + \lambda_o g\rho_w}{\lambda_w + \lambda_o}\right]$$

$$= -k\lambda_w\left[\frac{\lambda_o g\rho_w - \lambda_o g\rho_o - P_{cx}\lambda_o}{\lambda_w + \lambda_o}\right]$$

$$= -k\lambda_w\lambda_o\left[\frac{g(\rho_w - \rho_o) - P_{cx}}{\lambda_w + \lambda_o}\right]$$

$$\lambda_T = \lambda_w + \lambda_o \quad , \quad \Delta\rho = \rho_w - \rho_o$$

$$u_w = -k\lambda_w\lambda_o\left[\frac{g\Delta\rho - P_{cx}}{\lambda_T}\right]$$

$$u_w = -k\frac{\lambda_w\lambda_o}{\lambda_T}(g\Delta\rho - P_{cx})$$

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Task 3

(iii) Derive equation for water mass-balance:

$$\phi(\rho_w S_w)_t + (\rho_w u_w)_x = 0$$

since ρ_w is assumed constant

$$\phi S_{w\,t} + u_{w\,x} = 0$$

Inserting expression for u_w

$$\phi S_{w\,t} + \left[-k\frac{\lambda_w \lambda_o}{\lambda_T}(g\Delta\rho - P_{c\,x})\right]_x = 0$$

$$\phi S_{w\,t} + \left(k\frac{\lambda_w \lambda_o}{\lambda_T}P_{c\,x} - k\frac{\lambda_w \lambda_o}{\lambda_T}g\Delta\rho\right)_x$$

(iv) Setting the following:

$$P_c = 0 \quad , \quad x = x_D L \quad , \quad t = t_D \frac{L\phi\mu_o}{gk\Delta\rho} \quad S = S_w$$

$$\phi\frac{\partial S}{\partial t} - kg\Delta\rho\frac{\partial \lambda_w \lambda_o}{\lambda_T \partial x} = 0$$

$$\phi\frac{\partial S}{\partial t_D}\cdot\frac{gk\Delta\rho}{L\phi\mu_o} - gk\Delta\rho\cdot\frac{\partial \lambda_w \lambda_o}{\lambda_T \partial x_D}\cdot\frac{1}{L} = 0$$

Multiply both sides by $\frac{L\mu_o}{gk\Delta\rho}$:

$$S_t + \left(\frac{-\mu_o \lambda_w \lambda_o}{\lambda_T}\right)_x = 0$$

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Task 3 (iv) cont..

$$g(s) = -\frac{\mu_o \lambda_w \lambda_o}{\lambda_w + \lambda_o} = -\frac{\mu_o k_{rw} k_{ro}}{\mu_w \mu_o \left(\frac{k_{rw}}{\mu_w} + \frac{k_{ro}}{\mu_o} \right)}$$

$$= -\frac{k_{rw} k_{ro}}{k_{rw} + k_{ro} \frac{\mu_w}{\mu_o}} = -\frac{k_{rw} k_{ro}}{k_{rw} + k_{ro} M}$$