Assignment on Intro to Algorithms

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Following are the three NP-Complete problems.

1. Hamiltonian Path Problem

Hamiltonian Path refers to a path in undirected or directed graph that visits each vertex exactly once. Hamiltonian Path problem is a problem of finding if such paths exist in a given graph.

Proof:
The possible Hamiltonian Path of a given graph G should contain n vertices of the graph. So, There should be **n!** possible Hamiltonian paths for a graph G. Let's pick a path from those n! possibilities non-deterministically. If we traverse through this path and make sure that each vertex in the path is visited only once, we can verify that the path is Hamiltonian Path of the graph G. This can be obviously done in polynomial time as we have to check n vertices and n edges to traverse through the path. Which makes this problem a NP class problem.

2. Clique Problem

Clique refers to a subgraph C of a given graph G= (V, E), such that each vertices in C are adjacent to each other. Clique problem is a problem of finding if such cliques exist in a given graph.

Proof:
As we would be choosing k vertices from possible n, there would be total permutation of P(n, k) which is equal to **n!/(n-k)!**. The possible solution of Clique problem for given G should contain k(< n) vertices. So, there would be k vertices to be tested with themselves if they are adjacent or not. This can be verified in O(n^2) time. Since, the solution can be verified in polynomial time, the Dominating Set problem is NP class problem.

3. Dominating Set Problem

In graph theory, a dominating set D for a graph G = (V, E) is a subset of V in such a way that every vertex which is not in D is adjacent to at least one member of D. Dominating set problem refers to finding if such subset exists in the given graph.

Proof:
As we would be choosing k vertices for D from possible n vertices of G, there would be total permutation of P(n, k) which is equal to **n!/(n-k)!.** The possible solution of Dominating Set problem for given G should contain k(< n) vertices. So, there would be (n-k) remaining vertices to be tested with these k vertices if they are adjacent or not. This can be verified in O(n^2) time. Since, the solution can be verified in polynomial time, the Dominating Set problem is NP class problem.