

1. (100 points) This question is about how to calculate a potential future exposure for a 5-year interest rate swap. Suppose the par swap rate is 4.5%, and both floating leg and fixed leg are paid quarterly. To simplify our process of getting information and pricing an interest rate swap, we assume that the interest rate swap is based on USD treasury yield, and you can use implied forward yield for the floating side payment calculation, and no day count is used by assuming the payment is made exactly on a quarterly basis.
  - (a) (10 points) Download the yield curve from April 10, 2003 to April 12, 2024, and calculate the daily change in percentage, and study the volatility and correlation structure, and state your observations.
  - (b) (20 points) Suppose we could use a simple short rate model to describe the interest rate movement, and assume that one month yield is the short rate, which can be described as follow:

$$dr = \alpha(\beta - r)dt + \sigma dW_t.$$

Use the one-month downloaded yield to estimate the parameter  $\alpha$ ,  $\beta$  and  $\sigma$ .

- (c) (5 points) Based on the April 12, 2024 data, calculate the spread between any other points with the one month yield.
- (d) (25 points) Come up with a Monte Carlo simulation engine for the future yield curve based on the following: 1. one month yield is based on the above estimated stochastic differential equation; 2. Any other points on the yield curve are assumed to maintain the same spread as of April 12, 2024.
- (e) (15 points) Price the interest rate swap today (April 13, 2024), and assume you are the fixed rate receiver, and the notional amount is \$ 10 million USD.
- (f) (20 points) Price the interest rate swap using the simulated yield curves at years 0.25, 0.5, 1.0, . . . , 4.5, 5, and present the potential credit exposure at 95% percentile, and plot them on a graph.
- (g) (5 points) Calculate the potential future exposure as the maximum of the potential credit exposure at future times.