FACETS OF THE UNION-CLOSED POLYTOPE

A Dissertation Presented

by

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ABSTRACT

Facets of the Union-Closed Sets Polytope

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In the haze of the 1970s, a conjecture was born to unknown parentage...the union-closed sets conjecture. Given a family of sets $\mathcal{F}$, we say that $\mathcal{F}$ is union-closed if for every two sets $S, T \in \mathcal{F}$, we have $S \cup T \in \mathcal{F}$. The union-closed sets conjecture states that there is an element in at least half of the sets of any (non-empty) union-closed family. In 2016, Pulaj, Raymond, and Theis [27] reinterpreted the conjecture as an optimization problem that could be formulated as an integer program. This thesis is concerned with the study of the polytope formed by taking the convex hull of the integer points satisfying the integer program. We find several facets and describe some small cases of this complicated polytope in full.

TABLE OF CONTENTS

LIST OF FIGURES

CHAPTER 1

INTRODUCTION

The *union-closed sets conjecture* first entered the literature in the 1980s, for example in [34] and [1], but there are several comments in early papers that make mention of it as a conjecture that was widely known. The conjecture is synonymously called Frankl's conjecture after Péter Frankl ([12]) who is credited with popularizing the conjecture. Let $\mathcal{P}([n])$ be the power set of $[n] := \{1, \ldots, n\}$ and let $\mathcal{F} \subseteq \mathcal{P}([n])$ be a family of sets with the property that if $S, T \in \mathcal{F}$, then $S \cup T \in \mathcal{F}$. We call such collections of sets $\mathcal{F}$ *union-closed families.*

Conjecture 1.0.1 (The union-closed sets conjecture or Frankl's conjecture)**.** *If $\mathcal{F} \subseteq \mathcal{P}([n])$ is a (non-empty) union-closed family, then there exists an element in $[n]$ such that this element is contained in at least half of the sets in $\mathcal{F}$.*

There has been a relatively steady march of results showing that the union-closed sets conjecture is true for larger and larger values of n. The most recent and best result was obtained by Vučković and Živković in [40].

Theorem 1.0.2 (Vučković-Živković, 2017)**.** *If $\mathcal{F} \subseteq \mathcal{P}([n])$ is a union-closed family where $n \leq 12$, then the union-closed sets conjecture holds for $\mathcal{F}$.*

On top of there being relatively few cases where the union-closed sets conjecture is known to be true, no constant bound was known until very recently. In other words, there was no known constant c such that, given any union-closed family $\mathcal{F}$, one could prove that there exists an element in at least $c|\mathcal{F}|$ sets. The best bound for a long time was due to Knill in [19].

Theorem 1.0.3 (Knill, 1994). *In any union-closed family $\mathcal{F}$, there exists an element in at least $\frac{|\mathcal{F}|-1}{\log_2 |\mathcal{F}|}$ sets.*

In the fall of this past year, Gilmer gave a novel use of entropy to achieve the first known constant bound in [14].

Theorem 1.0.4 (Gilmer, 2022). *Let $\mathcal{F} \subseteq \mathcal{P}([n])$ be a union-closed family with $\mathcal{F} \neq \emptyset$, then there exists an element that appears in at least $0.01|\mathcal{F}|$ sets in $\mathcal{F}$.*

This result was immediately improved upon by Sawin in [35] who confirmed a better bound conjectured in Gilmer's original paper.

Theorem 1.0.5 (Sawin, 2022). *Let $\mathcal{F} \subseteq \mathcal{P}([n])$ be a union-closed family with $\mathcal{F} \neq \emptyset$. Then there exists an element contained in at least $\frac{3-\sqrt{5}}{2}|\mathcal{F}| \approx 0.382|\mathcal{F}|$ sets.*

Among the many different interpretations of the union-closed sets conjecture, Pulaj, Raymond and Theis in [27] stated the following equivalent version in terms of optimization. Our work revolves around this polyhedral version of the conjecture. In what follows, for a union-closed family $\mathcal{F} \subseteq \mathcal{P}([n])$ and $i \in [n]$, we let $\mathcal{F}_i := \{S \in \mathcal{F} | i \in S\}$.

Conjecture 1.0.6 (Maximization Conjecture). *For any positive integers a and n, let*

$$\mathfrak{F}(a,n) = \{\mathcal{F} \subseteq \mathcal{P}([n]) \,|\mathcal{F} \textit{ is a non-empty union-closed family}$$
$$\textit{and } \max_{i\in[n]} |\mathcal{F}_i| \leq a\}.$$

Then $\max_{\mathcal{F}\in\mathfrak{F}(a,n)} |\mathcal{F}| \leq 2a$ for all $a, n \in \mathbb{Z}_{>0}$.

To see that this is equivalent to the union-closed sets conjecture, let $\mathcal{F}$ be a counterexample to the Maximization Conjecture. Then for every element i in the groundset of $\mathcal{F}$, we have $|\mathcal{F}_i| < \frac{|\mathcal{F}|}{2}$. In other words, there is a union-closed family such that $|\mathcal{F}| > 2a$ where a is the maximum frequency of all elements in the groundset of $\mathcal{F}$.

In [27], the authors studied the Maximization Conjecture with the following integer program.

$$
\begin{aligned}
f(n,a) := \max & \sum_{S \in \mathcal{P}([n])} x_S \\
\text{such that } & x_S + x_T \leq 1 + x_{S \cup T} && \text{for all } S, T \in \mathcal{P}([n]) \\
& \sum_{\substack{S \in \mathcal{P}([n]): \\ i \in S}} x_S \leq a && \text{for all } i \in [n] \\
& x_S \in \{0,1\} && \text{for all } S \in \mathcal{P}([n])
\end{aligned}
$$

The variables x_S represent whether some set $S \in \mathcal{P}([n])$ is within a particular family or not. The first set of inequalities enforce the union-closed condition for all families considered, and the second set of inequalities enforce the fixed maximum frequency a. We let $P(n,a)$ denote the polytope defined as the convex hull of the integer points of this integer program, i.e.,

$$f(n,a) = \max_{\mathbf{x} \in P(n,a)} \sum_{S \in \mathcal{P}([n])} x_S.$$

We call $P(n,a)$ the *union-closed polytope.*

Much of the novel work in this thesis is concerned with studying the polytope $P(n,a)$ in Chapter 4. We find several classes of facets of $P(n,a)$ and conjecture some valid inequalities which are particularly helpful in the calculation of $f(n,a)$.

Some of the other results are computational. We are particularly interested in understanding $P(n,n)$ because of the following reasons. In [27], the authors gave a different conjecture which, though very much related to the union-closed sets conjecture, does not imply and is not implied by it.

Conjecture 1.0.7 (Pulaj-Raymond-Theis, 2016)**.** *Fix $a \in \mathbb{Z}_{>0}$. Then $f(n,a) = f(n+1,a)$ for every $n \in \mathbb{Z}_{>0}$ such that $n \geq \lceil \log_2 a \rceil + 1$.*

The authors proved this conjecture for $n \geq a$ which implies that one does not need to study $f(n,a)$ for $a > n$. Moreover, since $f(n,a) \leq f(n+1,a)$, it makes sense to focus the study of $P(n,a)$ for the case when $n = a$. The polytope $P(n,n)$ quickly escalates in complexity as n grows. Previously, the only values for which $P(n,n)$ was known were $n \leq 3$. By taking advantage of symmetry as $\mathfrak{S}_n$ acts on the ground set $[n]$, we give what we believe is a complete facet description of $P(4,4)$ as well as many different classes of facets for $P(5,5)$. Each facet listed actually represents a class which can be identified under the following equivalence.

Definition 1.0.8. Two union-closed families $\mathcal{F}$ and $\mathcal{G}$ with $\mathcal{F}, \mathcal{G} \subseteq \mathcal{P}([n])$ are called *isomorphic* if there exists some permutation $\sigma \in \mathcal{S}_n$ such that $\sigma(\mathcal{F}) = \mathcal{G}$.

For instance, this means that the family $\{\{1\},\{2\},\{1,2\}\}$ is equivalent to the family $\{\{1\},\{3\},\{1,3\}\}$. This equivalence carries on to the level of inequalities. The inequality $x_{\{1\}} + x_{\{2\}} \leq 1 + x_{\{1,2\}}$ is equivalent to $x_{\{1\}} + x_{\{3\}} \leq 1 + x_{\{1,3\}}$.

The computational results, found in the Appendix, were exceedingly helpful for finding and describing several classes of facets. The facets found all have similar proofs and stem from taking conical combinations of the union-closed constraints and rounding down their right-hand side, i.e., inequalities that have *Chvátal rank 1*. Then one only needs to find enough linearly independent vertices that are tight with the inequality to prove that it is facet-defining.

Theorem 1.0.9 (G., 2023). *The inequality*

$$x_{A \setminus S} + x_{B_1} + x_{B_2} \leq 1 + x_A + x_{B_1 \cup B_2}$$

where $A, B_1, B_2, S \in \mathcal{P}([n])$ with $B_1, B_2 \subset A$ and $S \subset B_1, B_2$ is facet-defining for $P(n, a)$ when $n, a \geq 5$.

Theorem 1.0.10 (G., 2023). *Let $A, B, C \in \mathcal{P}([n])$ be distinct sets such that $A \cup B$, $A \cup C$ and $B \cup C$ are distinct from one another and from A, B, C. Then the inequality*

$$x_A + x_B + x_C \leq 1 + x_{A \cup B} + x_{A \cup C} + x_{B \cup C}$$

is valid and facet-defining for $P(n, a)$ with $n, a \geq 3$.

Theorem 1.0.11 (G., 2023). *Let $A, B, C \in \mathcal{P}([n])$ be distinct sets such that $D := A \cup B = A \cup C = B \cup C$ and such that D is distinct from A, B, C. Then the inequality*

$$x_A + x_B + x_C \leq 1 + 2x_D$$

is valid and facet-defining for $P(n, a)$ with $n, a \geq 3$.

The following facet-defining inequalities do not have rank 1, but were found by looking at our computations and proven in similar ways to the ones before.

Theorem 1.0.12 (G., 2023). *The inequality*

$$\sum_{i=1}^{n} x_{[n] \setminus \{i\}} - (n-1)x_{[n]} \leq 1$$

is valid and facet-defining for $P(n, n)$.

The next conjecture and evidence that follows differs in that we are using the frequency constraints which are unique to the optimization formulation. These constraints prove more tricky to generalize because they depend heavily on a for $P(n, a)$.

Conjecture 1.0.13. *Fix a positive integer i. If $n = \sum_{k=i-1}^{m} \binom{m}{k}$ for some positive integer m and $n \geq i+1$, then the constraint $\sum_{\substack{S \in \mathcal{P}([n]): \\ |S|=i}} x_S \leq \binom{m+1}{i}$ is a tight valid inequality for $P(n,n)$.*

Furthermore, if $\sum_{k=i-1}^{m-1} \binom{m-1}{k} < n < \sum_{k=i-1}^{m} \binom{m}{k}$ for some positive integer m and $n \geq i+1$, then the constraint $\sum_{\substack{S \in \mathcal{P}([n]): \\ |S|=i}} x_S \leq \binom{m+1}{i} - 1$ is a valid inequality for $P(n,n)$ though it might not be tight.

The evidence for the conjecture is found in the following extreme versions of it and computations provided in the appendix.

Theorem 1.0.14 (G., 2023)**.** *The inequality $\sum_{\substack{S \in \mathcal{P}(n): \\ |S|=1}} x_S \leq \lfloor \log_2 a + 1 \rfloor$ is valid for $P(n,a)$.*

Theorem 1.0.15 (G., 2023)**.** *The inequality $\sum_{\substack{S \in \mathcal{P}(n): \\ |S|=n-2}} x_S \leq n-1$ is valid for $P(n,n)$.*

This thesis proceeds as follows. In Chapter 2, we review important ideas of linear and integer programming that will be useful to understand the polyhedral point of view of the union-closed conjecture. In Chapter 3, we describe some of the many interpretations of the union-closed sets conjecture and results previously found. In Chapter 4, we present new results on Frankl's conjecture from a polyhedral point of view.

CHAPTER 2

Background on Linear and Integer Programming

2.1 Complexity

One of the great difficulties in solving combinatorial problems from a computational perspective is the amount of time it takes to actually compute an answer to a question. We now give a lightening fast intro to complexity theory.

Let Σ be a finite set (typically $\Sigma = \{0, 1\}$) called the *alphabet*. Elements of Σ are letters and an ordered finite sequence of symbols is called a *word*. Σ^* is the collection of all words of symbols from Σ. A *problem* is a subset Π of $\Sigma^* \times \Sigma^*$ where we can think of a word z as the input of a problem and y is the solution, forming $(z, y) \subset \Pi$.

Furthermore, an algorithm A is a set of instructions which *solves* a problem Π if given any input z of Π, A will find a solution y or a certificate that no such y exists in a finite number of steps. The *runtime* of an algorithm is informally the number of operations needed to solve any instance of a problem. To each algorithm we can define a running time function $f(\sigma) := \max_{z,\text{size}(z)\leq\sigma}(\text{running time of } A \text{ for input } z)$ for $\sigma \in \mathbb{Z}_{\geq 0}$. We say that an algorithm is *polynomial-time* if the function f for an algorithm A is bounded by some polynomial $g \geq f$. In this case we would write A has running time $O(g)$.

For our purposes we consider two possibly different classes of problems, $\mathcal{P}$ and $\mathcal{NP}$. In practice, a problem is in $\mathcal{P}$ if a solution can be found and verified in polynomial time, i.e. there exists an algorithm with runtime $O(f)$ for some polynomial f. A problem is in $\mathcal{NP}$ if a solution can be verified but not necessarily found in polynomial time. We will expound more on the distinction made here when we discuss integer programming.

Two of the main tools used throughout this thesis are linear programming and integer programming. A *linear program* (LP) is an optimization problem

$$\begin{aligned} &\min \mathbf{cx} \\ &\text{such that } A\mathbf{x} \leq \mathbf{b} \\ &\qquad\qquad \mathbf{x} \in \mathbb{R}^n \end{aligned}$$

where $\mathbf{b} \in \mathbb{R}^m, \mathbf{c} \in (\mathbb{R}^n)^*,$ and $A \in \mathbb{R}^{m \times n}$.

Some quick terminology used throughout linear programming: The components of the vector $\mathbf{x} \in \mathbb{R}^n$ are called the *decision variables.* The vector $\mathbf{c} \in (\mathbb{R}^n)^*$ is called the *objective function.* Any vector $\mathbf{x} \in \mathbb{R}^n$ for which the inequalities $A\mathbf{x} \leq \mathbf{b}$ is satisfied is called a *feasible solution.*

The geometry of the feasible region for a linear program is a polyhedron.

Definition 2.1.1. Let P be the intersection of some half-spaces $\mathbf{a}_i\mathbf{x} \leq b_i$ where $i \in [m]$, i.e., $P = \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} \leq \mathbf{b}\}$ where $A \in \mathbb{R}^{m \times n}$, $\mathbf{a}_i$ are the rows of A and $\mathbf{b} \in \mathbb{R}^m$. Then P is called a *polyhedron.* If P is a bounded polyhedron, then it is called a *polytope.*

Therefore, the set of feasible solutions to a linear program describes a polyhedron $P := \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} \leq b\}$.

One final definition for linear programming:

Definition 2.1.2. Given an LP with constraint matrix $A = \begin{pmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \vdots \\ \mathbf{a}_m \end{pmatrix}$, a vector $\mathbf{x} \in \mathbb{R}^n$ is called a *basic feasible solution* (BFS) of the LP if it is a a feasible solution and there is a an index set $I := \{i_1, i_2, ..., i_k\}$ with $k \leq m$ such that the matrix $A_I := \begin{pmatrix} \mathbf{a}_{i_1} \\ \vdots \\ \mathbf{a}_{i_k} \end{pmatrix}$ has full rank and $A_I\mathbf{x} = \mathbf{b}$. This is equivalent to $\mathbf{x}$ being a vertex of P, the polyhedron of feasible solutions.

We will present two algorithms for solving LPs, the simplex method and the ellipsoid method. They are helpful for understanding the underlying geometry behind LPs and are the sources of inspiration for the techniques that modern solvers use. The simplex method was first described by Dantzig while working for the Pentagon and first published in 1951 [9]. The simplex method is fairly intuitive and performs well in most practical cases, but has exponential runtime in the worst case [18]. The next algorithm we will present is the ellipsoid method, found by Kachiyan in 1978 [17] The upside of the ellipsoid method is that it gave the theoretical confirmation that LP's can be solved in polynomial time; however, there has never been a practical implementation for linear programs [15]. The story of linear programming does not end with these two algorithms; however, they provide a sufficient introduction to the field.

2.1.1 The simplex method

Rather than first describing the full simplex algorithm in all its details, we will give a geometric description which is more illustrative. Let $P = \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} \leq b\}$

be the polyhedron where $A \in \mathbb{R}^{m \times n}$ and we assume that P is nonempty. The nonempty condition is a non-trivial assumption which is commonly addressed by the *big-M method* [37]. Solving the question of whether or not a polytope is empty was the initial motivation behind the ellipsoid algorithm. However, the non-empty condition is a non-issue for the union-closed conjecture as we will see later on.

Since P is non-empty, we can start the simplex algorithm by finding a vertex $\mathbf{x}_0$ of P. Then we can look at the neighbors of $\mathbf{x}_0$, defined as vertices of P that are connected by an edge with $\mathbf{x}_0$, and we evaluate the objective function at each of its neighbors. If $\mathbf{x}_0$ has the minimal value amongst all the vertices, we are done and $\mathbf{x}_0$ is optimal, otherwise, choose any neighbor $\mathbf{x}$ of $\mathbf{x}_0$ such that $\mathbf{cx} < \mathbf{cx}_0$ and repeat. With a few not onerous conditions this produces an optimal solution.

The simplex algorithm

Input:

1. $A \in \mathbb{R}^{m \times n}$, $\mathbf{b} \in \mathbb{R}^m$, $c \in (\mathbb{R}^m)^*$

2. A BFS $\mathbf{v}$ and the row index set I such that $\mathbf{v} = A_I^{-1}\mathbf{b}_I$

Output: An optimal solution or certificate of unboundedness.

Algorithm: Compute $\mathbf{u} := \mathbf{c}A_I^{-1}$

1. If $\mathbf{u} \geq 0$, then $\mathbf{v}$ is optimal.

2. If $\mathbf{u} < 0$, then choose $i \in I$ such that $u_i < 0$ and define $\mathbf{d} := -A_I^{-1}\mathbf{e}_i$.

3. Let λ^* be the largest real number λ such that $A(\mathbf{v} + \lambda\mathbf{d}) \leq \mathbf{b}$.

   (a) If λ^* is finite, then

   $$\lambda^* = \min\left\{\frac{b_j - \mathbf{a}_j\mathbf{v}}{\mathbf{a}_j\mathbf{d}} | j = 1, ..., m,\ \mathbf{a}_j\mathbf{d} > 0\right\}.$$

   Let k denote the index where we achieve the minimum. Then $I' := I \setminus \{i\} \cup \{k\}$ denotes the index set of new BFS. Restart algorithm with I'.

   (b) λ^* is infinite. Then the objective function is unbounded.

One of the most important parts in the simplex algorithm is the choice of a pivoting rule in step 3a. There are many different pivoting rules that can be used and there are a number of advantages and disadvantages to each. We will not go into depth about pivoting rules, but it is worth noting that for almost all known pivoting rules, examples exist where the number of iterations is exponential in $n + m$ [15]. Many of these examples are variants on the Klee-Minty cube [18]

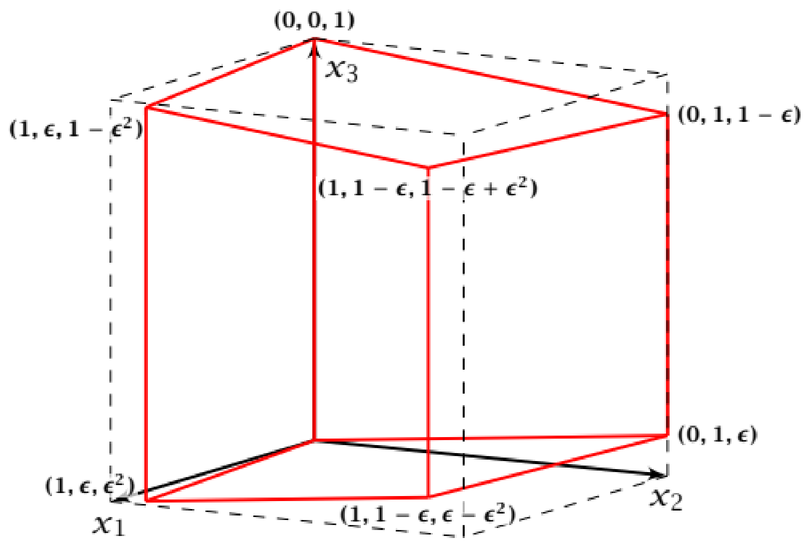

Figure 1: Klee-Minty Cube (picture from [28])

shown in figure 1 where we can see that the feasible region is a slight deformation of the cube.

To illustrate the simplex algorithm, let us consider the following

Example 2.1.3. Consider the LP

$$\begin{aligned} &\min -x-y \\ \text{such that } &y-.5x\geq 0 \\ &y-.7x\geq -1 \\ &y-.9x\geq -2.5 \\ &y-.3x\leq 2 \\ &y\geq 1 \\ &x\geq 0. \end{aligned}$$

We begin solving the LP via the simplex method by choosing the point $\mathbf{x}_0 := (0,1)$, the intersection of the last two constraints, then we evaluate our objective function on the neighbors of $\mathbf{x}_0$ given by $N(\mathbf{x}_0) = \{\mathbf{x}_1, \mathbf{x}_4\}$. We choose $\mathbf{x}_1$ as our next point in the algorithm since $(1,1) \cdot \mathbf{x}_1 < (1,1) \cdot \mathbf{x}_4$. Then we iterate this process until we arrive at $\mathbf{x}_3$ and notice that $(1,1) \cdot \mathbf{x}_2 > (1,1) \cdot \mathbf{x}_3$ and $(1,1) \cdot \mathbf{x}_4 > (1,1) \cdot \mathbf{x}_3$. Thus $\mathbf{x}_3$ is the optimal solution. The feasible region created by the constraints can be seen in Figure 2.

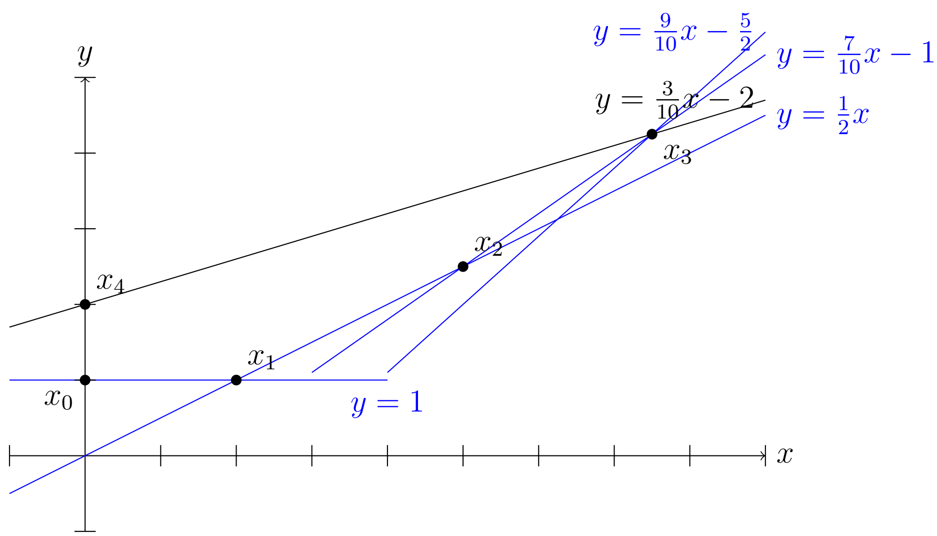

Figure 2: Feasible region of the linear program in Example 2.1.3

The ellipsoid method

Intuitively, the ellipsoid algorithm is used to determine if a convex body, P, is nonempty. The method starts with an ellipsoid E_0 which contains P. If the center $\mathbf{x}_0$ of E_0 is in P, then we are done. Otherwise, we find a cutting plane $\mathbf{cx} \leq \mathbf{b}$ which is valid for all points $\mathbf{x} \in P$ but not $\mathbf{x}_0$. Then we take the minimal ellipsoid which contains $E_0 \cap \{\mathbf{x} | \mathbf{cx} \leq \mathbf{b}\}$ and repeat.

Definition 2.1.4. The ellipsoid $E(\mathbf{x}_0, D)$ is defined as $\{\mathbf{x} | (\mathbf{x} - \mathbf{x}_0)^T D^{-1} (\mathbf{x} - \mathbf{x}_0) \leq 1\}$ where $\mathbf{x}_0$ is the center and D is a positive definite matrix.

The ellipsoid algorithm

Input:

1. A polytope P with $\mathcal{H}$-description $\{\mathbf{x} \in \mathbb{R}^n | \mathbf{a}_i^T \mathbf{x} \leq \mathbf{b}_i,\ i = 1, ..., m\}$
2. A bound $v \in \mathbb{R}$ such that either P is empty if $Vol(P) > v$.
3. A ball $E_0 = (\mathbf{x}_0, r^2 I)$ with $Vol(E_0) < v$ and $P \subset E_0$.

Output:

Either a point $\mathbf{x} \in P$ or a certificate that P is empty.

Algorithm

1. Initial Step:

   Let $t^* = \lceil 2(n+1)\log(V/v) \rceil$, $E_0 = E(\mathbf{x}_0, r^2 I)$, $\mathbf{D}_0 = r^2 I$, and $t = 0$.

2. Iterative Step

   (a) If $t^* = t$ stop, P is empty.

   (b) If $\mathbf{x}_t \in P$ stop, P is nonempty.

   (c) If $\mathbf{x}_t \notin P$, find an $\mathbf{a}_i$ such that $\mathbf{a}_i^T \mathbf{x}_t > \mathbf{b}_i$.

   (d) Let $E_{t+1} = E(\mathbf{x}_{t+1}, \mathbf{D}_{t+1})$ be the ellipsoid containing $E_t \cap \{\mathbf{x} \in \mathbb{R}^n | \mathbf{a}_i^T \mathbf{x} \leq \mathbf{b}_i\}$. Where

   $$\mathbf{x}_{t+1} = \mathbf{x}_t + \frac{\mathbf{D}_t \mathbf{a}_i}{(n+1)\sqrt{\mathbf{a}_i^T \mathbf{D}_t \mathbf{a}_i}}$$

   $$\mathbf{D}_{t+1} = \frac{n^2}{n^2-1}\left(\mathbf{D}_t - \frac{2\mathbf{D}_t \mathbf{a}_i \mathbf{a}_i^T \mathbf{D}_t}{(n+1)\mathbf{a}_i^T \mathbf{D}_t \mathbf{a}_i}\right)$$

   (e) t:=t+1

The following is a standard theorem that shows that the new ellipsoid E_{t+1} in part (d) has smaller volume than E_t and actually contains $E_t \cap H$ for some hyperplane H.

Theorem 2.1.5. *Let $E = E(\mathbf{z}, \mathbf{D})$ be an ellipsoid in $\mathbb{R}^n$, and let $\mathbf{a} \in \mathbb{R}^n$ be non-zero. Let $H = \{\mathbf{x} \in \mathbb{R}^n | \mathbf{a}^T\mathbf{x} \geq \mathbf{a}^T\mathbf{z}\}$ and*

$$\mathbf{z}' = \mathbf{z} + \frac{\mathbf{Da}}{(n+1)\sqrt{\mathbf{a}^T\mathbf{Da}}}$$

$$\mathbf{D}' = \frac{n^2}{n^2-1}\left(\mathbf{D} - \frac{2\mathbf{Daa}^T\mathbf{D}}{(n+1)\mathbf{a}^T\mathbf{Da}}\right)$$

The matrix $\mathbf{D}'$ is symmetric and positive definite, therefore we get a new ellipsoid $E' = E(\mathbf{z}', \mathbf{D}')$. Furthermore,

1. $E \cap H \subset E'$

2. $\frac{Vol(E')}{Vol(E)} < \frac{1}{e^{2(n+1)}}$

Two main issues which arise are the lack of an explicit description of the polyhedron P and a description with exponentially many constraints. Both of these issues are resolved with a *separation oracle*. The separation oracle for P does the following: given $\mathbf{x}^* \in \mathbb{R}^n$ decide if $\mathbf{x}^* \in P$ or find an inequality $\mathbf{a}^T\mathbf{x} \leq \mathbf{b}$ valid for P with $\mathbf{a}^T\mathbf{x}^* > \mathbf{b}$. If the separation oracle runs in polynomial time, then the ellipsoid algorithm will find an optimum solution in polynomial time.

While the ellipsoid method provides a theoretical result showing that linear programs can be solved in polynomial time, it is far too slow in practice. The simplex algorithm which was invented over twenty years prior and has an exponential runtime in the worst case for most known pivot rules, is far faster on average. There are many things to love about a linear program. It can be solved in polynomial time via the ellipsoid method, but, interestingly, the standard algorithm used to solve a given LP is the simplex algorithm with a pivot rule which does not have polynomial runtime in the worst case, but in practice is faster than the ellipsoid method.

2.2 Integer Programming methods

Many problems in combinatorics can be stated when solutions are restricted to take on only integer values. However, the techniques discussed previously are typically inadequate to solve with an integrality constraint. In order to conquer the task of integer programming we will introduce Chvátal-Gomory cuts and the branch and cut procedure. Consider

$$\begin{aligned} \min \mathbf{cx} \\ \text{such that } A\mathbf{x} = \mathbf{b} \\ \mathbf{x} \in \mathbb{Z}^n \end{aligned}$$

where $\mathbf{b} \in \mathbb{R}^m, \mathbf{c} \in (\mathbb{R}^n)^*, A \in \mathbb{R}^{m \times n}$. In order to access the integer valued solutions Gomory in 1958 introduced the cutting plane method which extended the simplex method to solve integer linear programs. A *cutting plane* of a polyhedron P is an inequality that is valid for the integer hull $P_I := \text{conv}(P \cap \mathbb{Z}^n)$ but not necessarily for P.

A general cutting plane algorithm: [3]

1. Solve the linear relaxation for an optimal solution $\mathbf{x}^*$.
2. If $\mathbf{x}^*$ is ineger-valued, then $\mathbf{x}^*$ is optimal for the IP.
3. If not, add a constraint to the linear relaxation that every integer solution satisfies but $\mathbf{x}^*$ does not. Go to step 1.

We use Schrijver's textbook chapters in integer programming as a source for the following further explanation [37]. In order to define the Chvátal-Gomory procedure, we need a few definitions. A *Hilbert basis* of a rational polyhedral cone $K \subset \mathbb{R}^n$ is a set $\{\mathbf{h}_1, ..., \mathbf{h}_t\} \subset K \cap \mathbb{Z}^n$ such that if $\mathbf{k} \in K \cap \mathbb{Z}^n$ then $\mathbf{k} = \sum_{i=1}^t c_i \mathbf{h}_i$ where $c_i \in \mathbb{N}$. The Hilbert basis is the tool that will allow us to get closer approximations to the integer hull and eventually describe it exactly. Let $P = \{\mathbf{x} \in \mathbb{R}^n | A\mathbf{x} \leq \mathbf{b}\}$ and $\mathcal{A}_{\mathbf{v}} := \{\mathbf{a}_i | \mathbf{a}_i^T \mathbf{v} = b_i\}$. Then define $P^{(1)}$ to be the polyhedron cut out by the inequalities $\mathbf{h}^T \mathbf{x} \leq \lfloor \mathbf{h}^T \mathbf{v} \rfloor$ for every vertex $v \in P$ and every $\mathbf{h} \in Hilb(\mathcal{A}_{\mathbf{v}})$. From this we get the chain $P_I \subset P^{(1)} \subset P$; moreover, we can define this process iteratively to get $P^{(i)} := (P^{(i-1)})^{(1)}$. The *Chvátal rank* of P is the smallest t such that $P^{(t)} = P_I$. Chvátal originially proved that there is such a t for rational polyhedra, and we have this extension thanks to Schrijver.

Theorem 2.2.1 (Schrijver, 1980 [36])**.** *For each polyhedron P, there exists a natural number t such that $P^{(t)} = P_I$.*

While the Chvátal rank is finite, it can be arbitrarily large. The classic example due to Chvátal is the polytope $P_k := \text{conv}\{(0,0), (0,1), (k, 1/2)\}$ where k is some positive integer. Since $P_I = \text{conv}\{(0,0),(0,1)\}$ the Chvátal rank of P_k is at least k.

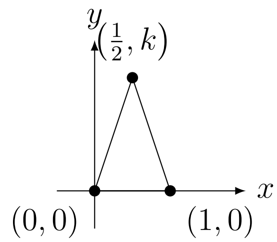

Figure 3: The polytope P_k

From 2000 we have the following theorem from Eisenbrand [10]:

Theorem 2.2.2. *Optimization over the first Chvátal closure is NP-hard.*

Despite this distressing theorem, we have another result from Eisenbrand that limits the Chvátal rank for polytopes contained in the $[0, 1]$ cube.

Theorem 2.2.3 (Eisenbrand, Schulz 2003 [11])**.** *If $P \subset [0, 1]^n$, then the Chvátal rank of P is bounded by a function in $O(n^2 \log n)$.*

Next we have the branch and bound method. The idea for this method is to partition the feasible set of solutions to the given optimization problem, then we find a lower bound for the subproblems. There are several ways a branch and bound algorithm can be implemented with the main choices being: how bounding solutions are computed, how the partitioning of the feasible region is created, and what order to traverse the branching.

Branch and bound

1. Find initial solution x^*. If x^* is integral, the solution is optimal.
2. Break F into subproblems $\{F_1, ..., F_n\}$.
3. If F_i is infeasible, delete the subproblem.
4. If $b(F_i) \geq x^*$ delete F_i.
5. If $b(F_i) < x^*$, repeat with $x^* := b(F_i)$.

Example 2.2.4. Suppose we are given the integer program

$$\begin{aligned} \max\ & 2x + 3y \\ \text{such that } & y + \frac{3}{4}x \leq 4 \\ & y - 2x \leq 0 \\ & 3x - y \leq 7 \\ & x, y \geq 0 \\ & x, y \in \mathbb{Z} \end{aligned}$$

To solve this via the branch and bound method, we first solve the linear relaxation.

The optimal solution is (1.455,2.909) in Figure 4. So we branch on the y variable and create two subproblems: one with the constraint $y \geq 3$ in Figure 5, and the other with $y \leq 2$ in Figure 7. With the constraint $y \geq 3$ added, we get an infeasible problem and terminate this branch. Solve the linear relaxation with the added constraint $y \leq 2$ to get an optimal solution of $(2.667, 2)$. Then we branch again. Here we add the constraint $x \geq 3$ in Figure 8 and get an infeasible solution. We prune this branch. We add the constraint $x \leq 2$ in Figure 10 and solve the linear relaxation for an optimal solution of $(2, 2)$. This solution is integral, so we

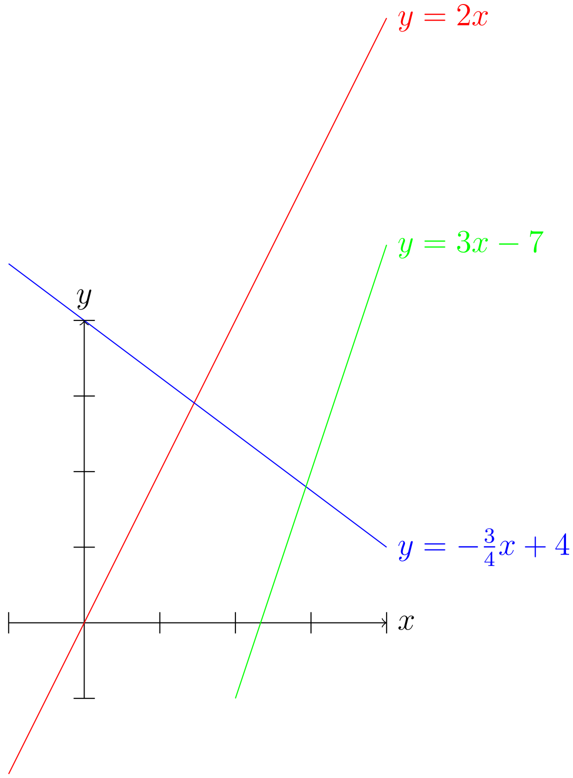

Figure 4: Initial LP

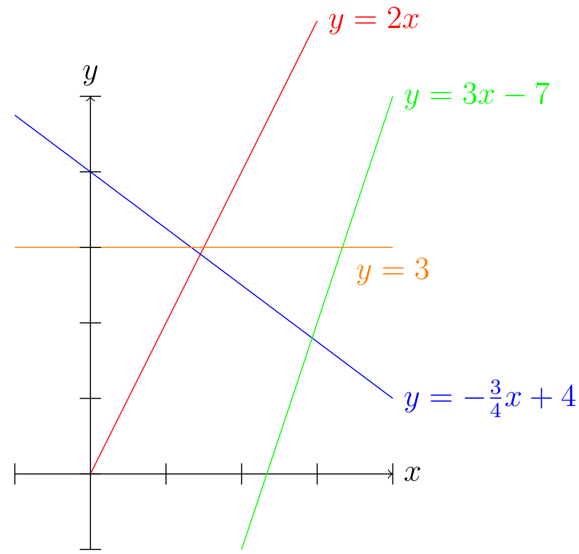

Figure 5: Equations on the x-y plane with $y \geq 3$.

Figure 6: Bound the original LP with $y \geq 3$

have successfully solved the original IP. While the branch and bound method does terminate, it can be made substantially more efficient if we add in cuts to the picture. When we do so, the method is called *branch and cut*.

The example above gets greatly simplified when we add in the cut $x + y \leq 4$ in Figure 11. We terminate after the first branching with an optimal solution.

However, finding valid cutting planes is not an easy task, and for optimal performance, knowledge of the combinatorial structure of the underlying problem is often required.

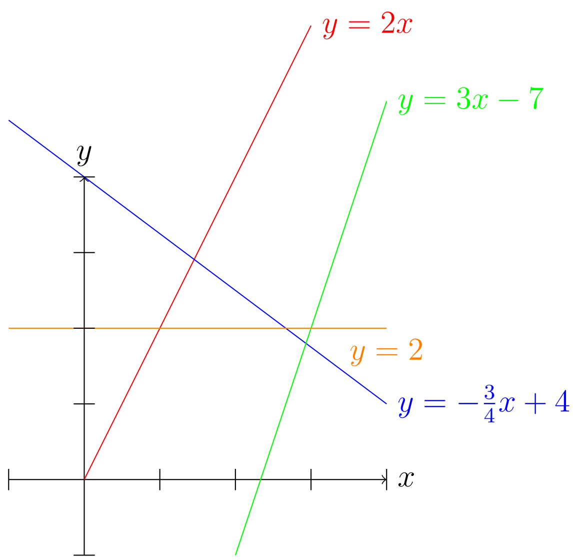

Figure 7: Bound with $y \le 2$.

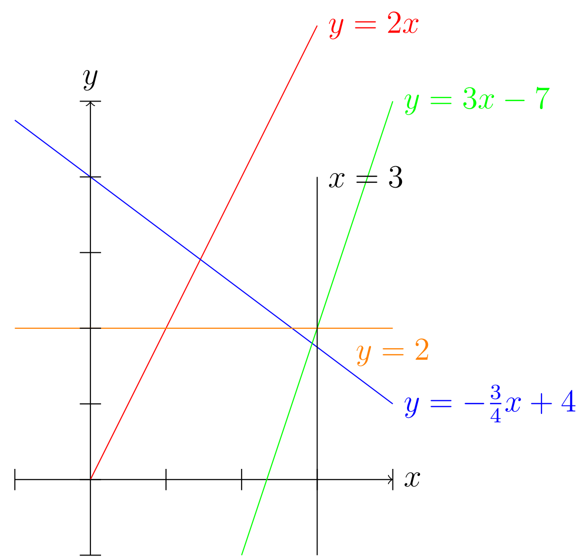

Figure 8: Bound with $x \ge 3$

Figure 9: Stages 2 and 3 of branch and bound.

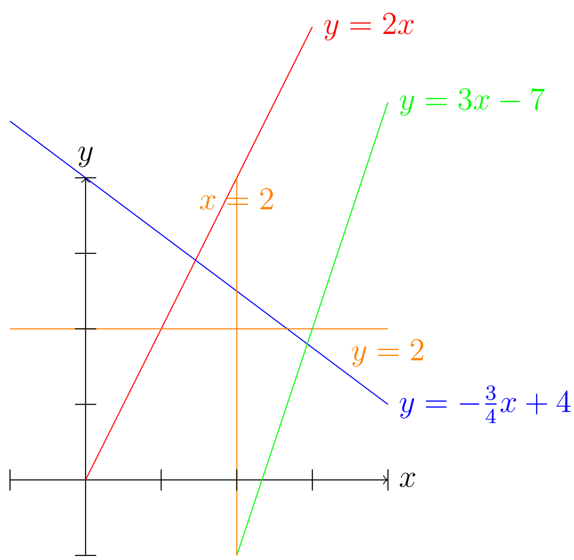

Figure 10: Bound with $x \le 2$.

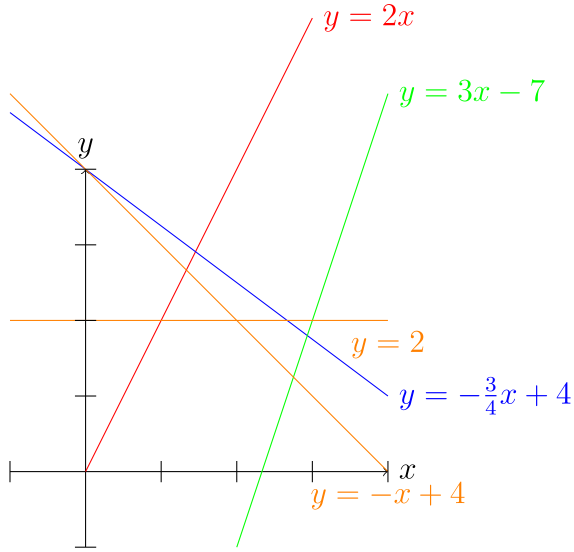

Figure 11: Branch and cut with cut $y \leq -x + 4$

2.3 Polarity

In Section 4.1, we use the concept of polarity applied to polytopes. The following standard definition and theorems can be found in [41].

Definition 2.3.1. For any subset $P \subseteq \mathbb{R}^d$, the *polar set* is defined by

$$P^{\Delta} := \{\mathbf{c} \in (\mathbb{R}^d)^* \mid \mathbf{c}\mathbf{x} \leq 1 \text{ for all } \mathbf{x} \in P\} \subseteq (\mathbb{R}^d)^*.$$

Theorem 2.3.2. *If P is a polytope and $(0, 0, ..., 0) \in P$, then $P = P^{\Delta\Delta}$. If $P = conv(V)$ with $V \subseteq \mathbb{R}^d$ is a polytope with $(0, 0, ..., 0)$ in its interior, then*

$$P^{\Delta} = \{\mathbf{a} \in (\mathbb{R}^d)^* \mid \mathbf{a}\mathbf{v} \leq 1 \text{ for all } \mathbf{v} \in V\}.$$

The final theorem we need for this is the following.

Theorem 2.3.3. *For a polytope P, the face lattice of P^{Δ} is the opposite of the face lattice of P.*

This means that vertices and facets in P^{Δ} correspond to facets and vertices, respectively, in P.

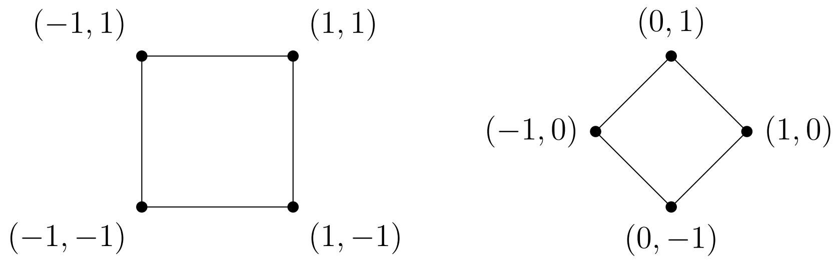

Figure 12: The square (left) and the two-dimensional cross-polytope (right) are polars of one another.

Here is a classic example of these concepts.

Example 2.3.4. Let $P = \text{conv}(\{(1,1),(1,-1),(-1,-1),(-1,1)\}$ be the $[-1,1]^2$ square. Then $P^\Delta = \{(a_1,a_2) : (a_1,a_2)v \leq 1 \text{ for all } v \in V\}$. The vertices of the polar, which turns out to be the cross-polytope, are $\{(1,0),(0,1),(-1,0),(0,-1)\}$, which correspond to the normal vectors of the facets describing the square. See Figure 12.

CHAPTER 3

Literature review of Frankl's Conjecture

In this section, we go over different results about Frankl's conjecture. Recall from the introduction that $\mathcal{P}([n])$ denotes the power set of $[n] := \{1, \ldots, n\}$. Let $\mathcal{F}$ be a subset of $\mathcal{P}([n])$ with the property that if $S, T \in \mathcal{F}$, then $S \cup T \in \mathcal{F}$. We call such collections of sets $\mathcal{F}$ *union-closed families* and $[n]$ is the *ground set* of $\mathcal{F}$. Let $\mathcal{F}_i := \{S \in \mathcal{F} | i \in S\}$ and let $a(\mathcal{F}) := \max_{i \in [n]} |\mathcal{F}_i|$ be the maximum number of sets in $\mathcal{F}$ containing any element, i.e., the *maximum frequency* of the family. Finally, let $\mathcal{F}_{\bar{i}} := \{S \in \mathcal{F} | i \notin S\}$.

3.1 Optimization Version

The large majority of the work in this thesis comes from the viewpoint of the optimization version of Frankl's conjecture which was first formulated by Pulaj, Raymond, and Theis in [27]. They gave the following equivalent formulations of Frankl's conjecture.

Conjecture 3.1.1 (Maximization Conjecture)**.** *For any positive integers a and n, let*

$$\mathfrak{F}(a, n) = \{\mathcal{F} \subseteq \mathcal{P}([n]) \, | \mathcal{F} \textit{ is a non-empty union-closed family and } a(\mathcal{F}) \leq a\}.$$

Then $\max_{\mathcal{F}\in\mathfrak{F}(a,n)} |\mathcal{F}| \leq 2a$ *for all* $a, n \in \mathbb{N}^+$.

To see that this is equivalent to the union-closed sets conjecture, let $\mathcal{F}$ be a counterexample to the Maximization Conjecture. Then for every element i in the groundset of $\mathcal{F}$, we have $|\mathcal{F}_i| < \frac{|\mathcal{F}|}{2}$. In other words, there is a union-closed family such that $|\mathcal{F}| > 2a$ where a is the maximum frequency of all elements in the groundset of $\mathcal{F}$.

Conjecture 3.1.2 (Minimization Conjecture). *For any positive integers* m *and* n, *let*

$$\mathscr{G}(m,n) := \{\mathcal{F} \subseteq \mathcal{P}([n]) \mid \mathcal{F} \text{ is a non-empty union-closed family, and } |\mathcal{F}| = m\}.$$

Then $\min_{\mathcal{F}\in\mathscr{G}(m,n)} a(\mathcal{F}) \geq \frac{m}{2}$ *for all* $m, n \in \mathbb{N}^+$.

Similarly, to see that this is equivalent to the union-closed sets conjecture, let $\mathcal{F}$ be a counterexample to the Minimization Conjecture. Then the most frequent element in $\mathcal{F}$ is there less than $\frac{|\mathcal{F}|}{2}$ times, and so this is also a counterexample to the original union-closed sets conjecture.

In [27], the authors studied the Maximization Conjecture with the following integer program.

$$f(n,a) := \max \sum_{S\in\mathcal{P}([n])} x_S \tag{3.1}$$

$$\text{such that } x_S + x_T \leq 1 + x_{S\cup T} \qquad \text{for all } S,T \in \mathcal{P}([n]) \tag{3.2}$$

$$\sum_{\substack{S\in\mathcal{P}([n]):\\ i\in S}} x_S \leq a \qquad \text{for all } i \in [n] \tag{3.3}$$

$$x_S \in \{0,1\} \qquad \text{for all } S \in \mathcal{P}([n]) \tag{3.4}$$

Solutions to this integer program correspond to union-closed families where each element appears in at most a sets of the family. The variables x_S act as indicator variables for the sets in the family, i.e., x_S has value 1 if S is in the family and value 0 otherwise. The first set of constraints enforces the union-closed condition; indeed, if S and T are both in a family, then $x_S = x_T = 1$, which means that $x_{S\cup T} = 1$ for the constraint to be valid, which implies that $S \cup T$ is also in the family. The second set of constraints ensures that no element can appear more than a times; indeed, at most a of the variables corresponding to sets containing some element i can take value 1 and thus be present in the family. The optimal solution is thus a union-closed family where every element appears at most a times that has as many sets as possible. We let $P(n, a)$ denote the polytope defined as the convex hull of the integer points of this integer program, i.e.,

$$f(n, a) = \max_{\mathbf{x}\in P(n,a)} \sum_{S\in\mathcal{P}([n])} x_S.$$

One of the most important features of $P(n, a)$ is that all of its lattice points are vertices because it is a 0/1-polytope and each of the vertices corresponds to a union-closed family by construction.

The Minimization Conjecture was also studied in [27] with the following integer program.

$$
\begin{aligned}
g(n,m) := \min & \sum_{\substack{S \in \mathcal{P}([n]):\\ 1 \in S}} x_S & \\
\text{such that } & x_S + x_T \le 1 + x_{S \cup T} & \text{for all } S, T \in \mathcal{P}([n]) \\
& \sum_{\substack{S \in \mathcal{P}([n]):\\ i \in S}} x_S \ge \sum_{\substack{S \in \mathcal{P}([n]):\\ i+1 \in S}} x_S & \text{for all } 1 \le i \le n-1 \\
& \sum_{S \in \mathcal{P}([n])} x_S = m & \\
& x_S \in \{0,1\} & \text{for all } S \in \mathcal{P}([n])
\end{aligned}
$$

The variables x_S act the same as before and the first set of constraints, as before, enforces the union-closed property. The second set of constraints ensures that the elements are ordered in a non-increasing fashion according to their frequency, which in particular implies that the element 1 is most common element in the family. The final constraint guarantees that the total number of sets in the family is m. The optimal solution is thus a union-closed family with m sets where the most common element appears as seldom as possible.

One of the most interesting observations that came from [27] was a pair of conjectures that are not equivalent to Frankl's conjecture for union-closed families, but that if true, the original conjecture would have a constant bound that beats even the recently discovered constant bound.

Conjecture 3.1.3 (*f-conjecture*, Pulaj-Raymond-Theis, 2016 [27])**.** *Fix $a \in \mathbb{N}^+$. Then $f(n,a) = f(n+1,a)$ for every $n \in \mathbb{N}^+$ such that $n \ge \lceil \log_2 a \rceil + 1$.*

Conjecture 3.1.4 (*g-conjecture*, Pulaj-Raymond-Theis, 2016 [27])**.** *Fix $m \in \mathbb{N}^+$. Then $g(n,m) = g(n+1,m)$ for every $n \in \mathbb{N}^+$ such that $n \ge \lceil \log_2 m \rceil$.*

These conjectures are essentially saying that $f(n,a)$ and $g(n,m)$ do not depend on n so long as one picks a reasonable value of n. For $g(n,m)$, if $n < \lceil \log_2 m \rceil$, then $|P([n])| = 2^n < m$, and it not possible to have m sets. For $f(n,a)$, if $n < \lceil \log_2 a \rceil + 1$, then each element appears in $\mathcal{P}([n])$ less than a times, so $f(n,a) = 2^n$. Pulaj, Raymond and Theis first proved the equivalence between these two conjectures.

Theorem 3.1.5 (Pulaj-Raymond-Theis, 2016 [27])**.** *The f- and g-conjectures are equivalent, specifically, $f(n,a) = f(n+1,a)$ for every $a, n \in \mathbb{N}^+$ such that $n \geq \lceil \log_2 a \rceil + 1$ if and only if $g(n',m) = g(n'+1,m)$ for every $m, n' \in \mathbb{N}^+, m \geq 2$ such that $n' \geq \lceil \log_2 m \rceil$.*

Frankl's conjecture does not imply the f- or g-conjecture. If Frankl's conjecture is true, then

$$\max_n f(n,a) \leq 2a$$

for any $a \in \mathbb{N}^+$. Pulaj, Raymond and Theis showed that $f(n,a) \leq f(n+1,a)$, so Frankl's conjecture says that $f(n,a)$ must eventually stabilize as n grows. However, the f-conjecture states that the function stabilizes as soon as $n \geq \lceil \log_2 a \rceil + 1$, which the Frankl's conjecture says nothing about. Similarly, Frankl's conjecture implies that

$$\min_{n \geq \lceil \log_2 m \rceil} g(n,m) \geq m/2$$

for any $m \in \mathbb{N}^+$. Given that Pulaj, Raymond and Theis also showed that $g(n,m) \geq g(n+1,m)$, Frankl's conjecture says that $g(n,m)$ must eventually stabilize as n grows, but again it does not say when it stabilizes whereas the g-conjecture states that it happens immediately when $n \geq \lceil \log_2 m \rceil$.

While Frankl's conjecture and the $f-$ and $g-$conjectures are not equivalent, the $f-$ and $g-$conjectures do provide a fairly good constant bound.

Theorem 3.1.6 (Pulaj-Raymond-Theis, 2016 [27])**.** *If the $f-$ and $g-$conjectures* *hold, then any union-closed family with m sets contains an element in at least $\frac{6}{13}m$ sets of the family.*

Furthermore, Pulaj, Raymond and Theis were able to show that the conjectures hold for n big enough.

Theorem 3.1.7 (Pulaj-Raymond-Theis, 2016 [27]). *We have that $f(n-1, a) = f(n, a)$ for all $n > a$.*

They proved a similar result for g. Because of this, the polytope $P(n, n)$ plays a major role in this thesis. Indeed, since f is non-decreasing as n grows, showing that $f(n, n) \leq 2n$ for every $n \in \mathbb{N}^+$ would prove Frankl's conjecture. We will study different features of $P(n, n)$ in the next section.

3.2 Local Configurations

One idea that has led to many results on Frankl's conjecture is that of *local configurations*: finding some local conditions of union-closed families that ensure that the conjecture hold for such families. We go over some of these results in this sections. It is interesting to note that a foundational result in this area (Theorem 3.2.6) is due to Poonen ([23]) and relies on a different formulation of Frankl's conjecture that also uses polyhedral theory to verify whether specific sets could generate families for which Frankl's conjecture was guaranteed to hold. Poonen also proved the following general results which are useful to keep in mind before we go more deeply into local configurations.

Lemma 3.2.1 (Poonen, 1992 [23]). *If $\mathcal{F}$ is a finite union-closed family, i.e., if $|\mathcal{F}| < \infty$, we can assume that the sets in $\mathcal{F}$ are finite and thus that the ground set is also finite.*

Moreover, Poonen in [23] also pointed out that the conjecture does not have to hold if we allow the number of sets in the family to be infinite. We can see this by constructing the union-closed family $\mathcal{F} := \{S_k\}_{k=1}^{\infty}$ where $S_k = \{k, k+1, k+2, ...\}$ and the ground set is $\mathbb{N}^+$. Then each element $k \in \mathbb{N}^+$ is in finitely many sets of $\mathcal{F}$, namely in k sets, but since there are infinitely many sets in $\mathcal{F}$, no element is in at least half of the sets and Frankl's conjecture fails.

The first known result regarding local conditions comes from folklore.

Theorem 3.2.2. *If $\mathcal{F}$ is a union-closed family containing a singleton, then $\mathcal{F}$ satisfies Frankl's conjecture.*

Proof: Let $\mathcal{F}$ be a union-closed family that contains the singleton $\{x\}$. We have that $\mathcal{F} = \mathcal{F}_{\bar{x}} \cupdot \mathcal{F}_x$. Because $\mathcal{F}$ is union-closed, adding x to any set that doesn't contain x must yield a set in $\mathcal{F}$, i.e., $\{S \cup \{x\} | S \in \mathcal{F}_{\bar{x}}\} \subseteq \mathcal{F}_x$ which implies that $|\mathcal{F}_{\bar{x}}| \leq |\mathcal{F}_x|$. In particular, this means that $|\mathcal{F}| = |\mathcal{F}_{\bar{x}}| + |\mathcal{F}_x| \leq 2|\mathcal{F}_x|$, and so element x is in at least half of the sets of $\mathcal{F}$. $\square$

Renaud and Sarvate proved a similar result for doubletons in [34]. The proof given here is similar to the proof for singletons but there are more details needed to work out the size of $\mathcal{F}_{\bar{x}}$.

Theorem 3.2.3 (Renaud-Sarvate, 1989)**.** *If $\mathcal{F}$ contains a set of size two, then $\mathcal{F}$ satisfies Frankl's conjecture. In particular, if $\{x, y\} \in \mathcal{F}$, then x or y is contained in at least half of the sets of $\mathcal{F}$.*

Proof: Let $\mathcal{F}$ be a union-closed family. If $\mathcal{F}$ contains a singleton, we are done by Theorem 3.2.2. So assume that the smallest cardinality of a set in $\mathcal{F}$ is two, and let $\{x, y\} \in \mathcal{F}$. Without loss of generality, we can assume $|\mathcal{F}_x| \geq |\mathcal{F}_y|$. As in the previous proof, we examine $\mathcal{F} = \mathcal{F}_x \cupdot \mathcal{F}_{\bar{x}}$ and we want to show $|\mathcal{F}_x| \geq |\mathcal{F}_{\bar{x}}|$. Note that sets in $\mathcal{F}_{\bar{x}}$ don't contain x, but some contain y whereas others don't, i.e.,

$$\mathcal{F}_{\bar{x}} = (\mathcal{F}_y \setminus \mathcal{F}_x) \cupdot \mathcal{F}_{\overline{xy}}$$

where $\mathcal{F}_{\overline{xy}} := \{S \in \mathcal{F} | \{x, y\} \cap S = \emptyset\}$.

The union of $\{x, y\}$ with every set in $\mathcal{F}_{\bar{x}}$ must be in $\mathcal{F}$ (and $\mathcal{F}_x$) since $\mathcal{F}$ is union-closed; however, by doing so, some sets may be repeated twice, namely sets $S \in \mathcal{F}_y \backslash \mathcal{F}_x$ such that $S \backslash \{y\} \in \mathcal{F}_{\overline{xy}}$. Suppose that $|\{S \backslash \{y\} | S \in \mathcal{F}_y \backslash \mathcal{F}_x\} \cap \mathcal{F}_{\overline{xy}}| = m$, i.e., that m sets get repeated twice when taking the union of $\{x, y\}$ with every set in $\mathcal{F}_{\bar{x}}$.

Now, note that $\mathcal{F}_x$ is partitioned into sets containing x but not y, i.e., sets in $\mathcal{F}_x \backslash \mathcal{F}_y$, and sets containing x and y. Observe that sets in $\{S \cup \{x, y\} | S \in \mathcal{F}_y \backslash \mathcal{F}_x\}$ and $\{S \cup \{x, y\} | S \in \mathcal{F}_{\overline{xy}}\}$ are among the sets containing x and y (there could be others), and that m of these sets are double counted. So we have that

$$|\mathcal{F}_x| \geq |\mathcal{F}_x \setminus \mathcal{F}_y| + |\mathcal{F}_y \setminus \mathcal{F}_x| + |\mathcal{F}_{\overline{xy}}| - m.$$

We assumed at the beginning that $|\mathcal{F}_y \setminus \mathcal{F}_x| \leq |\mathcal{F}_x \setminus \mathcal{F}_y|$. Moreover, we have that $m \leq \min\{|\mathcal{F}_{\overline{xy}}|, |\mathcal{F}_y \setminus \mathcal{F}_x|\}$, and thus $|\mathcal{F}_y \setminus \mathcal{F}_x| \geq m$. So

$$\begin{aligned} |\mathcal{F}_x| &\geq |\mathcal{F}_x \setminus \mathcal{F}_y| + |\mathcal{F}_y \setminus \mathcal{F}_x| + |\mathcal{F}_{\overline{xy}}| - m \\ &\geq |\mathcal{F}_y \setminus \mathcal{F}_x| + |\mathcal{F}_y \setminus \mathcal{F}_x| + |\mathcal{F}_{\overline{xy}}| - |\mathcal{F}_y \setminus \mathcal{F}_x| \\ &\geq |\mathcal{F}_y \setminus \mathcal{F}_x| + |\mathcal{F}_{\overline{xy}}|. \end{aligned}$$

Since $|\mathcal{F}_{\bar{x}}| = |\mathcal{F}_y \backslash \mathcal{F}_x| + |\mathcal{F}_{\overline{xy}}|$, this implies that $|\mathcal{F}_x| \geq |\mathcal{F}_{\bar{x}}|$ which yields $|\mathcal{F}_x| \geq \frac{1}{2}|\mathcal{F}|$. □

Unfortunately, Theorems 3.2.2 and 3.2.3 do not carry on to sets of size three: for a union-closed family $\mathcal{F}$ where the smallest nonempty set in $\mathcal{F}$ has size three, it is not guaranteed that one of the elements in the 3-set is in as least half of the sets of $\mathcal{F}$. Renaud and Sarvate produced an example of this claim in [34], which is the union-closed family with 28 sets in the following example. To lighten the notation, we first introduce the following definition.

Definition 3.2.4. For collections of sets $\mathcal{A}$ and $\mathcal{B}$, we define $\mathcal{A} \uplus \mathcal{B} := \{A \cup B | A \in \mathcal{A},\ B \in \mathcal{B}\}$.

Example 3.2.5. Let

$$\mathcal{G}_i = \{\{\emptyset\}, \{i\}, \{1,2,3\}\}, \text{ and}$$

$$\mathcal{H}_i = \{\{4,5,6,7,8,9\} \setminus \{2i+2\}, \{4,5,6,7,8,9\} \setminus \{2i+3\}\},$$

for $i \in \{1,2,3\}$, and let

$$\mathcal{F} = \{\{\emptyset\}, \{1,2,3\}\} \cup (\mathcal{G}_1 \uplus \mathcal{H}_1) \cup (\mathcal{G}_2 \uplus \mathcal{H}_2) \cup (\mathcal{G}_3 \uplus \mathcal{H}_3) \cup (\mathcal{P}([3]) \uplus \{\{4,5,6,7,8,9\}\}).$$

$\mathcal{F}$ is a union-closed family with 28 sets, the smallest nonempty set is $\{1,2,3\}$, and each of the elements in $\{1,2,3\}$ is in exactly $13 < \frac{28}{2}$ of the sets in $\mathcal{F}$.

As mentioned earlier, Poonen in [23] gave an exact (though non-explicit) description of when the presence of a union-closed subfamily within a larger union-closed family guarantees Frankl's conjecture for the latter.

Theorem 3.2.6 (Poonen, 1992)**.** *Suppose $\mathcal{F}'$ is a union-closed family containing $\emptyset$ whose largest set A has k elements. Then the following are equivalent:*

1. *For every union-closed family $\mathcal{F}$ containing $\mathcal{F}'$, there exists $i \in A$ such that $|\mathcal{F}_i| \geq |\mathcal{F}|/2$.*

2. *There exist non-negative real numbers $c_1, ..., c_k$ with sum 1 such that for every union-closed family $\mathcal{G} \subseteq \mathcal{P}(A)$ with $\mathcal{F}' \uplus \mathcal{G} = \mathcal{G}$,*

$$\sum_{i=1}^{k} c_i |\mathcal{G}_i| \geq |\mathcal{G}|/2.$$

Poonen's theorem gives an exact way to check whether the presence of a given union-closed family $\mathcal{F}'$ in a union-closed family $\mathcal{F}$ will guarantee that Frankl's conjecture holds for the family $\mathcal{F}$. The second condition of the theorem more or less states that for every union-closed family that is closed under taking unions with sets in $\mathcal{F}'$, there is a convex combination of the sizes of the $\mathcal{G}_i$'s that is at least half the number of sets in that family. This can easily be translated into the language of linear programming because there are finitely many families $\mathcal{G}$ for a fixed $\mathcal{F}'$ and A.

Here is an example to help shed some light on how Theorem 3.2.6 can be used and to show that, even though the presence of a single 3-set does not guarantee that a family will be union-closed, the presence of three 3-sets spanning four elements does. This also highlights the fact that the theorem, though extremely powerful, quickly becomes computationally challenging to use.

Example 3.2.7. Let $\mathcal{H} := \{\{1,2,3\},\{1,2,4\},\{1,3,4\}\}$. Let $\mathcal{F}$ be a union-closed family containing $\mathcal{H}$ and $\emptyset$. We will show that $1,2,3$ or 4 is in at least half of the sets of $\mathcal{F}$. Because $\mathcal{F}$ is union-closed and $\mathcal{H} \subseteq \mathcal{F}$, $\mathcal{F}$ also contains the family $\mathcal{F}' = \{\emptyset,\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\}$. Letting $A = \{1,2,3,4\}$, we use Theorem 3.2.6 to create the following linear program.

$$
\begin{aligned}
&\max 0 \\
&\text{s.t.} \quad \sum_{i=1}^{4} c_i = 1 \\
&\qquad\quad \sum_{i=1}^{4} c_i|\mathcal{G}_i| \geq \frac{|\mathcal{G}|}{2} \qquad \text{for all union-closed } \mathcal{G} \subseteq \mathcal{P}([4]) \text{ such that } \mathcal{F}' \uplus \mathcal{G} = \mathcal{G} \\
&\qquad\quad c_1, c_2, c_3, c_4 \geq 0.
\end{aligned}
$$

Note that this linear program is really a feasibility problem because finding any solution would suffice for Theorem 3.2.6; that is why we maximize zero—any objective function would do. The explicit linear program is given below—we put the corresponding $\mathcal{G}$ in parenthesis below each inequality of the second type.

In this example, there are 1592 different union-closed families $\mathcal{G} \subseteq \mathcal{P}(\mathcal{A})$ such that $\{\emptyset, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\} \uplus \mathcal{G} = \mathcal{G}$, so there are 1592 constraints of the second type. For instance, $\mathcal{G}$ could be $\{\{1\}, \{2\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\}$ which yields the constraint $7c_1+6c_2+5c_3+3c_4 \geq \frac{9}{2}$ or $\mathcal{G}$ could be $\{\{1,2,3\}, \{1,2,3,4\}\}$ which yields the constraint $2c_1+2c_2+2c_3+c_4 \geq \frac{2}{2}$. The full linear program is in Appendix A.1. Using a solver such as Gurobi, we find the feasible solution $c_1 = c_2 = c_3 = c_4 = 1/4$.

On the other hand, if we let $\mathcal{H}_2 := \{\{1,2,3\}\}$ and $\mathcal{F}_2' = \{\emptyset, \{1,2,3\}\}$ with $A_2 = \{1,2,3\}$, then the corresponding linear program is not feasible since as we saw in Example 3.2.5, there exists union-closed families that contain $\mathcal{F}_2'$ where neither 1, 2 or 3 appears in at least half of the sets. Infeasibility is easy to check in this case. One can take $\mathcal{G}$ to be $\{\{\}, \{1\}, \{1,2,3\}\}$ or $\{\{\}, \{2\}, \{1,2,3\}\}$ or $\{\{\}, \{3\}, \{1,2,3\}\}$, which respectively yield the inequalities $2c_1 + c_2 + c_3 \geq \frac{3}{2}$ and $c_1 + 2c_2 + c_3 \geq \frac{3}{2}$ and $c_1 + c_2 + 2c_3 \geq \frac{3}{2}$, thus implying by adding them together that $c_1 + c_2 + c_3 \geq \frac{9}{8}$, but we also have that $c_1 + c_2 + c_3 = 1$, so the linear program is infeasible.

Poonen's theorem was used in this way to prove Frankl's conjecture for families containing various small finite families. Since the cases for union-closed families containing 1-sets and 2-sets was so nice, and the case for 3-sets had an immediate counterexample to the pattern, the study of union-closed families containing several 3-sets or bigger sets became a topic of interest as illustrated by the previous example. Vaughan in [38] defined union-closed families such as $\mathcal{F}'$ in Theorem 3.2.6 to be *Frankl Complete*, often shortened to FC. There is one further definition that Morris [21] gave based on this idea.

Definition 3.2.8 (Morris, 2006 [21])**.** Let $FC(k,n)$ denote the smallest positive integer m such that any m of the k-sets in $[n]$ generate an FC-family.

For example, we have that $FC(1,n) = 1$ and $FC(2,n) = 1$ from Theorems 3.2.2 and 3.2.3. From Example 3.2.5, we know that there exists a union-closed family with a single 3-set $\{1,2,3\}$ such that $1,2,3$ all have frequency less than $1/2$. From Example 3.2.7, we know that having any three of the 3-sets in [4] generates an FC-family. This is equivalent to saying that $1 < FC(3,4) \leq 3$. In fact, we can translate a result from Poonen in [23] to this language to state the following.

Theorem 3.2.9 (Poonen, 1992 [23])**.** *We have that* $FC(3,4) = 3$*.*

In general, $FC(k,n)$ always exists when $n \geq 2(k-1)$ because of the following theorem by Gao and Yu in [13].

Theorem 3.2.10 (Gao-Yu, 1998 [13])**.** *For all* $k \geq 1$ *and* $n \geq 2k-2$*, the union-closed family* $\mathcal{F} \subset \mathcal{P}([n])$ *generated by all the* k*-sets in* $[n]$ *is an* FC*-family, and therefore* $FC(k,n) \leq \binom{n}{k}$*.*

It took some time for values of $FC(3,n)$ to be understood for $n > 4$ with results building on one another in [38], [39], [21] and [24].

Theorem 3.2.11 (Vaughan, 2003 [38])**.** *Any union-closed family that contains a family of sets isomorphic to* $\{135, 236, 456\}$ *satisfies Frankl's conjecture.*

Theorem 3.2.12 (Vaughan, 2004 [39])**.** *Any union-closed family that contains three* 3*-sets with a common element satisfies Frankl's conjecture.*

Theorem 3.2.13 (Morris, 2006 [21]). *The following statements hold:*

1. $FC(3,5) = 3$,
2. $FC(3,6) = 4$,
3. $FC(3,7) \leq 6$,
4. $FC(3,n) \geq \lfloor \frac{n}{2} \rfloor + 1$.

The question of understanding $FC(3,n)$ in general was finally settled in 2021.

Theorem 3.2.14 (Pulaj, 2021 [24]). *We have that* $FC(3,n) = \lfloor \frac{n}{2} \rfloor + 1$ *for all* $n \geq 4$.

To prove this result, Pulaj described a polytope that led to the development of a cutting plane algorithm to deal with the fact that the polytope behind Theorem 3.2.6 can have exponentially many constraints. Given some union-closed family $\mathcal{F}' \subseteq \mathcal{P}([n])$ and $\mathbf{c} \in \mathbb{R}^{2^n}$, consider the 0/1-polyhedron $X(\mathcal{F}', \mathbf{c})$ below which Pulaj proved that, when empty, implies that $\mathcal{F}'$ is an FC-family. Further, he shows that a non-empty $X(\mathcal{F}', \mathbf{c})$ corresponds to a violated inequality $\sum_{i=1}^{k} c_i |\mathcal{G}_i| \geq \frac{|\mathcal{G}|}{2}$ in Theorem 3.2.6.

$$
\begin{aligned}
X(\mathcal{F}', \mathbf{c}) := \{\mathbf{x} \in \mathbb{R}^{2^n} \,|\, & x_S + x_T \leq 1 + x_{S \cup T} \quad \forall S \in \mathcal{P}([n]), \forall T \in \mathcal{P}([n]) \\
& \sum_{S \in \mathcal{P}([n])} \left(\sum_{i \in S} c_i - \sum_{i \notin S} c_i \right) x_S + 1 \leq 0 \\
& x_S \leq x_{S \cup T} \quad \forall S \in \mathcal{P}([n]), \forall T \in \mathcal{F}' \\
& x_S \in \{0,1\} \quad \forall S \in \mathcal{P}([n])\}.
\end{aligned}
$$

Similar to the polyhedron in the work of Pulaj, Raymond, and Theis in [27], we know that any feasible point in $X(\mathcal{F}', \mathbf{c})$ corresponds to a union-closed family because of the first set of constraints. Moreover, the third set of constraints ensures that for any union-closed family $\mathcal{G}$ in $X(\mathcal{F}', \mathbf{c})$, $\mathcal{G} \uplus \mathcal{F}' = \mathcal{G}$ as in Poonen's theorem. Finally, the second set of constraints are essential to show that if $X(\mathcal{F}', \mathbf{c})$ is empty, then $\mathcal{F}'$ is an FC-family. With these tools, Pulaj was able to prove the following theorem.

Theorem 3.2.15 (Pulaj, 2021 [24]). *Let $\mathcal{F}'$ be a union-closed family such that $\emptyset \in \mathcal{F}'$ and let $\mathbf{c} \in \mathbb{Z}^n_{\geq 0}$ such that $\sum_{i \in [n]} c_i \geq 1$. Then $X(\mathcal{F}', \mathbf{c})$ is nonempty if and only if there exists an inequality $\sum_{i=1}^k c_i |\mathcal{G}_i| \geq \frac{|\mathcal{G}|}{2}$ for some union-closed family $\mathcal{G}$ such that $\mathcal{F}' \uplus \mathcal{G} = \mathcal{G}$ that is violated by $\bar{\mathbf{y}} := \frac{\mathbf{c}}{|\mathbf{c}|}$.*

This led to the following cutting-plane algorithm which Pulaj proved in [26] is correct.

Input: A union-closed family $\mathcal{F}' \subseteq \mathcal{P}([n])$ such that $[n] \in \mathcal{F}'$ and $\emptyset \in \mathcal{F}'$.

Output: $\mathcal{F}'$ is an FC-family or $\mathcal{F}'$ is not an FC-family.

1. $H := \{\mathbf{y} \in \mathbb{R}^n \mid \sum_{i \in [n]} y_i = 1,\ y_i \geq 0\ \forall i \in [n]\}$
2. **while** there exists $\bar{\mathbf{y}} \in H$ such that $\bar{\mathbf{y}} = (\frac{a_1}{b_1}, \dots, \frac{a_n}{b_n}) \in \mathbb{Q}^n_{\geq 0}$
   (a) set $g := lcm(b_1, b_2, ..., b_n)$
   (b) set $\mathbf{c} := g\bar{\mathbf{y}}$
   (c) If there exists a feasible point in $X(\mathcal{F}, c)$ corresponding to some family $\mathcal{G}$, then update $H := H \cap \{\mathbf{y} \in \mathbb{R}^n \mid \sum_{i \in [n]} y_i |\mathcal{G}_i| \geq |\mathcal{G}|/2\}$.
   (d) Else, return "$\mathcal{F}'$ is an FC-family."
3. Return "$\mathcal{F}'$ is not an FC family."

In [25], using this cutting plane algorithm alongside an exact computational solver, Pulaj and Wood computed several new values and better bounds for $FC(k, n)$.

Theorem 3.2.16 (Pulaj-Wood, 2023 [25]). *The following upper bounds hold:*

1. $FC(4,n) \leq 1 + \lceil \frac{11}{1680} n(n-1)(n-2)(n-3) \rceil$ *for* $n > 8$,
2. $FC(5,n) \leq 1 + \lceil \frac{13}{2520} n(n-1)(n-2)(n-3)(n-4) \rceil$ *for* $n > 7$,
3. $FC(6,n) \leq 1 + \lceil \frac{5}{4032} n(n-1)(n-2)(n-3)(n-4)(n-5) \rceil$ *for* $n > 8$.

Theorem 3.2.17 (Pulaj, Wood 2023). *The following statements hold:*

1. $FC(4,7) = 10$,
2. $FC(4,8) = 12$,
3. $FC(5,7) = 14$,
4. $FC(6,8) = 26$.

3.3 Lattice

There is an equivalent formulation of the union-closed sets conjecture in terms of intersection-closed families. If $\mathcal{F} \subseteq \mathcal{P}([n])$ is a union-closed family, then we can take the complement of all the sets in the family to get the family $\mathcal{D} := \{[n] \setminus S \mid S \in \mathcal{F}\}$ which is *intersection-closed*, i.e. if $T, U \in \mathcal{D}$, then $T \cap U \in \mathcal{D}$. This reformulation yields an equivalent conjecture that was already mentioned in [31].

Conjecture 3.3.1 (Intersection-closed sets conjecture). *Any finite intersection-closed family with at least two sets has an element that is contained in at most half of the sets in the family.*

The intersection-closed sets conjecture lends itself to other interpretations. Specifically, we get a version of the conjecture in terms of both graphs and lattices that have led to proofs of Frankl's conjecture for specific types of lattices and graphs.

To give that lattice formulation of Frankl's conjecture (as presented in [6]), we briefly review some lattice terminology. A *finite lattice* is a finite poset $(L, \leq)$ in which

1. every pair of elements $a, b \in L$ has a unique greatest lower bound denoted by $a \wedge b$ called the *meet* of a and b, and

2. every pair of elements $a, b \in L$ has a unique smallest upper bound denoted by $a \vee b$ called the *join*.

The unique minimal element is denoted by 0 and the unique maximal element is denoted by 1. A non-zero element $a \in L$ is said to be *join-irreducible* if $a = b \vee c$ implies $a = b$ or $a = c$. Finally, we let $[a) := \{x \in L | x \geq a\}$.

Given an intersection-closed family $\mathcal{F} \subseteq \mathcal{P}([n])$, we can assume that $[n] \in \mathcal{F}$. Then $(\mathcal{F}, \subset)$ forms a lattice. To see this, let $S, T \in \mathcal{F}$. Then the unique greatest lower bound is $S \wedge T = S \cap T \in \mathcal{F}$. The presence of ground set $[n]$ ensures that any two sets always have a smallest upper bound.

Conjecture 3.3.2 (Lattice formulation of the conjecture)**.** *Let L be a finite lattice with at least two elements. Then there is a join-irreducible element a with $|[a)| \leq \frac{1}{2}|L|$.*

Theorem 3.3.3 (Bruhn-Schaudt, 2013 [6])**.** *The above conjecture holds if and only if the union-closed sets conjecture is true.*

Example 3.3.4. Let $\mathcal{F}$ be the intersection-closed family $\{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,5\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,2,5\}, \{1,3,4\}, \{1,3,5\},$

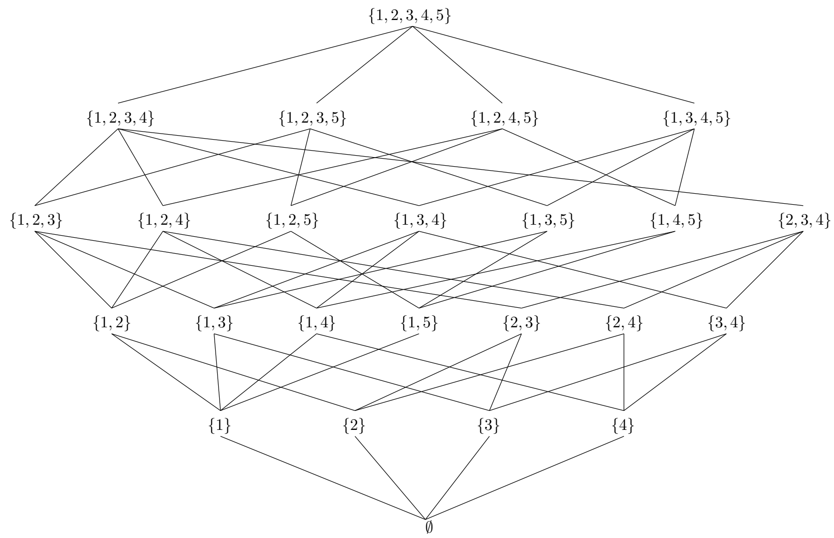

Figure 13: Lattice of an intersection-closed family where elements $\{1\},\{2\},\{3\},\{4\},\{1,5\}$ are join-irreducible and $|[\{2\})| = |[\{3\})| = |[\{4\})| = 12 = |L|/2$ and $|[\{1,5\})| = 8 < \frac{|L|}{2}$.

$\{1,4,5\},\{2,3,4\},\{1,2,3,4\},\{1,2,3,5\},\{1,2,4,5\},\{1,3,4,5\},\{1,2,3,4,5\}\}$. There are 24 sets, and element 1 appears in 16 of them, element 2 in 12, 3 in 12, 4 in 12, and 5 in 8, and so the intersection-closed sets conjecture hold.

Note that $\mathcal{F}$ also yields the lattice in Figure 13. The join-irreducible elements in the lattice in are $\{1\},\{2\},\{3\},\{4\},\{1,5\}$ and $|[\{2\})| = |[\{3\})| = |[\{4\})| = 12 = \frac{|L|}{2}$ and $|[\{1,5\})| = 8 < \frac{|L|}{2}$, thus satisfying the lattice formulation of the conjecture.

The lattice conjecture is known to hold for different types of lattices.

Definition 3.3.5. A lattice L is called a *distributive lattice* if

$$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$$

for all $x, y, z \in L$.

Theorem 3.3.6 (Poonen, 1992 [23])**.** *The lattice conjecture holds for distributive lattices.*

Definition 3.3.7. 1. Let x, y be elements of a lattice L with $x < y$. We say x is a *lower cover* of y is $x \leq z \leq y$ implies $x = z$ or $y = z$ for all elements z.

2. A lattice L is *lower semimodular* if $x \wedge y$ is a lower cover of $x \in L$ whenever $y \in L$ is a lower cover of $x \vee y$.

3. A lattice L is *atomic* if every element is the join of some set of atoms.

4. A lattice L is *geometric* if it is both atomic and upper semimodular (which is the dual of lower semimodular).

Theorem 3.3.8 (Poonen,1992 [23])**.** *The lattice conjecture holds for geometric lattices.*

Theorem 3.3.9 (Reinhold, 2000 [30])**.** *The lattice conjecture holds for lower semimodular lattices.*

3.4 Graph Formulation

The intersection-closed sets conjecture also lends itself to a graph interpretation. We first introduce some terminology.

Definition 3.4.1. Let $G = (V, E)$ be a graph, we call a set $S \subseteq V$ a *stable set* if $\{u, v\} \notin E$ for every $u, v \in S$, i.e., if no two vertices of S are adjacent. We say that S is a *maximal stable set* if $S \cup u$ is no longer a stable set for every $u \in V \setminus S$. In other words, for every $u \in V \setminus S$, there is some $v \in S$ such that $\{u, v\} \in E$.

Conjecture 3.4.2 (Graph formulation of the conjecture)**.** *Let G be a finite graph with at least one edge. Then there will be two adjacent vertices each belonging to at most half of the maximal stable sets.*

This version of the union-closed sets conjecture was first made by Bruhn, Charbit, Schaudt, and Telle in [5]. The authors quickly observed that the conjecture is true for all non-bipartite graphs. Therefore the above conjecture is equivalent to the conjecture below. As a reminder, a bipartite graph is a graph $G = (V, E)$ where there is a partition of the vertices $V = U \uplus W$ such that all edges have exactly one vertex in U and one vertex in W.

Conjecture 3.4.3 (Graph formulation II of the conjecture)**.** *Let G be a finite bipartite graph with at least one edge. Then each of the two bipartition classes contains a vertex belonging to at most half of the maximal stable sets.*

Theorem 3.4.4 (Bruhn-Charbit-Schaudt-Telle, 2015 [5])**.** *Conjecture 3.4.3 is equivalent to the intersection-closed sets conjecture which implies that it is equivalent also to the union-closed sets conjecture.*

The main idea of the proof for the equivalence of the conjectures is as follows. Let $\mathcal{F} \subseteq \mathcal{P}([n])$ be some intersection-closed family where $[n] \in \mathcal{F}$. Then let G be the bipartite graph with partition $[n] \uplus \mathcal{F}$ where there is an edge between $S \in \mathcal{F}$ and $i \in [n]$ if $i \in S$. Then $\mathcal{F} = \{T \cap [n] \mid T \text{ is a maximal stable set of } G\}$. Thus if Conjecture 3.4.3 holds, there is a vertex of G in $[n]$ that appears in at most half of the maximal stable sets of G, and so that element of $[n]$ appears in a most half of the sets of $\mathcal{F}$. Conversely, consider a bipartite graph $G = (V, E)$ with $V = U \uplus W$. Without loss of generality, it suffices to show that there is an element of U that is in at most half of the stable sets of G. Let $\mathcal{F} := \{T \mid T \text{ is a maximal stable set of } G\}$. Note that $\{S \cap U \mid S \in \mathcal{F}\}$ is intersection-closed. So if the intersection-closed sets conjecture holds, then there is some element in $\{S \cap U \mid S \in \mathcal{F}\}$ that appears in at most half of the sets, which means that the corresponding vertex in G appears in at most half of the maximal stable sets of G.

The graph formulation of the union-closed conjecture allowed the authors to prove the conjecture for certain subclasses of graphs. A bipartite graph G is said to be *chordal* if it contains no induced cycles of length greater than or equal to six.

Theorem 3.4.5 (Bruhn-Charbit-Schaudt-Telle, 2015 [5]). *Conjecture 3.4.3 holds for every chordal bipartite graph.*

Similar to the local configuration arguments seen earlier in this thesis, the proof of this theorem relies on the local structure of chordal bipartite graphs. In particular, the authors show that if one can find two adjacent vertices x, y in a bipartite graph $G = (V, E)$ such that $N^2(x) \subseteq N(y)$ (i.e., the set containing the neighbors of the neighbors of x (including x) is contained in the set containing the neighbors of y), then y is contained in less than half of the maximal stable sets of G.

Hammer, Maffray, and Preissmann in [16], as well as Pelsmajer, Tokaz, and West [22] proved that chordal bipartite graphs always contain such a vertex y which finishes the proof of the theorem.

For the next theorem, we say that a graph $G = (V, E)$ is *reduced* if there is no vertex v of G whose neighborhood is equal to the union of neighborhoods of some other vertices. In other words, for every $u \in V$, there does not exist $\{v_1, ..., v_k\} \subseteq V$ such that $N(u) = \bigcup_{i=1}^{k} N(v_i)$.

Theorem 3.4.6 (Bruhn-Charbit-Schaudt-Telle, 2015 [5]). *For any bipartite graph G, there is a reduced induced subgraph G' so that G satisfies Conjecture 3.4.3 if G' satisfies Conjecture 3.4.3.*

Proof. Let $u, v_1, v_2, ..., v_k$ be distinct vertices such that $N(u) = \bigcup_{i=1}^{k} N(v_i)$. Let $\mathcal{A}_u$ (respectively $\mathcal{A}_{\{v_1,...,v_k\}}$) be the set of maximal stable sets containing u (respectively $\{v_1, ..., v_k\}$). Since the neighborhoods are equal, we have that $\mathcal{A}_u = \mathcal{A}_{\{v_1,...,v_k\}}$. By

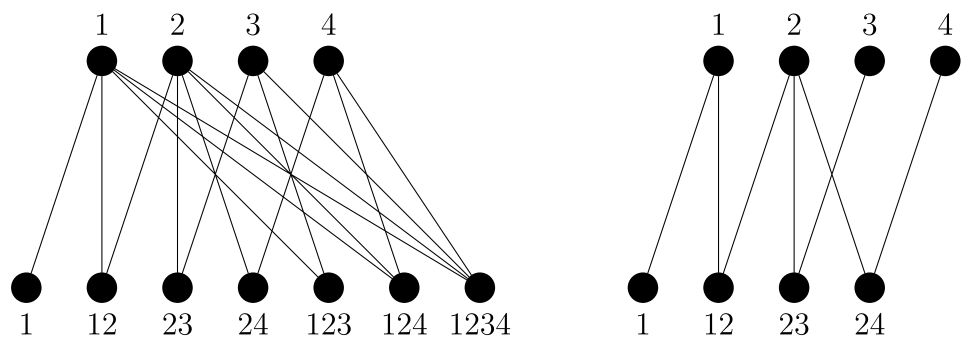

Figure 14: Example of an unreduced graph G (left) and a reduced induced subgraph G' in G (right).

this equality we have that a maximal stable set A of G gives a maximal stable set $A \setminus u$ of $G \setminus u$. In the other direction, a maximal stable set A' of $G \setminus u$ is a maximal stable set in G if $\{v_1, ..., v_k\} \not\subseteq A'$. If $\{v_1, ..., v_k\} \subseteq A'$, then $A' \cup u$ is a maximal stable set of G. This implies that a vertex in at most half the number of stable sets in $G \setminus u$ also satisfies this condition in G. Iteratively deleting vertices u finishes the proof. $\square$

See Figure 14 for an example of Theorem 3.4.6.

A graph is said to be *subcubic* if every vertex has degree at most three.

Theorem 3.4.7 (Bruhn-Charbit-Schaudt-Telle, 2015 [5])**.** *Every subcubic bipartite graph satisfies Conjecture 3.4.3.*

The proof of this theorem again relies on analyzing the local structure of the graph. First, the authors deal with the cases where the graph contains vertices of degree one or two. Then they appeal to a graph version of Theorem 3.2.12 to finish the proof.

Finally, a *p-random bipartite graph* is a graph $G = (V, E)$ with $V = U \cupdot W$ such that any pair of vertices $u \in U$ and $w \in W$ forms an edge with probability p for some $p \in (0, 1)$. It turns out that one can show that Conjecture 3.4.3 almost holds for random bipartite graphs. More precisely, the following can be shown.

Theorem 3.4.8 (Bruhn-Schaudt, 2013 [7]). *Fix $p \in (0,1)$. For every $\delta > 0$, almost every p-random bipartite graph contains a vertex in each of its bipartition classes that appears in at most $1/2 + \delta$ times the total number of maximal stable sets.*

3.5 Probabilistic results

The most substantial progress on Frankl's conjecture was recently made by Gilmer in [14] when he gave the first constant lower bound on the frequency of an element in a union-closed family. His result was quickly improved by Sawin in [35]. Prior to that, there was no known constant c such that, given any union-closed family $\mathcal{F}$, one could prove that there exists an element in at least $c|\mathcal{F}|$ sets. The best bound for a long time was due to Knill in [19].

Theorem 3.5.1 (Knill, 1994). *In any union-closed family $\mathcal{F}$, there always exists an element in at least $\frac{|\mathcal{F}|-1}{\log_2 |\mathcal{F}|}$ sets, that is, $a(\mathcal{F}) \geq \frac{|\mathcal{F}|-1}{log_2|\mathcal{F}|}$.*

One of the key methods used in all probabilistic results is examining the average frequency of all the elements in a given union-closed family. A union-closed family $\mathcal{F} \subseteq \mathcal{P}([n])$ satisfies Frankl's conjecture if we have that

$$\frac{1}{n}\sum_{i\in[n]} |\mathcal{F}_i| \geq \frac{1}{2}|\mathcal{F}|.$$

To make use of this fact, without having to examine individual $\mathcal{F}_i$'s directly, one can make use of the identity

$$\sum_{i\in[n]} |\mathcal{F}_i| = \sum_{S\in\mathcal{F}} |S|.$$

This identity is used throughout much of the literature. In particular, Reimer in [29] provided the following theorem bounding the average size of sets in a union-closed family.

Theorem 3.5.2 (Reimer, 2003)**.** *For any union-closed family $\mathcal{F} \subseteq \mathcal{P}([n])$, we have that*

$$\frac{1}{|\mathcal{F}|}\sum_{S\in\mathcal{F}}|S| \geq \frac{1}{2}\log_2(|\mathcal{F}|).$$

Equivalently,

$$\frac{1}{n}\sum_{i\in[n]}|\mathcal{F}_i| \geq \frac{\log_2(|\mathcal{F}|)}{n}\frac{|\mathcal{F}|}{2}.$$

Reimer's theorem relies on an idea called the *up-compression* of a union-closed family. Let $\mathcal{F} \subseteq \mathcal{P}([n])$ be a union-closed family, let $i \in [n]$ and $S \in \mathcal{F}$.The *up-compression* of S with respect to i is defined as

$$u_i(F) := \begin{cases} F \cup i & \text{if } F \cup i \notin \mathcal{F}, \text{ and} \\ F & \text{otherwise.} \end{cases}$$

We denote the up-compression of $\mathcal{F}$ with respect to i as $u_i(\mathcal{F}) := \{u_i(S) \mid S \in \mathcal{F}\}$. Reimer in [29] showed that $u_i(\mathcal{F})$ is still union-closed for any $i \in [n]$. By applying this function for every $i \in [n]$, we obtain what is called the *up-set* of $\mathcal{F}$, namely $u(\mathcal{F}) := u_n \circ u_{n-1} \circ \ldots \circ u_1(\mathcal{F})$. Moreover, $u(\mathcal{F}) \subseteq \mathcal{P}([n])$ has the property that if $S \in u(\mathcal{F})$ and $S \subset T \subset [n]$, then $T \in u(\mathcal{F})$.

Example 3.5.3. Let $\mathcal{F} = \{\{1\}, \{1,2\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,2,3,4\}\}$. Then

$$u_2 \circ u_1(\mathcal{F}) = u_1(\mathcal{F}) = \{\{1\}, \{1,2\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,2,3,4\}\},$$

$$u_3 \circ u_2 \circ u_2(\mathcal{F}) = \{\{1,3\}, \{1,2\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,2,3,4\}\}, \text{ and}$$

$$u(\mathcal{F}) = \{\{1,3,4\}, \{1,2\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, \{1,2,3,4\}\}.$$

Observe that since $\{1,2\} \in u(\mathcal{F})$, so are $\{1,2,3\}$ and $\{1,2,4\}$. Similarly, since $\{1,3\} \in u(\mathcal{F})$, so are $\{1,2,3\}$ and $\{1,3,4\}$.

Reimer proved that the difference between a family $\mathcal{F}$ and its up-set $u(\mathcal{F})$ is not too great [29], i.e.,

Lemma 3.5.4 (Reimer, 2003 [29]). *Let $\mathcal{F}$ be a union-closed family on ground set $[n]$, then*

$$\sum_{F \in \mathcal{F}} |u(F) - F| \leq |\mathcal{F}|(m - \log_2(|\mathcal{F}|)).$$

After establishing this bound, they exploited the simpler structure of the up-set to finish the proof of Theorem 3.5.2.

Probabilistic methods were also used in the following two theorems about families that have a lot of sets with respect to their ground sets in [2] and [6].

Theorem 3.5.5 (Balla-Bollobás-Eccles, 2013). *The union-closed conjecture holds for any family $\mathcal{F} \subseteq \mathcal{P}([n])$ where $|\mathcal{F}| \geq \frac{2}{3}2^n$.*

Theorem 3.5.6 (Bruhn-Schaudt, 2013). *Let $\mathcal{F} \subseteq \mathcal{P}([n])$ be any union-closed family such that $2^{n-1} < |\mathcal{F}| \leq 2^n$. Then $a(\mathcal{F}) \geq \frac{6}{13}|\mathcal{F}|$, i.e., there exists an element in at least $\frac{6}{13}$ of the sets of the family.*

To understand the newer results of [14] and [35], we need to introduce some ideas from information theory.

Definition 3.5.7. Let X be a discrete random variable taking values in some set S. For each $s \in S$, let $p_s := P(X = s)$. We define the *entropy* of X to be

$$H(X) := \sum_{s \in S} -p_s \log_2 p_s.$$

The entropy of a random variable is often described as a measure of the "surprise" in the randomness of X. Another way to think of entropy is the amount of information contained in a random variable X. This is made precise in Shannon's source coding theorem which can be seen in [20] chapter 4.

Here are a few quick properties of entropy that are needed for the proof of Gilmer's main theorem:

1. For random variables X and Y, we write $H(X, Y)$ for the entropy of joint random variables (X, Y), i.e.,

$$H(X,Y) = \sum_{(x,y)} -P(X = x, Y = y) \log_2 P(X = x, Y = y)$$

   .

2. If X and Y are independent random variables, then $H(X, Y) = H(X)+H(Y)$.

3. There is a chain rule for entropy. Given a sequence of random variables $X_1, ..., X_n$, denote $X_{<i} = (X_1, ..., X_{i-1})$. Then $H(X_1, ..., X_n) = \sum_i H(X_i|X_{<i})$.

4. Finally, for random variables X and Y and a function $f(Y)$, we have the inequality $H(X|Y) \leq H(X|f(Y))$.

Theorem 3.5.8 (Gilmer, 2022 [14])**.** *Let A and B denote independent samples from a distribution over subsets of $[n]$. Assume that for all $i \in [n]$, $P(i \in A) \leq 0.01$. Then $H(A \cup B) \geq 1.26H(A)$.*

This theorem involves creating a chain where the bits of information contained in $A \cup B$ are revealed sequentially. This sequential construction relies entirely on the chain rule as stated earlier.

Theorem 3.5.9 (Gilmer, 2022 [14]). *Let $\mathcal{F} \subseteq \mathcal{P}([n])$ be a union-closed family with $\mathcal{F} \neq \emptyset$. Then there exists $i \in [n]$ that is contained in at least $0.01|\mathcal{F}|$ sets of the family.*

The proof of this theorem follows by noting that Theorem 3.5.8 implies that if $H(A) > 0$, we have $H(A \cup B) > H(A)$. But, if A, B are sampled independently and uniformly at random from a union-closed family $\mathcal{F}$, then $H(A \cup B) \leq H(A)$. This is because A is uniform and entropy is maximized by the uniform distribution [8].

Sawin in [35] immediately improved the results of Gilmer by proving a bound Gilmer conjectured.

Theorem 3.5.10 (Sawin, 2022). *Let $\mathcal{F} \subseteq \mathcal{P}([n])$ be a union-closed family with $\mathcal{F} \neq \emptyset$. Then there exists $i \in [n]$ contained in at least $\frac{3-\sqrt{5}}{2}|\mathcal{F}|$ sets of the family.*

We will not include the proof of Sawin's theorem, but note that it is heavily computational in nature and Sawin's contribution was proving the bound Gilmer conjectured. An interesting fact that Sawin proved is that the bound of $\frac{3-\sqrt{5}}{2}$ is the best possible. In other words, the entropy method as it is currently applied cannot prove the Frankl conjecture.

3.6 Current best computational results

Different computational results have been achieved through the years in [32], [33], [4] and [40]. Most of these use results presented in the previous subsections to reduce the size of computations.

Theorem 3.6.1 (Roberts, 1992 [32]). *The inequality $|\mathcal{F}| < 4n - 1$ holds for any union-closed family $\mathcal{F} \subseteq \mathcal{P}([n])$ that is a minimum counterexample to the Frankl conjecture.*

Theorem 3.6.2 (Roberts-Simpson, 2010 [33]). *The Frankl conjecture is true for any family* $|\mathcal{F}| \leq 46$.

Theorem 3.6.3 (Bošnjak-Marković, 2008 [4]). *The Frankl conjecture is true for any family* $\mathcal{F} \subseteq \mathcal{P}([n])$ *with* $n \leq 11$.

Theorem 3.6.4 (Vučković-Živković, 2012 [40]). *The Frankl conjecture is true for any family* $\mathcal{F} \subseteq \mathcal{P}([n])$ *such that* $n \leq 12$ *and* $|\mathcal{F}| \leq 50$.

CHAPTER 4

New results for $P(n, a)$

In Section 4.1, we compute many equivalence classes of facets for $P(4, 4)$, $P(5, 5)$ and $P(6, 6)$. We also explain how the frequency constraints add complexity to $P(n, a)$ as a decreases and how valid inequalities for $P(n, a)$ can be carried over to valid inequalities of $P(n + 1, a)$. In Section 4.2, we prove that certain inequalities are valid and facet-defining for $P(n, a)$ for various values of n and a. Finally, in Section 4.3, we consider a wide class of inequalities that we conjecture are always valid and which, though not facet-defining, are very helpful when added to the linear relaxation of $f(n, a)$ to give a tighter relaxation. We prove the conjecture for a few cases.

4.1 Computing facets of $P(n, n)$

The majority of the work in this section focuses on the polytope $P(n, n)$ defined by Pulaj, Raymond, and Theis in [27] and described earlier in Section 3.1. Theorem 3.1.7 proves the *f-conjecture* for $n > a$, and this provided the original motivation for inspecting the polytope $P(n, n)$ (as opposed to the more general $P(n, a)$).

A full description of $P(3, 3)$ was previously known because softwares such as Polymake and Sagemath are powerful enough to compute it; however, the softwares failed to compute $P(4,4)$ and above. We give a description of $P(4,4)$ which we have strong evidence to believe is complete, but we cannot be certain without further computational power. Additionally, we give 26862 classes of facet-defining inequalities for $P(5,5)$ and 2566 classes for $P(6,6)$. It is worth recalling that we call two union-closed families $\mathcal{F} = \{F_1, ..., F_m\}$ and $\mathcal{G} = \{G_1, ..., G_m\}$ equivalent on a ground set $[n]$ when there exists an element $\sigma \in \mathfrak{S}_n$ such that $\{\sigma(F_1), ..., \sigma(F_m)\} = \{G_1, ..., G_m\}$. For example, $\{\{1\}, \{3\}, \{1,3\}\}$ is equivalent to $\{\{1\}, \{2\}, \{1,2\}\}$. This equivalence carries on over to inequalities describing the polytope $P(n,n)$. So when we list an inequality in Table 1 and in the tables in the Appendix, each inequality is actually a representative of a class of inequalities that can be stated by letting $\mathfrak{S}_n$ act on the ground set. Again, to be explicit, this means that when the inequality $x_{\{1\}} + x_{\{2\}} - x_{\{1,2\}} \leq 1$ is listed as a facet, we also have the facet $x_{\{1\}} + x_{\{3\}} - x_{\{1,3\}} \leq 1$.

In the Appendix are 2001 classes of facets for $P(4,4)$ which are representing 38011 facets, and 26862 classes of facets for $P(5,5)$ representing 2742316 facets. In contrast, as can be seen in Table 1, there are 21 classes of facet-defining inequalities for $P(3,3)$, including 12 classes that are the original inequalities (union-closed inequalities, frequency inequalities, and inequalities to bound variables). The complexity of $P(n,n)$ grows quickly as n increases.

In order to compute some facets of $P(4,4)$, $P(5,5)$ and $P(6,6)$, we use the concept of polarity (which was introduced in Section 2.3) and the optimization solver Gurobi.

The reason for moving to the polar of $P(n,n)$ is that it allows us to leverage the computational power of Gurobi. Indeed, recall from Theorem 2.3.2 that for a polytope that contains the origin in its interior, the facets of the polar are in correspondence with the vertices of the original polytope. Furthermore, from Theorem 2.3.3, the vertices of the polar correspond to facets of the original polytope.

Gurobi (and other similar softwares) is traditionally used as an optimization solver, i.e., it is very powerful at optimizing a linear constraint over a linear or integer program. It cannot directly compute the facet-defining inequalities of a polytope given as a convex hull of vertices. However, it is also surprisingly effective at finding all lattice points of a polytope behind an integer program. Indeed, one can ask Gurobi to list the x best solutions of an integer program. By letting x be big enough, it will give all lattice points in the polytope. In this way, we can use Gurobi to calculate all the lattice points of $P(n,n)$, which correspond to union-closed families. Note that naïve code to enumerate all union-closed families fails when $n > 4$ whereas Gurobi can do so for bigger values of n; for example, Gurobi easily found the 1,567,982 vertices in $\{0,1\}^{64}$ of $P(6,6)$. Moreover, observe that since $P(n,n)$ is a 0/1-polytope, all lattice points are vertices.

Although the origin corresponds to a union-closed family (namely the empty one) and is thus in $P(n,n)$, it is not in its interior: it is a vertex of $P(n,n)$. So we translate $P(n,n)$ (by translating all its vertices) to obtain $P'(n,n)$ which contains the origin in its interior. For example, since $\mathbf{e}_S \in P(n,n)$ for every $S \in \mathcal{P}([n])$ because any family consisting of a single subset of $[n]$ is a union-closed family, we have that the convex combination $\frac{1}{2^n}\sum_{S\in\mathcal{P}([n])}\mathbf{e}_S$ is in the interior of $P(n,n)$. So we can translate every vertex $\mathbf{v}$ of $P(n,n)$ to $\mathbf{v} - \frac{1}{2^n}\sum_{S\in\mathcal{P}([n])}\mathbf{e}_S$ to ensure that the origin is in the interior of $P'(n,n)$. Since we have all the vertices of $P'(n,n)$, we know the full facet description of $P'(n,n)^\Delta$. We can then use Gurobi in a more traditional way by optimizing in various directions to find vertices of $P'(n,n)^\Delta$, which we know correspond to facets of $P'(n,n)$: each vertex $\mathbf{w}$ of $P'(n,n)^\Delta$ found corresponds to a facet $\mathbf{wx} \leq 1$ in $P'(n,n)$. In this case, we unfortunately can't use Gurobi to list all vertices as $P'(n,n)^\Delta$ is not an integral polytope anymore. One could try to first dilate $P(n,n)$ first so that one could then translate it by an integral vector to get the origin in its interior. However, Gurobi would then find all lattice points, including those that aren't vertices, and the size of the data would make it hard to separate the vertices (corresponding to facets in the original polytope) from non-vertices. Still by optimizing in many different directions, one can get a lot of vertices of $P'(n,n)^\Delta$, and thus facets of $P'(n,n)$. Translating back, we have that $\mathbf{w}(\mathbf{x} - \frac{1}{2^n}\mathbf{1}) \leq 1$ is a facet for $P(n,n)$ or, equivalently, that $\mathbf{wx} \leq 1 + \frac{1}{2^n}||\mathbf{w}||$ is a facet for $P(n,n)$.

Finding vertices in $(P'(n,n))^\Delta$

Input: Vertex set of $P'(n,n)$

Output: Some vertices of $(P'(n,n))^\Delta$

Algorithm:

Let $V^\Delta = \{\emptyset\}$. While $i < N$ for some big N:

1. Let $\mathbf{c} \in \mathbb{R}^{2^n}$ be a random vector (different techniques can be used here to investigate different directions).
2. Solve the linear program $\max\{\mathbf{cy} \mid \mathbf{vy} \leq 1 \text{ for all vertices } \mathbf{v} \in P'(n,n)\}$, and let $\mathbf{w}$ be the solution. If $\mathbf{w} \in V^\Delta$, pass.
3. If $\mathbf{w} \notin V^\Delta$, calculate the orbit under the action by $\mathfrak{S}_n$ on the indices as we have done previously and add all vertices to V^Δ.
4. Return to step 2 and increase i by 1.

Return V^Δ.

It is important to note that there is not a theoretical explanation to say how big N should be and what is the best way of randomly choosing directions. For $P(4,4)$ we tested on the order of a million random objective functions (using different ways of creating random vectors). As i grew, V^{Δ} would grow more slowly until it stopped growing. We chose N to be twice as big as the last iteration when V^{Δ} grew.

Numerical issues were also present: we know that each facet inequality for $P(n,n)$ can be written with integer coefficients, however, $\mathbf{wx} \leq 1 + \frac{1}{2^n}||\mathbf{w}||$ is written with decimal points up to some precision determined by Gurobi, and correctly translating that to a facet inequality with integer coefficients is non-trivial. There are a few (respectively many) facet inequalities we have found for $P(5,5)$ (respectively $P(6,6)$) that we were not able to translate.

After we obtain the facets, we compute their Chvátal ranks. Associated to any integer program is its linear relaxation. In our case, the linear relaxation of $f(n,n)$ defines the polytope

$$
\begin{aligned}
R(n,n) := \{\mathbf{x} \in \mathbb{R}^{2^n} \; | x_S + x_T \leq 1 + x_{S\cup T} \text{ for all } S,T \in \mathcal{P}([n]) \\
\sum_{\substack{S\in\mathcal{P}([n]):\\ i\in S}} x_S \leq n \text{ for all}\, i \in [n] \\
0 \leq x_S \leq 1 \text{ for all } S \in \mathcal{P}([n])\}.
\end{aligned}
$$

Different algorithms exist to go from $R(n,n)$ to $P(n,n)$. One of them is a cutting-plane algorithm called the Chvátal-Gomory procedure which is applied recursively on $R(n,n)$ as described in Section 2.2. The rank of a particular inequality is the smallest number of times the procedure needs to be applied for this inequality to be found. Rank 0 inequalities are those defining $R(n,n)$, namely the union-closed constraints, the frequency constraints and the variable bound constraints. An inequality has rank 1 if it can be obtained in one iteration of the algorithm. In the Chvátal-Gomory procedure, this means that an inequality can be obtained by taking a conical combination of the rank 0 inequalities, and rounding down the right-hand side. An inequality has rank 2 if it can be obtained by taking a conical combination of the rank 0 and rank 1 inequalities, and rounding down the right-hand side, and so on.

Note that the algorithm below only gives an upper bound on the Chvátal rank for the facets we found. Indeed, since some facet inequalities might be missing, it could be that certain facets would have a smaller Chvátal rank had the missing ones been present.

Chvátal rank calculation

Input: The set $\mathcal{H}$ of facets of $P(n, n)$ found by the previous algorithm, and the set $\mathcal{L}_0$ containing the original constraints: union-closed constraints, frequency constraints, and variable bounds.

Output: An upper bound on the Chvátal rank of each facet found by the previous algorithm.

Algorithm:

Set $i = 0$.

While $\mathcal{H} \neq \emptyset$:

> Set $\mathcal{L}_{i+1} = \mathcal{L}_i$.
>
> For every equivalence class of constraints in $\mathcal{H}$:
>
> > Pick a constraint $\mathbf{cx} \leq b$ in the equivalence class. Solve the linear program maximizing $\mathbf{cx}$ subject to all the constraints in $\mathcal{L}_i$. Let b' be the optimal value. If $\lfloor b' \rfloor = b$, then add all the constraints in the equivalence class to $\mathcal{L}_{i+1}$, and remove them all from $\mathcal{H}$.
>
> If $\mathcal{L}_{i+1} = \mathcal{L}_i$, break and return $\{\mathcal{L}_0, \mathcal{L}_1, \ldots, \mathcal{L}_i\}$.
>
> Else, increase i by 1.

Return $\{L_0, L_1, \ldots, L_{i+1}\}$

Note that if $\mathcal{L}_{i+1} = \mathcal{L}_i$ while $\mathcal{H} \neq \emptyset$, this means that we are not able to compute the rank of certain facet-defining inequalities because other inequalities are missing. This occurs in the case of $P(5, 5)$, which implies that the list of facet-defining inequalities is incomplete.

Table 1 below gives the rank of all facet-defining inequalities for $P(3, 3)$—though

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	b
Rank 0								
0	0	0	1	0	1	1	1	3
0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	1	0	1
0	0	0	0	0	1	1	-1	1
0	0	0	1	1	0	0	-1	1
0	0	0	-1	0	0	0	0	0
0	0	0	1	0	0	0	0	1
0	0	1	1	0	0	-1	0	1
-1	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	1
Rank 1								
0	1	1	1	-1	-1	-1	0	1
0	1	0	1	1	0	0	0	2
0	0	0	1	0	1	1	0	2
0	1	0	0	1	1	2	-2	2
0	0	0	0	1	1	1	-2	1
Rank 2								
0	1	1	2	1	0	0	0	3
0	1	0	1	1	1	2	-1	3
0	1	0	1	2	1	2	-2	3
Rank 3								
0	1	1	1	1	1	1	0	3

Table 1: $P(3,3)$ by Chvátal rank of facet inequalities $c_1x_\emptyset + c_2x_{\{1\}} + c_3x_{\{2\}} + c_4x_{\{3\}} + c_5x_{\{1,2\}} + c_6x_{\{1,3\}} + c_7x_{\{2,3\}} + c_8x_{\{1,2,3\}} \leq b$

the facet description was previously known, the computations of their ranks is new to the best of our knowledge. In the Appendix, Tables A.3, A.5, A.6 give the facets we found respectively for $P(4,4)$, $P(5,5)$ and $P(6,6)$ as well as their ranks in cases when we were able to compute them.

4.1.1 Increasing complexity of $P(n,a)$ as a decreases

Recall that for a union-closed family $\mathcal{F}$, $\mathcal{F}_i = \{S \in \mathcal{F} | i \in S\}$. Let $x(\mathcal{F}_i) := \sum_{S\in\mathcal{F}_i} x_S$. Then we can rewrite the frequency constraints in the integer program

$\emptyset$	$\{1\}$	$\{2\}$	$\{3\}$	$\{4\}$	$\{1,2\}$	$\{1,3\}$	$\{1,4\}$	$\{2,3\}$	$\{2,4\}$	$\{3,4\}$	$\{1,2,3\}$	$\{1,2,4\}$	$\{1,3,4\}$	$\{2,3,4\}$	$[4]$
1	1	1	1	0	1	1	0	1	0	0	1	1	1	1	1
1	1	1	0	0	1	1	0	1	0	1	1	1	1	1	1
1	1	1	0	0	1	1	0	0	1	1	1	1	1	1	1
1	1	0	0	0	1	1	0	1	1	1	1	1	1	1	1
1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1

Table 2: Classes of optimal solutions of $f(4,7)$.

for $f(n,a)$ as $x(\mathcal{F}_i) \leq a$ for each $i \in [n]$. The introduction of these constraints substantially increases the complexity of $P(n,a)$ as a decreases. This can be seen computationally from the facet structure of $P(4,8)$, $P(4,7)$ and $P(4,4)$, or directly by calculation. We have that $f(4,8) = 16$, where there is a unique optimal solution given by the power set $\mathcal{P}([4])$. Since every element of the ground set appears exactly 8 times in $\mathcal{P}([4])$, every union-closed family $\mathcal{F} \subseteq \mathcal{P}([4])$ is such that $x(\mathcal{F}_i) \leq 8$ for every $i \in [4]$—the frequency inequalities are thus redundant in this case. Polymake and Sage could compute the full facet description of $P(4,8)$; the 63 facet-defining classes of inequalities are in Table A.1 in the Appendix.

As soon as a is lowered to $a = 7$, we get $f(4,7) = 12$ and there are five equivalence classes of optimal solutions listed as incidence vectors in Table 2. Each vector can be symmetrized under $\mathfrak{S}_4$ to obtain other optimal solutions to $f(4,7)$. The full facet description of $P(4,7)$ could still be computed by Polymake and sage; the 277 classes of facet-defining inequalities are in Table A.2 in the Appendix. This is still far away from the 2001 classes of facet-defining inequalities for $P(4,4)$ we found, and which could not be computed by Polymake or Sage.

We see this increase in complexity mirrored in the Chvátal ranks of facet-defining inequalities present in $P(4,8)$, $P(4,7)$ and $P(4,4)$. In $P(4,8)$, the highest rank is 2 versus 4 for $P(4,7)$ and 6 for $P(4,4)$.

Some of the facet-defining inequalities for $P(4,8)$—which really only come from applying the Chvátal procedure to the union-closed inequalities and the variable bounds constraints—still appear in $P(4,7)$ and $P(4,4)$. As we will see in Section 4.2, we found it easier to prove that such inequalities are facet-defining compared to inequalities where one needs to use frequency constraints although those inequalities are more helpful to bound $f(n,a)$.

4.1.2 Inequalities in $P(n,a)$ related to inequalities in $P(n+1,a)$

One of the difficulties in producing examples for $P(n,a)$ is the doubly exponential nature of the problem. We are dealing with subsets of the power set of n. So it would be beneficial if we have some way of taking valid inequalities for $P(n,a)$ and showing that they are valid for $P(n+1,a)$.

Here is an easy example of going in the opposite direction from $P(n+1,a)$ to $P(n,a)$. Take the frequency inequality $x(\mathcal{F}_i) \leq a$ for some $i \in [n+1]$ which is valid for $P(n+1,a)$. By adding $-x_S \leq 0$ for every $S \in \mathcal{P}([n+1])$ such that $\{i, n+1\} \subseteq S$, we obtain the inequality

$$\sum_{\substack{S \in P([n+1]): \\ i \in S, n+1 \notin S}} x_S \leq a$$

which is valid for $P(n,a)$. However, instead of showing the validity of a constraint for $P(n,a)$ coming from a constraint for $P(n+1,a)$, we would like to show the validity of a constraint for $P(n+1,a)$ coming from a constraint for $P(n,a)$.

Theorem 4.1.1. *Let $\sum_{S \in \mathcal{P}([n])} c_S x_S \leq b$ be a valid inequality for $P(n,a)$. Then we get valid inequalities for $P(n+1,a)$ given by*

$$\sum_{S \in \mathcal{P}([n])} c_{\sigma(S)} x_{\sigma(S)} \leq b$$

for $\sigma \in \mathfrak{S}_{n+1}$.

Proof. Suppose for the sake of contradiction that $\sum_{S\in\mathcal{P}([n])} c_{\sigma(S)}x_{\sigma(S)} \leq b$ is not valid for some union-closed family $\mathcal{F} \subseteq \mathcal{P}([n+1])$. Under the action of $\mathfrak{S}_{n+1}$, we can assume without loss of generality that the constraint that $\mathcal{F}$ violates is $\sum_{S\in\mathcal{P}([n])} c_S x_S \leq b$. Let $\mathcal{G} := \{S \in \mathcal{F} | n+1 \notin S\}$. Then $\mathcal{G} \subseteq \mathcal{P}([n])$ is a union-closed family in $P(n,a)$ which violates the valid inequality $\sum_{S\in\mathcal{P}([n])} c_S x_S \leq b$, a contradiction. $\square$

Remark 4.1.2. This theorem means that every class of facet-defining inequalities in $P(n,a)$ gives us a class of valid inequalities for $P(n+1,a)$. Unfortunately, facets do not directly translate to facets in a similarly simple manner.

4.2 Some new facet-defining inequalities for $P(n,a)$

Theorem 4.2.1. *The inequality*

$$x_{A\setminus S} + x_{B_1} + x_{B_2} \leq 1 + x_A + x_{B_1\cup B_2}$$

is facet-defining for $P(n,a)$ for $n \geq 4$ and $a \geq 5$ where $B_1, B_2 \subset A$, $S \subset B_1, B_2$ and the sets $A, A\setminus S, B_1,\ B_2, B_1 \cup B_2$ are distinct.

Proof. First note that to satisfy the condition that the sets need to be distinct implies that $S \neq \emptyset$, which in turn implies that $|B_1| \geq 2$ and $|B_2| \geq 2$. It then follows that $|A|, |B_1 \cup B_2| \geq 3$, giving us that $n \geq 4$. If we do not have the distinct condition, the inequality still holds, but it requires a different argument.

Now we prove that the proposed inequality is valid by writing it as the following conical combination of defining inequalities of $P(n,a)$:

$$
\begin{array}{rcccccl}
\frac{1}{2}(& x_{A\setminus S} & +x_{B_1} & & -x_A & & \leq 1) \\
+\frac{1}{2}(& x_{A\setminus S} & & +x_{B_2} & -x_A & & \leq 1) \\
+\frac{1}{2}(& & x_{B_1} & +x_{B_2} & & -x_{B_1\cup B_2} & \leq 1) \\
+\frac{1}{2}(& & & & & -x_{B_1\cup B_2} & \leq 0) \\
\hline
 & x_{A\setminus S} & +x_{B_1} & +x_{B_2} & -x_A & -x_{B_1\cup B_2} & \leq \frac{3}{2}.
\end{array}
$$

Since we know $x_S \in \{0,1\}$ for all $S \in \mathcal{P}([n])$, we can round down the right-hand side to achieve the proposed inequality. Now to prove that the inequality is facet-defining we must find 2^n linearly independent union-closed families that are tight with the inequality. We do this by partitioning $\mathcal{P}([n])$.

Let $\mathcal{S}_0 := \{\{A\setminus S\}, \{B_1\}, \{B_2\}, \{A\setminus S, B_1, A\}, \{B_1, B_2, B_1\cup B_2\}\}$. These union-closed families correspond to the matrix

$$
J := \begin{array}{c}
\begin{array}{ccccc} A\setminus S & B_1 & B_2 & A & B_1\cup B_2 \end{array} \\
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 1
\end{pmatrix}
\end{array}
\begin{array}{c}
\\
\{A\setminus S\} \\
\{B_1\} \\
\{B_2\} \\
\{A\setminus S, B_1, A\} \\
\{B_1, B_2, B_1\cup B_2\}
\end{array}
$$

with the two block identity matrices showing independence. Now we will partition the rest of $\mathcal{P}([n])$.

1. If $T \subsetneq A\setminus S$ or B_1 or B_2 and $F \in \{A\setminus S, B_1, B_2\}$, then $\{T, F\}$ where $T \subset F$ is union-closed, tight with the inequality, and linearly independent from all of the previous families.

2. Let T contain exactly one of $A\setminus S, B_1, B_2$ and $T \notin \{A, B_1\cup B_2\}$. Then, as before, the set $\{T, F\}$, where $F \subset T$ and $F \in \{A\setminus S, B_1, B_2\}$, is union-

closed, tight with the inequality, and linearly independent for each distinct such $T \in \mathcal{P}([n])$.

3. Let T contain exactly two of $A \setminus S, B_1, B_2$ and $T \neq B_1 \cup B_2$. First note that if $T \supset A \setminus S \cup B_1$, then $T \supset A \setminus S \cup B_2$ since $A \setminus S \cup B_1 = A$. So we only need to consider $T \supset B_1 \cup B_2$. For this case, consider the union-closed family $\{T, B_1, B_2, B_1 \cup B_2\}$.

4. Next, let $T \supset A \setminus S \cup B_1 \cup B_2$ and $T \neq A$. Here we take the union-closed family $\{A \setminus S, B_1, B_2, A, B_1 \cup B_2, T\}$.

5. Finally, we can have a set $T \in \mathcal{P}([n])$ that is not contained in and does not contain any of the sets in $\{A \setminus S, B_1, B_2, A, B_1 \cup B_2\}$. We have the associated tight union-closed family $\{A \setminus S, T, T \cup A \setminus S\}$.

This gives the partition $\mathcal{P}([n]) = \mathcal{S}_0 \cup \mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{S}_3 \cup \mathcal{S}_4 \cup \mathcal{S}_5$ where $\mathcal{S}_i = \{T \in \mathcal{P}([n]) \mid T$ is as described in case i for $i \in [5]\}$. Then using this ordering we get the block matrix

$$\begin{pmatrix} J & 0 & 0 & 0 & 0 & 0 \\ M_1 & I & 0 & 0 & 0 & 0 \\ M_2 & 0 & I & 0 & 0 & 0 \\ M_4 & 0 & 0 & I & 0 & 0 \\ M_7 & 0 & 0 & 0 & I & 0 \\ M_{11} & M_{12} & M_{13} & M_{14} & M_{15} & I \end{pmatrix}$$

since for $T \in \mathcal{S}_5$, $T \cup A \setminus S \in \mathcal{S}_2$ or $\mathcal{S}_4$. This shows that the chosen families are linearly independent. Finally, note that in the union-closed families produced in case 4, there is an element guaranteed to be in at least five of those sets (six if $S \subset B_1 \cap B_2$, which is why $a \geq 5$. $\square$

Example 4.2.2. Let $A = \{1,2,3,4,5\}$ with $S = \{4,5\}$, $B_1 = \{1,4,5\}$ and $B_2 = \{2,4,5\}$. Then we have a facet-defining inequality

$$x_{\{1,2,3\}} + x_{\{1,4,5\}} + x_{\{2,4,5\}} \leq 1 + x_{\{1,2,4,5\}} + x_{\{1,2,3,4,5\}}$$

for $P(n,a)$ for any $a \geq 5$.

Theorem 4.2.3. *Let $A, B, C \in \mathcal{P}([n])$ be distinct sets such that $A \cup B$, $A \cup C$ and $B \cup C$ are distinct from one another and from A, B, C. Then the inequality*

$$x_A + x_B + x_C \leq 1 + x_{A\cup B} + x_{A\cup C} + x_{B\cup C}$$

is valid and facet-defining for $P(n,a)$ with $n, a \geq 3$.

Proof. We first show the validity of the inequality by considering the following conical combination of defining inequalities of $P(n,a)$:

$$\begin{array}{rcccccccl}
\frac{1}{2}(& x_A & +x_B & & -x_{A\cup B} & & & \leq 1) \\
+\frac{1}{2}(& x_A & & +x_C & & -x_{A\cup C} & & \leq 1) \\
+\frac{1}{2}(& & x_B & +x_C & & & -x_{B\cup C} & \leq 1) \\
+\frac{1}{2}(& & & & -x_{A\cup B} & & & \leq 0) \\
+\frac{1}{2}(& & & & & -x_{A\cup C} & & \leq 0) \\
+\frac{1}{2}(& & & & & & -x_{B\cup C} & \leq 0) \\
\hline
& x_A & +x_B & +x_C & -x_{A\cup B} & -x_{A\cup C} & -x_{B\cup C} & \leq \frac{3}{2}
\end{array}$$

Since we know $x_S \in \{0,1\}$ for all $S \in \mathcal{P}([n])$, we can round down the right-hand side, and thus obtain that $x_A + x_B + x_C - x_{A\cup B} - x_{A\cup C} - x_{B\cup C} \leq 1$ as desired.

We now show that this inequality is facet-defining by producing 2^n linearly independent union-closed families that are tight with this inequality. Take the three union-closed families $\{A\}, \{B\}, \{C\}$; these are tight with the inequality. For each set $S \in \mathcal{P}([n]) \backslash \{A, B, C\}$ that is contained in or contains at least one of A,

B or C, if $S \notin \{A \cup B, A \cup C, B \cup C\}$, take a union-closed family containing S and one of A, B and C that S contains or is contained in (for example, if $S \subseteq A$, take the union-closed family $\{A, S\}$, or if $B \subseteq S$, take $\{B, S\}$). Such families are also tight with the inequality. If $S \in \{A \cup B, A \cup C, B \cup C\}$, take the union-closed family containing S and the two sets among A, B, C that give S by taking their union so that this family is tight with our inequality. The next case is if $S \in \mathcal{P}([n])$ and S is not contained in and does not contain any of the sets A, B, C, and $S \cup A, S \cup B, S \cup C \in \{A \cup B, A \cup C, B \cup C\}$. First we claim that in this case $S \cup A, S \cup B$, and $S \cup C$ cannot all be distinct. Suppose that they were. Then, up to renaming the sets, there are three cases that arise.

1. The first scenario is when $S \cup A = B \cup C$, $S \cup B = A \cup C$, and $S \cup C = A \cup B$. Since these unions are all distinct and they cannot each contain each other, there must be some element present in one union that isn't present in another, say without loss of generality that $i \in A \cup B$ such that $i \notin A \cup C$. This implies that $i \in B$ and $i \notin A, C$. But we have $i \in S \cup B = A \cup C$, a contradiction.

2. The second scenario is when $S \cup A = B \cup C$, $S \cup B = A \cup B$, and $S \cup C = A \cup C$. Since these unions are all distinct, either there exists $i \in A \cup B = S \cup B$ such that $i \notin A \cup C = S \cup C$ or there exists $i \in A \cup C = S \cup C$ such that $i \notin A \cup B = S \cup B$. In the former, this implies that $i \in B$ but i is not in A, C or S, which contradicts the fact that $S \cup A = B \cup C$. In the latter, this implies that $i \in C$, but i is not in A, B or S, again contradicting $S \cup A = B \cup C$.

3. The final scenario is when $S \cup A = A \cup B$, $S \cup B = B \cup C$, and $S \cup C = A \cup C$. From these equalities we get the following three equalities $S \setminus A = B \setminus A,\ S \setminus B = C \setminus B,\ S \setminus C = A \setminus C$. Since all sets in the inequality are distinct and they cannot each contain each other, there must be some element present in

one union that isn't present in another, say without loss of generality that there exists $i \in A \cup B$ such that $i \notin A \cup C$, which implies $i \in B$ and $i \notin A, C$. Since $S \setminus A = B \setminus A$, we have that $i \in S$. But this gives the contradiction that $i \in A \cup C = S \cup C$.

This shows that the sets $S \cup A$, $S \cup B$, and $S \cup C$ cannot all be distinct. Therefore, at least two of $S \cup A, S \cup B$, and $S \cup C$ must be equal. Without loss of generality, suppose $S \cup A = S \cup B$. Again, up to renaming sets, three cases arise.

1. Suppose that $S \cup A = S \cup B = A \cup B$. Then, take the family containing $A, B, S, A \cup B$, which is union-closed and tight with the inequality. Note that this family requires $a \geq 4$.

2. Suppose that $S \cup A = S \cup B = A \cup C$ and $S \cup C = A \cup B$. Then since $S \cup B = A \cup C$, we know that $B \subseteq A \cup C$. Similarly, since $S \cup C = A \cup B$, we know that $C \subseteq A \cup B$. This implies that $A \cup B = A \cup C$, a contradiction to the unions being distinct.

3. Suppose that $S \cup A = S \cup B = A \cup C$ and $S \cup C = B \cup C$. Then $S \cup B = A \cup C$ implies that $B \subseteq A \cup C$. Moreover, $S \cup C = B \cup C$ implies that S contains no element in $A \setminus (B \cup C)$. Since $S \cup B = A \cup C$, and the left-hand side contains no element in $A \setminus (B \cup C)$, neither does the right-hand side. Therefore, $A \subseteq B \cup C$. This implies that $A \cup C = B \cup C$, a contradiction to the unions being distinct.

Note that having $S \cup A = S \cup B = S \cup C = A \cup C$ would put us back in the first case which is why we did not consider it. This concludes our analysis for the third type of set $S \in \mathcal{P}([n])$.

For our fourth and final type of set $S \in \mathcal{P}([n])$, let S be such that S is not contained in and does not contain any of the sets A, B, C, and where we additionally require that $S \cup F \notin \{A \cup B, A \cup C, B \cup C\}$ for some $F \in \{A, B, C\}$. Then we take the union-closed family $\{S, F, S \cup F\}$ which is tight with the inequality.

This gives a partition $\mathcal{P}([n]) = \mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{S}_3 \cup \mathcal{S}_4$ where $\mathcal{S}_1 := \{A, B, C\}$, $\mathcal{S}_2 := \{S \in \mathcal{P}([n]) \backslash \{A, B, C\} \mid S \text{ is contained in or contains at least one of } A, B, \text{ or } C\}$,

$$\begin{aligned}\mathcal{S}_3 := \{S \in \mathcal{P}([n]) \mid &S \text{ is not contained in any of } A, B, C,\\ &\text{and does not contain } A, B, \text{ or } C,\\ &\text{and } S \cup A, S \cup B, S \cup C \in \{A \cup B, A \cup C, B \cup C\}\},\end{aligned}$$

and

$$\begin{aligned}\mathcal{S}_4 := \{S \in \mathcal{P}([n]) \mid &S \text{ is not contained in any of } A, B, C,\\ &\text{and does not contain } A, B, \text{ or } C,\\ &\text{and } S \cup F \text{ is not in } \{A \cup B, A \cup C, B \cup C\} \text{ for some } F \in \mathcal{S}_1\}.\end{aligned}$$

Then one can see that the tight families described above form the following matrix (keeping the families and the partition in the order described)

$$\begin{pmatrix} I & 0 & 0 & 0 \\ M_1 & I & 0 & 0 \\ M_2 & M_3 & I & 0 \\ M_4 & M_5 & 0 & I \end{pmatrix}$$

since if $S \in \mathcal{S}_4$, then $F \in \mathcal{S}_1$ and $S \cup F \in \mathcal{S}_2$. This shows that the tight families chosen are linearly independent, and proves the result. $\square$

Example 4.2.4. For example, $x_{\{1\}} + x_{\{2\}} + x_{\{3\}} - x_{\{1,2\}} - x_{\{1,3\}} - x_{\{2,3\}} \leq 1$ is facet-defining for $P(n, n)$ for $n \geq 3$, and $x_{\{1\}} + x_{\{2\}} + x_{\{3,4\}} - x_{\{1,2\}} - x_{\{1,3,4\}} - x_{\{2,3,4\}} \leq 1$ as well as $x_{\{1\}} + x_{\{2,4\}} + x_{\{3,4\}} - x_{\{1,2,4\}} - x_{\{1,3,4\}} - x_{\{2,3,4\}} \leq 1$ and many others are facet-defining for $P(n, n)$ for $n \geq 4$. (They're also facet-defining for any value of

$a \geq 4$ since all tight families used in the proof have at most four sets, and thus the frequency of any element is trivially at most four.)

Theorem 4.2.5. *Let $A, B, C \in \mathcal{P}([n])$ be distinct sets such that $D := A \cup B = A \cup C = B \cup C$ and such that D is distinct from A, B, C. Then the inequality*

$$x_A + x_B + x_C \leq 1 + 2x_D$$

is valid and facet-defining for $P(n, a)$ with $n, a \geq 3$.

Proof. We first show the validity of the inequality by considering the following conical combination of defining inequalities of $P(n, a)$:

$$\begin{array}{rccccl}
\frac{1}{2}(& x_A & +x_B & & -x_D & \leq 1) \\
+\frac{1}{2}(& x_A & & +x_C & -x_D & \leq 1) \\
+\frac{1}{2}(& & x_B & +x_C & -x_D & \leq 1) \\
+\frac{1}{2}(& & & & -x_D & \leq 0) \\
\hline
 & x_A & +x_B & +x_C & -2x_D & \leq \frac{3}{2}
\end{array}$$

Since we know $x_S \in \{0, 1\}$ for all $S \in \mathcal{P}([n])$, we can round down the right-hand side, and thus obtain that $x_A + x_B + x_C - 2x_D \leq 1$ as desired.

We now need to show that this inequality is facet-defining by producing 2^n linearly independent union-closed families that are tight with this inequality. First we take the families $\{A\}, \{B\}, \{C\}$. For each set $S \in \mathcal{P}([n]) \backslash \{A, B, C\}$ that is contained in or contains at least one of A, B or C, if $S \neq D$, take a union-closed family containing S and one of A, B and C that S contains or is contained in (for example, if $S \subseteq A$, take the union-closed family $\{A, S\}$, or if $B \subseteq S$, take $\{B, S\}$). If $S = D$, take the union-closed family containing S and the union-closed family $\{A, B, C, S\}$; this family requires $n, a \geq 3$. Next, if S is neither contained in nor contains A, B, or C, then we can take the union closed family $\{A, S, A \cup S\}$.

This gives us the partition $\mathcal{P}([n]) = \mathcal{S}_1 \cup \mathcal{S}_2 \cup \mathcal{S}_3$ where $\mathcal{S}_1 = \{\{A\}, \{B\}, \{C\}\}$, $\mathcal{S}_2 = \{S \in \mathcal{P}([n]) | S \text{ is contained in or contains } A, B, \text{ or } C\}$, and $\mathcal{S}_3 = \{S \in \mathcal{P}([n]) | S \text{ is neither contained in or containing } A, B \text{ or } C\}$. This partition gives the below matrix where columns are indexed by the partition and rows correspond to the above union-closed families.

$$\begin{pmatrix} I & 0 & 0 \\ M_1 & I & 0 \\ M_2 & M_3 & I \end{pmatrix}$$

□

Example 4.2.6. For example, $x_{\{1,2,4\}} + x_{\{1,3,4\}} + x_{\{2,3,4\}} - 2x_{\{1,2,3,4\}} \leq 1$ and $x_{\{3,4\}} + x_{\{1,2,3\}} + x_{\{1,2,4\}} - 2x_{\{1,2,3,4\}} \leq 1$ are facet-defining for $P(n, n)$ for $n \geq 4$. (They're also facet-defining for any value of $a \geq 3$ since all tight families used in the proof have at most three sets, and thus the frequency of any element is trivially at most three.)

Looking at our calculations for $P(4, 4)$ and $P(5, 5)$, we noticed that $x_{\{1,2,3\}} + x_{\{1,2,4\}} + x_{\{1,3,4\}} + x_{\{2,3,4\}} - 3x_{\{1,2,3,4\}} \leq 1$ and $x_{\{1,2,3,4\}} + x_{\{1,2,3,5\}} + x_{\{1,2,4,5\}} + x_{\{1,3,4,5\}} + x_{\{2,3,4,5\}} - 4x_{\{1,2,3,4,5\}} \leq 1$ are respectively rank-2 facets for those two polytopes, and $x_{\{1,2\}} + x_{\{1,3\}} + x_{\{2,3\}} - 2x_{\{1,2,3\}} \leq 1$ is a rank-1 facet for $P(3, 3)$. We show that the generalizations of these inequalities are always facet-defining for $P(n, n)$.

Theorem 4.2.7. *The inequality*

$$\sum_{i=1}^{n} x_{[n]\setminus\{i\}} - (n-1)x_{[n]} \leq 1$$

is valid and facet-defining for $P(n, n)$.

Proof. We first show the validity of the inequality. Consider a union-closed family $\mathcal{F}$ containing k sets of size $n-1$; note that $0 \leq k \leq n$ since the ground set has

n elements. If $k \in \{0,1\}$, then the inequality holds trivially since $x_{[n]} \in \{0,1\}$. If $k \in \{2,3,\ldots,n\}$, then $[n] \in \mathcal{F}$, so $x_{[n]} = 1$ and the inequality thus holds.

To show that this inequality is facet-defining, we again produce 2^n linear independent union-closed families that are tight with this inequality. For $S \in \mathcal{P}([n])$ such that $0 \leq |S| \leq n-2$, take the union-closed family $\{S,T\}$ for any T such that $S \subseteq T$ and $|T| = n-1$. Furthermore, also take the union-closed family that contains all sets of size $n-1$ as well as $[n]$; note that this family does not break the frequency inequality since each element appears exactly n of those sets. Finally, for $S \in \mathcal{P}([n])$ such that $|S| = n-1$, take the union-closed family containing only S. By changing the order of the sets of $\mathcal{P}([n])$ so that $[n]$ comes before the sets of size $n-1$, we can see that these tight families (in that order) form the matrix

$$\begin{pmatrix} I & M_1 \\ 0 & I \end{pmatrix}$$

where the bottom-right identity matrix is $n \times n$. This shows that the tight families chosen are linearly independent, and proves the result. □

4.3 Set size inequalities

None of the facet-defining inequalities we found in the previous section involved frequency inequalities in their conical combinations to prove their validity. We have found it to be substantially harder to generalize constraints that do involve frequency constraints even though those are necessary to narrow down the integrality gap between $P(n,n)$ and $R(n,n)$ in the direction that we are interested in, namely the all-ones direction, $\mathbf{1}$.

Conjecture 4.3.1. *Fix a positive integer i. If $n = \sum_{k=i-1}^{m} \binom{m}{k}$ for some positive integer m and $n \geq i+1$, then the constraint $\sum_{\substack{S \in \mathcal{P}([n]):\\ |S|=i}} x_S \leq \binom{m+1}{i}$ is a tight valid inequality for $P(n,n)$.*

Furthermore, if $\sum_{k=i-1}^{m-1} \binom{m-1}{k} < n < \sum_{k=i-1}^{m} \binom{m}{k}$ for some positive integer m and $n \geq i+1$, then the constraint $\sum_{\substack{S \in \mathcal{P}([n]):\\ |S|=i}} x_S \leq \binom{m+1}{i} - 1$ is a valid inequality for $P(n,n)$ though it might not be tight.

First, for $i \in [n]$, we define

$$f(n,a,i) := \max\{ \sum_{\substack{S \in \mathcal{P}([n]):\\ |S|=i}} x_S | \mathbf{x} \in P(n,a)\}.$$

The conjecture states that $f(n,n,i) = \binom{m+1}{i}$ for positive integers m and i such that $n = \sum_{k=i-1}^{m} \binom{m}{k} \geq i+1$. Furthermore, the conjecture implies that $f(n,n,i) \leq \binom{m+1}{i} - 1$ if $\sum_{k=i-1}^{m-1} \binom{m-1}{k} < n < \sum_{k=i-1}^{m} \binom{m}{k}$ for some positive integer m and $n \geq i+1$. Note that the latter is just an upper bound and $f(n,n,i)$ might be smaller as we will see in the example below.

Example 4.3.2. Let $n = a = 4$ and $i = 2$. Using any optimization solver, we can compute that $f(4,4,2) = 3$, which implies that $x_{\{1,2\}} + x_{\{1,3\}} + x_{\{1,4\}} + x_{\{2,3\}} + x_{\{2,4\}} + x_{\{3,4\}} \leq 3$ is a tight valid inequality for $P(4,4)$. Alternatively, one can check that this is a valid inequality for $P(4,4)$ as follows. Let $\mathcal{G} \subset \{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}$ with $|\mathcal{G}| = 4$. Then up to symmetry, we have two cases.

1. If $\mathcal{G} = \{\{1,2\},\{1,3\},\{2,3\},\{1,4\}\}$, then the union-closure of $\mathcal{G}$ is

   $$\{\{1,2\},\{1,3\},\{2,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\},$$

   which contains element 1 more than 4 times.

2. If $\mathcal{G} = \{\{1,2\},\{1,3\},\{2,4\},\{3,4\}\}$, then the union-closure of $\mathcal{G}$ is

$$\{\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\},$$

which also contains element 1 more than 4 times.

Thus any union-closed family $\mathcal{F}$ which contains four (or more) sets among $\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}$ is such that $x(\mathcal{F}_1) > 4$, and thus cannot correspond to a point in $P(4,4)$. Thus $x_{\{1,2\}}+x_{\{1,3\}}+x_{\{1,4\}}+x_{\{2,3\}}+x_{\{2,4\}}+x_{\{3,4\}} \leq 3$ is valid for $P(4,4)$. One can show that this is a tight valid inequality by considering the union-closed family $\{\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$ which corresponds to a lattice point in $P(4,4)$.

The conjecture in this case only gives an upper bound to $f(4,4,2)$. Indeed, note that $\binom{2}{1}+\binom{2}{2} < 4 < \binom{3}{1}+\binom{3}{2}+\binom{3}{3}$, so the conjecture states that $f(4,4,2) \leq \binom{4}{2} - 1 = 5$.

n \ i	1	2	3	4	5	6	7	8	9	10
2	2									
3	2	3								
4	3	3	4							
5	3	3	4	5						
6	3	4	4	5	6					
7	3	6	4	5	6	7				
8	4	6	5	6	6	7	8			
9	4	6	6	7	6	7	8	9		
10	4	6	8	7	7	8	8	9	10	
11	4	6	10	8	7	10	8	9	10	11

Table 3: Optimal values $f(n,n,i)$ for $n \leq 11$ and $1 \leq i \leq n-1$. In red are the entries that Conjecture 4.3.1 predicts exactly.

Table 3 records $f(n,n,i)$ for $n \leq 10$ and $i \leq 9$ whereas Table 4 records the upper bounds for $f(n,n,i)$ given by Conjecture 4.3.1. In both tables, the red entries are the values of n and i for which the conjecture gives a tight bound. For example, for

n \ i	1	2	3	4	5	6	7	8	9	10
2	2									
3	2	3								
4	3	5	4							
5	3	5	9	5						
6	3	5	9	14	6					
7	3	6	9	14	20	7				
8	4	9	9	14	20	27	8			
9	4	9	9	14	20	27	35	9		
10	4	9	9	14	20	27	35	44	10	
11	4	9	10	14	20	27	35	44	54	11

Table 4: Upper bound for $f(n,n,i)$ for $n \leq 11$ and $1 \leq i \leq n-1$ given by Conjecture 4.3.1. In red are the bounds the conjecture claims to be tight.

$n = 11$ and $i = 3$, we have that $n = \binom{4}{2} + \binom{4}{3} + \binom{4}{4}$ (i.e., $m = 4$), and the conjecture states accurately that $f(11, 11, 3) = \binom{4+1}{3} = 10$. For $i = 4$, the next entry in the column that would be red would be $n = 16 = \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$ with a value of $\binom{5+1}{4} = 15$. We see that the behavior between red entries in Table 3 is much more complicated than in Table 4. Understanding that behavior better and refining the conjecture (and proving it!) is a very interesting question for future work. Indeed, by adding the cuts $\sum_{\substack{S\in\mathcal{P}([n]):\\ |S|=i}} x_S \leq f(n,n,i)$ to the polytope $R(n,n)$ behind the linear relaxation of $f(n,n)$, the optimal value of the linear relaxation improves significantly as can be seen in Table 5. Indeed, even though these inequalities do not correspond to facets we found for $P(n,n)$, they are helpful in narrowing the integrality gap between $R(n,n)$ and $P(n,n)$ in the direction that interests us—more so than the facet-defining inequalities we found in the previous section, which could be recovered by running the Chvátal-Gomory procedure solely on the union-closed inequalities and variable bound constraints (without the frequency inequalities).

We now present some evidence for Conjecture 4.3.1 by proving some edge cases.

First, notice that the main diagonal in Tables 3 and 4 is red. This corresponds

n	$f(n,n)$	relaxation with cuts	relaxation without cuts
3	5	6.167	6.5
4	8	9.25	9.6
5	9	12	13.558
6	10	15.25	18.482
7	12	19.4	24.188
8	16	23.8	30.636
9	17	27.8	37.931
10	18	32.4	46.068

Table 5: Comparison of the optimal value obtained when maximizing $\sum_{S\in\mathcal{P}([n])} x_S$ over $P(n,n)$ versus $R(n,n)$ with the added cuts $\sum_{\substack{S\in\mathcal{P}([n]):\\|S|=i}} x_S \leq f(n,n,i)$ versus $R(n,n)$.

to $i = n-1$ and so $m = i = n-1$ (i.e., $n = \binom{i}{i-1} + \binom{i}{i} = i+1$). In that case, the conjecture is true.

Theorem 4.3.3. *For the polytope $P(n,n)$, the inequality $\sum_{\substack{S\in\mathcal{P}([n]):\\|S|=n-1}} x_S \leq n$ is a tight valid inequality.*

Proof. Certainly, since there are n subsets of size $n-1$ in $[n]$, the inequality is trivially true. We can see that it is tight because of the union-closed family containing all sets of size $n-1$ as well as $[n]$. Each element is in exactly n sets, so this family corresponds to a point in $P(n,n)$ which is tight with the inequality. $\square$

On the other end of the conjecture, we can also easily prove the case when $i = 1$. Here, the conjecture actually gives a tight bound for all values of n, not just those equal to $\sum_{k=0}^{m}\binom{m}{k} = 2^m$ for some m. In general, we will have $m = \lfloor \log_2 n \rfloor$.

Theorem 4.3.4. *The constraint $\sum_{\substack{S\in\mathcal{P}([n])\\|S|=1}} x_S \leq \binom{\lfloor \log_2 n\rfloor+1}{1}$ is valid and tight for $P(n,n)$.*

Proof. Consider a union-closed family $\mathcal{F}$ that contains $j \geq \lfloor \log_2 n \rfloor + 2$ sets of size 1. Since $\mathcal{F}$ is union-closed, it must contain the union-closure of all those sets of size 1. Note that the union-closure of those sets is isomorphic to $\mathcal{P}([j])$, and that each element is in $2^{j-1} \geq 2^{\lfloor \log_2 n \rfloor + 1} > n$ sets of the union-closure, and thus each of those j elements is in more than n sets in $\mathcal{F}$. Therefore, such a family $\mathcal{F}$ cannot correspond to a point in $P(n,n)$, which means that $\sum_{\substack{S \in \mathcal{P}([n]) \\ |S|=1}} x_S \leq \binom{\lfloor \log_2 n \rfloor + 1}{1}$ is valid for $P(n,n)$.

To show that the inequality is tight, consider the union-closed family $\mathcal{F} = \mathcal{P}([\lfloor \log_2 n \rfloor + 1])$. Such a family corresponds to a point in $P(n,n)$ and contains $\lfloor \log_2 n \rfloor + 1$ sets of size 1, so it is tight with the inequality.

$\square$

Besides the two extreme cases $i = 1$ and $i = n-1$, we are also able to prove the conjecture in the case when $i = n-2$. Then, we have that $n - 1 = \binom{i}{i-1} + \binom{i}{i} < n < \binom{i+1}{i-1} + \binom{i+1}{i} + \binom{i+1}{i+1} = n + \binom{n-1}{n-3}$. So in the notation of Conjecture 4.3.1, $m = i+1 = n-1$ and we need to show that $\sum_{\substack{S \in \mathcal{P}([n]): \\ |S|=n-2}} x_S \leq \binom{n}{n-2} - 1$. We actually do more and actually show that $f(n,n,n-2)$ is equal to $n-1$, which implies that the previous inequality is also true (though not tight).

Theorem 4.3.5. *We have that $f(n,n,n-2) = n-1$ which implies that the inequality $\sum_{\substack{S \in \mathcal{P}([n]): \\ |S|=n-2}} x_S \leq n-1$ is valid and tight for $P(n,n)$.*

Proof. Suppose that $\mathcal{F}$ is a union-closed family that contains $j > n-1$ sets of size $n-2$. Since $\mathcal{F}$ is union-closed, it contains the union-closure of those j sets. First observe that $[n]$ is in the union-closure because a set of size $n-1$ set contains $n-1$ sets of size $n-2$, and $j > n-1$, so we cannot only have subsets of some set of size $n-1$. Furthermore, for that same reason, there must exist two distinct elements i_1, i_2 such that $[n]\backslash\{i_1\}$ and $[n]\backslash\{i_2\}$ are both contained in the union-closure. Finally, note that each element is on average in $\frac{j(n-2)}{n} \geq n-2$ of the j sets of size $n-2$. If there is an element in $[n]\backslash\{i_1, i_2\}$ that is present in at least $n-2$

of the j sets, then that element appears in a least $n-2+3 = n+1$ sets of $\mathcal{F}$, and so then $\mathcal{F}$ cannot correspond to a point of $P(n,n)$. Otherwise, if all elements in $[n]\backslash\{i_1, i_2\}$ appear in at most $n-3$ of the j sets, at least one of i_1 or i_2 must appear in at least $n-1$ of the j sets (because of the average), as well as in two of the three extra sets that we know are in the union-closure of the j sets. This implies that i_1 or i_2 appears in at least $n-1+2 = n+1$ sets, and in that case too $\mathcal{F}$ cannot correspond to a point of $P(n,n)$. Therefore, this means that $\sum_{\substack{S\in\mathcal{P}([n]):\\ |S|=n-2}} x_S \leq n-1$ is a valid inequality for $P(n,n)$.

To show that this inequality is tight, consider the union-closed-family $\{S \subseteq [n-1] \mid |S| = n-2\} \cup \{[n-1]\}$. This family contains $n-1$ sets of size $n-2$, and each element appears in exactly $n-1$ sets of the family, so it corresponds to a point of $P(n,n)$ which is tight with the inequality. $\square$

There is still a lot of interesting future work to do with this conjecture. Moreover, from computational experimentation, we observe that a similar conjecture can be made in general for $f(n,a,i)$ where $a \neq n$.

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APPENDIX A

Appendix

A.1 Poonen's linear program for Example 3.2.7

This is the linear program one can write based on Theorem 3.2.6 to show that any union-closed family that contains $\{\{i,j,k\},\{i,j,l\},\{i,k,l\}\}$ for some distinct elements i,j,k,l is union-closed. The optimal value is 0, i.e., this linear program is feasible, and one solution is $c_1 = c_2 = c_3 = c_4 = \frac{1}{4}$.

In the linear program, I included the family $\mathcal{G}$ in parenthesis before each corresponding constraint.

$$
\begin{aligned}
&\max 0\\
&\text{s.t. } c_1 + c_2 + c_3 + c_4 = 1\\
&\quad (\{\{\},\{1\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})\\
&\quad 6c_1 + 4c_2 + 3c_3 + 3c_4 \geq 7/2\\
&\quad (\{\{\},\{1\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})\\
&\quad 6c_1 + 5c_2 + 4c_3 + 4c_4 \geq 8/2\\
&\quad (\{\{\},\{1\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})\\
&\quad 7c_1 + 4c_2 + 4c_3 + 3c_4 \geq 8/2\\
&\quad (\{\{\},\{1\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})\\
&\quad 7c_1 + 5c_2 + 5c_3 + 4c_4 \geq 9/2\\
&\quad (\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})\\
&\quad 8c_1 + 4c_2 + 4c_3 + 4c_4 \geq 9/2\\
&\quad (\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})\\
&\quad 8c_1 + 5c_2 + 5c_3 + 5c_4 \geq 10/2\\
&\quad (\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})\\
&\quad 8c_1 + 5c_2 + 5c_3 + 4c_4 \geq 10/2
\end{aligned}
$$

$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 4c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 4c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 3c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 3c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 5c_4 \geq 10/2$

$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 3c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 3c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 3c_3 + 3c_4 \geq 6/2$
$(\{\{\},\{1\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{1\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{\},\{1\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 4c_3 + 4c_4 \geq 8/2$

$(\{\{\},\{1\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 3c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{1\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 5c_4 \geq 8/2$

$(\{\{\}, \{1\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\}, \{1\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 4c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{1\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{1,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 3c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{1\}, \{1,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 3c_4 \geq 8/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 4c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 3c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 4c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 5c_2 + 4c_3 + 4c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 6c_2 + 5c_3 + 5c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 6c_2 + 5c_3 + 4c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 5c_4 \geq 12/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 8c_2 + 6c_3 + 6c_4 \geq 13/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 8c_2 + 7c_3 + 7c_4 \geq 14/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 6c_4 \geq 13/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 6c_2 + 4c_3 + 5c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 7c_2 + 5c_3 + 6c_4 \geq 12/2$

$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 3c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 4c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 5c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 7c_3 + 6c_4 \geq 13/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 5c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 3c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 5c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 3c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 4c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 3c_4 \geq 9/2$

$(\{\{\},\{1\},\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 5c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 3c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 4c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 4c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 5c_4 \geq 13/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 7c_3 + 6c_4 \geq 14/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 8c_3 + 7c_4 \geq 15/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 8c_3 + 6c_4 \geq 14/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 3c_4 \geq 11/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 4c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 7c_3 + 5c_4 \geq 13/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 8c_3 + 6c_4 \geq 14/2$
$(\{\{\},\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 8c_3 + 5c_4 \geq 13/2$
$(\{\{\},\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 8c_3 + 8c_4 \geq 16/2$
$(\{\{\},\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 6c_3 + 7c_4 \geq 14/2$
$(\{\{\},\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 7c_3 + 8c_4 \geq 15/2$
$(\{\{\},\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 4c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 5c_3 + 7c_4 \geq 13/2$

$(\{\{\}, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 8c_4 \geq 14/2$
$(\{\{\}, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 8c_2 + 5c_3 + 7c_4 \geq 13/2$
$(\{\{\}, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 8c_4 \geq 14/2$
$(\{\{\}, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 6c_2 + 3c_3 + 6c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 7c_2 + 4c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 8c_4 \geq 13/2$
$(\{\{\}, \{1\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{\}, \{1\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\}, \{1\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{\}, \{1\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{1\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 3c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 4c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 4c_2 + 5c_3 + 4c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 5c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 5c_2 + 6c_3 + 4c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 6c_2 + 7c_3 + 5c_4 \geq 12/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 6c_4 \geq 13/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 7c_2 + 8c_3 + 7c_4 \geq 14/2$
$(\{\{\}, \{1\}, \{3\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$8c_1 + 6c_2 + 8c_3 + 6c_4 \geq 13/2$

$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 7c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 4c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 3c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 4c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 5c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 8c_3 + 6c_4 \geq 13/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 8c_3 + 5c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 5c_3 + 3c_4 \geq 8/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 8c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 8c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 6c_4 \geq 11/2$

$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 3c_4 \geq 9/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 8c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 8c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{1\},\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 8c_3 + 8c_4 \geq 15/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 8c_3 + 7c_4 \geq 14/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 7c_3 + 8c_4 \geq 14/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 4c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 7c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 8c_3 + 8c_4 \geq 14/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 8c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 8c_4 \geq 13/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 4c_3 + 4c_4 \geq 7/2$

$(\{\{\},\{1\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 4c_2 + 4c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 8c_4 \geq 14/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 7c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 4c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 5c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 8c_4 \geq 13/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 4c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 3c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 8c_4 \geq 13/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 3c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 8c_4 \geq 12/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 7c_4 \geq 11/2$

$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 3c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 4c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 5c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 6c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 6c_2 + 7c_3 + 8c_4 \geq 13/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 3\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 5c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 4c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 5c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 5c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 3c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 3\}, \{1, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$7c_1 + 4c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 3c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 4c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 5c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 6c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 6c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{2, 3\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 5c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 4c_2 + 3c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{2, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 5c_2 + 4c_3 + 7c_4 \geq 10/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{2, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 5c_2 + 5c_3 + 8c_4 \geq 11/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 3c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1\}, \{4\}, \{1, 4\}, \{3, 4\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\})$
$6c_1 + 4c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{\}, \{1, 2\}, \{1, 2, 3\}, \{1, 2, 4\}, \{1, 3, 4\}, \{1, 2, 3, 4\}\})$
$5c_1 + 4c_2 + 3c_3 + 3c_4 \geq 6/2$

$(\{\{\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+4c_4 \geq 7/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+3c_4 \geq 7/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+4c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+4c_3+4c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+5c_3+5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+5c_3+4c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+6c_3+5c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+6c_3+6c_4 \geq 11/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+7c_3+7c_4 \geq 12/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+7c_3+6c_4 \geq 11/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+4c_3+5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+5c_3+6c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+6c_3+7c_4 \geq 11/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+5c_3+5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+6c_3+6c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+3c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+4c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+6c_3+5c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+6c_4 \geq 11/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+5c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+4c_3+4c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+6c_4 \geq 10/2$

$(\{\{\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 4c_4 \geq 7/2$

$(\{\{\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+5c_4 \geq 8/2$
$(\{\{\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+3c_3+3c_4 \geq 5/2$
$(\{\{\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+4c_3+4c_4 \geq 6/2$
$(\{\{\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+4c_3+3c_4 \geq 6/2$
$(\{\{\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+4c_4 \geq 7/2$
$(\{\{\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+4c_3+4c_4 \geq 7/2$
$(\{\{\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+5c_3+5c_4 \geq 8/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+5c_3+4c_4 \geq 8/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+5c_4 \geq 9/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+6c_4 \geq 10/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+7c_4 \geq 11/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+6c_4 \geq 10/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+5c_4 \geq 8/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+6c_4 \geq 9/2$
$(\{\{\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+7c_4 \geq 10/2$
$(\{\{\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+5c_3+5c_4 \geq 8/2$
$(\{\{\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+6c_3+6c_4 \geq 9/2$
$(\{\{\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+3c_4 \geq 7/2$
$(\{\{\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+4c_4 \geq 8/2$
$(\{\{\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+5c_4 \geq 9/2$
$(\{\{\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+6c_4 \geq 10/2$
$(\{\{\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+7c_3+5c_4 \geq 9/2$
$(\{\{\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+4c_3+4c_4 \geq 7/2$

$(\{\{\}, \{1,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{1,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 3c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{\}, \{1,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 3c_2 + 3c_3 + 4c_4 \geq 6/2$
$(\{\{\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 3c_3 + 5c_4 \geq 7/2$
$(\{\{\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 7c_4 \geq 9/2$
$(\{\{\}, \{1,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 3c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{\}, \{1,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{\}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 3c_3 + 3c_4 \geq 7/2$
$(\{\{\}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\}, \{2\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 3c_4 \geq 8/2$
$(\{\{\}, \{2\}, \{1,2\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 4c_4 \geq 9/2$
$(\{\{\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{\}, \{2\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 4c_4 \geq 10/2$

$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+6c_3+5c_4\geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+6c_3+6c_4\geq 12/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+7c_3+7c_4\geq 13/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+7c_3+6c_4\geq 12/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+4c_3+5c_4\geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+5c_3+6c_4\geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+6c_3+7c_4\geq 12/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+6c_3+6c_4\geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+3c_4\geq 9/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+6c_3+4c_4\geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+6c_3+5c_4\geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+6c_4\geq 12/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+5c_4\geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+4c_3+4c_4\geq 9/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+5c_3+5c_4\geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+6c_3+6c_4\geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+5c_4\geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+3c_3+4c_4\geq 8/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+4c_3+5c_4\geq 9/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+4c_3+4c_4\geq 9/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+5c_3+5c_4\geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+5c_3+6c_4\geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+6c_3+7c_4\geq 12/2$

$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 3c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 4c_3 + 3c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 8c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 8c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{2\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 3c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 3c_3 + 3c_4 \geq 6/2$
$(\{\{\},\{2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{\},\{2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$

$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+7c_4 \geq 12/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+6c_4 \geq 11/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+4c_3+5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+7c_4 \geq 11/2$
$(\{\{\},\{2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+3c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+4c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+5c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+6c_4 \geq 11/2$
$(\{\{\},\{2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+5c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+4c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+3c_3+4c_4 \geq 7/2$
$(\{\{\},\{2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+5c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+4c_4 \geq 8/2$
$(\{\{\},\{2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+5c_4 \geq 9/2$
$(\{\{\},\{2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+5c_3+6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+7c_4 \geq 11/2$
$(\{\{\},\{2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+6c_4 \geq 10/2$
$(\{\{\},\{2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+3c_3+5c_4 \geq 8/2$

$(\{\{\},\{2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+4c_3+6c_4\geq 9/2$
$(\{\{\},\{2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+7c_4\geq 10/2$
$(\{\{\},\{2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+6c_4\geq 9/2$
$(\{\{\},\{2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+4c_3+3c_4\geq 7/2$
$(\{\{\},\{2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+4c_4\geq 8/2$
$(\{\{\},\{2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+5c_3+5c_4\geq 9/2$
$(\{\{\},\{2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+6c_3+6c_4\geq 10/2$
$(\{\{\},\{2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+5c_4\geq 9/2$
$(\{\{\},\{2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+3c_3+4c_4\geq 7/2$
$(\{\{\},\{2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+4c_3+5c_4\geq 8/2$
$(\{\{\},\{2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+6c_4\geq 9/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+6c_3+4c_4\geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+7c_3+5c_4\geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+7c_3+6c_4\geq 13/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+8c_3+7c_4\geq 14/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+8c_3+6c_4\geq 13/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+3c_4\geq 10/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+4c_4\geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+5c_4\geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+8c_3+6c_4\geq 13/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+5c_4\geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+4c_4\geq 10/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+6c_3+5c_4\geq 11/2$

$(\{\{\},\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+6c_3+6c_4 \geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+7c_4 \geq 13/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+6c_4 \geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+3c_4 \geq 9/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+4c_4 \geq 10/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+8c_2+6c_3+5c_4 \geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+8c_2+7c_3+6c_4 \geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+5c_4 \geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+4c_4 \geq 10/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+5c_4 \geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+6c_4 \geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+7c_4 \geq 13/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+8c_3+6c_4 \geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+3c_4 \geq 9/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+4c_4 \geq 10/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+5c_4 \geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+8c_3+6c_4 \geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+8c_3+5c_4 \geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+4c_4 \geq 9/2$
$(\{\{\},\{2\},\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+5c_4 \geq 10/2$
$(\{\{\},\{2\},\{3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+6c_4 \geq 11/2$
$(\{\{\},\{2\},\{3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+7c_4 \geq 12/2$
$(\{\{\},\{2\},\{3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+6c_4 \geq 11/2$

$(\{\{\},\{2\},\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+3c_4 \geq 8/2$
$(\{\{\},\{2\},\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+4c_4 \geq 9/2$
$(\{\{\},\{2\},\{3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+6c_3+5c_4 \geq 10/2$
$(\{\{\},\{2\},\{3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+7c_3+6c_4 \geq 11/2$
$(\{\{\},\{2\},\{3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+7c_3+5c_4 \geq 10/2$
$(\{\{\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+8c_3+8c_4 \geq 15/2$
$(\{\{\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+8c_3+7c_4 \geq 14/2$
$(\{\{\},\{2\},\{3\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+8c_4 \geq 14/2$
$(\{\{\},\{2\},\{3\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+8c_2+7c_3+7c_4 \geq 13/2$
$(\{\{\},\{2\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+8c_4 \geq 14/2$
$(\{\{\},\{2\},\{3\},\{4\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+8c_3+7c_4 \geq 13/2$
$(\{\{\},\{2\},\{3\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+8c_4 \geq 13/2$
$(\{\{\},\{2\},\{3\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+7c_3+7c_4 \geq 12/2$
$(\{\{\},\{2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+5c_4 \geq 8/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+6c_3+7c_4 \geq 13/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+7c_3+8c_4 \geq 14/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+4c_3+6c_4 \geq 11/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+5c_3+7c_4 \geq 12/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+6c_3+8c_4 \geq 13/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+6c_3+6c_4 \geq 12/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+7c_4 \geq 13/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+4c_3+5c_4 \geq 10/2$
$(\{\{\},\{2\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+5c_3+6c_4 \geq 11/2$

$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{1,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 8c_2 + 5c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 8c_2 + 6c_3 + 8c_4 \geq 13/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 3c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 7c_2 + 4c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 8c_4 \geq 12/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 8c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 8c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 3c_3 + 5c_4 \geq 9/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 7c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{2\}, \{4\}, \{1,2\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 8c_4 \geq 13/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 7c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{2\}, \{4\}, \{1,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{2\}, \{4\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{2\}, \{4\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 8c_4 \geq 12/2$

$(\{\{\},\{2\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+3c_3+6c_4 \geq 9/2$
$(\{\{\},\{2\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+4c_3+7c_4 \geq 10/2$
$(\{\{\},\{2\},\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+8c_4 \geq 11/2$
$(\{\{\},\{2\},\{4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+5c_3+6c_4 \geq 10/2$
$(\{\{\},\{2\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+6c_3+7c_4 \geq 11/2$
$(\{\{\},\{2\},\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+3c_3+5c_4 \geq 8/2$
$(\{\{\},\{2\},\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+4c_3+6c_4 \geq 9/2$
$(\{\{\},\{2\},\{4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+7c_4 \geq 10/2$
$(\{\{\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+4c_3+3c_4 \geq 6/2$
$(\{\{\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+4c_4 \geq 7/2$
$(\{\{\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+5c_4 \geq 8/2$
$(\{\{\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+6c_4 \geq 9/2$
$(\{\{\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+5c_4 \geq 8/2$
$(\{\{\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+3c_3+4c_4 \geq 6/2$
$(\{\{\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+4c_3+5c_4 \geq 7/2$
$(\{\{\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+6c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+4c_3+3c_4 \geq 7/2$
$(\{\{\},\{3\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+4c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+5c_3+3c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+4c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+5c_3+4c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+6c_3+5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+6c_3+4c_4 \geq 10/2$

$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 8c_3 + 7c_4 \geq 13/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 8c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 3c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 4c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 8c_3 + 6c_4 \geq 12/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 8c_3 + 5c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 7c_4 \geq 12/2$

$(\{\{\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 3c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 4c_3 + 3c_4 \geq 6/2$
$(\{\{\},\{3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 5c_3 + 3c_4 \geq 7/2$
$(\{\{\},\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$

$(\{\{\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 8c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 8c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 6c_3 + 3c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 4c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 8c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 8c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 7c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$

$(\{\{\},\{3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+7c_4 \geq 10/2$
$(\{\{\},\{3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+5c_3+5c_4 \geq 8/2$
$(\{\{\},\{3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+6c_3+6c_4 \geq 9/2$
$(\{\{\},\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+5c_3+3c_4 \geq 7/2$
$(\{\{\},\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+4c_4 \geq 8/2$
$(\{\{\},\{3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+5c_4 \geq 9/2$
$(\{\{\},\{3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+7c_3+6c_4 \geq 10/2$
$(\{\{\},\{3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+7c_3+5c_4 \geq 9/2$
$(\{\{\},\{3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+5c_4 \geq 8/2$
$(\{\{\},\{3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+6c_4 \geq 9/2$
$(\{\{\},\{3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+5c_3+4c_4 \geq 7/2$
$(\{\{\},\{3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+6c_3+5c_4 \geq 8/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+8c_3+8c_4 \geq 14/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+8c_3+7c_4 \geq 13/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+7c_3+8c_4 \geq 13/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+6c_3+6c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+7c_3+7c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+7c_4 \geq 13/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+8c_3+6c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+7c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+6c_3+5c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+6c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+8c_4 \geq 13/2$

$(\{\{\},\{3\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{4\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 8c_3 + 8c_4 \geq 13/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 8c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 8c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 8c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 8c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{3\},\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 8c_4 \geq 12/2$
$(\{\{\},\{3\},\{4\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 5c_3 + 6c_4 \geq 9/2$

$(\{\{\},\{3\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{3\},\{4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{3\},\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{3\},\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 4c_3 + 4c_4 \geq 6/2$
$(\{\{\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 5c_3 + 5c_4 \geq 7/2$
$(\{\{\},\{4\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{\},\{4\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 8c_4 \geq 13/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 7c_4 \geq 11/2$

$(\{\{\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 8c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 7c_4 \geq 11/2$

$(\{\{\}, \{4\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{4\}, \{1,2\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{4\}, \{1,2\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{4\}, \{1,2\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{\}, \{4\}, \{1,2\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{4\}, \{1,2\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$4c_1 + 3c_2 + 3c_3 + 4c_4 \geq 6/2$
$(\{\{\}, \{4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$4c_1 + 4c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 3c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 3c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 8c_4 \geq 12/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 3c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\}, \{4\}, \{1,3\}, \{1,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{\}, \{4\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{\}, \{4\}, \{1,3\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$

$(\{\{\},\{4\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 3c_3 + 5c_4 \geq 7/2$
$(\{\{\},\{4\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{\},\{4\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 3c_3 + 6c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 7c_4 \geq 9/2$
$(\{\{\},\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 8c_4 \geq 10/2$
$(\{\{\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 7c_4 \geq 9/2$
$(\{\{\},\{4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{\},\{4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{\},\{4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{\},\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 3c_3 + 5c_4 \geq 7/2$

$(\{\{\},\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{\},\{4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 5c_3 + 7c_4 \geq 9/2$
$(\{\{\},\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{\},\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 3c_3 + 3c_4 \geq 6/2$
$(\{\{1\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 4c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 4c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 3c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 5c_4 \geq 10/2$

$(\{\{1\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{1\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{1\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$

$(\{\{1\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{1\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{1\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 3c_3 + 3c_4 \geq 5/2$
$(\{\{1\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 4c_4 \geq 6/2$
$(\{\{1\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 4c_3 + 3c_4 \geq 6/2$
$(\{\{1\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{1\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{1\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 3c_4 \geq 7/2$

$(\{\{1\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+4c_4 \geq 8/2$
$(\{\{1\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+5c_4 \geq 9/2$
$(\{\{1\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+6c_4 \geq 10/2$
$(\{\{1\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+5c_4 \geq 9/2$
$(\{\{1\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+4c_4 \geq 7/2$
$(\{\{1\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+5c_4 \geq 8/2$
$(\{\{1\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+6c_4 \geq 9/2$
$(\{\{1\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+5c_3+4c_4 \geq 7/2$
$(\{\{1\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+6c_3+5c_4 \geq 8/2$
$(\{\{1\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+3c_3+4c_4 \geq 6/2$
$(\{\{1\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+5c_4 \geq 7/2$
$(\{\{1\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+4c_4 \geq 7/2$
$(\{\{1\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+5c_4 \geq 8/2$
$(\{\{1\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+6c_4 \geq 9/2$
$(\{\{1\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+7c_4 \geq 10/2$
$(\{\{1\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+6c_4 \geq 9/2$
$(\{\{1\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+3c_3+5c_4 \geq 7/2$
$(\{\{1\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+4c_3+6c_4 \geq 8/2$
$(\{\{1\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+7c_4 \geq 9/2$
$(\{\{1\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+4c_3+5c_4 \geq 7/2$
$(\{\{1\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+5c_3+6c_4 \geq 8/2$
$(\{\{1\},\{2\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+3c_3+3c_4 \geq 7/2$
$(\{\{1\},\{2\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+4c_3+4c_4 \geq 8/2$

$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 3c_4 \geq 8/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 5c_2 + 4c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 5c_3 + 4c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 6c_3 + 6c_4 \geq 12/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 7c_3 + 7c_4 \geq 13/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 4c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 3c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 4c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 5c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 7c_3 + 6c_4 \geq 12/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 5c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 3c_3 + 4c_4 \geq 8/2$

$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 5c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 3c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 3c_4 \geq 8/2$
$(\{\{1\},\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 5c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{2\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 3c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{2\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{2\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 6c_3 + 4c_4 \geq 11/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 7c_3 + 5c_4 \geq 12/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 7c_3 + 6c_4 \geq 13/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 8c_3 + 7c_4 \geq 14/2$

$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 8c_3 + 6c_4 \geq 13/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 3c_4 \geq 10/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 4c_4 \geq 11/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 7c_3 + 5c_4 \geq 12/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 8c_3 + 6c_4 \geq 13/2$
$(\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 8c_3 + 5c_4 \geq 12/2$
$(\{\{1\},\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 8c_3 + 8c_4 \geq 15/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 6c_3 + 7c_4 \geq 13/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 8c_2 + 7c_3 + 8c_4 \geq 14/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1 + 6c_2 + 4c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 5c_3 + 7c_4 \geq 12/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1 + 7c_2 + 6c_3 + 8c_4 \geq 13/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 5c_3 + 7c_4 \geq 12/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 8c_4 \geq 13/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 3c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 4c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 8c_4 \geq 12/2$
$(\{\{1\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 3c_4 \geq 6/2$
$(\{\{1\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{1\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 3c_3 + 4c_4 \geq 6/2$

$(\{\{1\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+5c_4 \geq 7/2$
$(\{\{1\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+6c_4 \geq 8/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+5c_3+3c_4 \geq 8/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+6c_3+4c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1+4c_2+5c_3+4c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+5c_2+6c_3+5c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1+5c_2+6c_3+4c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+7c_3+5c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+7c_2+7c_3+6c_4 \geq 12/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+7c_2+8c_3+7c_4 \geq 13/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+8c_3+6c_4 \geq 12/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+6c_3+6c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+7c_3+7c_4 \geq 12/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1+4c_2+6c_3+5c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+5c_2+7c_3+6c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+6c_3+3c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+7c_3+4c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+7c_3+5c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+8c_3+6c_4 \geq 12/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+8c_3+5c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+6c_3+5c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+7c_3+6c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+6c_3+4c_4 \geq 9/2$

$(\{\{1\},\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 5c_3 + 3c_4 \geq 7/2$
$(\{\{1\},\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 8c_3 + 7c_4 \geq 12/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 8c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 3c_4 \geq 8/2$
$(\{\{1\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 4c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 8c_3 + 6c_4 \geq 11/2$
$(\{\{1\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 8c_3 + 5c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1\},\{3\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{1\},\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 7c_3 + 5c_4 \geq 9/2$

$(\{\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+7c_2+8c_3+8c_4 \geq 14/2$
$(\{\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+8c_3+7c_4 \geq 13/2$
$(\{\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+7c_3+8c_4 \geq 13/2$
$(\{\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1+4c_2+6c_3+6c_4 \geq 11/2$
$(\{\{1\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+5c_2+7c_3+7c_4 \geq 12/2$
$(\{\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+8c_3+8c_4 \geq 13/2$
$(\{\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+8c_3+7c_4 \geq 12/2$
$(\{\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+7c_3+8c_4 \geq 12/2$
$(\{\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+3c_2+6c_3+6c_4 \geq 10/2$
$(\{\{1\},\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+7c_3+7c_4 \geq 11/2$
$(\{\{1\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+4c_3+4c_4 \geq 6/2$
$(\{\{1\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+5c_4 \geq 7/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1+4c_2+4c_3+5c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+5c_2+5c_3+6c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+6c_3+6c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+7c_2+6c_3+7c_4 \geq 12/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+7c_2+7c_3+8c_4 \geq 13/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+7c_3+7c_4 \geq 12/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1+5c_2+4c_3+6c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+5c_3+7c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+6c_2+6c_3+8c_4 \geq 12/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$8c_1+4c_2+5c_3+6c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$8c_1+5c_2+6c_3+7c_4 \geq 11/2$

$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 3c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 8c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 8c_4 \geq 12/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 3c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 3c_3 + 5c_4 \geq 7/2$

$(\{\{1\},\{4\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{1\},\{4\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{1\},\{4\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 3c_3 + 6c_4 \geq 8/2$
$(\{\{1\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 7c_4 \geq 9/2$
$(\{\{1\},\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 8c_4 \geq 10/2$
$(\{\{1\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{1\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 7c_4 \geq 9/2$
$(\{\{1,2\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 2c_3 + 2c_4 \geq 4/2$
$(\{\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 3c_3 + 3c_4 \geq 5/2$
$(\{\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 4c_4 \geq 6/2$
$(\{\{1,2\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 3c_3 + 3c_4 \geq 5/2$
$(\{\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 3c_4 \geq 6/2$
$(\{\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 6c_4 \geq 10/2$

$(\{\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 3c_4 \geq 7/2$
$(\{\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 5c_4 \geq 9/2$
$(\{\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 3c_3 + 4c_4 \geq 6/2$
$(\{\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 5c_4 \geq 7/2$

$(\{\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 7c_4 \geq 9/2$
$(\{\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 3c_3 + 2c_4 \geq 5/2$
$(\{\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 3c_4 \geq 6/2$
$(\{\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 4c_3 + 3c_4 \geq 6/2$
$(\{\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 7c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 2c_3 + 3c_4 \geq 5/2$
$(\{\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 3c_3 + 4c_4 \geq 6/2$
$(\{\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 3c_3 + 4c_4 \geq 6/2$
$(\{\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 4c_4 \geq 6/2$
$(\{\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 5c_4 \geq 7/2$
$(\{\{1,2,3\},\{1,2,3,4\}\})$
$2c_1 + 2c_2 + 2c_3 + 1c_4 \geq 2/2$
$(\{\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$3c_1 + 3c_2 + 2c_3 + 2c_4 \geq 3/2$
$(\{\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 3c_3 + 3c_4 \geq 4/2$
$(\{\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 4c_3 + 4c_4 \geq 5/2$

$(\{\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1 + 4c_2 + 3c_3 + 3c_4 \geq 4/2$
$(\{\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1 + 2c_2 + 3c_3 + 2c_4 \geq 3/2$
$(\{\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1 + 3c_2 + 4c_3 + 3c_4 \geq 4/2$
$(\{\{1,2,3\},\{2,3,4\},\{1,2,3,4\}\})$
$2c_1 + 3c_2 + 3c_3 + 2c_4 \geq 3/2$
$(\{\{1,2,3,4\}\})$
$1c_1 + 1c_2 + 1c_3 + 1c_4 \geq 1/2$
$(\{\{1,2,4\},\{1,2,3,4\}\})$
$2c_1 + 2c_2 + 1c_3 + 2c_4 \geq 2/2$
$(\{\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1 + 2c_2 + 2c_3 + 3c_4 \geq 3/2$
$(\{\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1 + 3c_2 + 3c_3 + 4c_4 \geq 4/2$
$(\{\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$2c_1 + 3c_2 + 2c_3 + 3c_4 \geq 3/2$
$(\{\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 4c_3 + 3c_4 \geq 5/2$
$(\{\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 4c_4 \geq 6/2$
$(\{\{1,3\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 2c_2 + 4c_3 + 2c_4 \geq 4/2$
$(\{\{1,3\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 5c_3 + 3c_4 \geq 5/2$
$(\{\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 4c_3 + 4c_4 \geq 6/2$
$(\{\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 5c_4 \geq 7/2$
$(\{\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 7c_4 \geq 10/2$
$(\{\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 6c_4 \geq 9/2$
$(\{\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 7c_4 \geq 9/2$

$(\{\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 5c_3 + 5c_4 \geq 7/2$
$(\{\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 6c_4 \geq 8/2$
$(\{\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 3c_4 \geq 6/2$
$(\{\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 4c_4 \geq 7/2$
$(\{\{1,3\},\{2,3\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 5c_3 + 2c_4 \geq 5/2$
$(\{\{1,3\},\{2,3\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 6c_3 + 3c_4 \geq 6/2$
$(\{\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 6c_4 \geq 9/2$
$(\{\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 5c_4 \geq 8/2$
$(\{\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 7c_3 + 4c_4 \geq 7/2$
$(\{\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 4c_4 \geq 6/2$
$(\{\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 5c_4 \geq 7/2$
$(\{\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 6c_4 \geq 8/2$
$(\{\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 5c_3 + 4c_4 \geq 6/2$
$(\{\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 6c_3 + 5c_4 \geq 7/2$
$(\{\{1,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 2c_2 + 5c_3 + 3c_4 \geq 5/2$
$(\{\{1,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 6c_3 + 4c_4 \geq 6/2$
$(\{\{1,3,4\},\{1,2,3,4\}\})$
$2c_1 + 1c_2 + 2c_3 + 2c_4 \geq 2/2$
$(\{\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$2c_1 + 2c_2 + 3c_3 + 3c_4 \geq 3/2$
$(\{\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 3c_3 + 4c_4 \geq 5/2$
$(\{\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 5c_4 \geq 6/2$
$(\{\{1,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 2c_2 + 2c_3 + 4c_4 \geq 4/2$
$(\{\{1,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 3c_3 + 5c_4 \geq 5/2$

$(\{\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+4c_3+4c_4 \geq 6/2$
$(\{\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+5c_4 \geq 7/2$
$(\{\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+6c_4 \geq 8/2$
$(\{\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+7c_4 \geq 9/2$
$(\{\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+6c_4 \geq 8/2$
$(\{\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+3c_3+5c_4 \geq 6/2$
$(\{\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+6c_4 \geq 7/2$
$(\{\{1,4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+2c_3+5c_4 \geq 5/2$
$(\{\{1,4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+3c_3+6c_4 \geq 6/2$
$(\{\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+7c_4 \geq 8/2$
$(\{\{1,4\},\{2,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+4c_3+7c_4 \geq 7/2$
$(\{\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+4c_3+5c_4 \geq 6/2$
$(\{\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+6c_4 \geq 7/2$
$(\{\{1,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+2c_2+3c_3+5c_4 \geq 5/2$
$(\{\{1,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+4c_3+6c_4 \geq 6/2$
$(\{\{2\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+2c_3+2c_4 \geq 5/2$
$(\{\{2\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+3c_3+3c_4 \geq 6/2$
$(\{\{2\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+4c_3+4c_4 \geq 7/2$
$(\{\{2\},\{1,2\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+3c_3+3c_4 \geq 6/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+4c_3+3c_4 \geq 7/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+4c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+4c_3+4c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+5c_3+5c_4 \geq 9/2$

$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 5c_3 + 4c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 3c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 6c_3 + 5c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{2\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 5c_3 + 6c_4 \geq 10/2$

$(\{\{2\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 3c_3 + 2c_4 \geq 6/2$
$(\{\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 7c_2 + 4c_3 + 3c_4 \geq 7/2$
$(\{\{2\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 8c_2 + 5c_3 + 5c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 8c_2 + 4c_3 + 4c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 8c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 2c_3 + 3c_4 \geq 6/2$
$(\{\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{2\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 7c_2 + 3c_3 + 4c_4 \geq 7/2$
$(\{\{2\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{2\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{2\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$3c_1 + 4c_2 + 2c_3 + 2c_4 \geq 4/2$
$(\{\{2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 3c_3 + 3c_4 \geq 5/2$
$(\{\{2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 4c_3 + 4c_4 \geq 6/2$

$(\{\{2\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+5c_2+3c_3+3c_4 \geq 5/2$
$(\{\{2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+4c_3+3c_4 \geq 6/2$
$(\{\{2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+4c_4 \geq 7/2$
$(\{\{2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+4c_4 \geq 7/2$
$(\{\{2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+5c_4 \geq 8/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+4c_4 \geq 8/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+5c_4 \geq 9/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+6c_3+6c_4 \geq 10/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+7c_4 \geq 11/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+6c_4 \geq 10/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+4c_3+5c_4 \geq 8/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+6c_4 \geq 9/2$
$(\{\{2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+7c_4 \geq 10/2$
$(\{\{2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+6c_4 \geq 9/2$
$(\{\{2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+3c_4 \geq 7/2$
$(\{\{2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+4c_4 \geq 8/2$
$(\{\{2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+5c_4 \geq 9/2$
$(\{\{2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+6c_4 \geq 10/2$
$(\{\{2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+5c_4 \geq 9/2$
$(\{\{2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+4c_4 \geq 7/2$
$(\{\{2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+5c_4 \geq 8/2$
$(\{\{2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+6c_4 \geq 9/2$
$(\{\{2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+5c_4 \geq 8/2$

$(\{\{2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+3c_3+4c_4 \geq 6/2$
$(\{\{2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+5c_4 \geq 7/2$
$(\{\{2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+4c_4 \geq 7/2$
$(\{\{2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+5c_4 \geq 8/2$
$(\{\{2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+5c_3+6c_4 \geq 9/2$
$(\{\{2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+7c_4 \geq 10/2$
$(\{\{2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+6c_4 \geq 9/2$
$(\{\{2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+3c_3+5c_4 \geq 7/2$
$(\{\{2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+4c_3+6c_4 \geq 8/2$
$(\{\{2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+7c_4 \geq 9/2$
$(\{\{2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+6c_4 \geq 8/2$
$(\{\{2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$3c_1+5c_2+3c_3+2c_4 \geq 5/2$
$(\{\{2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+4c_3+3c_4 \geq 6/2$
$(\{\{2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+4c_4 \geq 7/2$
$(\{\{2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+6c_2+4c_3+3c_4 \geq 6/2$
$(\{\{2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+5c_3+5c_4 \geq 8/2$
$(\{\{2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+7c_2+4c_3+4c_4 \geq 7/2$
$(\{\{2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+6c_3+6c_4 \geq 9/2$
$(\{\{2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+5c_4 \geq 8/2$
$(\{\{2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$3c_1+5c_2+2c_3+3c_4 \geq 5/2$
$(\{\{2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+3c_3+4c_4 \geq 6/2$
$(\{\{2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+4c_3+5c_4 \geq 7/2$
$(\{\{2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+6c_2+3c_3+4c_4 \geq 6/2$

$(\{\{2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+6c_4 \geq 8/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+6c_3+4c_4 \geq 10/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+7c_3+5c_4 \geq 11/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+7c_3+6c_4 \geq 12/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+8c_3+7c_4 \geq 13/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+8c_3+6c_4 \geq 12/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+3c_4 \geq 9/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+4c_4 \geq 10/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+5c_4 \geq 11/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+8c_3+6c_4 \geq 12/2$
$(\{\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+5c_4 \geq 11/2$
$(\{\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+5c_3+4c_4 \geq 9/2$
$(\{\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+6c_3+5c_4 \geq 10/2$
$(\{\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+6c_3+6c_4 \geq 11/2$
$(\{\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+7c_4 \geq 12/2$
$(\{\{2\},\{3\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+6c_4 \geq 11/2$
$(\{\{2\},\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+3c_4 \geq 8/2$
$(\{\{2\},\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+4c_4 \geq 9/2$
$(\{\{2\},\{3\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+8c_2+6c_3+5c_4 \geq 10/2$
$(\{\{2\},\{3\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+8c_2+7c_3+6c_4 \geq 11/2$
$(\{\{2\},\{3\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+5c_4 \geq 10/2$
$(\{\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+4c_4 \geq 9/2$
$(\{\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+5c_4 \geq 10/2$

$(\{\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+6c_4\geq 11/2$
$(\{\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+7c_4\geq 12/2$
$(\{\{2\},\{3\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+8c_3+6c_4\geq 11/2$
$(\{\{2\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+3c_4\geq 8/2$
$(\{\{2\},\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+4c_4\geq 9/2$
$(\{\{2\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+5c_4\geq 10/2$
$(\{\{2\},\{3\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+8c_3+6c_4\geq 11/2$
$(\{\{2\},\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+8c_3+5c_4\geq 10/2$
$(\{\{2\},\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+4c_4\geq 8/2$
$(\{\{2\},\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+5c_4\geq 9/2$
$(\{\{2\},\{3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+6c_4\geq 10/2$
$(\{\{2\},\{3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+7c_4\geq 11/2$
$(\{\{2\},\{3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+6c_4\geq 10/2$
$(\{\{2\},\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+3c_4\geq 7/2$
$(\{\{2\},\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+4c_4\geq 8/2$
$(\{\{2\},\{3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+6c_3+5c_4\geq 9/2$
$(\{\{2\},\{3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+7c_2+7c_3+6c_4\geq 10/2$
$(\{\{2\},\{3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+7c_3+5c_4\geq 9/2$
$(\{\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+8c_2+8c_3+8c_4\geq 14/2$
$(\{\{2\},\{3\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+8c_3+7c_4\geq 13/2$
$(\{\{2\},\{3\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+8c_2+7c_3+8c_4\geq 13/2$
$(\{\{2\},\{3\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+8c_2+7c_3+7c_4\geq 12/2$
$(\{\{2\},\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+8c_4\geq 13/2$

$(\{\{2\},\{3\},\{4\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 8c_3 + 7c_4 \geq 12/2$
$(\{\{2\},\{3\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 7c_3 + 8c_4 \geq 12/2$
$(\{\{2\},\{3\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 7c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 5c_3 + 5c_4 \geq 7/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 6c_3 + 7c_4 \geq 12/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 8c_2 + 7c_3 + 8c_4 \geq 13/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 4c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 6c_3 + 6c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 7c_3 + 7c_4 \geq 12/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 5c_4 \geq 9/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 5c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 8c_2 + 6c_3 + 8c_4 \geq 12/2$
$(\{\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 3c_3 + 6c_4 \geq 9/2$
$(\{\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 4c_3 + 7c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 8c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 8c_2 + 5c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 8c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 3c_3 + 5c_4 \geq 8/2$
$(\{\{2\},\{4\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 4c_3 + 6c_4 \geq 9/2$

$(\{\{2\},\{4\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 8c_4 \geq 12/2$
$(\{\{2\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 6c_4 \geq 9/2$
$(\{\{2\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{2\},\{4\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{2\},\{4\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{2\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{2\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 3c_3 + 6c_4 \geq 8/2$
$(\{\{2\},\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 4c_3 + 7c_4 \geq 9/2$
$(\{\{2\},\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 8c_4 \geq 10/2$
$(\{\{2\},\{4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 7c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{2\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 7c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{2\},\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 5c_2 + 3c_3 + 5c_4 \geq 7/2$
$(\{\{2\},\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{2\},\{4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 6c_2 + 5c_3 + 7c_4 \geq 9/2$
$(\{\{2,3\},\{1,2,3\},\{1,2,3,4\}\})$
$2c_1 + 3c_2 + 3c_3 + 1c_4 \geq 3/2$
$(\{\{2,3\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$3c_1 + 4c_2 + 3c_3 + 2c_4 \geq 4/2$

$(\{\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+4c_3+3c_4\geq 5/2$
$(\{\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+4c_4\geq 6/2$
$(\{\{2,3\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+5c_2+4c_3+3c_4\geq 5/2$
$(\{\{2,3\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+4c_3+2c_4\geq 4/2$
$(\{\{2,3\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+5c_3+3c_4\geq 5/2$
$(\{\{2,3\},\{1,2,3\},\{2,3,4\},\{1,2,3,4\}\})$
$2c_1+4c_2+4c_3+2c_4\geq 4/2$
$(\{\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+5c_4\geq 7/2$
$(\{\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+6c_2+4c_3+4c_4\geq 6/2$
$(\{\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+6c_4\geq 8/2$
$(\{\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+5c_4\geq 7/2$
$(\{\{2,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+6c_3+4c_4\geq 6/2$
$(\{\{2,3,4\},\{1,2,3,4\}\})$
$1c_1+2c_2+2c_3+2c_4\geq 2/2$
$(\{\{2,4\},\{1,2,3\},\{1,2,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+2c_3+3c_4\geq 4/2$
$(\{\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+3c_3+4c_4\geq 5/2$
$(\{\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+4c_3+5c_4\geq 6/2$
$(\{\{2,4\},\{1,2,3\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+5c_2+3c_3+4c_4\geq 5/2$
$(\{\{2,4\},\{1,2,4\},\{1,2,3,4\}\})$
$2c_1+3c_2+1c_3+3c_4\geq 3/2$
$(\{\{2,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+2c_3+4c_4\geq 4/2$
$(\{\{2,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+3c_3+5c_4\geq 5/2$
$(\{\{2,4\},\{1,2,4\},\{2,3,4\},\{1,2,3,4\}\})$
$2c_1+4c_2+2c_3+4c_4\geq 4/2$
$(\{\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+6c_4\geq 7/2$
$(\{\{2,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+4c_3+6c_4\geq 6/2$
$(\{\{3\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+4c_3+3c_4\geq 6/2$

$(\{\{3\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 3c_4 \geq 7/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 4c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 7c_3 + 6c_4 \geq 11/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 7c_2 + 8c_3 + 7c_4 \geq 12/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 8c_3 + 6c_4 \geq 11/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 3c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 4c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 5c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 8c_3 + 6c_4 \geq 11/2$
$(\{\{3\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 8c_3 + 5c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 7c_3 + 5c_4 \geq 9/2$

$(\{\{3\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 4c_4 \geq 7/2$
$(\{\{3\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 4c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{3\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 3c_4 \geq 7/2$
$(\{\{3\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 4c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 7c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{3\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 5c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{3\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{3\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 5c_3 + 4c_4 \geq 7/2$
$(\{\{3\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 4c_3 + 3c_4 \geq 5/2$
$(\{\{3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 4c_2 + 5c_3 + 4c_4 \geq 6/2$
$(\{\{3\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1 + 2c_2 + 4c_3 + 2c_4 \geq 4/2$

$(\{\{3\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+5c_3+3c_4 \geq 5/2$
$(\{\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+5c_3+3c_4 \geq 6/2$
$(\{\{3\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+6c_3+4c_4 \geq 7/2$
$(\{\{3\},\{1,3\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+2c_2+5c_3+2c_4 \geq 5/2$
$(\{\{3\},\{1,3\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+6c_3+3c_4 \geq 6/2$
$(\{\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+5c_3+4c_4 \geq 7/2$
$(\{\{3\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+6c_3+5c_4 \geq 8/2$
$(\{\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+6c_3+4c_4 \geq 8/2$
$(\{\{3\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+5c_4 \geq 9/2$
$(\{\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+6c_4 \geq 10/2$
$(\{\{3\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+8c_3+7c_4 \geq 11/2$
$(\{\{3\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+8c_3+6c_4 \geq 10/2$
$(\{\{3\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+6c_4 \geq 9/2$
$(\{\{3\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+7c_4 \geq 10/2$
$(\{\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+6c_3+5c_4 \geq 8/2$
$(\{\{3\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+7c_3+6c_4 \geq 9/2$
$(\{\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+6c_3+3c_4 \geq 7/2$
$(\{\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+7c_3+4c_4 \geq 8/2$
$(\{\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+6c_3+2c_4 \geq 6/2$
$(\{\{3\},\{1,3\},\{2,3\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+7c_3+3c_4 \geq 7/2$
$(\{\{3\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+5c_4 \geq 9/2$
$(\{\{3\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+8c_3+6c_4 \geq 10/2$
$(\{\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+8c_3+5c_4 \geq 9/2$

$(\{\{3\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+8c_3+4c_4 \geq 8/2$
$(\{\{3\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+5c_4 \geq 8/2$
$(\{\{3\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+7c_3+6c_4 \geq 9/2$
$(\{\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+6c_3+4c_4 \geq 7/2$
$(\{\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+7c_3+5c_4 \geq 8/2$
$(\{\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+2c_2+6c_3+3c_4 \geq 6/2$
$(\{\{3\},\{1,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+7c_3+4c_4 \geq 7/2$
$(\{\{3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+4c_3+4c_4 \geq 6/2$
$(\{\{3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+5c_4 \geq 7/2$
$(\{\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+4c_4 \geq 7/2$
$(\{\{3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+5c_4 \geq 8/2$
$(\{\{3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+6c_4 \geq 9/2$
$(\{\{3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+7c_4 \geq 10/2$
$(\{\{3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+7c_3+6c_4 \geq 9/2$
$(\{\{3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+6c_4 \geq 8/2$
$(\{\{3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
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$(\{\{3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+5c_3+5c_4 \geq 7/2$
$(\{\{3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+6c_3+6c_4 \geq 8/2$
$(\{\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+5c_3+3c_4 \geq 6/2$
$(\{\{3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+4c_4 \geq 7/2$
$(\{\{3\},\{2,3\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+5c_3+2c_4 \geq 5/2$
$(\{\{3\},\{2,3\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+6c_3+3c_4 \geq 6/2$
$(\{\{3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+5c_4 \geq 8/2$

$(\{\{3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+7c_3+6c_4 \geq 9/2$
$(\{\{3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+7c_3+5c_4 \geq 8/2$
$(\{\{3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+7c_3+4c_4 \geq 7/2$
$(\{\{3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+5c_4 \geq 7/2$
$(\{\{3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+6c_4 \geq 8/2$
$(\{\{3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+5c_3+4c_4 \geq 6/2$
$(\{\{3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+6c_3+5c_4 \geq 7/2$
$(\{\{3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+2c_2+5c_3+3c_4 \geq 5/2$
$(\{\{3\},\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+6c_3+4c_4 \geq 6/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+8c_3+8c_4 \geq 13/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+8c_3+7c_4 \geq 12/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+7c_3+8c_4 \geq 12/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+6c_3+6c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+7c_3+7c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+8c_3+7c_4 \geq 12/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+8c_3+6c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+7c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+6c_3+5c_4 \geq 9/2$
$(\{\{3\},\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+6c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+7c_2+7c_3+8c_4 \geq 12/2$
$(\{\{3\},\{4\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+7c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+8c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+5c_3+6c_4 \geq 9/2$

$(\{\{3\},\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+7c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+7c_3+7c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+6c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+7c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+5c_4 \geq 8/2$
$(\{\{3\},\{4\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+6c_3+6c_4 \geq 9/2$
$(\{\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+8c_3+8c_4 \geq 12/2$
$(\{\{3\},\{4\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+8c_3+7c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+8c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+6c_3+6c_4 \geq 9/2$
$(\{\{3\},\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+7c_3+7c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
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$(\{\{3\},\{4\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
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$(\{\{3\},\{4\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+7c_3+7c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+6c_3+5c_4 \geq 8/2$
$(\{\{3\},\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+7c_3+6c_4 \geq 9/2$
$(\{\{3\},\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+7c_3+8c_4 \geq 11/2$
$(\{\{3\},\{4\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+7c_3+7c_4 \geq 10/2$
$(\{\{3\},\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
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$(\{\{3\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+5c_3+6c_4 \geq 8/2$
$(\{\{3\},\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
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$(\{\{3\},\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
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$(\{\{3\},\{4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+7c_3+6c_4 \geq 9/2$

$(\{\{3\},\{4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+7c_4 \geq 9/2$
$(\{\{3\},\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+5c_3+5c_4 \geq 7/2$
$(\{\{3\},\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+6c_3+6c_4 \geq 8/2$
$(\{\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+4c_3+4c_4 \geq 5/2$
$(\{\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+5c_3+5c_4 \geq 6/2$
$(\{\{3,4\},\{1,2,3\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+2c_2+4c_3+3c_4 \geq 4/2$
$(\{\{3,4\},\{1,2,3\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+5c_3+4c_4 \geq 5/2$
$(\{\{3,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+2c_2+3c_3+4c_4 \geq 4/2$
$(\{\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+4c_3+5c_4 \geq 5/2$
$(\{\{3,4\},\{1,3,4\},\{1,2,3,4\}\})$
$2c_1+1c_2+3c_3+3c_4 \geq 3/2$
$(\{\{3,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$2c_1+2c_2+4c_3+4c_4 \geq 4/2$
$(\{\{4\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+3c_3+4c_4 \geq 6/2$
$(\{\{4\},\{1,2\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+4c_3+5c_4 \geq 7/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+4c_4 \geq 7/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+5c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+4c_2+4c_3+5c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+5c_3+6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+6c_3+6c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+6c_3+7c_4 \geq 11/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+7c_2+7c_3+8c_4 \geq 12/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+7c_3+7c_4 \geq 11/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1+5c_2+4c_3+6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1+6c_2+5c_3+7c_4 \geq 10/2$

$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 6c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$7c_1 + 4c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$7c_1 + 5c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 5c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 6c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 7c_3 + 7c_4 \geq 11/2$
$(\{\{4\},\{1,2\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 7c_3 + 6c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 5c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 5c_3 + 5c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 3c_3 + 5c_4 \geq 7/2$
$(\{\{4\},\{1,2\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 5c_3 + 7c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 7c_2 + 6c_3 + 8c_4 \geq 11/2$
$(\{\{4\},\{1,2\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 6c_3 + 7c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 3c_3 + 6c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 4c_3 + 7c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 6c_2 + 5c_3 + 8c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 4c_3 + 6c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 5c_3 + 7c_4 \geq 9/2$

$(\{\{4\},\{1,2\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+5c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+5c_3+6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+7c_2+6c_3+7c_4 \geq 10/2$
$(\{\{4\},\{1,2\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+6c_3+6c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+3c_3+5c_4 \geq 7/2$
$(\{\{4\},\{1,2\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+4c_3+6c_4 \geq 8/2$
$(\{\{4\},\{1,2\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+6c_2+5c_3+7c_4 \geq 9/2$
$(\{\{4\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+4c_3+5c_4 \geq 7/2$
$(\{\{4\},\{1,2\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+6c_4 \geq 8/2$
$(\{\{4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+3c_3+4c_4 \geq 5/2$
$(\{\{4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+4c_3+5c_4 \geq 6/2$
$(\{\{4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+2c_2+2c_3+4c_4 \geq 4/2$
$(\{\{4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+3c_3+5c_4 \geq 5/2$
$(\{\{4\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+4c_3+4c_4 \geq 6/2$
$(\{\{4\},\{1,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+5c_4 \geq 7/2$
$(\{\{4\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+3c_2+4c_3+5c_4 \geq 7/2$
$(\{\{4\},\{1,3\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+5c_3+6c_4 \geq 8/2$
$(\{\{4\},\{1,3\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+6c_3+6c_4 \geq 9/2$
$(\{\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+6c_3+7c_4 \geq 10/2$
$(\{\{4\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+6c_2+7c_3+8c_4 \geq 11/2$
$(\{\{4\},\{1,3\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+7c_3+7c_4 \geq 10/2$
$(\{\{4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1+4c_2+4c_3+6c_4 \geq 8/2$
$(\{\{4\},\{1,3\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1+5c_2+5c_3+7c_4 \geq 9/2$

$(\{\{4\},\{1,3\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 5c_2 + 6c_3 + 8c_4 \geq 10/2$
$(\{\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$6c_1 + 3c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{4\},\{1,3\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$6c_1 + 4c_2 + 6c_3 + 7c_4 \geq 9/2$
$(\{\{4\},\{1,3\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 5c_4 \geq 8/2$
$(\{\{4\},\{1,3\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 6c_4 \geq 9/2$
$(\{\{4\},\{1,3\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 7c_3 + 7c_4 \geq 10/2$
$(\{\{4\},\{1,3\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 7c_3 + 6c_4 \geq 9/2$
$(\{\{4\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 5c_4 \geq 7/2$
$(\{\{4\},\{1,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{4\},\{1,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 7c_4 \geq 9/2$
$(\{\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 5c_3 + 5c_4 \geq 7/2$
$(\{\{4\},\{1,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 6c_3 + 6c_4 \geq 8/2$
$(\{\{4\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 3c_2 + 3c_3 + 5c_4 \geq 6/2$
$(\{\{4\},\{1,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 4c_3 + 6c_4 \geq 7/2$
$(\{\{4\},\{1,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 2c_2 + 2c_3 + 5c_4 \geq 5/2$
$(\{\{4\},\{1,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 3c_3 + 6c_4 \geq 6/2$
$(\{\{4\},\{1,4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 5c_3 + 6c_4 \geq 8/2$
$(\{\{4\},\{1,4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 5c_3 + 7c_4 \geq 9/2$
$(\{\{4\},\{1,4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 6c_2 + 6c_3 + 8c_4 \geq 10/2$
$(\{\{4\},\{1,4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 6c_3 + 7c_4 \geq 9/2$
$(\{\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1 + 4c_2 + 3c_3 + 6c_4 \geq 7/2$
$(\{\{4\},\{1,4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1 + 5c_2 + 4c_3 + 7c_4 \geq 8/2$
$(\{\{4\},\{1,4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1 + 3c_2 + 2c_3 + 6c_4 \geq 6/2$

$(\{\{4\},\{1,4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+3c_3+7c_4 \geq 7/2$
$(\{\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+5c_2+5c_3+8c_4 \geq 9/2$
$(\{\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+4c_3+8c_4 \geq 8/2$
$(\{\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$5c_1+3c_2+4c_3+6c_4 \geq 7/2$
$(\{\{4\},\{1,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$5c_1+4c_2+5c_3+7c_4 \geq 8/2$
$(\{\{4\},\{1,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+2c_2+3c_3+6c_4 \geq 6/2$
$(\{\{4\},\{1,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+4c_3+7c_4 \geq 7/2$
$(\{\{4\},\{2,3\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+5c_4 \geq 7/2$
$(\{\{4\},\{2,3\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+5c_3+6c_4 \geq 8/2$
$(\{\{4\},\{2,3\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+6c_2+6c_3+7c_4 \geq 9/2$
$(\{\{4\},\{2,3\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+6c_3+6c_4 \geq 8/2$
$(\{\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+3c_3+5c_4 \geq 6/2$
$(\{\{4\},\{2,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+4c_3+6c_4 \geq 7/2$
$(\{\{4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+2c_3+5c_4 \geq 5/2$
$(\{\{4\},\{2,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+3c_3+6c_4 \geq 6/2$
$(\{\{4\},\{2,4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+5c_2+5c_3+7c_4 \geq 8/2$
$(\{\{4\},\{2,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+4c_2+4c_3+7c_4 \geq 7/2$
$(\{\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$4c_1+3c_2+4c_3+5c_4 \geq 6/2$
$(\{\{4\},\{3,4\},\{1,2,3\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$4c_1+4c_2+5c_3+6c_4 \geq 7/2$
$(\{\{4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{1,2,3,4\}\})$
$3c_1+2c_2+3c_3+5c_4 \geq 5/2$
$(\{\{4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\},\{1,2,3,4\}\})$
$3c_1+3c_2+4c_3+6c_4 \geq 6$
$c_1,c_2,c_3,c_4 \geq 0$

A.2 Facets of $P(4, 8)$

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	b
Rank 0																
-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1
0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	1
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	-1	1
0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	-1	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	1
0	1	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Rank 1																
0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	-2	1
0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	-2	1
0	0	0	0	0	0	0	0	1	0	1	0	1	0	-1	-1	1
0	0	0	0	0	0	0	1	0	1	1	0	-1	-1	-1	0	1
0	0	0	0	0	0	1	0	1	1	0	-1	0	0	-1	-1	1
0	0	0	0	0	0	1	1	0	0	1	0	0	-2	0	0	1
0	0	0	0	0	1	0	1	1	0	2	0	0	-1	-1	-1	2
0	0	0	0	0	1	0	1	1	1	1	0	-1	-1	-1	0	2
0	0	0	0	1	1	1	0	0	0	0	-1	-1	-1	0	0	1
0	0	0	0	1	1	1	0	0	1	1	0	-1	-1	-1	0	2
0	0	0	1	1	1	2	1	1	2	-1	-1	-1	-1	-1	0	4
0	0	1	1	1	0	0	0	-1	-1	-1	0	0	0	0	0	1
0	1	0	0	1	0	0	-1	1	0	0	-1	0	0	-1	0	1
0	1	0	1	0	1	-1	0	1	0	1	-1	0	-1	-1	0	2
0	1	1	1	0	0	-1	1	-1	0	1	0	-1	-1	0	0	2
0	1	1	1	1	1	-1	-1	1	-1	1	-1	0	0	-1	0	3
0	1	2	1	1	-1	0	0	-1	-1	0	0	0	-1	0	0	2
Rank 2																
0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-3	1
0	0	0	0	0	0	0	1	0	1	1	1	-1	-1	-1	-1	1

0	0	0	0	0	1	0	1	2	0	2	-1	0	-1	-2	-1	2
0	0	0	0	0	1	0	2	1	2	2	0	-3	-2	-3	-1	2
0	0	0	0	0	1	1	0	0	1	1	-1	-1	-1	-1	-1	1
0	0	0	0	0	1	1	1	1	0	0	-2	-1	-1	0	-1	1
0	0	0	0	0	1	1	1	1	0	1	-2	-1	-2	-1	-1	1
0	0	0	0	0	2	0	2	2	0	2	-1	-1	-1	-1	-1	3
0	0	0	0	0	3	3	1	0	2	2	-2	-2	-2	0	-1	4
0	0	0	0	1	0	1	1	1	1	1	0	-1	-2	-2	0	2
0	0	0	0	1	0	2	1	2	1	1	-1	0	-2	-2	0	3
0	0	0	0	1	1	1	2	1	2	2	0	-3	-3	-3	0	3
0	0	0	1	1	1	2	2	2	1	-1	-2	-1	-1	-1	0	4
0	0	0	1	1	1	2	2	2	2	-1	-2	-2	-1	-1	0	4
0	0	1	0	0	0	1	1	0	0	1	-1	-1	-2	-1	0	1
0	0	1	0	1	1	2	1	1	-1	1	-3	-1	-3	-1	0	2
0	0	1	1	1	1	1	1	-1	0	-1	-1	-2	-1	0	0	2
0	0	1	1	1	1	2	2	1	-1	-1	-3	-1	-2	-1	0	3
0	1	0	0	1	1	1	-1	0	1	1	-1	-1	-1	-1	0	2
0	1	0	0	1	2	1	0	3	2	1	-4	-3	-1	-4	0	3
0	1	0	1	0	0	-1	1	1	2	1	-1	-2	-1	-3	0	2
0	1	1	0	1	1	1	-1	1	-1	2	-2	-1	-1	-1	0	3
0	1	1	1	0	-1	-1	2	1	2	2	-1	-2	-2	-4	0	3
0	1	1	1	0	2	1	0	-1	3	2	-2	-3	-1	-2	0	4
0	1	1	1	1	-1	-1	-1	-1	-1	-1	0	0	0	0	0	1
0	2	2	2	2	-1	-1	-1	-1	-1	-1	0	0	0	0	0	3

Rank 3

0	0	0	0	0	1	1	1	1	1	1	-2	-2	-2	-2	-1	1
0	0	0	0	0	3	3	3	1	1	1	-4	-4	-4	0	-1	3
0	1	1	0	1	1	4	1	4	1	4	-6	-1	-6	-6	0	5

Table A.1: The 63 classes of facet-defining inequalities of $P(4,8)$ ordered by Chvátal rank. Each row corresponds to the inequality $c_1x_\emptyset + c_2x_{\{1\}} + c_3x_{\{2\}} + c_4x_{\{3\}} + c_5x_{\{4\}} + c_6x_{\{1,2\}} + c_7x_{\{1,3\}} + c_8x_{\{1,4\}} + c_9x_{\{2,3\}} + c_{10}x_{\{2,4\}} + c_{11}x_{\{3,4\}} + c_{12}x_{\{1,2,3\}} + c_{13}x_{\{1,2,4\}} + c_{14}x_{\{1,3,4\}} + c_{15}x_{\{2,3,4\}} + c_{16}x_{\{1,2,3,4\}} \leq b$.

A.3 Facets of $P(4,7)$

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	b
Rank 0																
-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1
0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	1
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	-1	1
0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	-1	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	1
0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1	7
0	1	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Rank 1																
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-2	1
0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	-2	1
0	0	0	0	0	0	0	0	1	0	1	0	1	0	-1	-1	1
0	0	0	0	0	0	0	0	1	1	1	0	0	0	-2	0	1
0	0	0	0	0	0	0	1	0	1	1	0	-1	-1	-1	0	1
0	0	0	0	0	0	0	1	1	0	1	0	0	-1	-1	-1	1
0	0	0	0	0	0	1	1	1	1	1	-1	0	-1	-1	0	2
0	0	0	0	0	1	0	1	2	0	1	-1	0	0	-1	-1	2
0	0	0	0	1	0	0	1	2	1	1	0	0	0	-2	0	3
0	0	0	0	1	0	2	1	0	1	1	1	0	-2	0	-1	3
0	0	0	0	1	1	1	0	0	1	1	0	-1	-1	-1	0	2
0	0	0	1	0	0	1	0	1	0	1	0	0	0	0	0	3
0	0	0	1	0	0	1	0	1	0	1	0	3	0	0	-3	3
0	0	0	1	0	1	0	0	0	1	0	-1	-1	0	-1	0	1
0	0	0	1	0	1	1	1	1	1	2	0	-1	-1	-1	0	4
0	0	0	1	0	1	1	2	2	1	1	-1	-1	-1	-1	0	4
0	0	0	1	1	0	0	1	0	1	0	0	0	0	0	0	3
0	0	0	1	1	1	1	1	1	0	0	-1	0	-1	0	0	3
0	0	1	0	1	1	0	0	1	0	1	0	0	1	-1	-1	3

0	0	1	0	1	1	1	2	2	0	1	-1	-1	-1	-1	0	4
0	0	1	0	1	1	2	0	1	0	0	-2	0	0	0	0	3
0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	3
0	1	0	0	0	1	1	1	0	1	2	0	-1	-2	0	0	3
0	1	0	0	1	1	0	-1	0	1	1	0	-1	-1	-1	0	2
0	1	0	0	1	1	1	0	0	0	0	0	0	0	2	-2	3
0	1	1	1	1	1	0	-1	0	0	1	0	1	1	0	-2	4
0	1	1	1	1	1	1	-1	-1	-1	1	-1	0	-1	0	0	3
0	1	1	1	1	2	-1	0	0	1	2	1	-1	1	-1	-2	5
0	1	1	2	1	0	-1	0	-1	0	-1	0	-1	0	0	0	2

Rank 2

0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-3	1
0	0	0	0	0	0	1	1	1	1	1	-1	-1	-2	-2	-1	1
0	0	0	0	0	0	1	2	1	2	0	0	-2	-1	-1	-1	2
0	0	0	0	0	1	0	0	1	1	1	-1	-1	0	-2	-1	1
0	0	0	0	0	1	0	1	1	0	1	-1	-1	-1	-1	-1	1
0	0	0	0	0	1	1	1	0	0	0	-1	-1	-1	1	-1	1
0	0	0	0	0	2	0	3	2	1	3	0	-2	-2	-2	-1	4
0	0	0	0	0	2	2	0	0	2	2	-1	-1	-1	-1	-1	3
0	0	0	0	0	2	2	2	0	1	1	-2	-3	-3	0	-1	2
0	0	0	0	1	3	0	2	3	2	2	-1	-3	0	-3	0	5
0	0	0	1	0	0	1	1	1	1	1	-1	0	-2	-2	0	2
0	0	0	1	0	1	0	1	0	1	0	-1	-2	-1	-1	0	1
0	0	0	1	0	1	1	2	2	2	1	-1	-1	-1	-2	0	4
0	0	0	1	0	1	1	3	1	1	2	0	-1	-3	-1	0	4
0	0	0	1	0	2	2	0	1	2	2	-2	0	-1	-2	0	4
0	0	0	1	0	2	2	1	1	3	3	-1	-1	-2	-3	0	5
0	0	0	1	1	1	4	4	4	4	1	-2	-2	-3	-3	0	8
0	0	0	1	1	3	4	4	6	6	1	-4	-4	-1	-5	0	10
0	0	1	0	0	2	1	1	2	2	1	-3	-3	0	-3	0	3
0	0	1	0	1	1	0	1	1	-1	1	-1	-1	-1	-1	0	2
0	0	1	0	1	1	0	1	1	1	1	0	0	0	0	0	5
0	0	1	0	1	1	0	1	1	1	1	2	0	0	0	-2	5
0	0	1	0	1	1	0	1	1	1	1	2	0	2	0	-4	5
0	0	1	0	1	1	0	1	1	1	2	2	0	0	-1	-2	5
0	0	1	0	1	1	0	1	2	1	2	0	0	0	-2	0	5
0	0	1	0	1	1	1	2	2	1	2	0	-1	-1	-2	0	5
0	0	1	0	1	1	2	2	3	1	2	-1	0	-2	-2	0	6
0	0	1	0	1	1	2	2	3	1	3	-1	0	-2	-3	0	6
0	0	1	0	1	1	3	2	3	1	2	-2	0	-2	-2	0	6
0	0	1	0	1	2	0	1	1	1	2	0	-1	0	-1	0	5
0	0	1	0	1	2	0	2	1	1	1	1	-2	0	0	-1	5

0	0	1	0	1	2	0	2	1	1	1	1	-2	1	0	-2	5
0	0	1	0	1	2	1	3	3	1	2	-1	-2	-1	-2	0	6
0	0	1	0	1	2	2	2	4	1	4	-2	0	-2	-4	0	7
0	0	1	0	1	2	3	2	1	0	1	-4	-3	-4	-1	0	3
0	0	1	0	1	3	1	3	4	1	3	-2	-2	-1	-3	0	7
0	0	1	0	1	3	2	3	5	1	4	-3	-1	-2	-4	0	8
0	0	1	1	0	1	1	2	-1	1	1	-1	-3	-3	-1	0	2
0	0	1	1	0	1	1	2	1	2	2	0	-1	-1	-2	0	5
0	0	1	1	0	2	1	0	1	2	2	-1	0	1	-2	-1	5
0	0	1	1	0	2	1	4	2	2	1	-1	-3	-1	-1	0	6
0	0	1	1	0	2	2	0	1	1	2	-2	0	0	-1	0	5
0	0	1	1	0	2	2	1	-1	1	2	-1	-1	-2	-1	0	4
0	0	1	1	0	2	3	1	1	2	1	-2	-1	-1	0	0	6
0	0	1	1	0	2	3	2	1	2	1	-2	-1	-2	0	0	6
0	0	1	1	0	3	1	1	1	2	3	-1	-1	-1	-2	0	6
0	0	1	1	0	3	3	1	1	2	1	-3	-1	-1	0	0	6
0	0	1	1	1	0	1	1	1	0	1	0	0	0	0	0	5
0	0	1	1	1	0	1	1	1	0	1	1	2	0	0	-3	5
0	0	1	1	1	1	0	2	0	1	1	2	-1	0	0	-2	5
0	0	1	1	1	1	1	0	1	0	1	0	2	0	0	-2	5
0	0	1	1	1	1	3	4	3	1	3	-1	0	-4	-2	0	8
0	0	1	1	1	1	4	1	1	2	2	-1	0	-2	-1	0	7
0	0	1	1	1	2	0	1	0	1	1	1	-1	0	0	-1	5
0	0	1	1	1	2	1	0	1	1	0	-1	0	0	0	0	5
0	0	1	1	1	3	1	1	0	1	1	-1	-2	0	0	0	5
0	0	1	1	1	3	2	1	1	2	2	-1	-2	0	-1	0	7
0	0	1	1	1	4	4	4	1	1	1	-6	-6	-6	-1	0	5
0	0	1	1	2	1	1	1	1	0	0	-1	0	0	0	0	5
0	0	1	1	2	1	1	1	1	0	0	0	-1	0	0	0	5
0	0	1	1	2	1	2	1	1	0	0	-1	0	-1	0	0	5
0	0	1	1	2	2	2	0	1	0	0	-2	0	0	0	0	5
0	0	1	2	2	1	2	1	1	1	0	0	-1	0	0	0	7
0	0	2	1	1	0	1	1	0	0	1	0	0	0	0	0	5
0	0	2	1	1	0	2	1	0	0	1	0	0	-1	0	0	5
0	1	0	0	1	0	1	-1	2	1	1	-2	-1	-1	-3	0	2
0	1	0	0	1	1	1	1	0	2	2	0	-1	-1	0	0	5
0	1	0	0	1	1	2	1	1	1	1	0	0	-1	-1	0	5
0	1	0	0	1	1	2	1	2	1	1	-2	0	-1	0	0	5
0	1	0	0	1	1	2	1	3	1	1	-2	0	-1	-1	0	5
0	1	0	0	1	2	1	1	2	1	2	-1	-1	-1	-1	0	5
0	1	0	0	1	2	2	1	0	1	1	0	-1	-1	2	-2	5
0	1	0	0	1	2	2	1	0	2	2	0	-2	-2	0	0	5

0	1	0	1	0	1	1	1	1	0	2	0	0	-1	0	0	5
0	1	0	1	0	1	1	1	1	0	2	0	0	-1	1	-1	5
0	1	0	1	0	1	1	1	1	0	2	0	2	-1	1	-3	5
0	1	0	1	0	1	1	2	1	1	2	0	0	-2	-1	0	5
0	1	0	1	0	2	1	1	1	0	2	-1	0	-1	1	-1	5
0	1	0	1	0	2	1	1	1	0	2	-1	1	-1	1	-2	5
0	1	0	1	0	3	1	1	2	2	3	-2	-1	-1	-2	0	6
0	1	0	1	0	3	1	3	2	1	4	-1	-1	-3	-2	0	7
0	1	0	1	1	1	0	1	0	1	1	0	0	0	1	-1	5
0	1	0	1	1	2	-1	-1	1	1	1	-1	-1	-1	-2	0	3
0	1	0	1	1	2	1	-1	0	3	2	-1	-2	-2	-3	0	4
0	1	0	1	1	2	1	0	1	0	1	-1	0	0	0	0	5
0	1	0	1	1	2	1	0	2	0	1	-2	1	0	0	-1	5
0	1	0	1	1	2	2	1	1	4	2	0	-2	-1	-2	0	7
0	1	0	1	1	3	1	2	1	2	2	0	-3	-1	0	0	7
0	1	0	1	1	5	2	1	2	2	3	-3	-2	-1	-1	0	8
0	1	0	1	1	6	3	1	2	3	3	-4	-3	-1	-1	0	9
0	1	0	2	1	3	2	5	6	3	2	-3	-2	-2	-3	0	11
0	1	1	0	0	-1	2	2	2	2	1	-1	-1	-2	-2	0	4
0	1	1	0	0	0	2	3	2	2	1	-1	-2	-2	-1	0	5
0	1	1	0	0	1	1	1	1	1	2	0	0	0	-2	0	5
0	1	1	0	0	1	1	1	1	1	4	0	0	-2	-2	0	5
0	1	1	0	0	1	2	1	1	2	1	-1	-1	0	-1	0	5
0	1	1	0	0	1	2	2	1	1	2	-1	-1	-2	0	0	5
0	1	1	0	1	-1	1	1	2	-1	2	-1	-1	-3	-2	0	3
0	1	1	0	1	1	1	0	1	1	2	0	0	0	-2	0	5
0	1	1	0	1	1	1	1	0	0	2	1	0	-1	0	-1	5
0	1	1	0	1	1	1	1	1	1	1	1	0	1	1	-3	6
0	1	1	0	1	1	2	0	1	1	0	-1	0	1	1	-2	5
0	1	1	0	1	1	2	0	2	1	0	-2	0	0	0	0	5
0	1	1	0	1	1	3	1	1	1	2	-1	0	-2	0	0	6
0	1	1	0	1	1	4	1	2	2	2	-2	0	-2	-1	0	7
0	1	1	0	2	1	1	0	1	0	0	0	0	1	0	-1	5
0	1	1	1	0	-1	1	2	-1	2	2	-1	-2	-4	-2	0	3
0	1	1	1	0	0	-1	1	-1	1	1	0	-2	-1	-1	0	2
0	1	1	1	0	2	1	1	2	2	4	-1	0	-1	-3	0	7
0	1	1	1	0	2	1	3	2	2	5	0	-1	-3	-3	0	8
0	1	1	1	0	2	2	1	1	1	3	-1	0	-2	0	0	7
0	1	1	1	0	2	2	1	1	1	3	-1	1	-2	0	-1	7
0	1	1	1	0	2	2	1	1	1	4	-1	1	-2	-1	-1	7
0	1	1	1	0	3	3	2	1	4	1	-2	-3	-1	0	0	8
0	1	2	0	1	0	1	1	2	0	1	-1	0	0	-1	0	5

0	1	2	0	1	0	2	1	0	0	1	0	0	-1	1	-1	5
0	2	0	1	2	2	2	1	1	2	1	-1	0	0	0	0	9
0	2	0	1	2	2	2	1	3	2	1	-3	0	0	0	0	9
0	2	1	0	0	2	3	3	1	1	5	-1	-1	-5	0	0	8
0	2	1	1	1	0	1	0	1	1	1	0	0	0	0	0	6
0	2	1	2	1	2	0	1	1	1	2	0	-1	0	0	0	8
0	2	2	2	2	-1	-1	-1	-1	-1	-1	0	0	0	0	0	3
0	2	2	2	2	1	0	-1	0	-1	2	0	2	0	0	-2	7
0	2	2	2	2	1	1	4	3	-1	-1	-2	-1	-1	0	0	9
0	2	2	2	2	1	3	-1	1	4	-1	-2	-1	2	-1	-2	9
0	2	2	2	2	3	1	0	-1	-1	2	-1	0	-1	2	-2	8

Rank 3

0	0	0	0	0	1	1	1	1	1	1	-2	-2	-2	-2	-1	1
0	0	0	0	0	3	1	1	3	3	1	-4	-4	0	-4	-1	3
0	0	1	2	3	8	2	3	1	3	2	-2	-6	0	0	0	12
0	0	2	1	3	3	9	3	2	3	4	-3	0	-6	-1	0	14
0	1	0	2	3	6	1	3	2	3	2	0	-6	0	0	0	12
0	1	1	0	1	1	1	1	1	1	1	1	0	0	0	-1	6
0	1	1	0	1	1	1	1	1	1	1	1	0	0	1	-2	6
0	1	1	0	1	1	1	1	2	1	1	0	0	1	-1	-1	6
0	1	1	0	1	1	1	1	3	1	1	-1	0	1	-1	-1	6
0	1	1	0	1	1	2	1	1	1	2	0	0	-1	-1	0	6
0	1	1	0	1	1	3	1	1	1	1	-1	0	-1	0	0	6
0	1	1	1	0	1	1	1	1	1	2	0	0	0	-1	0	6
0	1	1	1	0	1	1	2	1	1	2	0	0	-2	0	0	6
0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	6
0	1	1	1	1	2	1	1	1	2	2	0	-1	0	-1	0	7
0	1	1	1	1	2	3	1	1	2	3	-1	0	-2	-1	0	8
0	1	1	1	1	2	3	1	1	3	2	-1	-1	-1	-1	0	8
0	1	1	1	1	3	1	1	2	2	3	-1	-1	0	-2	0	8
0	1	1	1	1	3	3	1	1	3	4	-1	-1	-2	-2	0	9
0	1	1	3	1	1	0	1	0	1	0	0	0	0	0	0	6
0	1	2	0	2	2	1	2	5	2	5	0	0	0	-5	-1	11
0	1	2	0	3	2	7	4	2	3	3	0	-1	-6	1	-1	14
0	1	2	1	2	3	1	2	1	1	3	1	-1	-1	0	-1	10
0	1	2	2	1	2	2	1	1	1	3	0	0	-1	0	0	10
0	1	2	2	1	3	2	1	1	1	3	-1	0	-1	0	0	10
0	1	2	2	2	1	2	4	2	0	2	0	0	-3	0	0	10
0	1	2	2	2	1	3	5	3	1	3	0	0	-4	-1	0	12
0	1	2	2	2	3	4	1	2	2	1	-3	0	0	0	0	11
0	1	2	2	3	1	2	3	2	0	1	0	0	-2	0	0	10
0	1	2	2	4	2	2	1	2	0	0	-1	0	0	0	0	10

0	1	2	3	1	2	3	6	3	2	4	0	0	-5	-1	0	14
0	1	2	3	1	2	3	7	3	2	4	0	-1	-5	-1	0	14
0	1	2	3	2	2	4	8	5	2	5	-1	0	-6	-2	0	17
0	1	3	2	3	3	2	5	3	2	2	-1	-2	0	0	0	15
0	1	3	2	3	5	2	5	3	2	2	-1	-4	0	0	0	15
0	1	3	2	5	3	2	3	3	0	0	-1	-2	0	0	0	13
0	1	4	1	2	5	9	2	5	4	2	-8	-1	0	-1	0	17
0	2	0	1	2	2	2	1	1	2	1	-1	0	0	2	-2	9
0	2	0	1	2	2	2	1	1	3	1	-1	-1	0	0	0	9
0	2	0	1	2	2	2	1	1	3	1	-1	-1	0	1	-1	9
0	2	0	1	2	2	2	1	2	3	1	-2	-1	0	0	0	9
0	2	0	2	3	2	2	1	2	3	1	-2	0	0	0	0	11
0	2	0	3	1	2	3	2	3	9	4	1	-2	-1	-6	-1	14
0	2	1	0	3	2	2	3	9	4	3	-2	-1	0	-6	0	14
0	2	1	2	1	1	1	2	3	1	2	0	2	0	-1	-2	10
0	2	1	2	1	2	1	3	3	2	1	-1	-2	0	0	0	10
0	2	1	2	2	2	2	2	4	1	1	-2	1	0	0	-1	11
0	2	1	2	3	6	2	3	1	5	2	0	-6	0	0	0	14
0	3	1	2	3	6	2	3	1	6	3	1	-6	0	0	-1	16
0	3	1	2	3	6	3	3	2	6	3	0	-5	0	-1	0	17
0	3	2	0	1	3	3	4	3	2	7	0	-1	-6	-1	0	14
0	3	2	1	1	2	3	3	1	1	6	0	0	-5	0	0	12
0	3	2	1	3	3	7	3	2	4	7	-1	0	-6	-1	0	18
0	3	2	2	1	3	3	4	2	2	6	0	-1	-4	0	0	15
0	3	2	2	1	3	3	5	2	2	6	0	-1	-5	0	0	15
0	3	2	2	3	3	3	3	6	1	1	-4	0	0	0	0	15
0	3	2	3	1	3	3	6	2	1	3	0	0	-3	4	-4	16
0	3	3	2	1	3	2	6	3	6	1	0	-6	0	0	0	16
0	3	3	2	2	1	2	4	4	2	2	0	0	-1	-1	0	15

Rank 4

0	1	4	3	4	9	3	9	4	4	4	-1	-8	-1	0	0	23
0	2	3	3	1	3	3	2	3	4	6	0	-1	0	-3	0	17
0	5	4	5	2	5	5	10	4	3	10	0	-1	-9	0	0	28
0	5	5	2	4	5	10	4	10	4	2	-10	0	0	0	0	27

Table A.2: The 277 classes of facet-defining inequalities of $P(4,7)$ ordered by Chvátal rank. Each row corresponds to the inequality $c_1x_\emptyset + c_2x_{\{1\}} + c_3x_{\{2\}} + c_4x_{\{3\}} + c_5x_{\{4\}} + c_6x_{\{1,2\}} + c_7x_{\{1,3\}} + c_8x_{\{1,4\}} + c_9x_{\{2,3\}} + c_{10}x_{\{2,4\}} + c_{11}x_{\{3,4\}} + c_{12}x_{\{1,2,3\}} + c_{13}x_{\{1,2,4\}} + c_{14}x_{\{1,3,4\}} + c_{15}x_{\{2,3,4\}} + c_{16}x_{\{1,2,3,4\}} \leq b$.

A.4 Facets of $P(4,4)$

The following table records all 2001 classes of facet-defining inequalities

$$c_1 x_\emptyset + c_2 x_{\{1\}} + c_3 x_{\{2\}} + c_4 x_{\{3\}} + c_5 x_{\{4\}} + c_6 x_{\{1,2\}} + c_7 x_{\{1,3\}} + c_8 x_{\{1,4\}} + c_9 x_{\{2,3\}}$$

$$+ c_{10} x_{\{2,4\}} + c_{11} x_{\{3,4\}} + c_{12} x_{\{1,2,3\}} + c_{13} x_{\{1,2,4\}} + c_{14} x_{\{1,3,4\}} + c_{15} x_{\{2,3,4\}} + c_{16} x_{\{1,2,3,4\}} \leq b$$

up to symmetry for $P(4,4)$. Facets are listed according to their Chvátal rank.

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	c_{16}	b
Rank 0																
-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	-1	1
0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	1
0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	-1	1
0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	1
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	-1	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	1	0	0	-1	0	0	0	1
0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1	4
0	1	0	1	0	0	-1	0	0	0	0	0	0	0	0	0	1
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
Rank 1																
0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	-2	1
0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	-2	1
0	0	0	0	0	0	0	0	0	1	1	2	1	0	0	-2	2
0	0	0	0	0	0	0	2	1	1	1	0	0	0	0	0	3
0	0	0	0	0	0	1	0	0	1	1	0	0	-1	-1	-1	1
0	0	0	0	0	0	1	0	1	0	0	-1	1	0	0	-1	1
0	0	0	0	0	0	1	1	0	0	1	0	0	-2	0	0	1
0	0	0	0	0	1	0	0	0	0	0	1	1	0	1	-1	2
0	0	0	0	0	1	0	0	0	0	0	1	1	0	2	-2	2

0	0	0	0	0	1	0	1	0	0	1	0	0	0	1	-1	2
0	0	0	0	0	1	1	0	0	0	0	0	1	0	0	0	2
0	0	0	0	0	1	1	0	1	0	0	-1	0	0	1	0	2
0	0	0	0	0	1	1	1	0	0	0	-1	-1	-1	0	0	1
0	0	0	0	0	1	1	1	0	0	0	1	1	1	0	0	3
0	0	0	0	0	2	1	1	0	0	0	0	0	1	0	0	3
0	0	0	0	1	0	0	1	0	0	0	0	1	1	1	0	3
0	0	0	0	1	1	0	0	0	0	0	1	0	0	1	-1	2
0	0	0	0	1	1	2	0	0	0	1	-1	0	-1	1	0	3
0	0	0	1	0	0	0	0	1	2	0	0	0	2	-1	-1	3
0	0	0	1	0	0	1	0	1	0	0	1	0	1	2	0	4
0	0	0	1	0	1	1	0	1	0	0	0	0	1	1	1	4
0	0	0	1	0	1	1	0	1	0	1	-1	0	0	0	1	3
0	1	0	0	0	0	0	0	0	0	0	1	1	1	0	-1	2
0	1	0	0	0	0	0	0	0	1	1	0	-1	-1	-1	0	1
0	1	0	0	0	0	1	1	0	0	0	1	1	0	0	0	3
0	1	0	0	0	0	1	1	0	0	0	2	2	1	0	-1	4
0	1	0	0	0	1	0	0	0	1	0	1	0	1	0	0	3
0	1	0	0	0	1	1	0	0	1	1	-1	-1	-1	0	0	2
0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	1	3
0	1	0	1	0	1	-1	0	0	0	2	-1	1	0	0	-1	3
0	1	0	1	0	1	0	0	0	0	0	0	1	1	0	0	3
0	1	0	1	1	1	-1	-1	1	0	0	-1	0	0	-1	0	2
0	1	1	0	0	-1	0	0	0	0	1	0	0	-1	-1	0	1
0	1	1	0	0	0	1	1	0	0	0	0	0	0	0	1	3
0	1	1	0	1	0	1	-1	0	1	2	0	0	-1	-2	0	3
0	1	1	1	0	1	0	0	0	1	0	1	1	1	0	1	5
0	1	1	1	1	1	-1	1	1	-1	0	-1	-1	0	0	0	3
0	1	2	1	0	-1	0	1	-1	0	0	0	-1	0	1	0	3
0	2	0	0	1	0	0	-1	0	1	1	0	-1	-1	0	1	3
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0	2	1	1	1	-1	-1	-1	0	0	0	0	0	0	-1	0	2
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Rank 2

0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	-3	1
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Rank 3

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Rank 4

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0	4	0	0	3	1	4	-3	6	3	2	0	3	5	1	0	15
0	4	0	0	3	5	0	-3	7	3	3	1	5	4	1	0	17
0	4	0	0	5	4	3	-4	7	3	5	2	4	7	0	0	20
0	4	0	0	6	4	0	2	9	7	6	1	3	6	-2	6	26
0	4	0	1	3	1	1	-3	0	3	2	5	4	4	3	-4	12
0	4	0	2	2	0	-2	-2	1	1	1	3	3	3	1	-3	8
0	4	0	2	3	1	1	0	5	3	3	-1	1	1	-2	5	14
0	4	0	2	3	3	-2	-2	4	0	2	2	5	6	2	-4	13
0	4	0	2	3	4	2	-2	5	0	3	-2	6	3	2	-1	15
0	4	0	3	0	0	-3	4	3	6	3	4	1	4	0	0	15
0	4	0	3	0	1	-1	2	3	4	0	3	2	5	3	-2	13
0	4	0	3	0	2	-3	0	3	2	2	3	4	4	2	-2	12
0	4	0	3	0	2	-1	2	1	4	3	5	2	3	3	-1	14

0	4	0	3	2	1	-2	1	0	4	2	3	1	3	2	1	13
0	4	0	3	3	0	-2	-2	3	3	-3	4	4	4	3	-3	12
0	4	0	3	3	4	-3	-3	3	3	1	4	4	8	2	-4	15
0	4	0	4	4	4	-2	-2	4	4	1	1	1	6	-2	0	15
0	4	0	5	2	0	0	3	2	6	-2	2	-2	1	-2	2	13
0	4	0	6	4	7	-4	4	7	0	-4	3	4	11	10	-9	23
0	4	0	10	11	10	3	4	3	0	-10	3	4	11	19	-11	33
0	4	1	0	5	3	4	-1	3	0	2	3	6	4	5	-1	18
0	4	1	3	3	3	-2	-2	3	3	0	4	4	10	3	-5	16
0	4	1	4	0	4	1	4	1	6	1	1	-1	4	-1	5	18
0	4	1	7	2	2	0	5	3	9	3	4	-2	0	-4	5	21
0	4	2	0	0	1	3	3	0	2	5	3	1	-2	0	3	13
0	4	2	0	0	1	4	0	7	5	5	2	4	5	1	2	20
0	4	2	0	0	2	4	4	2	2	7	1	1	-4	0	4	15
0	4	2	0	4	1	0	-2	4	2	1	2	4	4	2	1	15
0	4	2	2	0	3	0	4	-2	3	5	10	3	4	9	-8	19
0	4	2	2	2	-2	-1	-1	2	2	-1	4	4	6	2	-5	11
0	4	2	2	2	0	1	-1	-2	2	3	5	1	1	3	-1	12
0	4	3	0	0	-3	0	2	2	3	2	6	3	4	2	-4	12
0	4	3	0	0	-3	1	1	3	3	0	4	4	6	3	-5	12
0	4	3	0	0	-3	4	4	6	0	4	6	12	4	7	-9	20
0	4	3	0	0	-2	1	0	0	3	3	4	4	4	3	-3	12
0	4	3	0	0	-1	0	0	2	3	2	4	3	6	2	-4	12
0	4	3	0	2	-3	1	0	3	1	0	3	4	4	2	-3	11
0	4	3	0	4	0	3	-2	3	-2	4	3	9	3	4	-4	16
0	4	3	3	0	-3	-2	0	-3	3	3	4	5	4	3	-4	12
0	4	3	4	7	4	4	-4	-2	0	-2	4	7	8	11	-9	21
0	4	3	4	8	4	3	-4	3	2	-4	3	7	13	8	-8	24
0	4	4	0	0	2	2	2	5	5	7	2	2	0	-4	6	19
0	4	4	0	6	1	3	-2	1	-4	0	1	5	3	7	-5	14
0	4	4	0	6	3	0	-4	6	-4	6	4	10	9	3	-8	21
0	4	4	0	7	2	4	-1	4	-1	1	4	7	7	7	-3	22
0	4	4	1	1	0	1	1	3	3	5	1	1	-1	-3	5	14
0	4	4	4	1	0	-2	4	-2	4	5	13	4	5	5	-6	21
0	4	6	0	4	-4	3	0	0	-1	0	6	6	4	6	-7	15
0	4	6	1	0	-4	4	4	-1	6	9	6	6	0	1	0	22
0	4	6	6	0	-4	0	3	-4	5	3	9	5	3	3	-4	19
0	5	0	0	0	1	1	1	4	4	4	5	5	5	4	-3	16
0	5	0	0	4	5	0	1	7	4	1	6	4	12	7	-7	23
0	5	0	6	2	5	-5	-2	0	2	1	6	5	8	6	-8	17
0	5	0	8	5	1	-1	0	0	5	-5	8	5	9	8	-9	21
0	5	1	0	4	3	3	-4	6	2	1	-5	2	4	-1	0	13

0	5	1	3	3	1	1	1	5	5	3	-1	-1	1	-3	7	17
0	5	2	0	0	-2	0	3	5	4	0	4	4	8	5	-6	16
0	5	2	0	5	-2	8	-5	5	5	5	9	15	9	5	-11	27
0	5	2	3	3	1	0	-1	5	-2	3	3	11	3	5	-5	19
0	5	3	0	0	-1	0	0	3	3	3	5	5	7	3	-5	15
0	5	3	0	3	1	0	-1	0	3	4	6	1	0	2	0	15
0	5	3	3	3	-3	-3	5	3	2	2	11	5	5	-3	-4	20
0	5	3	4	0	-1	-4	5	4	4	4	12	5	9	4	-7	23
0	5	4	2	0	-2	-2	3	2	0	5	6	5	2	2	-3	15
0	5	4	4	0	1	1	3	-4	4	4	7	1	1	2	1	18
0	5	4	7	2	0	-5	5	0	-2	-2	8	5	7	8	-9	19
0	5	4	8	5	1	-4	-1	-4	0	-2	5	1	4	4	-3	15
0	5	5	1	1	-2	2	-1	-1	2	5	5	5	2	2	-2	15
0	5	5	1	1	1	1	1	5	5	7	1	1	-1	-5	7	19
0	5	5	4	5	0	-4	-1	-4	-1	3	8	5	6	6	-7	18
0	5	5	5	3	-2	-2	2	-1	-3	-3	5	5	5	6	-6	15
0	5	5	8	0	1	-5	5	-1	0	0	8	5	8	9	-9	21
0	5	8	0	4	0	8	6	1	1	6	-1	1	-4	1	9	25
0	5	9	7	7	-5	0	-1	-4	-3	-3	5	4	2	4	-3	17
0	5	10	4	8	-2	1	-2	-4	-6	0	2	3	0	4	-1	16
0	8	1	6	1	5	0	5	1	9	3	5	-3	3	-1	7	26
0	8	1	7	7	5	-4	-4	4	4	0	5	5	12	3	-3	26
0	8	5	0	5	-5	1	-1	5	2	0	8	8	9	5	-8	22
0	8	5	5	5	-1	-5	-3	0	-1	1	5	4	6	1	-3	17
0	8	7	2	6	-4	0	-4	0	-2	1	2	4	2	1	0	15
0	8	7	9	0	0	-5	5	-4	4	6	9	0	1	2	0	24
0	8	8	0	11	-1	8	-7	0	-2	0	5	11	8	11	-10	28
0	9	0	6	6	0	-1	-6	0	6	0	10	9	10	6	-11	24
0	9	4	4	0	-3	-3	1	4	4	4	9	9	9	4	-7	24
0	9	4	8	0	-3	-6	2	1	6	0	7	2	7	1	-4	20
0	40	20	20	100	110	110	100	110	-20	-20	-40	100	100	220	-80	450

Rank 5

0	0	0	5	0	6	3	7	3	7	3	6	3	5	5	0	23
0	0	4	7	5	8	7	5	-3	5	-4	4	0	7	13	-5	27
0	0	5	0	0	3	8	9	5	5	9	8	7	3	5	0	29
0	0	5	9	6	7	6	3	-5	3	-6	5	3	9	13	-7	27
0	0	6	1	1	4	11	11	6	6	11	9	9	2	6	2	37
0	0	6	4	5	6	4	5	-4	-5	0	6	5	4	10	-4	22
0	0	6	4	5	6	4	5	-4	-2	3	9	5	4	13	-7	25
0	0	6	5	4	6	5	4	-5	-4	0	5	6	4	13	-7	22
0	0	6	5	5	8	5	5	-5	-5	2	8	8	5	15	-9	27
0	0	6	6	7	6	8	7	-6	4	7	22	6	7	18	-12	42

0	0	9	6	5	9	6	10	-6	-5	6	9	5	0	16	-7	33
0	0	14	8	10	9	11	5	-6	-8	5	8	14	5	21	-10	44
0	1	0	0	5	10	8	5	10	5	4	2	5	8	7	1	31
0	1	1	1	5	9	9	3	9	5	5	1	7	7	5	3	31
0	1	8	7	7	8	7	7	-4	-4	3	8	8	3	18	-6	35
0	2	9	4	8	9	11	-2	6	-6	8	-4	17	6	8	-5	37
0	3	0	0	0	5	5	0	5	6	6	5	9	9	6	-6	23
0	3	0	4	5	6	0	5	4	5	-4	3	0	11	4	0	24
0	3	1	5	0	3	3	4	5	12	10	3	4	10	2	6	35
0	3	4	4	4	5	2	4	5	-4	3	5	12	4	12	-5	28
0	4	1	4	4	7	-4	-4	4	4	2	7	7	13	4	-8	23
0	4	3	3	3	4	4	4	5	5	5	-2	-2	-2	-4	10	21
0	5	1	1	1	3	3	3	7	7	7	5	5	5	1	3	25
0	5	1	1	1	3	3	5	9	9	9	7	5	5	-1	3	29
0	5	1	3	3	1	1	1	5	5	3	-1	-1	3	-1	7	19
0	6	4	5	0	-4	-5	6	0	4	5	6	2	1	0	0	18
0	6	6	4	5	6	7	7	7	7	8	-3	-2	-4	-4	17	35
0	6	6	9	1	4	-3	6	-6	4	7	17	4	7	12	-9	33
0	7	0	9	8	7	-4	3	9	8	-5	5	0	15	4	-3	33
0	7	5	5	5	7	7	7	8	8	8	-3	-3	-3	-5	17	36
0	7	9	0	8	-4	7	0	9	-5	8	9	22	7	8	-10	37
0	7	9	7	7	9	11	11	9	9	10	-4	-4	-7	-5	23	48
0	8	13	8	0	-6	3	8	-6	2	3	13	8	3	13	-9	33
0	9	9	7	7	9	11	11	11	11	12	-4	-4	-7	-7	25	52
0	10	4	6	6	0	-6	-6	1	1	1	5	5	7	1	-3	20
0	10	7	0	7	-7	8	-7	5	2	7	10	17	8	5	-9	33
0	12	8	10	0	-8	-10	12	3	8	10	23	12	10	5	-11	44
0	15	12	2	7	-10	15	8	-2	12	18	26	12	-7	9	-7	58

Rank 6

0	0	10	8	12	7	5	9	0	-10	-8	0	2	4	12	-3	30
0	2	5	6	6	9	7	6	7	7	6	-5	-4	-2	-2	17	35
0	10	2	8	8	10	-8	-8	8	8	3	10	10	21	3	-9	40
0	14	20	1	14	-14	9	4	15	-14	14	20	33	9	15	-17	65
0	15	11	10	0	-11	-10	12	3	11	7	25	11	15	7	-13	49

Table A.3: The 2001 facet-defining inequalities $c_1 x_\emptyset + c_2 x_{\{1\}} + c_3 x_{\{2\}} + c_4 x_{\{3\}} + c_5 x_{\{4\}} + c_6 x_{\{1,2\}} + c_7 x_{\{1,3\}} + c_8 x_{\{1,4\}} + c_9 x_{\{2,3\}} + c_{10} x_{\{2,4\}} + c_{11} x_{\{3,4\}} + c_{12} x_{\{1,2,3\}} + c_{13} x_{\{1,2,4\}} + c_{14} x_{\{1,3,4\}} + c_{15} x_{\{2,3,4\}} + c_{16} x_{\{1,2,3,4\}} \leq b$ for $P(4,4)$.

The constraint $x_{\{1,2\}} + x_{\{1,3\}} + x_{\{1,4\}} + x_{\{2,3\}} + x_{\{2,4\}} + x_{\{3,4\}} + x_{\{1,2,3\}} + x_{\{1,2,4\}} + x_{\{1,3,4\}} + x_{\{2,3,4\}} \leq 4$ contains all optimal points for $f(4,4)$ and is found in the first inequality of level 4.

A.5 Facets of $P(5,5)$

Go to the following link to see the table that records the 26,862 known classes of facet-defining inequalities for $P(5,5)$:

`https://github.com/dgallagher02/frankl_facets/blob/main/P55table.pdf`

A.6 Facets of $P(6,6)$

Go to the following link to see the table that records the 2,566 known classes of facet-defining inequalities for $P(6,6)$:

`https://github.com/dgallagher02/frankl_facets/blob/main/P66table.pdf`