ACTU PS5841 Data Science in Finance and Insurance - Autumn 2023 (Y. Wang)
Assignment - 3
Assigned 9/28/23, Due 10/6/23

Life Expectancy

Let the variable T_x track the remaining life (in years) of a person of current age x. Since the timing of death is uncertain, T_x can be modeled as a random variable with a probability density $f_{T_x}(t)$. Suppose mortality follows the Makeham law, we have

$$f_{T_x}(t) = (A + Bc^{x+t}) \exp\Big(- At - \frac{B}{\log c} c^x (c^t - 1)\Big)$$

For this assignment, let's assume $A = 0.00022$, $B = 2.7e-6$, and $c = 1.124$. With these values, the probability of surviving beyond age 130 for anyone younger than 130 is practically zero.

The the expected future lifetime for a person of current age x is then

$$E(T_x) \approx \int_0^{130-x} t f_{T_x}(t) dt$$

The the expected age at death for a person of current age x is then

$$E(T_x) \approx x + \int_0^{130-x} t f_{T_x}(t) dt$$

$[a]$ Base on the above model, code the Repeated Simpson's Rule for numerical integration (with $n = 50$) to calculate life expectancy values for a person at each current age in
`np.linspace(0, 100, 11)` .
Let your code produce the following table.

	current age	expected future lifetime	expected age at death
0	0.0	85.564247	85.564247
1	10.0	75.745777	85.745777
2	20.0	65.913131	85.913131
3	30.0	56.079203	86.079203
4	40.0	46.277622	86.277622
5	50.0	36.591443	86.591443
6	60.0	27.209687	87.209687
7	70.0	18.510407	88.510407
8	80.0	11.103323	91.103323
9	90.0	5.652803	95.652803
10	100.0	2.400984	102.400984

The Repeated Simpson's Rule for numerical integration is

$$I = \int_a^b f(x) dx$$

$$I \approx \frac{h}{3}\Big(f(a) + 4\sum_{j=1}^{n} f(a + (2j-1)h) + 2\sum_{j=1}^{n-1} f(a + 2jh) + f(b)\Big)$$

$$h = \frac{b-a}{2n}$$

Please submit (*.ipynb and *.html) for your work

[b] Based on your estimation in [a], use `plotnine` to produce the following chart.