5.3 Lean production and quality management (HL)

Tool: Critical path analysis (HL)

Critical path analysis (CPA), also known as network analysis, is a project planning tool. It shows the critical path of a project, which is the minimum time period needed for the project to be completed. As with Gantt charts, critical path analysis can be used by project managers to gain an overview of:

- the tasks involved in a project
- how long each task is expected to take
- the order in which the tasks need to be done
- whether any tasks can be completed at the same time
- interim deadlines needed to keep the entire project on track
- whether any steps of the project have buffer time (also called a float time) in case of delays

This section is divided into several parts, to help you construct and interpret a network diagram and the critical path.

Exam tip

The IB Business Management syllabus states that you should be able to complete and analyse a critical path diagram, but that constructing a full critical path diagram is not expected.

Some activities in this section, however, ask you to construct a critical path. Practising this will ensure that you have a deep understanding of the structure and meaning of the numbers.

Critical path basics

Figure 1 shows a basic critical path (network) diagram for a fictional project – Project X – with some key components labelled. The letters in the diagram (A to F) represent the various tasks (or activities) of Project X. These are separated by circles, called nodes. The nodes represent the start and end of each task. Each node is divided into three parts, and you can number the nodes from left to right, for reference, using the left side of each node.

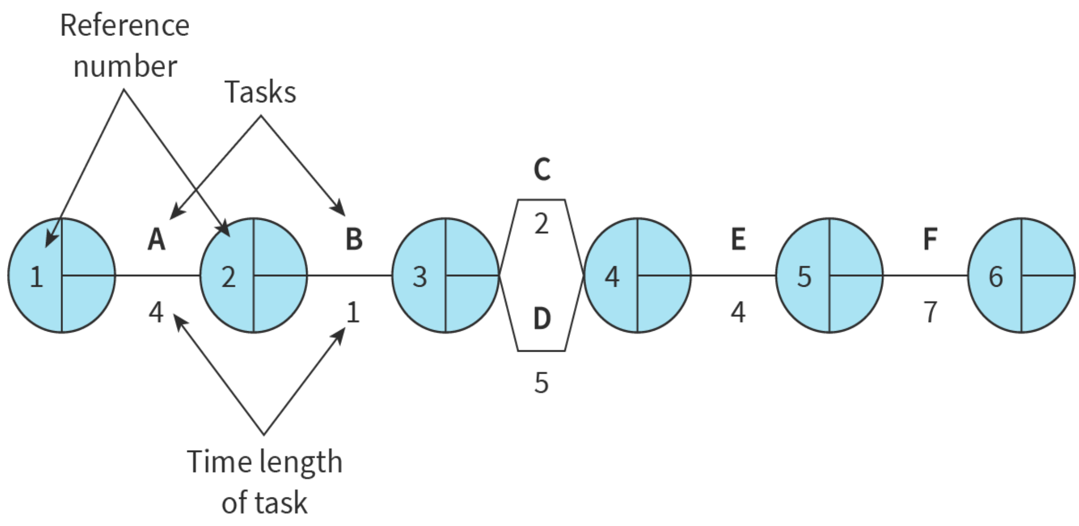

Figure 1. Critical path basics for Project X.

So, in **Figure 1**, the first task is A and the final task is F. The node to the left of task A (labelled 1) represents the start of task A. And the node to the right of task F (labelled 6) represents the end of the task F.

You will learn about the other parts of the node later. However, because some activities will be happening concurrently, the reference numbers for the nodes do not necessarily represent the order in which the activities take place.

The number written below the line underneath each task represents the time period for that task. For Project X, the times are in days. So the time period for task A is four days, the time period for task B is one day, and so on. Two tasks in Project X – tasks C and D – are completed at the same time, each with different time periods; task C takes two days and task D takes five days.

The critical path diagram is drawn from left to right. However, the end node for the entire project cannot be drawn until you are sure that all the interim activities are accounted for.

Exam tip

You should take care when constructing a critical path (network) diagram for the practice work in this section.

Use a ruler to make straight lines that extend from the nodes at the midpoint. The lines should not cross one another.

The information from **Figure 1** is summarised in **Table 1** below. In your exam, this is one way that you may be given information, from which you would be asked to draw a critical path (network) diagram.

Table 1. Summary of information for the critical path analysis (network diagram) for Project X.

Task	Preceded by	Duration (days)
A	--	4
B	A	1
C	B	2
D	B	5
E	C and D	4
F	E	7

Activity

Learner profile: Thinkers
Approaches to learning: Thinking skills (transfer)

Table 2 summarises the tasks involved in Project S, with their dependencies and durations. Use the information in the table to draw a simple critical path (network) diagram for Project S.

Table 2. Dependencies and duration of tasks for Project S.

Task	Preceded by	Duration (days)
A	--	2
B	A	3

Task	Preceded by	Duration (days)
C	A	7
D	B and C	1
E	D	5
F	E	4

Adding earliest start time and latest finish time to the nodes

Once you have drawn the basic critical path (network) diagram, you can add information about the earliest start time and latest finish time to each node. This will allow you to understand the time constraints of the project more clearly.

Earliest start time (EST)

The earliest start time (EST) indicates the earliest time that a task can begin. It is written in the upper right segment of the node and refers to the task just after the node, as shown in **Figure 3**.

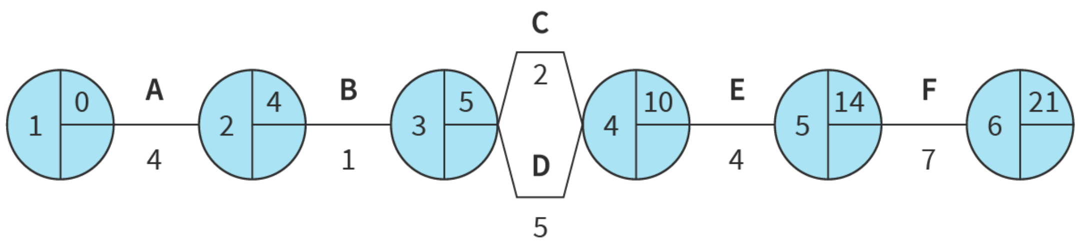

Figure 2. Adding earliest start times to the critical path (network) diagram for Project X.

Figure 3 shows that task A is the first task of Project X. It can start immediately, so 0 is written in the upper right segment of the node to the left of task A. Then, working from left to right you can see that:

- Task A will take four days to complete, so the earliest that task B can start is after day 4.
- Task B will take one day to complete. This means that tasks C and D can only start after day 5.
- Tasks C and D take different time periods and task E can only start after both task C and task D have been completed. In this case, you would select the higher of the two time durations for tasks C and D to insert as the earliest start time for task E. Thus, task E can only start after day 10.
- Task E takes four days to complete, so task F can only start after day 14.

The earliest start time for an activity is useful for businesses using just-in-time (JIT) strategies for lean production. Deliveries of resources can be planned to arrive just before the earliest start time for the activity that needs them. That way, storage costs for stock (inventory) can be reduced.

Latest finish time (LFT)

In **Figure 3**, the last node shows the end of Project X. The project should be completed in 21 days. This number is written in both the upper right and lower right segments of the last node. The lower right segment of the node refers to the latest finish time (LFT) that the preceding task should finish in order to keep Project X on track.

It is important to emphasise that the latest finish time (LFT) refers to the task to the **left** of the node. This is different from the earliest start time (EST), which always refers to the task to the **right** of the node.

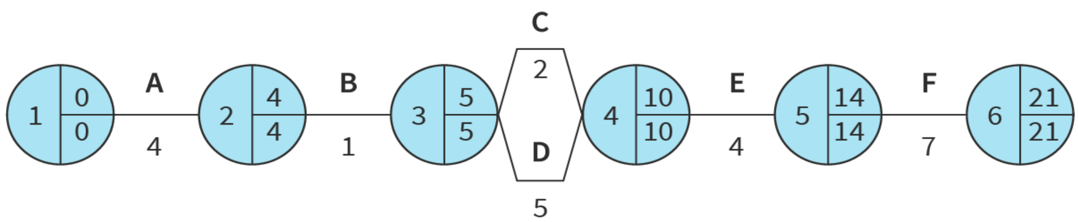

Figure 3. Adding latest finish times to the critical path (network) diagram for Project X.

Now, this time working from right to left, you can insert the latest finish times to each of the previous nodes and see that (as shown in **Figure 3**):

- The project should be completed in 21 days. Task F takes seven days to complete, so the latest finish time for task E is day 14.
- Task E takes four days to complete, so the latest time for tasks C and D to be finished is day 10.
- Tasks C and D take different amounts of time to complete. In this case, you would subtract the larger of the numbers. So the latest finish time for task B is day 5.
- Task B takes one day to complete, so the latest finish time for task A is day 4.

In this simple critical path diagram for Project X, the earliest start times (EST) and the latest finish times (LFT) are the same in each node. In more complex critical path diagrams, this will not always be the case. You will look at a more complex example later in this section.

Activity

Learner profile: Thinkers
Approaches to learning: Thinking skills (transfer)

Table 3 is a repeat of **Table 2** from the previous activity, summarising the tasks for Project S with their dependencies and durations. Use the critical path (network) diagram that you have already completed and add in the earliest start times (EST) and latest finish times (LFT) for Project S.

Table 3. Dependencies and duration of tasks for Project S.

Task	Preceded by	Duration (days)
A	--	2
B	A	3
C	A	7

Task	Preceded by	Duration (days)
D	C	1
E	D	5
F	E	4

Calculating free float and total float

The float time refers to the amount of time that a task or activity can overrun its time estimate, but not disrupt the estimated time for the other tasks or for the entire project. There are two calculations for float: free float and total float.

Free float is the amount of time that a task can overrun its time estimate, but not delay the **next** task. It is calculated as follows:

Free float of task Y = earliest start time (EST) of the next task – earliest start time (EST) of task Y – duration of task Y

Using the information from Project X, which is shown again in **Figure 4**, the free float for individual tasks can be calculated.

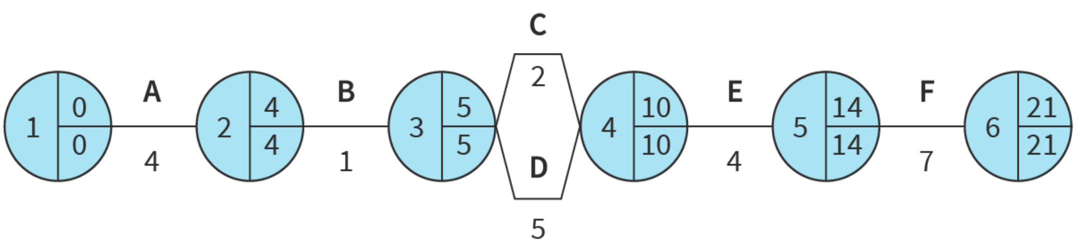

Figure 4. Critical path (network) diagram for Project X.

Free float of task B = EST of task C – EST of task B – duration of task B

= 5 days – 4 days – 1 day

= 0 days

Task B has 0 days free float. It therefore cannot overrun its time estimate without delaying task C.

Free float of task C = EST of task E – EST of task C – duration of task C

= 10 days – 5 days – 2 days

= 3 days

Task C has 3 days free float. It can therefore overrun its time estimate by three days without delaying task E.

Total float is the amount of time that a task or activity can overrun its time estimate, but not delay the **whole project**. It is calculated as follows:

Total float of task Y = latest finish time (LFT) of task Y – duration of task Y – earliest start time (EST) of task Y

Again, using the information from Project X, the total float for individual tasks can be calculated.

Total float of task E = LFT of task E – duration of task E – EST of task E

= 14 days – 10 days – 4 days

= 0 days

Task E has 0 days total float. This means that task E must finish on time or the entire project will be delayed.

Information about earliest start time (EST), latest finish time (LFT) and total float can be added to the information in **Table 1**. This information is included in **Table 4**. All tasks in Project X that have zero total float can then be identified. These tasks together are called the critical path because they must be completed on time. The critical path is also the minimum amount of time needed to complete the entire project. Note that, in **Table 4**, the total float is calculated using the following formula, with the subscript 'current task' indicating the current task being considered in the row:

$$\textbf{Total float} = \textbf{LFT}_{\textbf{current task}} - \textbf{EST}_{\textbf{current task}} - \textbf{duration}_{\textbf{current task}}$$

Table 4. Completed summary information for Project X.

Task	Preceded by	Duration (days)	EST	LFT	Total float (days)
A	--	4	0	4	4 – 0 – 4 = 0
B	A	1	4	5	5 – 4 – 1 = 0
C	B	2	5	10	10 – 5 – 2 = 3
D	B	5	5	10	10 – 5 – 5 = 0
E	C and D	4	10	14	14 – 10 – 4 = 0
F	E	7	14	21	21 – 14 – 7 = 0

According to the total float figures, tasks A, B, D, E and F form the critical path. If any of these activities run over their time estimates, the entire project will be delayed. It is important that project managers pay particular attention to these activities to keep them on track.

When you are asked to identify the critical path in the exam, there are two ways of doing this. You can either write out the critical path with arrows or you can use hash marks (//) to indicate the critical path on the diagram itself. Both of these methods are shown in **Figure 5.** Note also the key indicating the elements of the critical path diagram.

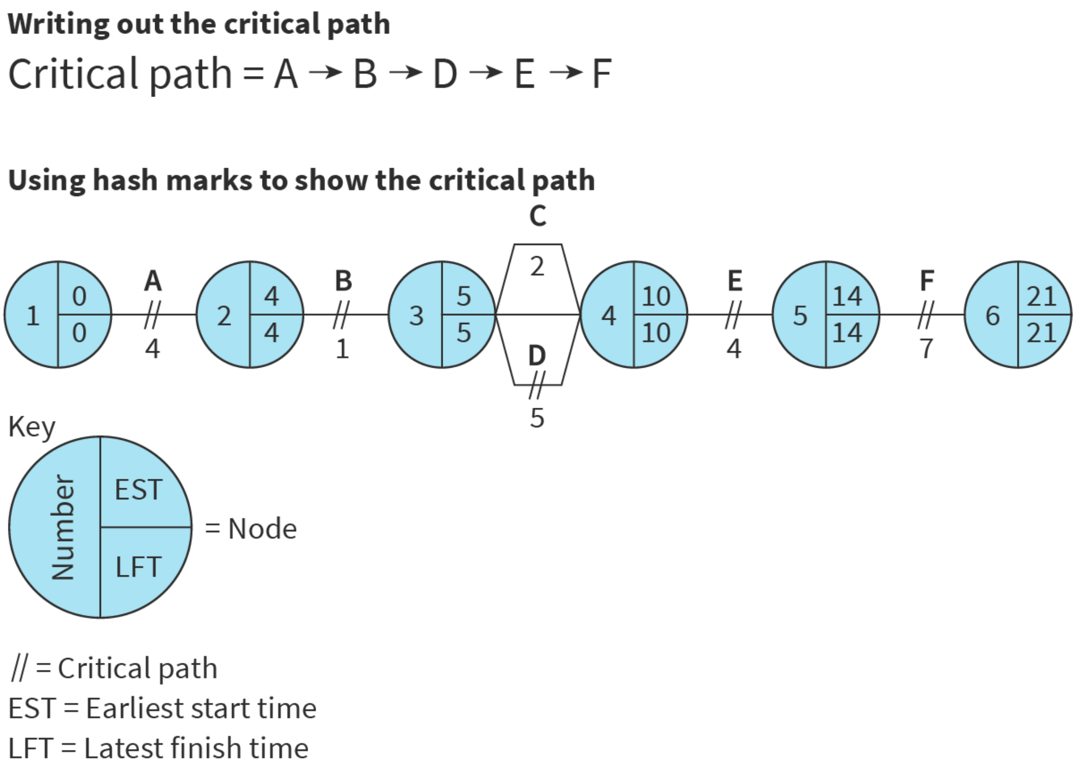

Figure 5. Two different ways of identifying the critical path for Project X.

Exam tip

You will likely be asked to identify the critical path in the exam. You must state the critical path explicitly or indicate it with hash marks on the diagram itself.

It is a good idea to double check your calculations. It can be very easy to mix up the EST and LFT for the current and next task when calculating free float and total float.

Complex critical path analysis

Until this point, you have been looking at a very simple critical path in order to learn the basics. In the exam, however, you will likely be given information for a more complex critical path analysis, such as the example given in **Figure 6** the following activity.

Activity

Learner profile: Thinkers
Approaches to learning: Thinking skills (transfer)

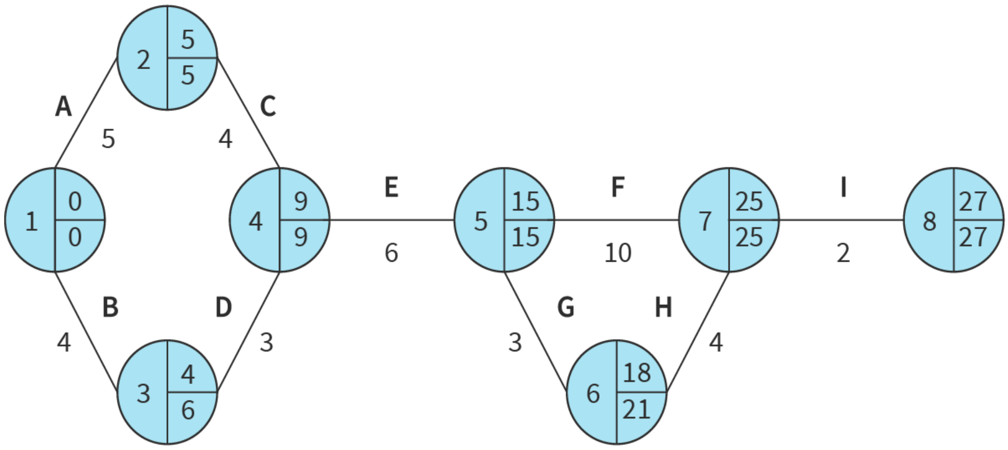

Figure 6. Critical path (network) diagram for Project Q.

See if you can answer each question using the information from **Figure 6**. You can reveal the solutions to check your understanding.

Questions

1. What is the total number of days that Project Q should take?
2. What is the earliest start time (EST) for task F?
3. What is the latest finish time (LFT) for task D?
4. Create a table to show the tasks, task dependencies, task durations, earliest start times, latest finish times, free float and total float (you will have to calculate these).
5. Identify the critical path for Project Q.

Exam tip

You may have noticed that – for tasks that are part of the critical path –the EST for the next task is the same as the LFT for the current task. This simply shows that the next task needs to start as soon as the previous task is finished; there is no float.

Activity

Learner profile: Thinkers
Approaches to learning: Thinking skills (critical thinking, creative thinking)

Bee-have (BH) is a small workshop that makes beehives for beekeepers using scrap wood from a local building supply store. There are eight tasks involved in constructing the beehives at BH, some of which can be executed concurrently. The tasks, task dependencies and task durations in hours are shown in **Table 5**.

Table 5. Tasks needed to complete a beehive at BH.

Task	Preceded by	Duration (hours)	EST	LFT	Free float (hours)	Total float (hours
A	-	1				
B	-	6				
C	A	2				
D	A	4				
E	C	3				
F	E and D	2				
G	E and D	6				
H	G and B	1				

Questions

1. Construct a critical path (network) diagram for the completion of a beehive at BH.
2. State the minimum number of hours needed to make a beehive.
3. Copy and complete the table with the missing information.
4. Identify the critical path for the construction of a beehive at BH.

Activity

Learner profile: Thinkers
Approaches to learning: Thinking skills (critical thinking, creative thinking)

You can really show a deep understanding of the critical path if you can create a problem yourself. Give it a try! Create a table of information about the tasks for a fictional project (Project R), their dependencies and their durations on a piece of paper and complete the critical path diagram for that information on the back of the piece of paper.

When you have done that:

- Swap your piece of paper with another student who has done the same and look at each other's tables.
- Each of you should create a critical path diagram from the other's table.
- Check your diagram against the other student's diagram. If there are discrepancies, see if you can resolve them to create fully aligned tables and diagrams.
- If your entire class does this, ask your teacher if the class can create a booklet from the work to share for practice.

Dummy activities

In more complex critical path diagrams, you may see a dotted line indicating a dummy activity. A dummy activity has no duration or cost. It is added to the diagram simply to show the relationships between real tasks.

The critical path diagram in **Figure 7** includes a dummy activity to show that task C must be preceded by both tasks A and B. For simplicity, the ESTs and LSTs have been left out; only the node numbers and activity letters are included. In this example, the dummy activity indicates that:

- Task C must come after tasks A and B are finished.
- Task D must come after task B (but not task A).

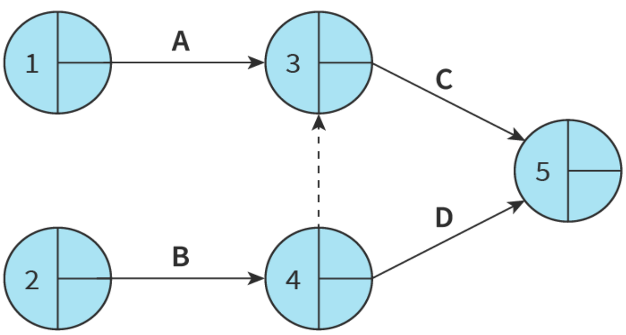

Figure 7. Critical path (network) diagram with a dummy activity included.

As noted in a previous exam tip, you will not be required to draw critical path diagrams or any dummy activities within them. However, if you see a dotted line in a critical path diagram in the exam, it is important that you know what it is.