2023 Spring, MATHGR5450 Credit Analytics: Assignment #1, Due Date: Feb 17, 2024

1. (15 points) We have a random variable X whose distribution function is $F(x)$ and whose density function is $f(x)$.
   (a) (5 points) Prove that $F(X)$ follows uniform distribution over $(0, 1]$.
   (b) (5 points) Take any uniform distribution U over $(0, 1]$, prove that $F^{-1}(U)$ follows the same distribution as X, where $F^{-1}()$ is the inverse function.
   (c) (5 points) Suppose that a random variable Y follows a standard normal distribution whose distribution function is $\Phi(y)$. Show that $\Phi^{-1}(F(X))$ follows the standard normal distribution.

2. (20 points) Suppose we have a random variable whose hazard rate function is $h(t) = \alpha e^{\beta t}$ where $\alpha > 0$ and $\beta > 0$.
   (a) (10 points) Show that the survival function is
   $$S(t) = e^{\frac{\alpha}{\beta}e^{\beta t} - \frac{\alpha}{\beta}}$$
   (b) (5 points) Derive the distribution function
   (c) (5 points) Derive the density function

3. (25 points) Suppose that we have a constant hazard rate h for a credit risk, and a constant instantaneous forward rate r over the period $[0, T]$.
   (a) (10 points) Suppose a contract which pays \$100 dollar when default occurs before T. Calculate the expected value and the variance of the present value for this contract.
   (b) (10 points) Suppose a contract which pays \$100 dollar per year if the underlying credit survives before T, and nothing after the credit defaults. Calculate the expected value and the variance of the present value for this contract.
   (c) (5 points) What is the relationship between the two present values above?

4. (45 points) Download the past 10-year (from Feb 2, 2012 to Feb 3, 2023) yield curve data from Fed St. Louis website. These rates are continuous zero rates. They are obtained by bootstrapping on the run treasury bond prices or yields. The zero rates are given at maturities: 1, 2, 3, 4 5,6 7,8,9 10, 15, 20, and 30 years.
   (a) (5 points) Calculate the covariance matrix for the daily change of the zero rates. Provide your observation on the standard deviations and the correlation structure of zero rates at different maturities,
   (b) (20 points) Perform a principal component analysis on the covariance matrix. Present the following information:
      - the total variation of the data
      - what percentage of the total variation is explained by the first one factor, two factors, and three factors? Use direct calculation to verify your results.

- Plot the factor loadings for the first three principal components. How would you explain these first three principal components?
- Calculate the standard deviations of the first three components

(c) (10 points) Since the first three components are independent, we could bump them up and down independently from each other. Suppose that we bump each component up or down to its 5th or 95th percentile, and assume that each component follows normal distribution. That is we stress each component to $\mu \pm 1.65\sigma$ where μ and σ are the mean and standard deviation of the component. Now we would have 8 possibilities if we use only up or down bumps on the first three components. List all eight combinations of the bumped values for the first three components.

(d) (10 points) Translate these bumps to the principal components to the bumps on the change of original 13 points on the yield curve by omitting the 4th principal component and above. Plot the exponential of the bumped value of 13 zero rate changes against its terms. In the future we could apply these bumps as a scale to the current zero rates to perform stress testing on the yield curve movement.

5. (40 points) Download ISDA CDS Standard Model Excel addin from ISDA website for CDS Model, and the pricing sheet provided in the Course work.

(a) (15 points) Use the standard model and a constant zero rate 5% to price a CDS deal with a maturity date of March 20, 2028, a running spread of 100 bps. The underlying credit has a flat CDS spread of 200 bps, and recovery rate of 40%. The valuation date is Feb 10, 2023. Present the value of the premium leg, loss leg, premium accrual, current MTM value, and a par credit default swap spread.

(b) (5 points) If the protection buyer of the above trade would like to close the above position, would should the buyer do?

(c) (20 points) Use the formulas provided in the lecture note, and use Python or Excel for the implementation of the formulas to check the numbers, and make comparison with the numbers from the Excel addin.