Assignment 3

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In [540…
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
```

Task 1

Estimate value and error variance

(a)

In [541…
```
x_u = np.array([[20, 20]]).T
loc_data = np.array([[0, 80]]).T
```

In [542…
```
dist_DD = np.abs(loc_data - loc_data.T)
dist_DD
```

Out[542]:
```
array([[ 0, 80],
       [80,  0]])
```

In [543…
```
loc_unknown = 30
dist_UD = np.abs(loc_unknown-loc_data)
dist_UD
```

Out[543]:
```
array([[30],
       [50]])
```

In [544…
```
mu = 18
```

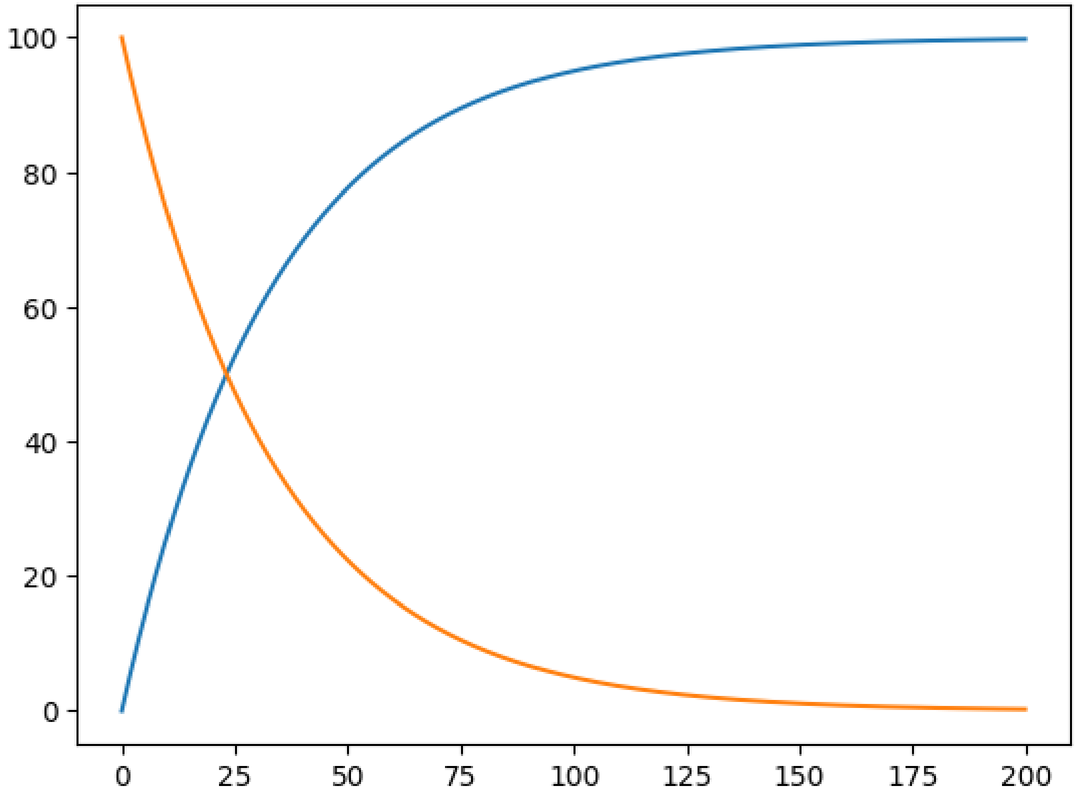
In [545…
```
def exp_var(h, r, n, s):
    gamma = n + s*(1-np.exp(-3*h/r))
    return gamma
```

In [546…
```
r = 100
n = 0
s = 100
```

In [547…
```
dist_plot = np.arange(0,200,0.1)
var_plot = exp_var(dist_plot, r, n, s)

plt.plot(dist_plot, var_plot)
plt.plot(dist_plot, s - var_plot)
```

Out[547]:
```
[<matplotlib.lines.Line2D at 0x20e88960b90>]
```



Data covariance from variogram model, $C_{DD} = sill - (\gamma(h_{DD}) - nugget)$

In [548…
```
gamma_DD = exp_var(dist_DD, r, n, s)
C_DD = s-(gamma_DD-n)
C_DD
```

Out[548]:
```
array([[100.        ,   9.07179533],
       [  9.07179533, 100.        ]])
```

Unknown-to-known data covariance from variogram model,
$C_{UD} = sill - (\gamma(h_{UD}) - nugget)$

In [549…
```
gamma_UD = exp_var(dist_UD, r, n, s)
C_UD = s-(gamma_UD-n)
C_UD
```

Out[549]:
```
array([[40.65696597],
       [22.31301601]])
```

$\lambda = \mathbf{C}_{DD}^{-1}\mathbf{C}_{UD}$

In [550…
```
lamb = np.dot(np.linalg.inv(C_DD), C_UD)
lamb
```

Out[550]:
```
array([[0.38953351],
       [0.18779248]])
```

$X(u_o)_{SK} = \lambda^T(\mathbf{X(u)} - \mu) + \mu$

In [551…
```
est = np.dot(lamb.T, (x_u-mu)) + mu
est
```

Out[551]: array([[19.15465198]])

$$\sigma^2_{SK} = sill - \lambda^T \mathbf{C}_{UD}$$

In [552…
```
var = s-np.dot(lamb.T, C_UD)
var
```

Out[552]: array([[79.97253276]])

(b)

In [553…
```
s = 200

gamma_DD = exp_var(dist_DD, r, n, s)
C_DD = s-(gamma_DD-n)

gamma_UD = exp_var(dist_UD, r, n, s)
C_UD = s-(gamma_UD-n)

lamb = np.dot(np.linalg.inv(C_DD), C_UD)

est = np.dot(lamb.T, (x_u-mu)) + mu
var = s-np.dot(lamb.T, C_UD)
print(est, var)
```

```
[[19.15465198]] [[159.94506552]]
```

As we can see the estimate stays the same and the variance is scaled by the sill.

Task 3

(a) Simple Kriging

Note: Showing Task 3 before Task 2 because I solved them in this order, so Task 3 is showing some more formulas that are not shown in 2.

In [554…
```
x_u = np.array([3, 4.5, 2]).T
dist_DD = np.array([[0, 2.5, 1.9], [2.5, 0, 2.8], [1.9, 2.8, 0]])
dist_UD = np.array([[1.2, 2, 0.9]]).T
mu = np.mean(x_u)
```

In [555…
```
def sph_var(h, r, n, s):
    gamma = n + s*(1.5*(h/r)-0.5*np.power((h/r),3))
    gamma[h>r] = n + s
    return gamma
```

In [556…
```
r = 10
n = 1.2
s = 6.8
```

In [557…
```
gamma_DD = sph_var(dist_DD, r, n, s)
C_DD = s-(gamma_DD-n)

gamma_UD = sph_var(dist_UD, r, n, s)
```

```
C_UD = s-(gamma_UD-n)

lamb = np.dot(np.linalg.inv(C_DD), C_UD)

est = np.dot(lamb.T, (x_u-mu)) + mu
var = s-np.dot(lamb.T, C_UD)
print(est, var)
```

```
[2.76370877] [[1.01191529]]
```

(b) Ordinary Kriging

$$C_{DD,OK} \rightarrow \begin{bmatrix} C_{DD,SK} & \mathbf{1} \\ \mathbf{1} & 0 \end{bmatrix}$$

In [558…

```
n_data = len(x_u)
OK_column = np.array([np.ones(n_data)]).T
C_DD_OK = np.c_[C_DD, OK_column]
OK_row = np.array([np.append(np.ones(n_data),0)])
C_DD_OK = np.r_[C_DD_OK, OK_row]
C_DD_OK
```

Out[558]:

```
array([[6.8      , 4.303125 , 4.8853206, 1.       ],
       [4.303125 , 6.8      , 4.0186368, 1.       ],
       [4.8853206, 4.0186368, 6.8      , 1.       ],
       [1.       , 1.       , 1.       , 0.       ]])
```

$$\mathbf{C}_{UD,OK} \rightarrow \begin{bmatrix} \mathbf{C}_{UD,SK} \\ 1 \end{bmatrix}$$

In [559…

```
C_UD_OK = np.r_[C_UD,np.array([[1]])]
C_UD_OK
```

Out[559]:

```
array([[5.5818752],
       [4.7872   ],
       [5.8844786],
       [1.       ]])
```

$$\lambda_{OK} = \mathbf{C}_{DD,OK}^{-1}\mathbf{C}_{UD,OK}$$

In [560…

```
lamb_OK = np.dot(np.linalg.inv(C_DD_OK), C_UD_OK)
lamb_OK
```

Out[560]:

```
array([[0.3196991 ],
       [0.17635466],
       [0.50394625],
       [0.18710623]])
```

In [561…

```
mu_OK = lamb_OK[-1]
```

$$X(u_o)_{OK} = \lambda^T\mathbf{X(u)}$$

In [562…

```
est_OK = np.dot(lamb_OK[:-1].T, x_u)
est_OK
```

Out[562]:

```
array([2.76058574])
```

$$\sigma^2_{OK} = sill - \lambda^T \mathbf{C}_{UD,SK} - \mu_{OK}$$

In [563…

```
var_OK = s - np.dot(lamb_OK[:-1].T, C_UD) - mu_OK
var_OK
```

Out[563]:

```
array([[1.0186674]])
```

Task 2

Ordinary Kriging Problem with Spatial Variations

(a) Estimate the value of a regionalized variable at point V

In [564…

```
x_u = np.array([20, 50, 30, 100]).T
a = np.sqrt(2)*50
b = 100
dist_DD = np.array([[0, a, a, b], [a, 0, b, a], [a, b, 0, a], [b, a, a, 0]])
dist_UD = np.array([[50, 50, 50, 50]]).T
mu = np.mean(x_u)

f = sph_var
r = 200
n = 0
s = 1
```

In [565…

```
def Kriging(x_u, dist_DD, dist_UD, mu, f, r, n, s):
    gamma_DD = f(dist_DD, r, n, s)
    C_DD = s-(gamma_DD-n)
    gamma_UD = f(dist_UD, r, n, s)
    C_UD = s-(gamma_UD-n)
    lamb = np.dot(np.linalg.inv(C_DD), C_UD)
    est = np.dot(lamb.T, (x_u-mu)) + mu
    var = s-np.dot(lamb.T, C_UD)

    n_data = len(x_u)
    OK_column = np.array([np.ones(n_data)]).T
    C_DD_OK = np.c_[C_DD, OK_column]
    OK_row = np.array([np.append(np.ones(n_data),0)])
    C_DD_OK = np.r_[C_DD_OK, OK_row]

    C_UD_OK = np.r_[C_UD,np.array([[1]])]
    lamb_OK = np.dot(np.linalg.inv(C_DD_OK), C_UD_OK)
    mu_OK = lamb_OK[-1]
    est_OK = np.dot(lamb_OK[:-1].T, x_u)
    var_OK = s - np.dot(lamb_OK[:-1].T, C_UD) - mu_OK
    return lamb_OK, est_OK, var_OK
```

In [566…

```
lamb_OK, est_OK, var_OK = Kriging(x_u, dist_DD, dist_UD, mu, f, r, n, s)
print(f'Weights ={lamb_OK[:-1]}')
print(f'Local mean = {mu}')
print(f'Estimate = {est_OK}')
print(f'Error variance = {var_OK}')
```

```
Weights =[[0.25]
 [0.25]
 [0.25]
 [0.25]]
Local mean = 50.0
Estimate = [50.]
Error variance = [[0.3083835]]
```

(b) add small nugget effect

In [567…

```
n = 0.25
lamb_OK, est_OK, var_OK = Kriging(x_u, dist_DD, dist_UD, mu, f, r, n, s)
print(f'Weights ={lamb_OK[:-1]}')
print(f'Local mean = {mu}')
print(f'Estimate = {est_OK}')
print(f'Error variance = {var_OK}')
```

```
Weights =[[0.25]
 [0.25]
 [0.25]
 [0.25]]
Local mean = 50.0
Estimate = [50.]
Error variance = [[0.3083835]]
```

(c) Two wells close together

In [568…

```
a = np.sqrt(2)*50
b = (25**2 + 100**2)**0.5
c = (50**2 + 100**2)**0.5
d = (25**2 + 50**2)**0.5
dist_DD = np.array([[0, a, a, b], [a, 0, 10, c], [a, 10, 0, c], [b, c, c, 0]])
dist_UD = np.array([[50, 50, 50, d]]).T

n = 0
```

In [569…

```
lamb_OK, est_OK, var_OK = Kriging(x_u, dist_DD, dist_UD, mu, f, r, n, s)
print(f'Weights ={lamb_OK[:-1]}')
print(f'Local mean = {mu}')
print(f'Estimate = {est_OK}')
print(f'Error variance = {var_OK}')
```

```
Weights =[[0.29292307]
 [0.17570215]
 [0.17570215]
 [0.35567262]]
Local mean = 50.0
Estimate = [55.48189562]
Error variance = [[0.31979971]]
```

(d) Anisotropy

Did not figure out how to do this

(e) Another configuration

In [570…
```
a = np.sqrt(2)*50
d = (25**2 + 50**2)**0.5
dist_DD = np.array([[0, a, 75, 100], [a, 0, d, a], [75, d, 0, 25], [100, a, 25,
dist_UD = np.array([[50, 50, 25, 50]]).T
```

In [571…
```
lamb_OK, est_OK, var_OK = Kriging(x_u, dist_DD, dist_UD, mu, f, r, n, s)
print(f'Weights ={lamb_OK[:-1]}')
print(f'Local mean = {mu}')
print(f'Estimate = {est_OK}')
print(f'Error variance = {var_OK}')
```

```
Weights =[[ 0.27809185]
 [ 0.13467022]
 [ 0.62901399]
 [-0.04177607]]
Local mean = 50.0
Estimate = [26.98816143]
Error variance = [[0.24377693]]
```

(f) Comments

- The small nugget effect that was added had no observable effect.
- Not sure what was meant by 2 and 3 being redundant in (c).
- In (c) we observe that 4 has a high weighting even though it is furthest away from estimation point, which leads to the estimate being higher than the mean.
- In (e) we observe that 4 receives a (small, but) negative weighting, which is probably due to it being "hidden" behind 3.

Task 4

Variogram fitting

In [572…
```
x_u = np.array([11, 12, 15, 17, 16, 19, 17])
loc_data = np.array([0, 10, 20, 30, 40, 50, 60])
```

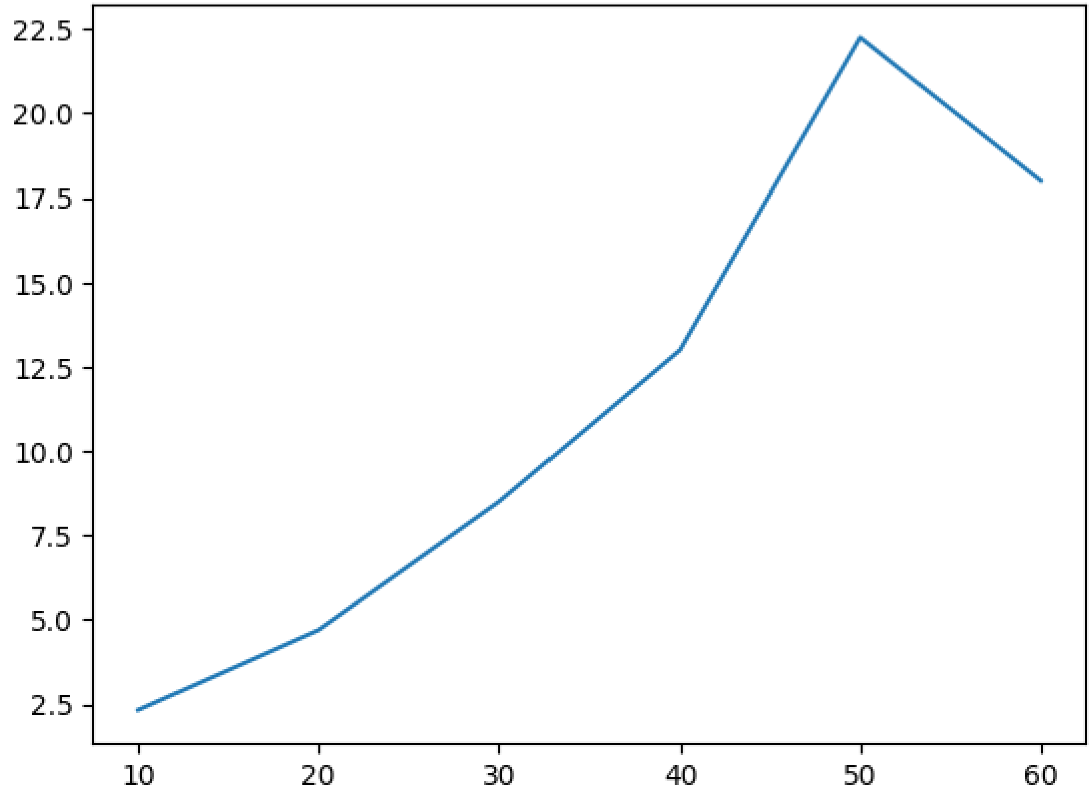
$$C(h) = \frac{1}{N_h}\sum_{i=1}^{N_h} x_i x_{i+h} - \bar{x}_i \bar{x_{i+h}}$$

In [573…
```
n_l = len(x_u)-1
E_var = np.zeros(n_l)

for l in range(1, n_l + 1):
    Nh = len(x_u) - l
    E_var[l-1] = (1/(2*Nh))*np.sum(np.power((x_u[l:] - x_u[:-l]), 2))
```

In [574…
```
plt.plot(loc_data[1:], E_var)
```

Out[574]:
```
[<matplotlib.lines.Line2D at 0x20e8896fe50>]
```



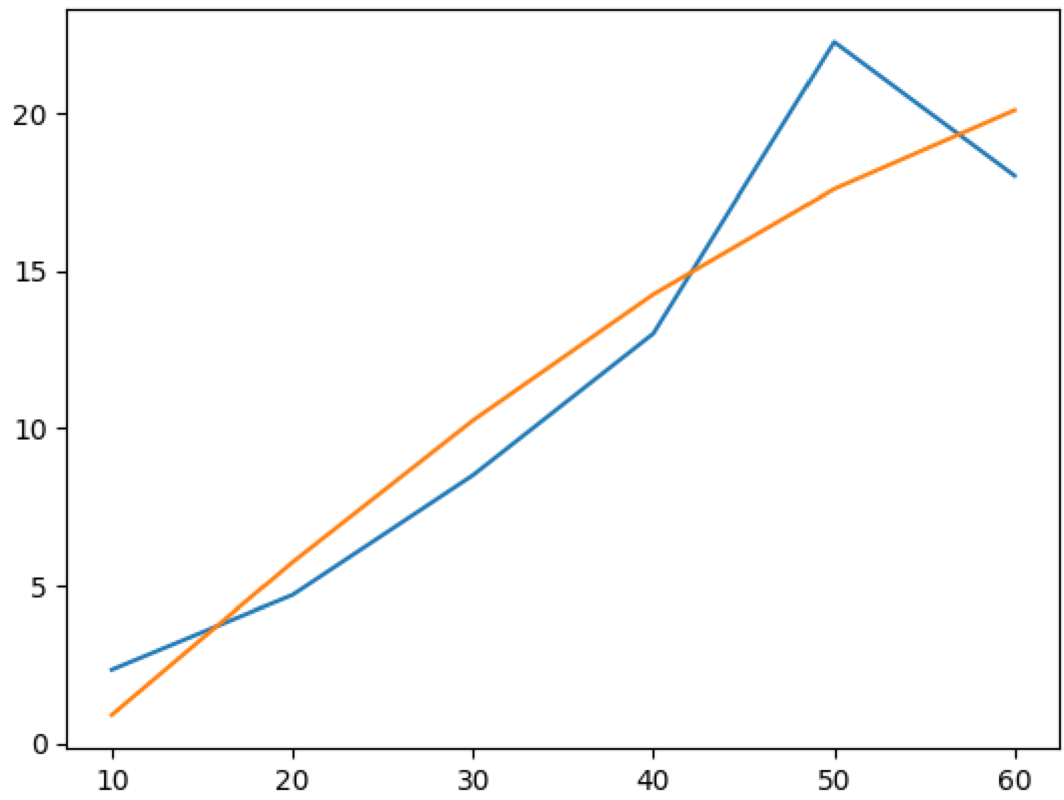
Values for p0 taken from visual inspection of above graph.

In [575…
```
sph_params,_ = curve_fit(sph_var, loc_data[1:], E_var, p0=[50,0,20])
sph_params
```

Out[575]: `array([77.61521397, -4.11293496, 26.05626909])`

In [576…
```
r, n, s = sph_params
Sph_var = sph_var(loc_data[1:], r, n, s)
plt.plot(loc_data[1:], E_var)
plt.plot(loc_data[1:], Sph_var)
```

Out[576]: `[<matplotlib.lines.Line2D at 0x20e88c410d0>]`



In [577…

```
exp_params,_ = curve_fit(exp_var, loc_data[1:], E_var, p0=[50,0,20])
exp_params
```

Out[577]:

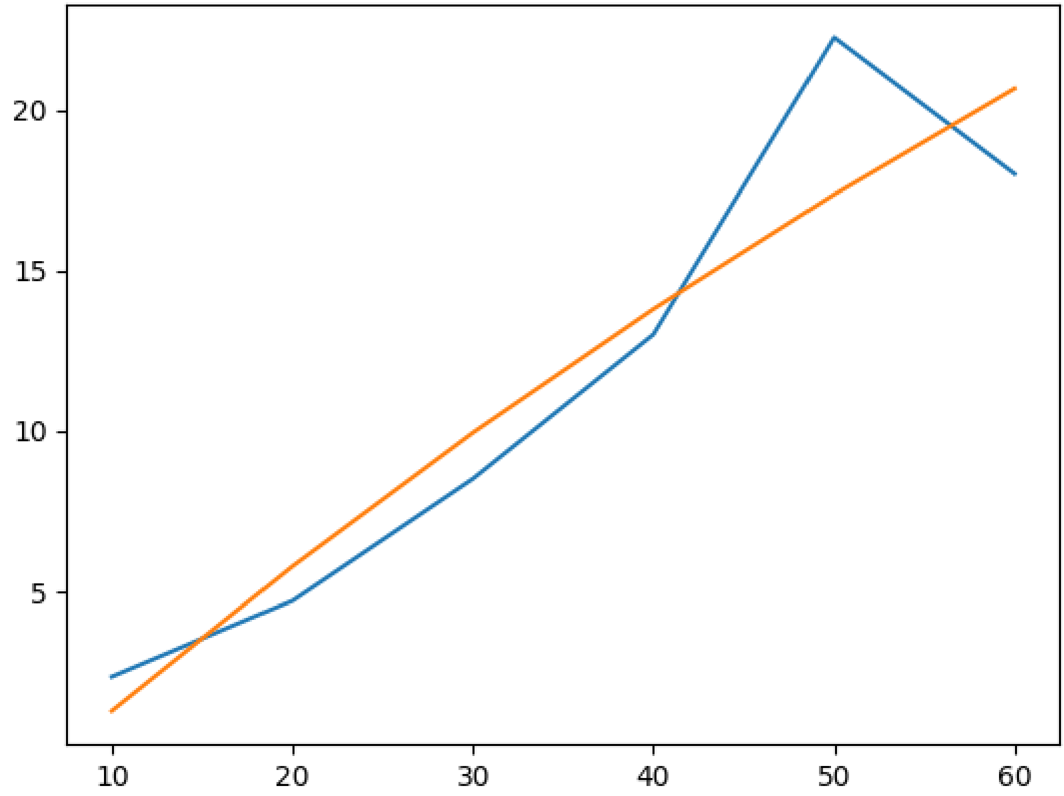
```
array([385.86471408,  -3.59846874,  65.06361688])
```

In [578…

```
r, n, s = exp_params
Exp_var = exp_var(loc_data[1:], r, n, s)
plt.plot(loc_data[1:], E_var)
plt.plot(loc_data[1:], Exp_var)
```

Out[578]:

```
[<matplotlib.lines.Line2D at 0x20e88cc9f90>]
```



In [579…

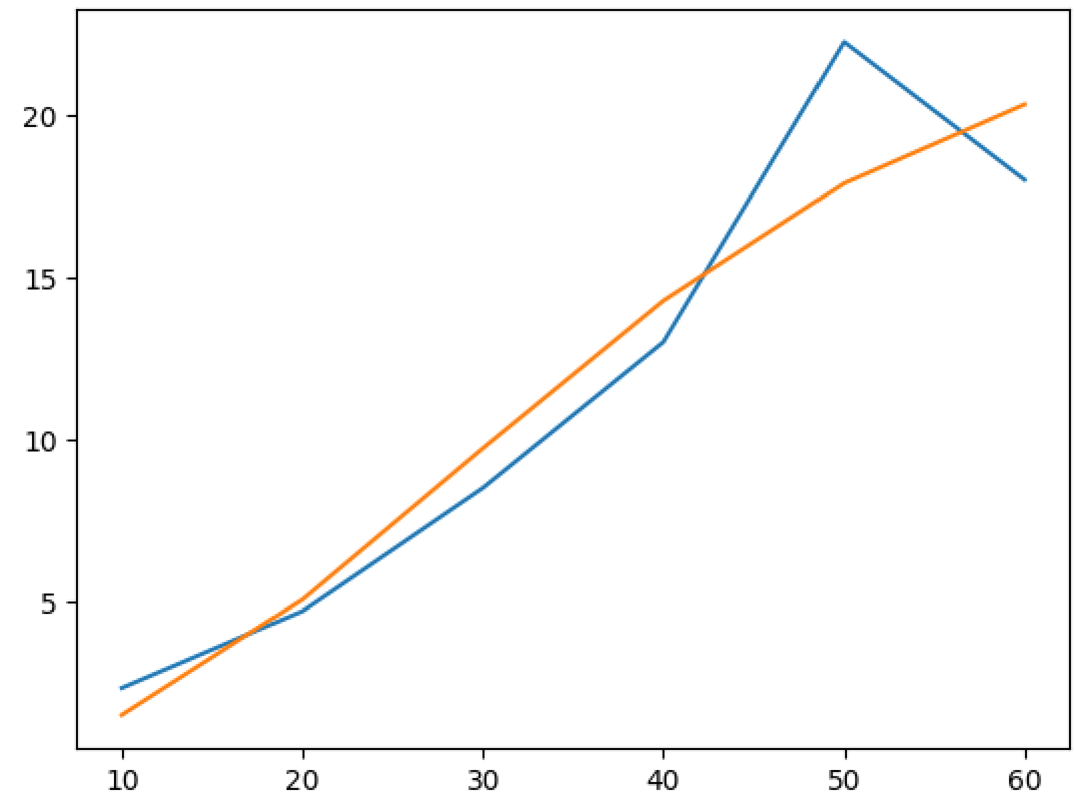
```
def gauss_var(h, r, n, s):
    gamma = n + s*(1-np.exp(-3*np.power(h/r, 2)))
    return gamma
```

In [580…

```
gauss_params,_ = curve_fit(gauss_var, loc_data[1:], E_var, p0=[50,0,20])
gauss_params
r, n, s = gauss_params
Gauss_var = gauss_var(loc_data[1:], r, n, s)
plt.plot(loc_data[1:], E_var)
plt.plot(loc_data[1:], Gauss_var)
```

Out[580]:

```
[<matplotlib.lines.Line2D at 0x20e88d26890>]
```



Finding solution for x = 15

In [581…
```
x_u = np.array([[11, 12, 15, 17, 16, 19, 17]]).T
loc_data = np.array([[0, 10, 20, 30, 40, 50, 60]]).T
```

In [582…
```
dist_DD = np.abs(loc_data[:3] - loc_data[:3].T)
dist_DD
```

Out[582]:
```
array([[ 0, 10, 20],
       [10,  0, 10],
       [20, 10,  0]])
```

In [583…
```
loc_unknown = 15
dist_UD = np.abs(loc_unknown-loc_data[:3])
dist_UD
```

Out[583]:
```
array([[15],
       [ 5],
       [ 5]])
```

In [584…
```
# Still using mean for full data set
mu = np.mean(x_u)
mu
```

Out[584]:
```
15.285714285714286
```

In [585…
```
r, n, s = exp_params
```

In [586…
```
gamma_DD = exp_var(dist_DD, r, n, s)
C_DD = s-(gamma_DD-n)

gamma_UD = exp_var(dist_UD, r, n, s)
C_UD = s-(gamma_UD-n)
```

```
lamb = np.dot(np.linalg.inv(C_DD), C_UD)

est = np.dot(lamb.T, (x_u[:3]-mu)) + mu
var = s-np.dot(lamb.T, C_UD)
print(est, var)
```

```
[[13.50134841]] [[2.52799208]]
```

Finding solution for x = 25

In [587…

```
dist_DD = np.abs(loc_data[1:4] - loc_data[1:4].T)

loc_unknown = 25
dist_UD = np.abs(loc_unknown-loc_data[1:4])

r, n, s = exp_params

gamma_DD = exp_var(dist_DD, r, n, s)
C_DD = s-(gamma_DD-n)

gamma_UD = exp_var(dist_UD, r, n, s)
C_UD = s-(gamma_UD-n)

lamb = np.dot(np.linalg.inv(C_DD), C_UD)

est = np.dot(lamb.T, (x_u[1:4]-mu)) + mu
var = s-np.dot(lamb.T, C_UD)
print(est, var)
```

```
[[15.99946064]] [[2.52799208]]
```