

Homework 9 - Predictive Modeling in Finance and Insurance

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cbeDat <- read.table("cbe.dat", header = TRUE)
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1. Exploratory Time Series Analysis

2. Sample Autocorrelation

The sample lag 2 autocorrelation formula is as follows:

$$\hat{\rho}_2 = \frac{\sum_{t=3}^T (y_t - \bar{y})(y_{t-2} - \bar{y})}{\sum_{t=1}^T (y_t - \bar{y})^2}$$

Note that $\bar{y} = \frac{1+1.5+1.6+1.4+1.5+1.7}{6} = 1.45$. So, I calculate, using the numbers given:

$$\begin{aligned}\hat{\rho}_2 &= \frac{(1.6 - 1.45)(1 - 1.45) + (1.4 - 1.45)(1.5 - 1.45) + (1.5 - 1.45)(1.6 - 1.45) + (1.7 - 1.45)(1.4 - 1.45)}{(1 - 1.45)^2 + (1.5 - 1.45)^2 + (1.6 - 1.45)^2 + (1.5 - 1.45)^2 + (1.7 - 1.45)^2} \\ &= \frac{-0.075}{0.2925} = -\mathbf{0.2564}\end{aligned}$$

3. Forecast error

Note that $y_{10} = y_0 + \sum_{i=1}^{10} c_i$. Since $\bar{c}_{10} = 2$, this means that $\sum_{i=1}^{10} c_i = 10 * \bar{c}_{10} = 10 * 2 = 20$. Therefore, $y_{10} = y_0 + 20$. From this, and the data given:

$$y_{19} = y_0 + \sum_{i=1}^{10} c_i + \sum_{i=11}^{19} c_i = y_0 + 20 + 26 = y_0 + 46$$

From the fact that all c_t values are positive, this seems to be a random walk model with drift. Therefore, to develop the estimate of \hat{y}_{19} from $\{y_i : i \in [10]\}$, I estimate the drift parameter δ :

$$\hat{\delta} = \mathbb{E}[\nabla y] = \bar{c}_{10} = 2$$

Therefore, as Forecast steps = $\ell = 19 - 10 = 9$, I predict \hat{y}_{19} from y_{10} :

$$\hat{y}_{19} = y_{10} + \ell * \hat{\delta} = y_0 + 20 + 9 * 2 = y_0 + 38$$

Thus, the forecast error is:

$$y_{19} - \hat{y}_{19} = (y_0 + 46) - (y_0 + 38) = \mathbf{8}$$

4. Forecast for AR(1) model

I am given that $\bar{y} = 8.01$. I am also given that $\hat{\alpha} = -0.79$ as the estimated parameter from the model. Note that the mean is subtracted before the model is fit; therefore:

$$\hat{y}_9 - \bar{y} = \hat{\alpha}^{9-7} (y_7 - \bar{y})$$

So, I solve for \hat{y}_9 :

$$\hat{y}_9 = \bar{y} + \hat{\alpha}^{9-7}(y_7 - \bar{y}) = 8.01 + (-.79)^2(8.5 - 8.01) = \mathbf{8.315809}$$

5. Time Series, AutoRegression, GLS