Functions

Summary week 2

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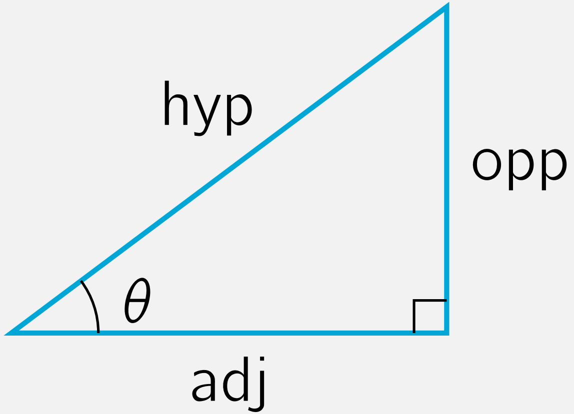
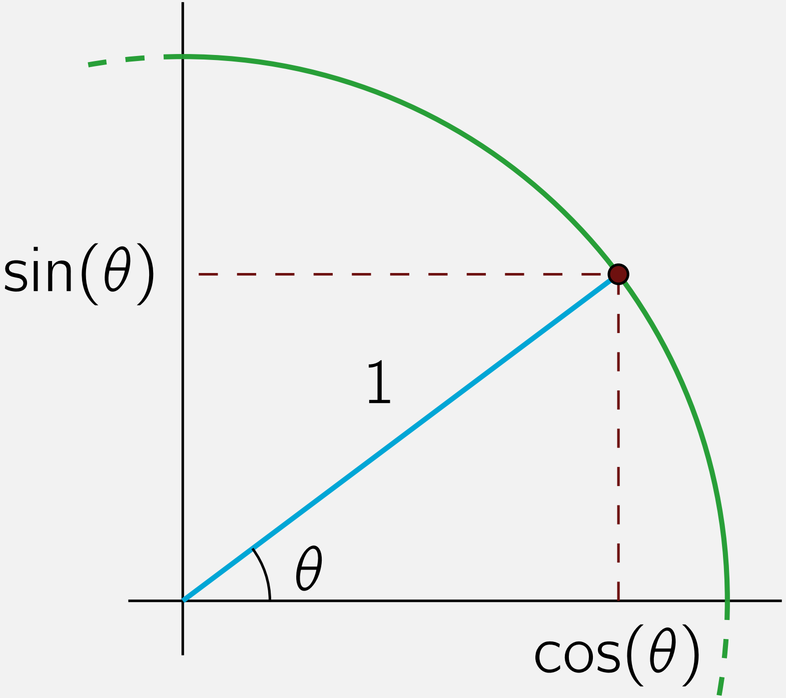
photo: Jorrit Lousberg

Trigonometric Functions





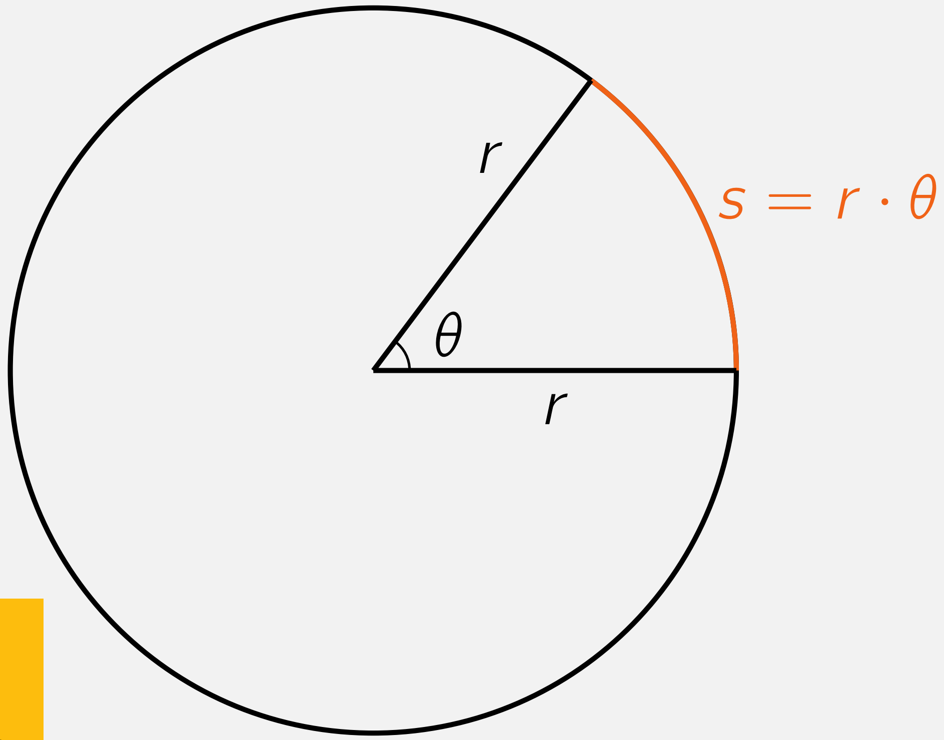
$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$ is y-coordinate on unit circle

$\cos(\theta) = \frac{\text{adj}}{\text{hyp}}$ is x-coordinate on unit circle

$\tan(\theta) = \frac{\text{opp}}{\text{adj}} = \frac{\sin(\theta)}{\cos(\theta)}$

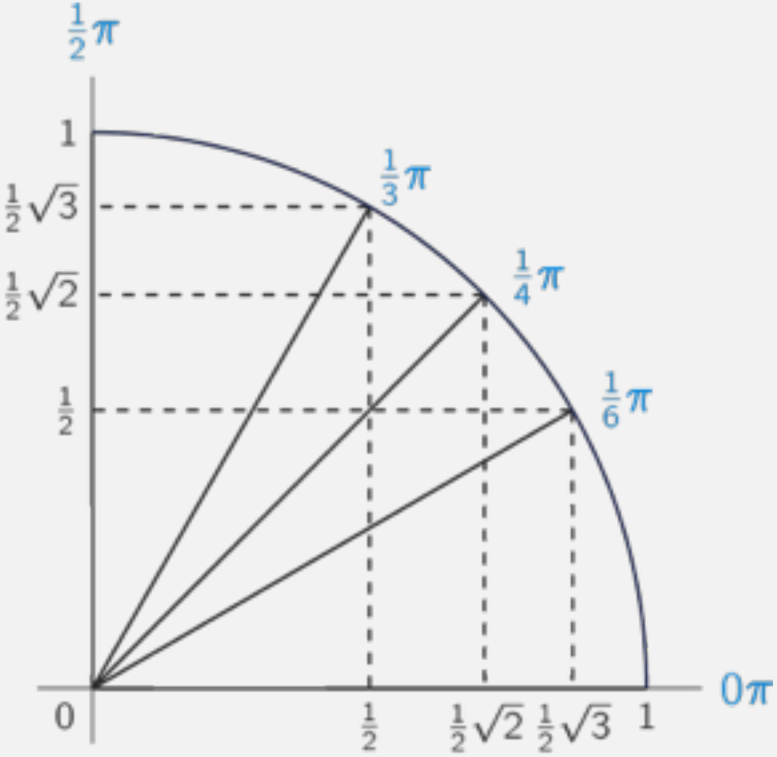
Radians

$$2\pi \text{ rad} = 360^\circ$$



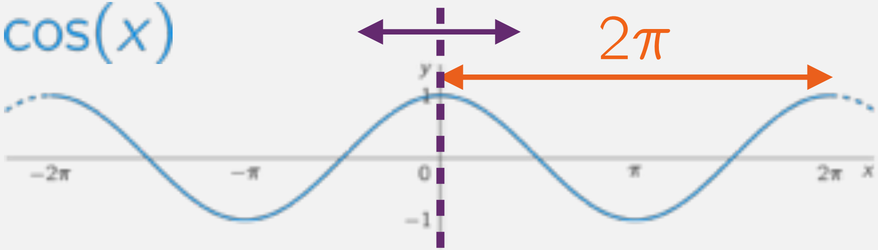
Radius $r = 1$:
angle θ in radians = distance s

Trigonometric functions – special values

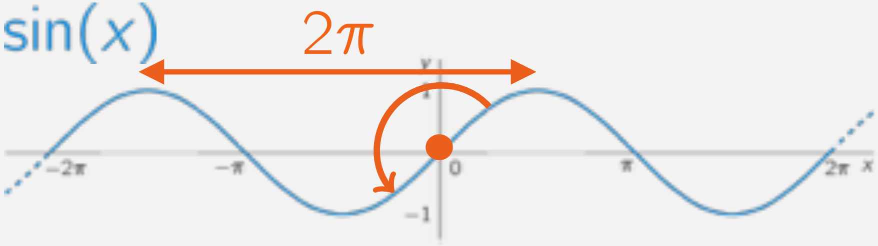


	0°	30°	45°	60°	90°
θ	0	$\frac{1}{6}\pi$	$\frac{1}{4}\pi$	$\frac{1}{3}\pi$	$\frac{1}{2}\pi$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1
$\cos(\theta)$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{3}\sqrt{3}$	1	$\sqrt{3}$	–

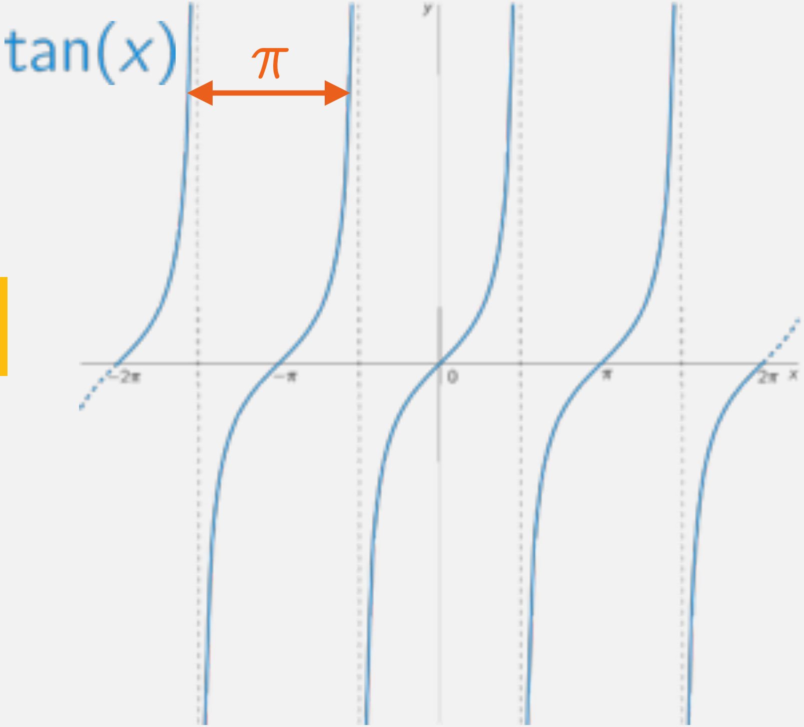
Graphs of Trigonometric Functions



$$\cos(x \pm 2\pi) = \cos(x) = \cos(-x)$$



$$\sin(x \pm 2\pi) = \sin(x) = -\sin(-x)$$



$$\tan(x \pm \pi) = \tan(x)$$

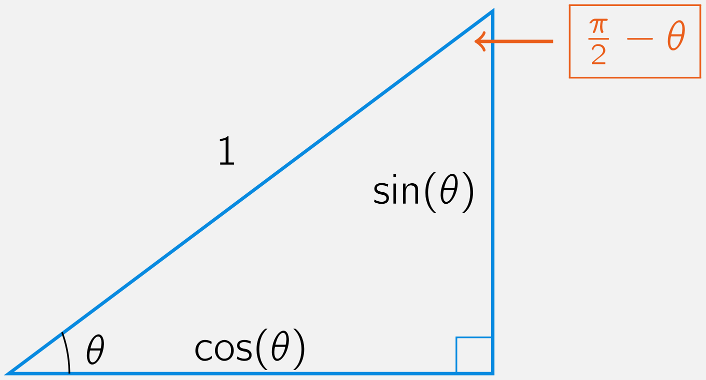
Other Trigonometric Identities

Pythagoras:

$$\sin(x)^2 + \cos(x)^2 = 1$$

$$\cos\left(\tfrac{\pi}{2} - \theta\right) = \sin(\theta)$$

$$\sin\left(\tfrac{\pi}{2} - \theta\right) = \cos(\theta)$$



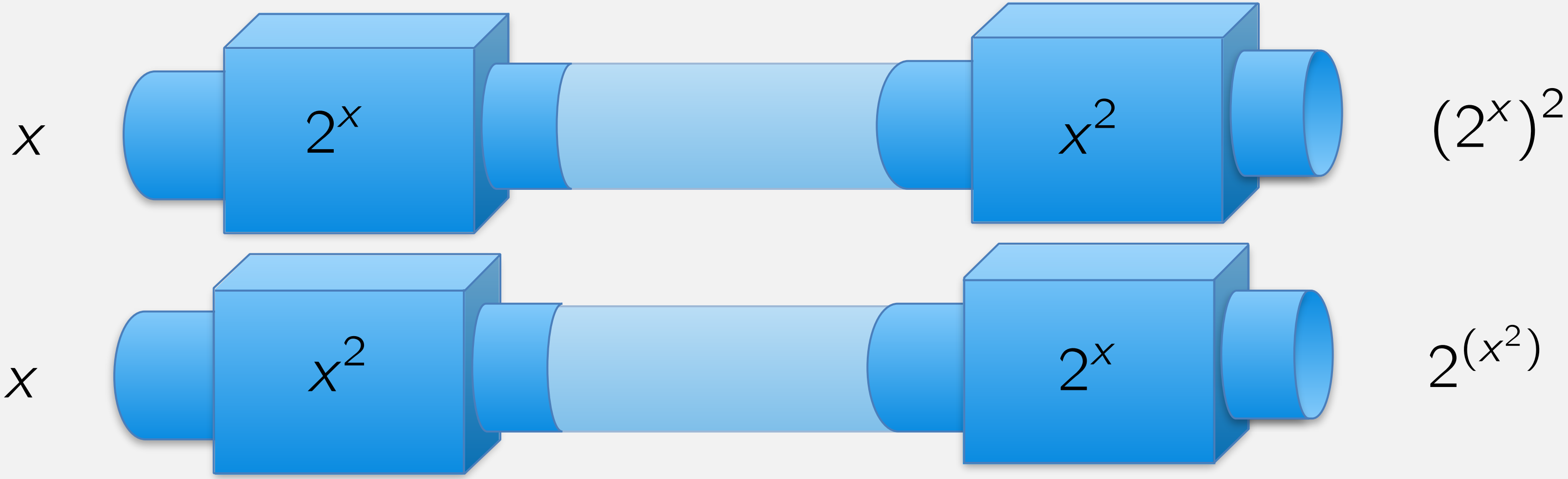
$$\sin(\alpha + \beta) = \cos(\alpha)\sin(\beta) + \sin(\alpha)\cos(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos(x)^2 - \sin(x)^2$$

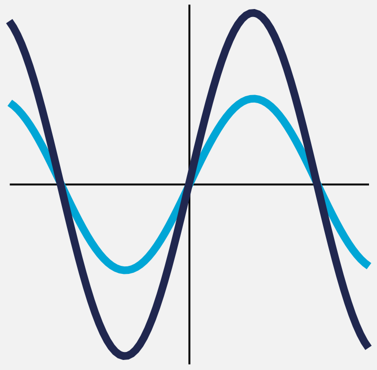
Composing Functions

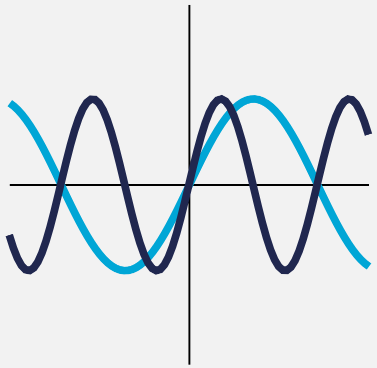


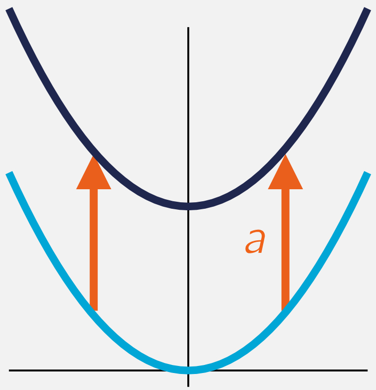
$$\text{At } x = 1: \quad \begin{aligned} 2^{(x^2)} &= 2^1 = 2 \\ (2^x)^2 &= 2^2 = 4 \end{aligned}$$

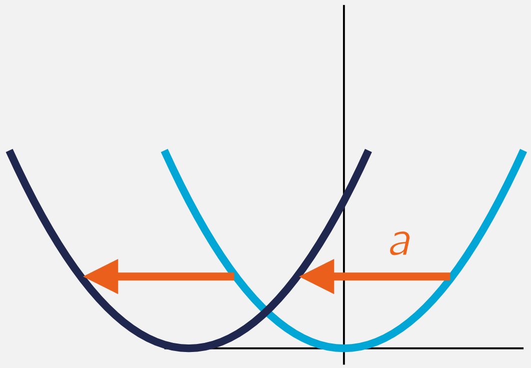
Composing with linear functions

$f(x) + a$	**Vertical shift upwards**
$f(x + a)$	**Horizontal shift to the left**
$af(x)$	**Vertical scaling by** a
$f(ax)$	**Horizontal scaling by** $1/a$



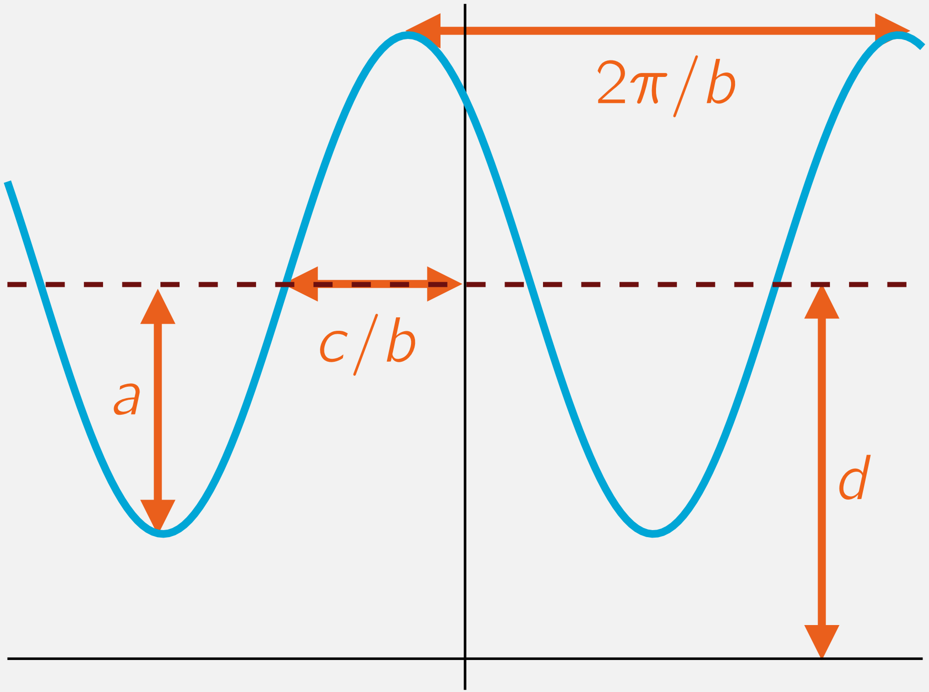






A wave function

$$y = a\sin(bt + c) + d$$



a	**Amplitude**
$2\pi/b$	**Period**
c	**Phase**
d	**Equilibrium height**

Exponential Functions

Form: $A\,b^x$

- b : base
- x : exponent

$$b^{x+y} = b^x b^y$$

$$b^{x-y} = \frac{b^x}{b^y}$$

$$(b^x)^y = b^{xy}$$

$$b^x c^x = (bc)^x$$

$b^x \quad b > 1$

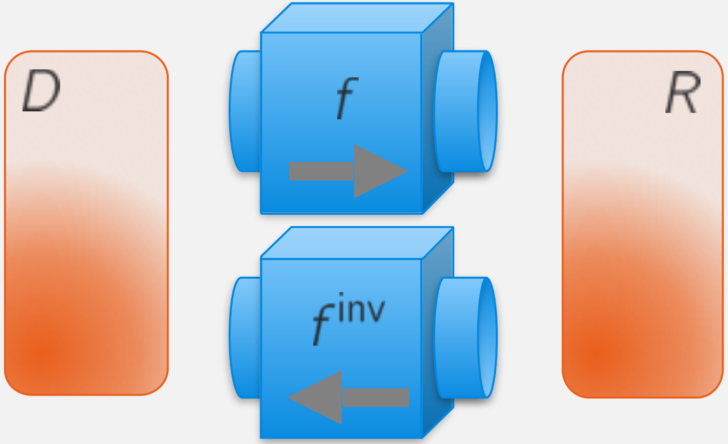
increasing

$b^x \quad b < 1$

decreasing

Inverse Functions

- $f^{\text{inv}}(y) = x \longleftrightarrow y = f(x)$
- Inverse exists $\longleftrightarrow$ injective
  - Horizontal line test
- f^{inv} has domain R and range D
- Graph: reflect in $y = x$



Logarithms

Logarithm is inverse of exponential:

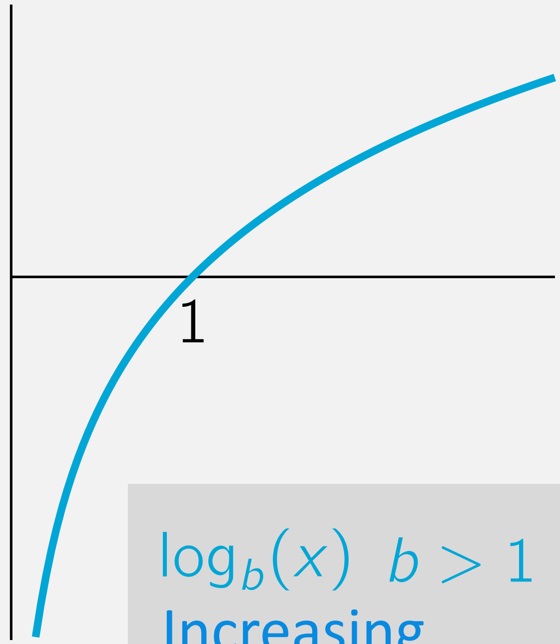
$$y = \log_b(x) \longleftrightarrow b^y = x$$

$$\log_b(xy) = \log_b(x) + \log_b(y)$$

$$\log_b\left(\tfrac{x}{y}\right) = \log_b(x) - \log_b(y)$$

$$\log_b(x^k) = k \log_b(x)$$

$$\log_b(x) = \frac{\log_c(x)}{\log_c(b)}$$



$\log_b(x)$ $b > 1$

Increasing

Domain is $(0, \infty)$

Common bases

Common bases b for exponentials and logarithms:

- $b = 10$
- $b = e = 2.71828\ldots$
- $b = 2$

$$\frac{d}{dx}e^x = e^x; \quad \log_e(x) = \ln(x)$$

Thank you for your attention!

photo: Jorrit Lousberg