Homework 9 - Predictive Modeling in Finance and Insurance

Dennis Goldenberg

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```
cbeDat <- read.table("cbe.dat", header = TRUE)
```

1. Exploratory Time Series Analysis

2. Sample Autocorrelation

The sample lag 2 autocorrelation formula is as follows:

$$\hat{\rho}_2 = \frac{\sum_{t=3}^{T}(y_t - \bar{y})(y_{t-2} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

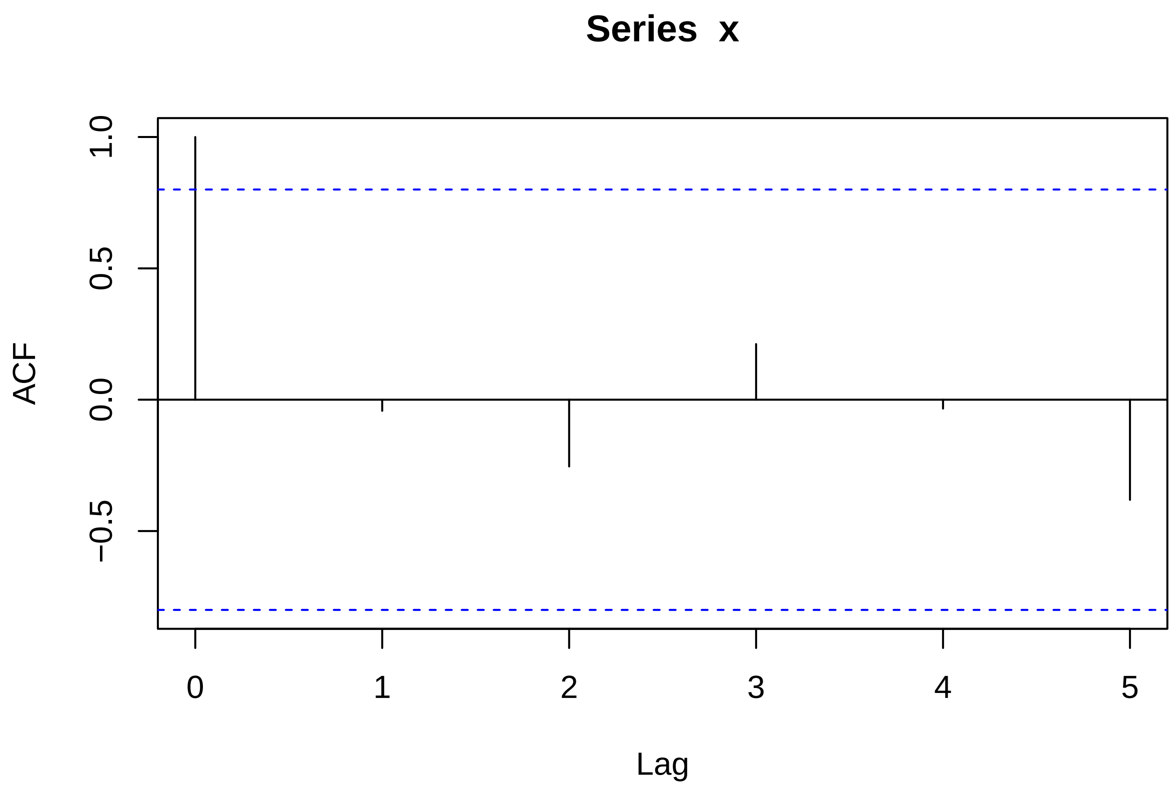
Note that $\bar{y} = \frac{1+1.5+1.6+1.4+1.5+1.7}{6} = 1.45$. So, I calculate, using the numbers given:

$$\hat{\rho}_2 = \frac{(1.6-1.45)(1-1.45)+(1.4-1.45)(1.5-1.45)+(1.5-1.45)(1.6-1.45)+(1.7-1.45)(1.4-1.45)}{(1-1.45)^2+(1.5-1.45)^2+(1.6-1.45)^2+(1.5-1.45)^2+(1.7-1.45)^2}$$
$$= \frac{-.075}{0.2925} = -\mathbf{0.2564}$$

```r
x <- c(1,1.5,1.6,1.4,1.5,1.7)
lagx2 <- c(1,1.5,1.6,1.4)
currx2 <- c(1.6,1.4,1.5,1.7)
print(cor(lagx2,currx2))
```

```
## [1] -0.4417926
```

```r
acf(x)[2]
```




```
##
## Autocorrelations of series 'x', by lag
##
##      2
## -0.254
```

3. Forecast error

Note that $y_{10} = y_0 + \sum_{i=1}^{10} c_i$. Since $\bar{c}_{10} = 2$, this means that $\sum_{i=1}^{10} c_i = 10 * \bar{c}_{10} = 10 * 2 = 20$. Therefore, $y_{10} = y_0 + 20$. From this, and the data given:

$$y_{19} = y_0 + \sum_{i=1}^{10} c_i + \sum_{i=11}^{19} c_i = y_0 + 20 + 26 = y_0 + 46$$

From the fact that all c_t values are positive, this seems to be a random walk model with drift. Therefore, to develop the estimate of $\hat{y}_{19}$ from $\{y_i : i \in [10]\}$, I estimate the drift parameter δ:

$$\hat{\delta} = \mathbb{E}\left[\nabla y\right] = \bar{c}_{10} = 2$$

Therefore, as Forecast steps $= \ell = 19 - 10 = 9$, I predict $\hat{y}_{19}$ from $y_1 0$:

$$\hat{y}_{19} = y_{10} + \ell * \hat{\delta} = y_0 + 20 + 9 * 2 = y_0 + 38$$

Thus, the forecast error is:

$$y_{19} - \hat{y}_{19} = (y_0 + 46) - (y_0 + 38) = \mathbf{8}$$

4. Forecast for AR(1) model

I am given that $\bar{y} = 8.01$. I am also given that $\hat{\alpha} = -0.79$ as the estimated parameter from the model. Note that the mean is subtracted before the model is fit; therefore:

$$\hat{y}_9 - \bar{y} = \hat{\alpha}^{9-7}\,(y_7 - \bar{y})$$

So, I solve for $\hat{y}_9$:

$$\hat{y}_9 = \bar{y} + \hat{\alpha}^{9-7}(y_7 - \bar{y}) = 8.01 + (-.79)^2(8.5 - 8.01) = \mathbf{8.315809}$$

5. Time Series, AutoRegression, GLS