Functions

Summary week 1

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photo: Jorrit Lousberg

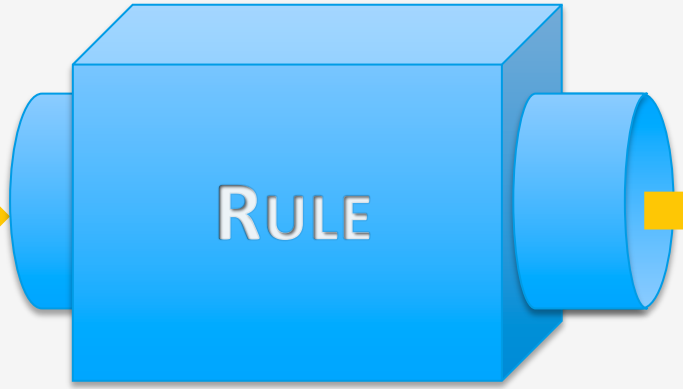
Functions: the definition

Domain

all
input values

Possible restriction:

- context
- defining formulas



Codomain

$\mathbb{R}$ *contains*

Output values

Range

all
attained values

Functions: description

Polynomials

Polynomial function:

Constructed from:

- variable x
- numbers
- addition
- multiplication

Standard form:

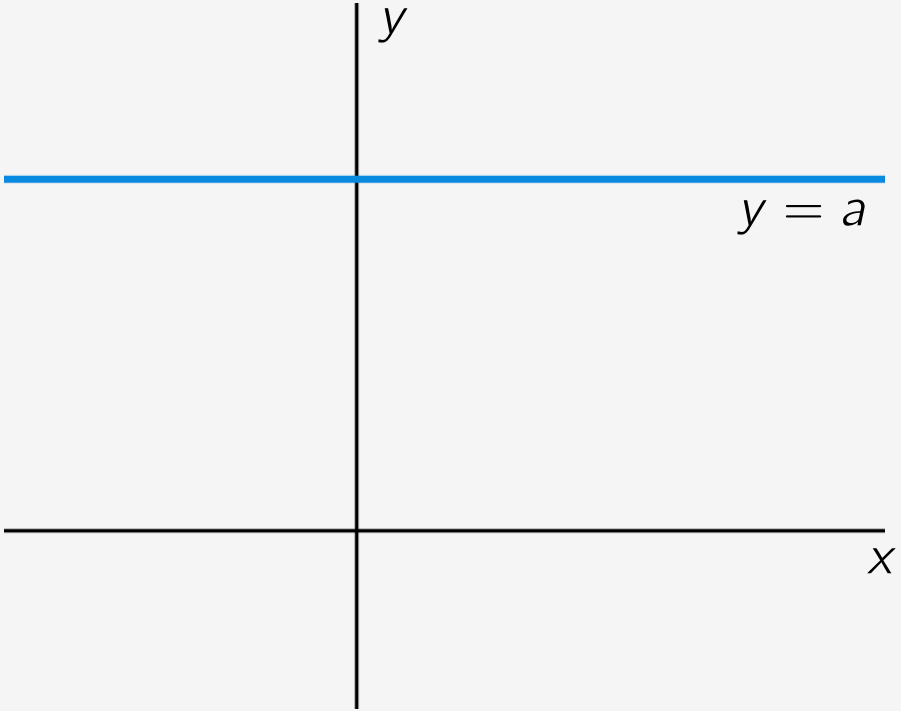
$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

Degree n

For $x \gg 0$ or $x \ll 0$: $\sim a_n x^n$

Max n intersections with straight line

Polynomial functions



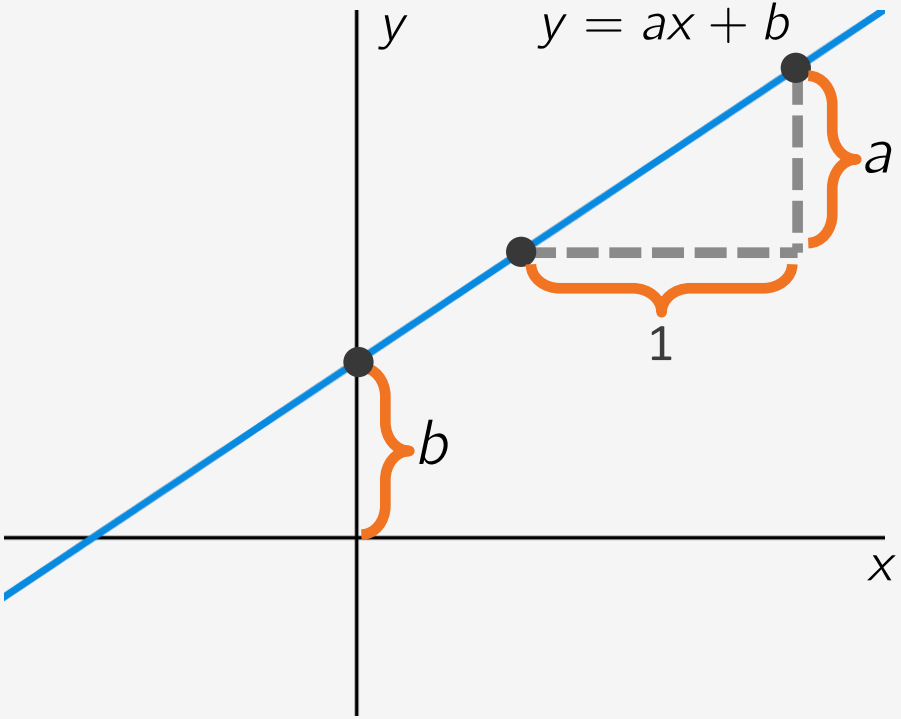
degree 0:

$p(x) = a$

degree 1:

degree 2:

Polynomial functions



degree 0:

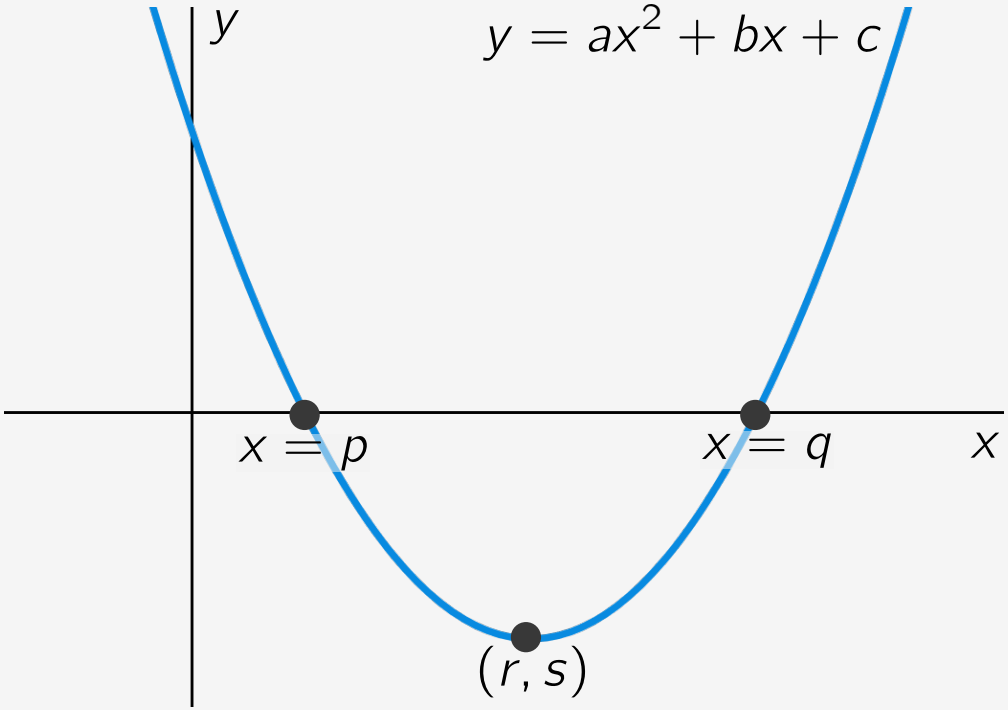
$p(x) = a$

degree 1:

$p(x) = ax + b$

degree 2:

Polynomial functions



degree 0:

$$p(x) = a$$

degree 1:

$$p(x) = ax + b$$

degree 2:

$$\begin{aligned} p(x) &= ax^2 + bx + c \\ &= a(x-p)(x-q) \\ &= a(x-r)^2 + s \end{aligned}$$

Rational functions

Rational function:

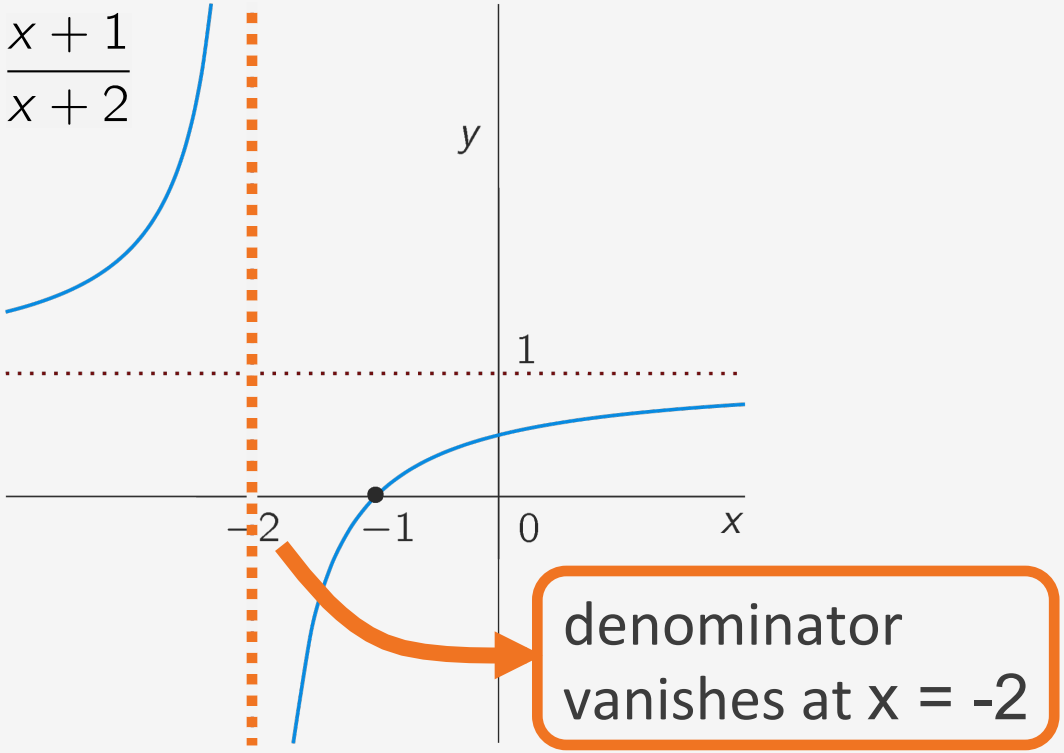
Constructed from:

- variable x
- numbers
- addition
- multiplication
- *division*

Can always be written as $\dfrac{P(x)}{Q(x)}$,

P, Q polynomial functions

Rational functions

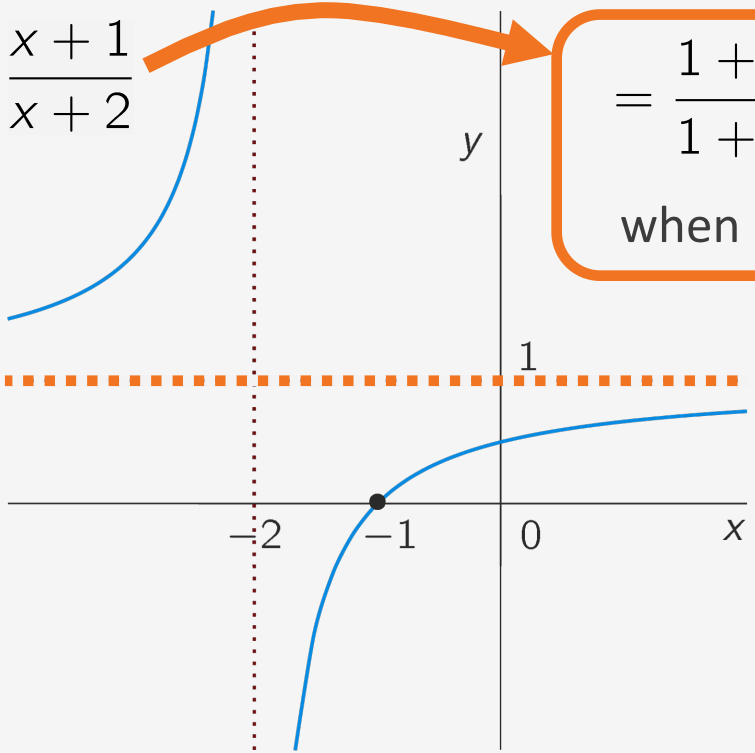


Vertical asymptotes

- function value to $\pm\infty$
- can occur when denominator $= 0$

Rational functions

$$\frac{x+1}{x+2}$$



$$= \frac{1+\frac{1}{x}}{1+\frac{2}{x}} \to 1$$

when $x \to \pm\infty$

Horizontal asymptotes

- function value to constant as x to $\pm\infty$
- divide by highest power of denominator

Power functions

$f(x) = x^a$, a constant

Domain issues:

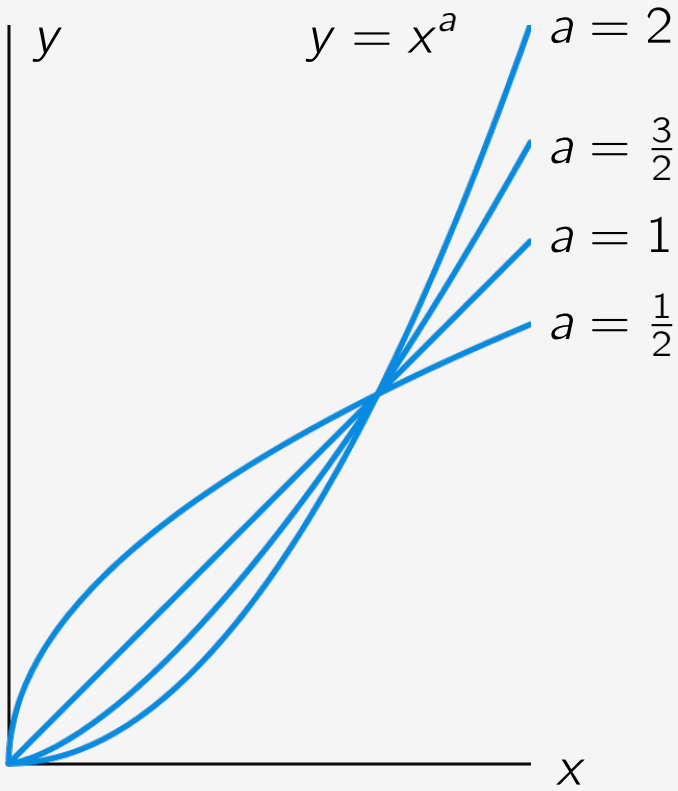
$x > 0$: defined for all a

$x = 0$: defined for $a > 0$

$x < 0$: defined for integer a

... and some non-integer a, such as $a = \frac{1}{3}, \frac{1}{5}, \ldots$

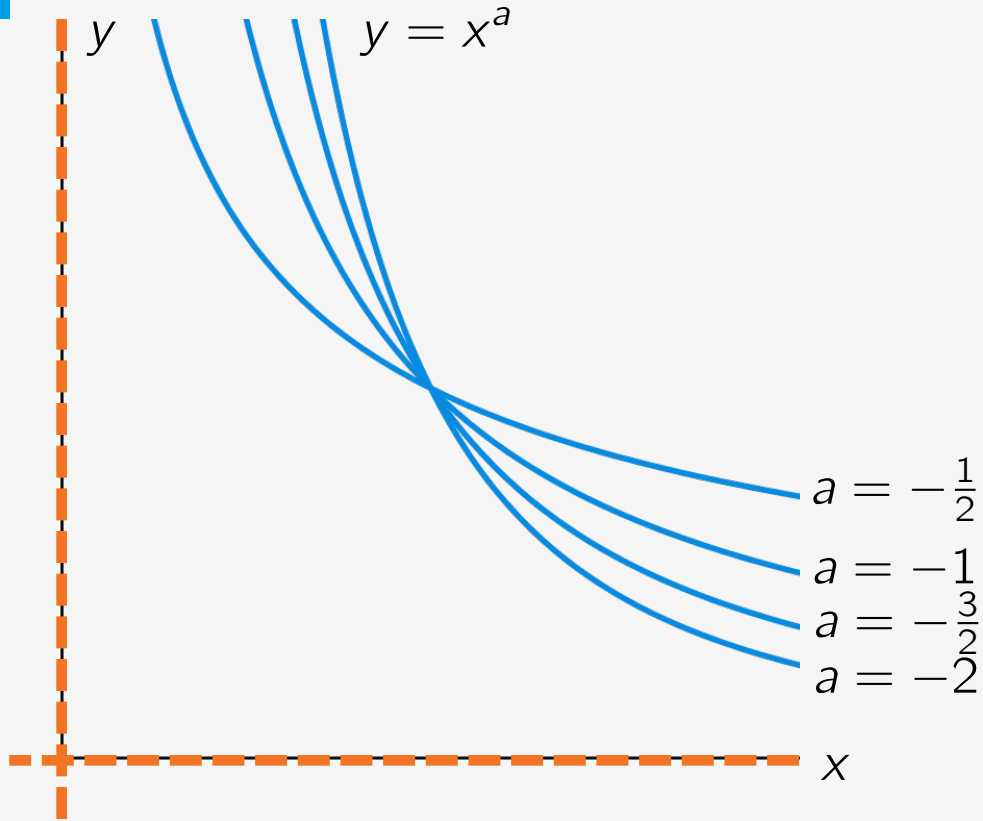
Power functions



Graphs $(x > 0)$

- $a > 0$: increasing

Power functions

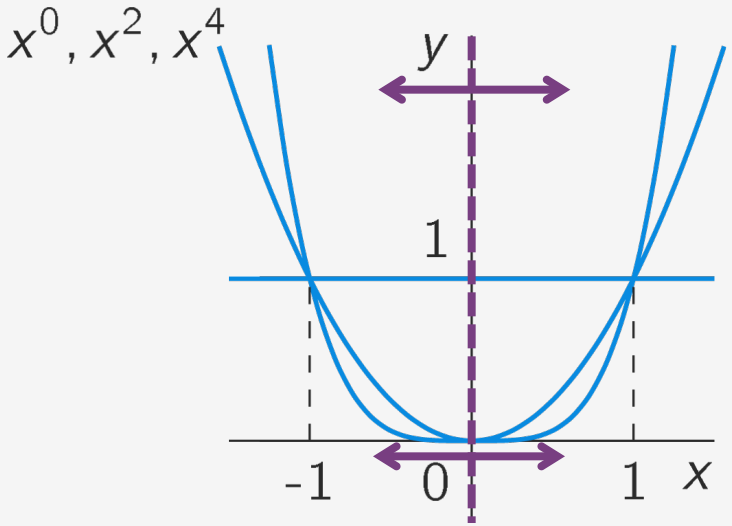


Graphs $(x > 0)$

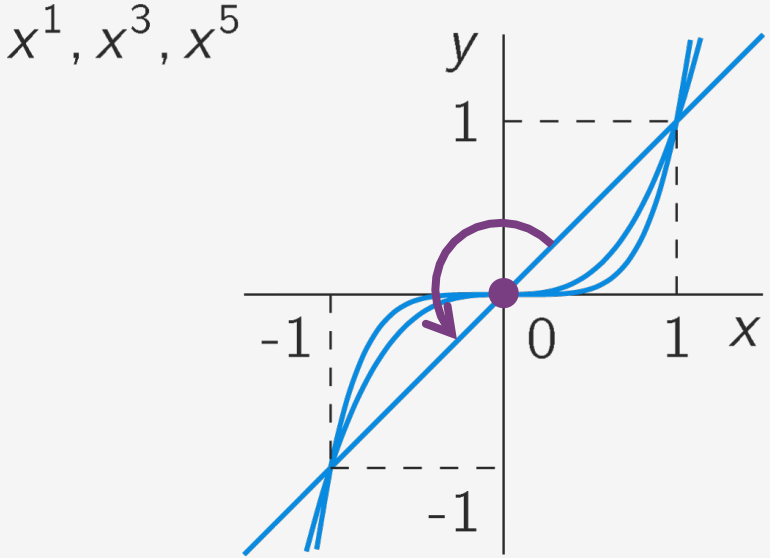
- $a > 0$: increasing
- $a < 0$: decreasing

  asymptotes at $x = 0$ and $y = 0$

Power functions



even power:
symmetric in *y*-axis



odd power:
symmetric in origin

Power functions

Rules of calculation

$$x^a x^b = x^{a+b}$$

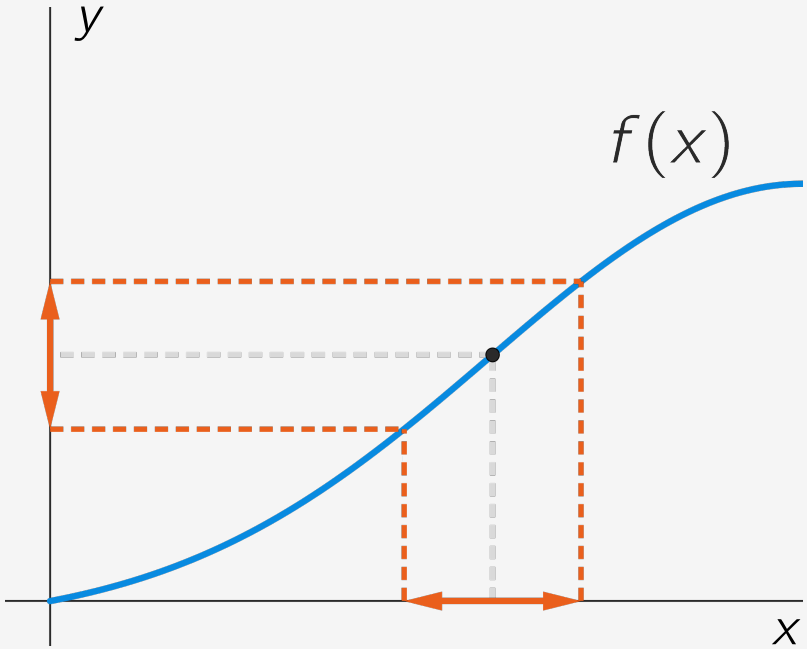
$$\frac{x^a}{x^b} = x^{a-b}$$

$$(x^a)^b = x^{a \cdot b}$$

$$(x\,y)^a = x^a\,y^a$$

hold whenever both sides are defined.

Continuous functions



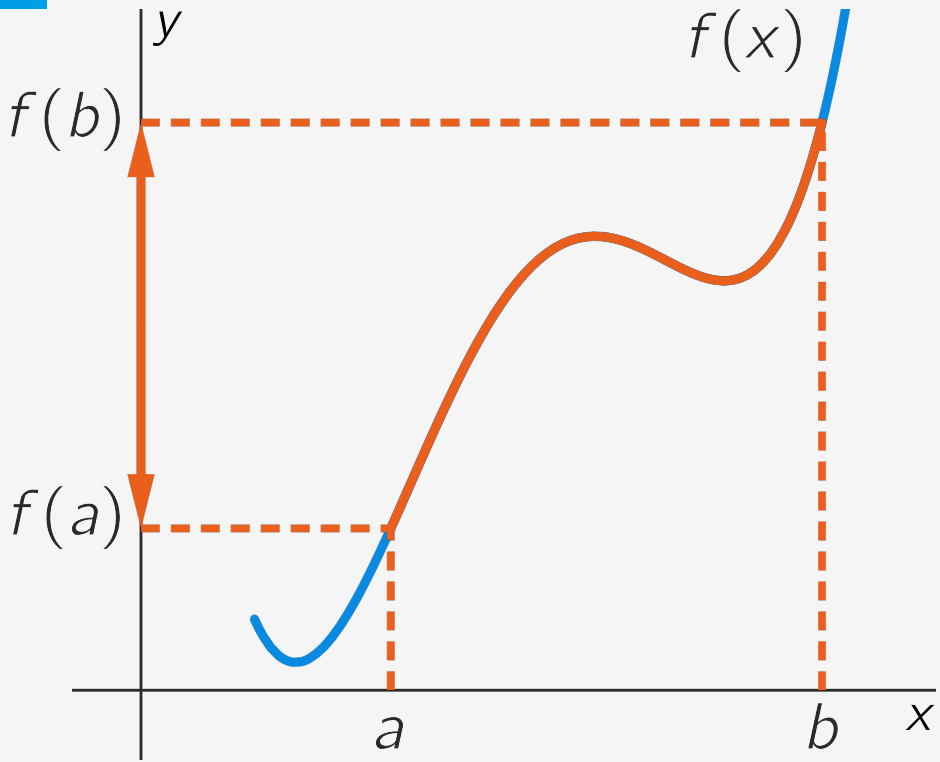
f is continuous:
A small change in x gives
a small change in $f(x)$

If *f* and *g* are continuous, then

- $f + g$
- $f \times g$
- $\frac{f}{g}$ $\quad g(x) \neq 0$

are also continuous.

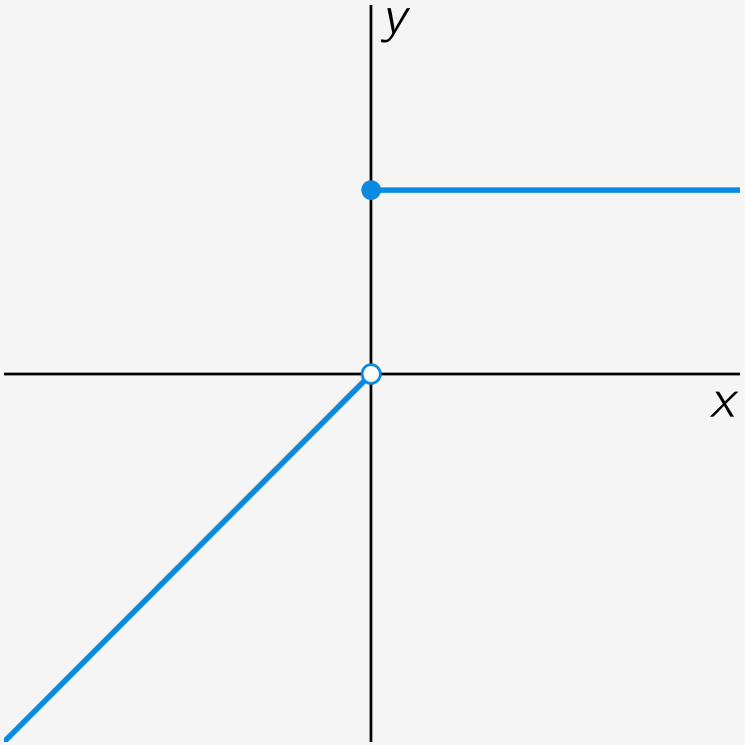
Continuous functions



Intermediate value property

If f is continuous on $[a,b]$, then it attains all values between $f(a)$ and $f(b)$.

Continuous functions



$$f(x) = \begin{cases} x & x < 0 \\ 1 & x \geq 0 \end{cases}$$

is not continuous

Good luck with your homework!

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