Programming Assignment 5: Minimum Spanning Trees

Revision: July 27, 2020

Introduction

Welcome to the third (and the last one) programming assignment of the Algorithms on Graphs class! In this programming assignment you will be practicing implementing algorithms computing minimum spanning trees.

Learning Outcomes

Upon completing this programming assignment you will be able to:

1. connect the given cities by roads of minimum total length such that there is a path between any two cities;
2. compute an optimal clustering of the given set of objects.

Passing Criteria: 1 out of 2

Passing this programming assignment requires passing at least 1 out of 2 programming challenges from this assignment. In turn, passing a programming challenge requires implementing a solution that passes all the tests for this problem in the grader and does so under the time and memory limits specified in the problem statement.

Contents

Graph Representation in Programming Assignments

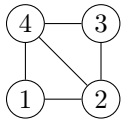
In programming assignments, graphs are given as follows. The first line contains non-negative integers n and m — the number of vertices and the number of edges respectively. The vertices are always numbered from 1 to n. Each of the following m lines defines an edge in the format `u v` where $1 \le u, v \le n$ are endpoints of the edge. If the problem deals with an undirected graph this defines an undirected edge between u and v. In case of a directed graph this defines a directed edge from u to v. If the problem deals with a weighted graph then each edge is given as `u v w` where u and v are vertices and w is a weight.

It is guaranteed that a given graph is simple. That is, it does not contain self-loops (edges going from a vertex to itself) and parallel edges.

Examples:

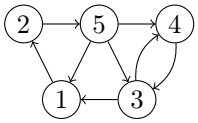
- An undirected graph with four vertices and five edges:

```
4 5
2 1
4 3
1 4
2 4
3 2
```



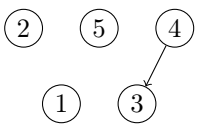
- A directed graph with five vertices and eight edges.

```
5 8
4 3
1 2
3 1
3 4
2 5
5 1
5 4
5 3
```



- A directed graph with five vertices and one edge.

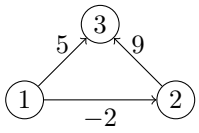
```
5 1
4 3
```



Note that the vertices 1, 2, and 5 are isolated (have no adjacent edges), but they are still present in the graph.

- A weighted directed graph with three vertices and three edges.

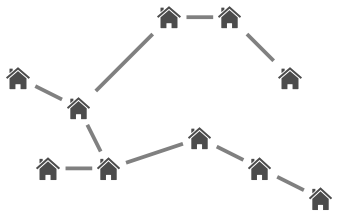
```
3 3
2 3 9
1 3 5
1 2 -2
```



1 Building Roads to Connect Cities

Problem Introduction

In this problem, the goal is to build roads between some pairs of the given cities such that there is a path between any two cities and the total length of the roads is minimized.



Problem Description

Task. Given n points on a plane, connect them with segments of minimum total length such that there is a path between any two points. Recall that the length of a segment with endpoints (x_1, y_1) and (x_2, y_2) is equal to $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Input Format. The first line contains the number n of points. Each of the following n lines defines a point (x_i, y_i).

Constraints. $1 \le n \le 200$; $-10^3 \le x_i, y_i \le 10^3$ are integers. All points are pairwise different, no three points lie on the same line.

Output Format. Output the minimum total length of segments. The absolute value of the difference between the answer of your program and the optimal value should be at most 10^{-6}. To ensure this, output your answer with at least seven digits after the decimal point (otherwise your answer, while being computed correctly, can turn out to be wrong because of rounding issues).

Time Limits.

language	C	C++	Java	Python	C#	Haskell	JavaScript	Ruby	Scala
time (sec)	2	2	3	10	3	4	10	10	6

Memory Limit. 512MB.

Sample 1.

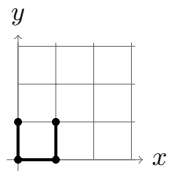
Input:

```
4
0 0
0 1
1 0
1 1
```

Output:

```
3.000000000
```

An optimal way to connect these four points is shown below. Note that there exists other ways of connecting these points by segments of total weight 3.



Sample 2.

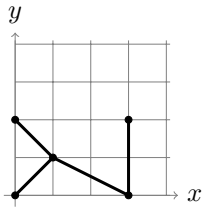
Input:

```
5
0 0
0 2
1 1
3 0
3 2
```

Output:

```
7.064495102
```

An optimal way to connect these five points is shown below.

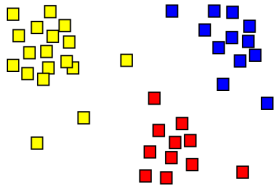


The total length here is equal to $2\sqrt{2} + \sqrt{5} + 2$.

2 Clustering

Problem Introduction

Clustering is a fundamental problem in data mining. The goal is to partition a given set of objects into subsets (or clusters) in such a way that any two objects from the same subset are close (or similar) to each other, while any two objects from different subsets are far apart.



Problem Description

Task. Given n points on a plane and an integer k, compute the largest possible value of d such that the given points can be partitioned into k non-empty subsets in such a way that the distance between any two points from different subsets is at least d.

Input Format. The first line contains the number n of points. Each of the following n lines defines a point (x_i, y_i). The last line contains the number k of clusters.

Constraints. $2 \le k \le n \le 200$; $-10^3 \le x_i, y_i \le 10^3$ are integers. All points are pairwise different.

Output Format. Output the largest value of d. The absolute value of the difference between the answer of your program and the optimal value should be at most 10^{-6}. To ensure this, output your answer with at least seven digits after the decimal point (otherwise your answer, while being computed correctly, can turn out to be wrong because of rounding issues).

Time Limits.

language	`C`	`C++`	`Java`	`Python`	`C#`	`Haskell`	`JavaScript`	`Ruby`	`Scala`
time (sec)	2	2	3	10	3	4	10	10	6

Memory Limit. 512MB.

Sample 1.

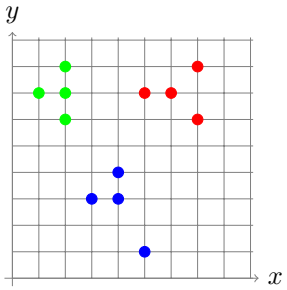
Input:

```
12
7 6
4 3
5 1
1 7
2 7
5 7
3 3
7 8
2 8
4 4
6 7
2 6
3
```

Output:

```
2.828427124746
```

The answer is $\sqrt{8}$. The corresponding partition of the set of points into three clusters is shown below.



Sample 2.

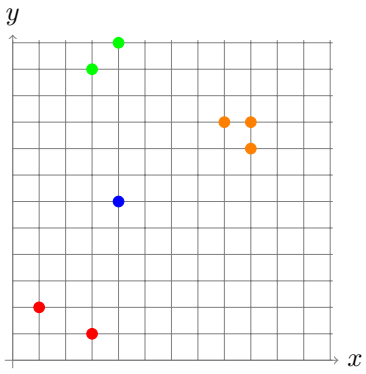
Input:

```
8
3 1
1 2
4 6
9 8
9 9
8 9
3 11
4 12
4
```

Output:

```
5.000000000
```

The answer is 5. The corresponding partition of the set of points into four clusters is shown below.



What To Do

Think about ways of adopting the Kruskal's algorithm for solving this problem.

3 Appendix

3.1 Compiler Flags

C (`gcc 7.4.0`). File extensions: `.c`. Flags:

```
gcc -pipe -O2 -std=c11 <filename> -lm
```

C++ (`g++ 7.4.0`). File extensions: `.cc`, `.cpp`. Flags:

```
g++ -pipe -O2 -std=c++14 <filename> -lm
```

If your `C/C++` compiler does not recognize `-std=c++14` flag, try replacing it with `-std=c++0x` flag or compiling without this flag at all (all starter solutions can be compiled without it). On Linux and MacOS, you most probably have the required compiler. On Windows, you may use your favorite compiler or install, e.g., `cygwin`.

C# (`mono 4.6.2`). File extensions: `.cs`. Flags:

```
mcs
```

Go (`golang 1.13.4`). File extensions: `.go`. Flags

```
go
```

Haskell (`ghc 8.0.2`). File extensions: `.hs`. Flags:

```
ghc -O2
```

Java (`OpenJDK 1.8.0_232`). File extensions: `.java`. Flags:

```
javac -encoding UTF-8
java -Xmx1024m
```

JavaScript (`NodeJS 12.14.0`). File extensions: `.js`. No flags:

```
nodejs
```

Kotlin (`Kotlin 1.3.50`). File extensions: `.kt`. Flags:

```
kotlinc
java -Xmx1024m
```

Python (`CPython 3.6.9`). File extensions: `.py`. No flags:

```
python3
```

Ruby (`Ruby 2.5.1p57`). File extensions: `.rb`.

```
ruby
```

Rust (`Rust 1.37.0`). File extensions: `.rs`.

```
rustc
```

Scala (`Scala 2.12.10`). File extensions: `.scala`.

```
scalac
```

3.2 Frequently Asked Questions

Why My Submission Is Not Graded?

You need to create a submission and upload the *source file* (rather than the executable file) of your solution. Make sure that after uploading the file with your solution you press the blue "Submit" button at the bottom. After that, the grading starts, and the submission being graded is enclosed in an orange rectangle. After the testing is finished, the rectangle disappears, and the results of the testing of all problems are shown.

What Are the Possible Grading Outcomes?

There are only two outcomes: "pass" or "no pass." To pass, your program must return a correct answer on all the test cases we prepared for you, and do so under the time and memory constraints specified in the problem statement. If your solution passes, you get the corresponding feedback "Good job!" and get a point for the problem. Your solution fails if it either crashes, returns an incorrect answer, works for too long, or uses too much memory for some test case. The feedback will contain the index of the first test case on which your solution failed and the total number of test cases in the system. The tests for the problem are numbered from 1 to the total number of test cases for the problem, and the program is always tested on all the tests in the order from the first test to the test with the largest number.

Here are the possible outcomes:

- `Good job!` Hurrah! Your solution passed, and you get a point!
- `Wrong answer.` Your solution outputs incorrect answer for some test case. Check that you consider all the cases correctly, avoid integer overflow, output the required white spaces, output the floating point numbers with the required precision, don't output anything in addition to what you are asked to output in the output specification of the problem statement.
- `Time limit exceeded.` Your solution worked longer than the allowed time limit for some test case. Check again the running time of your implementation. Test your program locally on the test of maximum size specified in the problem statement and check how long it works. Check that your program doesn't wait for some input from the user which makes it to wait forever.
- `Memory limit exceeded.` Your solution used more than the allowed memory limit for some test case. Estimate the amount of memory that your program is going to use in the worst case and check that it does not exceed the memory limit. Check that your data structures fit into the memory limit. Check that you don't create large arrays or lists or vectors consisting of empty arrays or empty strings, since those in some cases still eat up memory. Test your program locally on the tests of maximum size specified in the problem statement and look at its memory consumption in the system.
- `Cannot check answer. Perhaps the output format is wrong.` This happens when you output something different than expected. For example, when you are required to output either "Yes" or "No", but instead output 1 or 0. Or your program has empty output. Or your program outputs not only the correct answer, but also some additional information (please follow the exact output format specified in the problem statement). Maybe your program doesn't output anything, because it crashes.
- `Unknown signal 6 (or 7, or 8, or 11, or some other).` This happens when your program crashes. It can be because of a division by zero, accessing memory outside of the array bounds, using uninitialized variables, overly deep recursion that triggers a stack overflow, sorting with a contradictory comparator, removing elements from an empty data structure, trying to allocate too much memory, and many other reasons. Look at your code and think about all those possibilities. Make sure that you use the same compiler and the same compiler flags as we do.
- `Internal error: exception...` Most probably, you submitted a compiled program instead of a source code.

- `Grading failed.` Something wrong happened with the system. Report this through Coursera or edX Help Center.

May I Post My Solution at the Forum?

Please do not post any solutions at the forum or anywhere on the web, even if a solution does not pass the tests (as in this case you are still revealing parts of a correct solution). Our students follow the Honor Code: "I will not make solutions to homework, quizzes, exams, projects, and other assignments available to anyone else (except to the extent an assignment explicitly permits sharing solutions)."

Do I Learn by Trying to Fix My Solution?

My implementation always fails in the grader, though I already tested and stress tested it a lot. Would not it be better if you gave me a solution to this problem or at least the test cases that you use? I will then be able to fix my code and will learn how to avoid making mistakes. Otherwise, I do not feel that I learn anything from solving this problem. I am just stuck.

First of all, learning from your mistakes is one of the best ways to learn.

The process of trying to invent new test cases that might fail your program is difficult but is often enlightening. Thinking about properties of your program makes you understand what happens inside your program and in the general algorithm you're studying much more.

Also, it is important to be able to find a bug in your implementation without knowing a test case and without having a reference solution, just like in real life. Assume that you designed an application and an annoyed user reports that it crashed. Most probably, the user will not tell you the exact sequence of operations that led to a crash. Moreover, there will be no reference application. Hence, it is important to learn how to find a bug in your implementation yourself, without a magic oracle giving you either a test case that your program fails or a reference solution. We encourage you to use programming assignments in this class as a way of practicing this important skill.

If you have already tested your program on all corner cases you can imagine, constructed a set of manual test cases, applied stress testing, etc, but your program still fails, try to ask for help on the forum. We encourage you to do this by first explaining what kind of corner cases you have already considered (it may happen that by writing such a post you will realize that you missed some corner cases!), and only afterwards asking other learners to give you more ideas for tests cases.