LOWESS

⊙ SLR: Weighted Least Squares

minimize

$$S_w = \sum_i^n w_i(y_i - \beta_0 - \beta_1 x_i)^2$$

define

$$\bar{x}_w = \frac{\sum_i^n \omega_i x_i}{\sum_i^n \omega_i}, \qquad \bar{y}_w = \frac{\sum_i^n \omega_i y_i}{\sum_i^n \omega_i}$$

solution

$$\hat{\beta}_0 = \bar{y}_w - \hat{\beta}_1 \bar{x}_w$$

$$\hat{\beta}_1 = \frac{\sum_i^n \omega_i (x_i - \bar{x}_w)(y_i - \bar{y}_w)}{\sum_i^n \omega_i (x_i - \bar{x}_w)^2}$$

The solution for the non-weighted version is obtained by setting $w_i = 1$, $\forall i$.

⊙ MLR: Weighted Least Squares

Let X be the design matrix, and W be the diagonal matrix of weights.

$$\hat{\beta} = (X^T W X)^{-1} X^T W y$$

⊙ LOESS / LOWESS

LOESS: locally estimated scatterplot smoothing
LOWESS: locally weighted scatterplot smoothing

For the LOWESS estimate at location x_0

- identify the local neighborhood of x_0 as $x_1, ..., x_{n_0}$
- calculate distances $d_i = |x_i - x_0|$
- calculate scaled distances

$$d_i^* = \frac{d_i}{\max(d_1, \ldots, d_{n_0})}$$

- calculate weights

$$\omega_i = \text{tri}(d_i^*) = \begin{cases} (1 - |d_i^*|^3)^3, & |d_i^*| < 1 \\ 0, & \text{otherwise} \end{cases}$$

- fit a weighted least squared regression to observations in the local neighborhood
- use the fitted model to estimate the value at x_0