

# Homework 7 - Predictive Modeling in Finance and Insurance

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```
library(MASS)
library(ggplot2)
library(leaps)
Boston$chas <- factor(Boston$chas)
```

## 1. Generalized Linear Model

### a. Showing pdf belongs to exponential family

I can modify the p.d.f and show:

$$\begin{aligned} f(y; \theta) &= \theta y^{-\theta-1} \\ &= \theta e^{\ln(y^{-\theta-1})} \\ &= \theta e^{-\ln(y) * \theta - \ln(y)} \\ &= e^{\ln(\theta)} * e^{-\ln(y) * \theta - \ln(y)} \\ &= e^{-\ln(y)\theta + \ln(\theta) - \ln(y)} \end{aligned}$$

Thus, we have that  $a(y) = -\ln(y)$ ,  $b(\theta) = \theta$ ,  $c(\theta) = \ln(\theta)$ ,  $d(y) = -\ln(y)$ .

### b. Natural Exponential family or Exponential Dispersion Family

Note that  $a(y) \neq y$ ; thus the Pareto distribution is NOT part of the natural exponential family. However, if you restructure the p.d.f:

$$f(y; \theta) = e^{-\ln(y)\theta + \ln(\theta) - \ln(y)} = e^{-(\theta+1)\ln(y) + (1)\ln(\theta)}$$

You can note that the Pareto distribution is also part of the Exponential dispersion family, with  $\lambda = 1$ .

### c. Score statistic

To find the score statistic, I need log-likelihood function:

$$\ell(\theta; y) = \ln(f(y; \theta)) = -\ln(y) * \theta + \ln(\theta) - \ln(y)$$

### d. Pareto distribution and Exponential Family

## 2. Logistic Regression

### 3. Logistic Regression Part 2

- a. Coefficients from Models
- b. Probability Estimates from models
- c. Expected Values from 3 Models
- d. Pearson residuals
- e. Goodness of fit statistics

#### 4. Maximum Likelihood Estimation approximation

- a. Plot of number of cases against time period
- b. Plot against  $\log i$
- c. Fitting GLM

## 5. Deviance

- a. Calculating Deviance
- b. Calculating Pearson residual
- c. Calculating deviance residual

## 6. Nominal Regression

- a. Odds ratio
- b. Probability Calculation

## 7. Ordinal Regression