Differentiation

Summary

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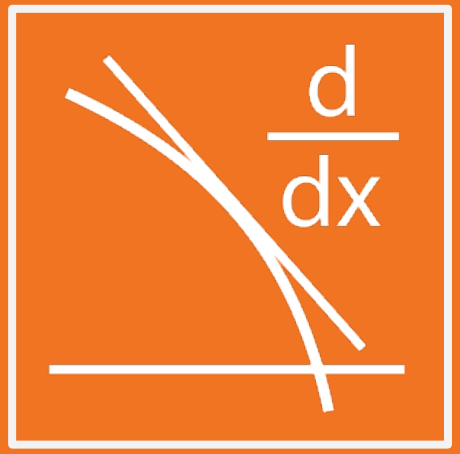
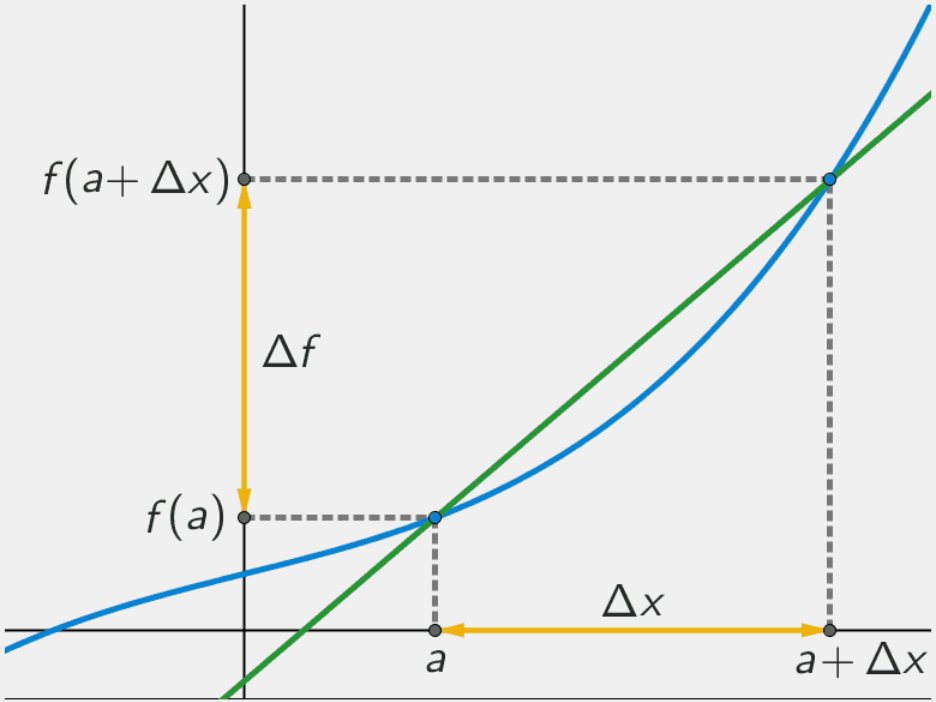
TU Delft



Definitions

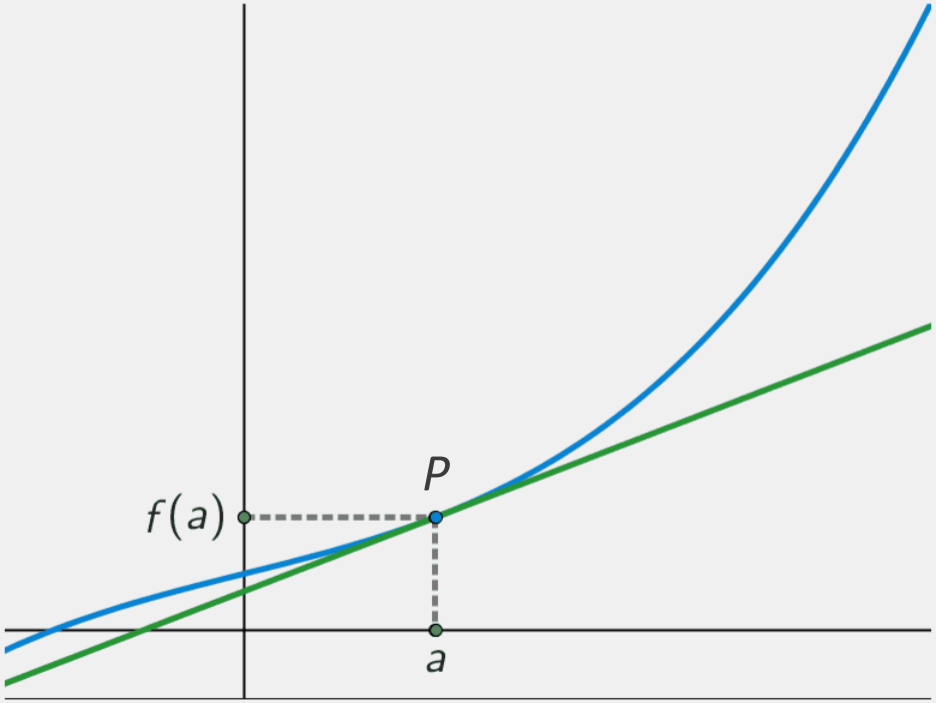
The difference quotient



Difference quotient:

$$\frac{f(a+\Delta x)-f(a)}{\Delta x}$$

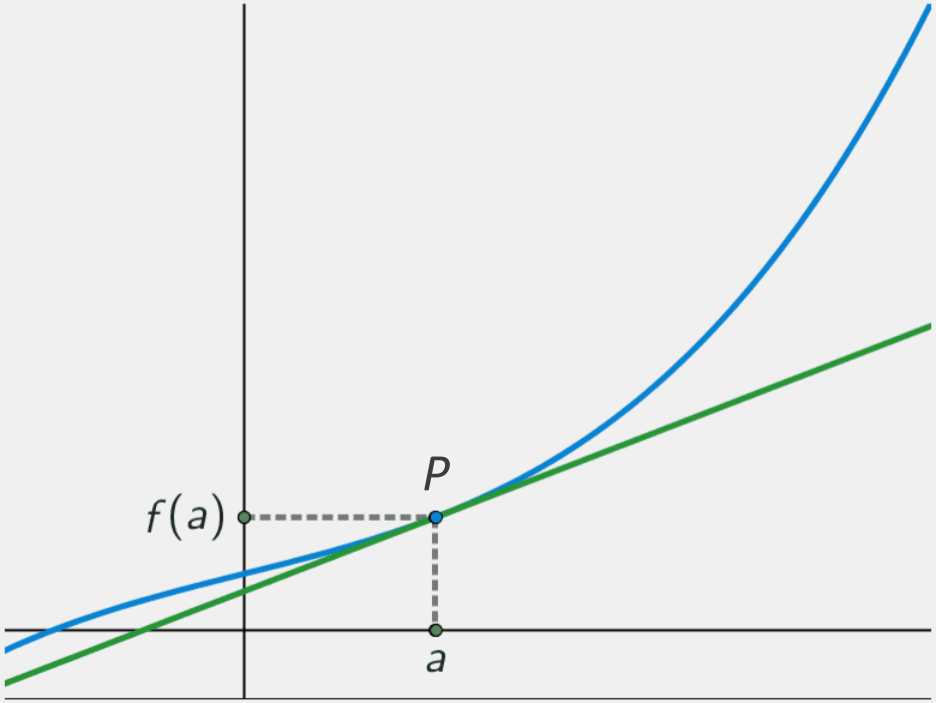
The tangent line



Derivative of f at $x = a$:

$$\lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

The tangent line



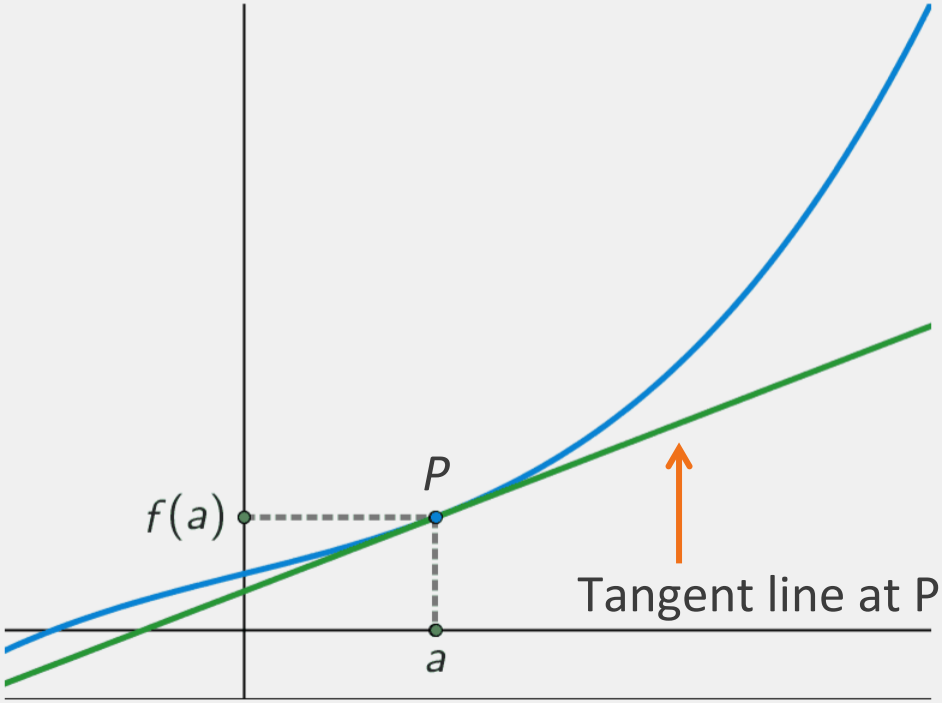
Derivative of f at $x = a$:

$$\lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$$

Notation:

$$f'(a) \quad \text{or} \quad \frac{df}{dx}(a)$$

The tangent line



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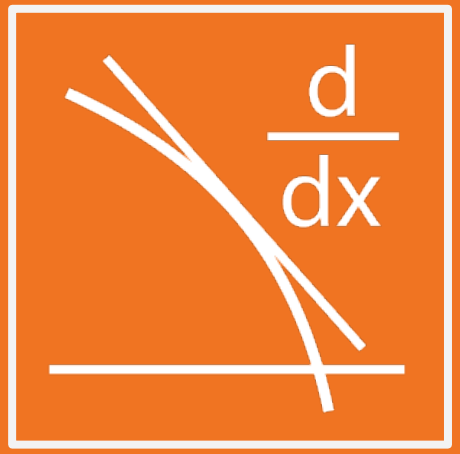


Derivatives of standard functions

Standard derivatives

$$\frac{d}{dx}x^p = px^{p-1}$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}a^x = a^x \ln(a)$$

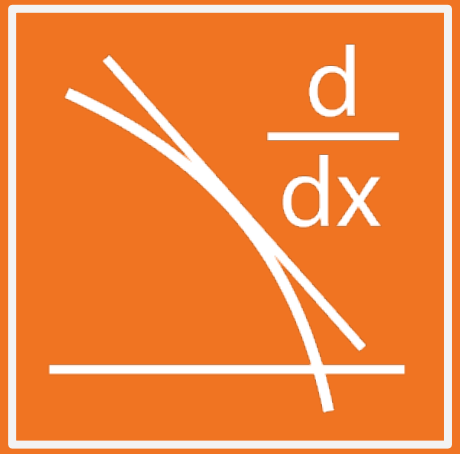
$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

Standard derivatives

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = \frac{1}{\cos(x)^2}$$



Rules of calculation

Rules of calculation

$$[cf(x)]' = cf'(x)$$

$$[f(x) + g(x)]' = f'(x) + g'(x)$$ Sum rule

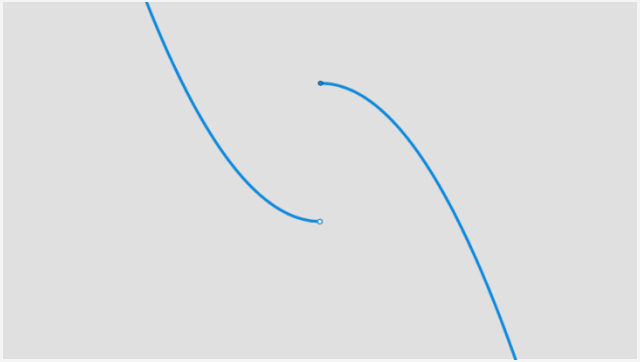
$$[f(x)g(x)]' = f'(x)g(x) + g'(x)f(x)$$ Product rule

$$[f(g(x))]' = f'(g(x)) \cdot g'(x)$$ Chain rule

Non-differentiable functions

Functions f that are not differentiable $x = a$:

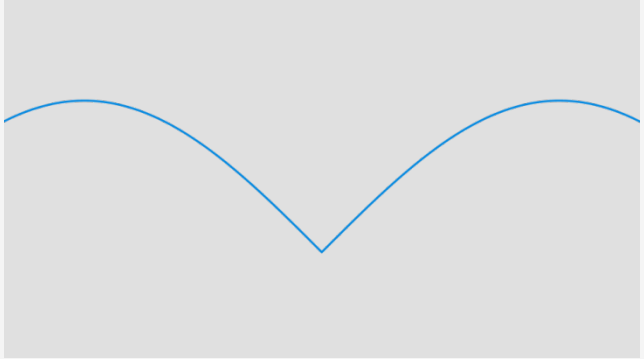
- f is discontinuous at $x = a$.



Non-differentiable functions

Functions f that are not differentiable $x = a$:

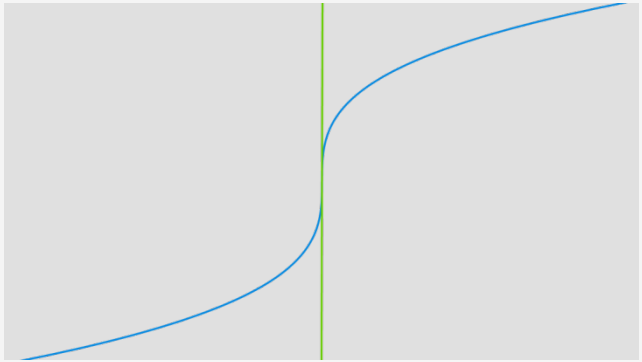
- f is discontinuous at $x = a$.
- The graph of f has a kink at $x = a$.



Non-differentiable functions

Functions f that are not differentiable $x = a$:

- f is discontinuous at $x = a$.
- The graph of f has a kink at $x = a$.
- The graph of f has a vertical tangent line at $x = a$.



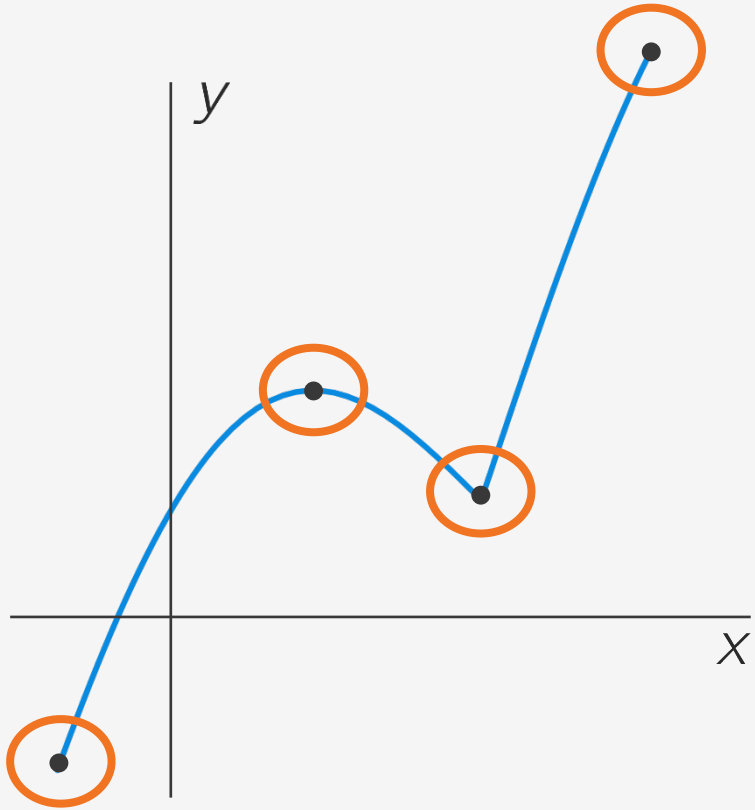
Finding minima and maxima

Given

- function f;
- point a such that $f(a)$ is local extremum;

Then

- $f'(a) = 0$, **critical point**
- or $f'(a)$ does not exist, **singular point**
- or a is a **boundary point**.



Thank you for your attention!