Equations

Summary week 3

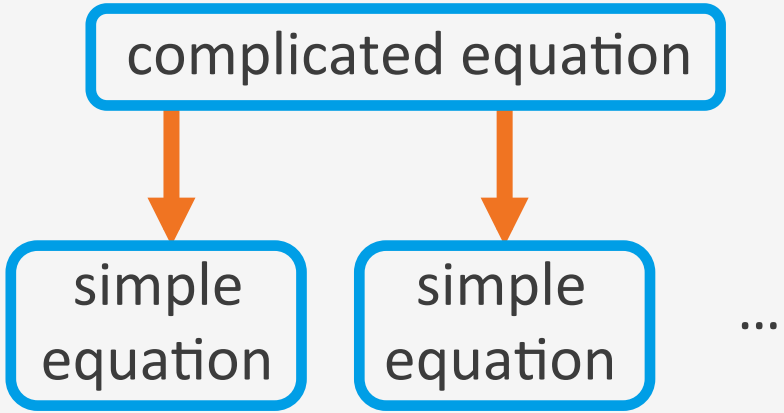
Pascal de koster

Solving equations

- Simplify ☐
- Solve ☐
- Check ☐

Solving equations

- Simplify ☐
- Solve ☐
- Check ☐



Simplify

- [x] Simplify
- [ ] Solve
- [ ] Check

Balance method

Be careful:

$A \cdot C = B \cdot C \rightarrow A = B$ or $C = 0$

$A^2 = B^2 \rightarrow A = B$ or $A = -B$

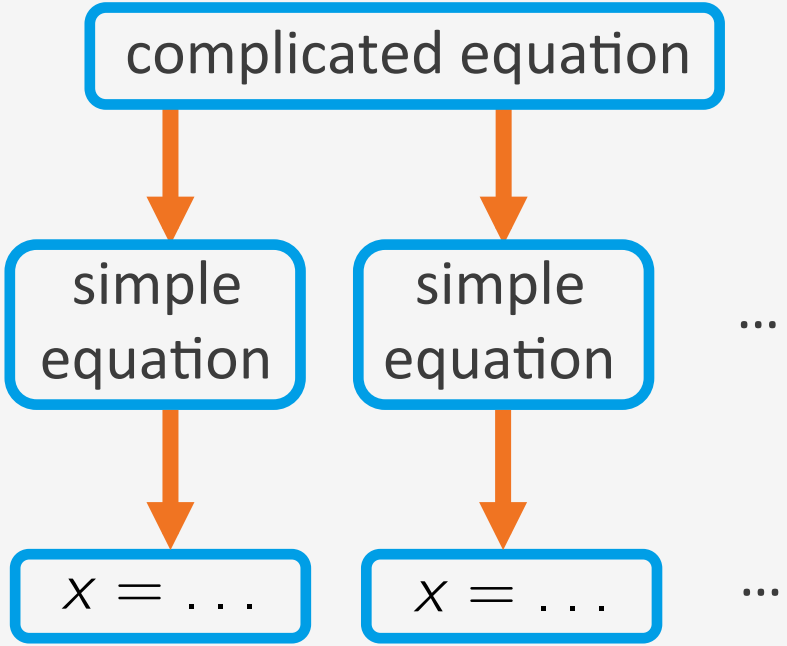
Factorization

$A \cdot B = 0 \rightarrow A = 0$ or $B = 0$

Substitution

Solve

- Simplify ☑
- Solve ☐
- Check ☐

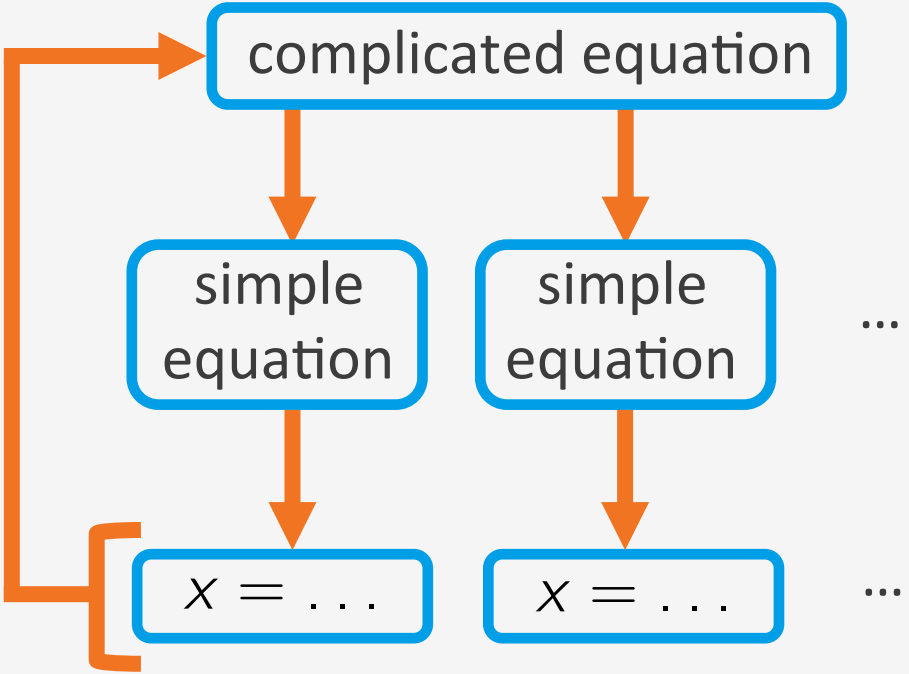


Check

Simplify ☑

Solve ☑

Check ☑



Polynomial equations

$p(x) = 0$, p polynomial function

Degree 1: $ax + b = 0 \quad (a \neq 0)$

Solution: $x = -\frac{b}{a}$

Polynomial equations

$p(x) = 0$, p polynomial function

Degree 2: $ax^2 + bx + c = 0 \quad (a \neq 0)$

Factorize:

$a(x-p)(x-q) = 0$

$x = p \quad \text{or} \quad x = q$

Complete the square:

$a(x-r)^2 + s = 0$

Solve!

Quadratic formula:

$D = b^2 - 4ac$

if $D \geqslant 0$,

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$$

Polynomial equations

$p(x) = 0$, p polynomial function

Higher degree: $p(x) = 0$

Factorize:

$$p(x) = q_1(x) \cdot q_2(x)$$

$$\begin{cases} q_1(x) = 0 \\ \text{or } \; q_2(x) = 0 \end{cases}$$

Guess solution $x = a$:

$$p(x) = (x - a)q(x)$$

$$\begin{cases} x = a \\ \text{or } \; q(x) = 0 \end{cases}$$

Substitute:

$$(\boxed{x^2})^2 + 3\boxed{x^2} + 2 = 0$$

$$\Big\downarrow \; x^2 = u$$

$$u^2 + 3u + 2 = 0$$

Polynomial equations

$p(x) = 0$, p polynomial function

Higher degree: $p(x) = 0$, p degree n

- at most n solutions
- n odd? At least 1 solution

Rational functions and roots

Get rid of rational functions and roots!

Multiply
by denominators

- Isolate root
- Raise to power

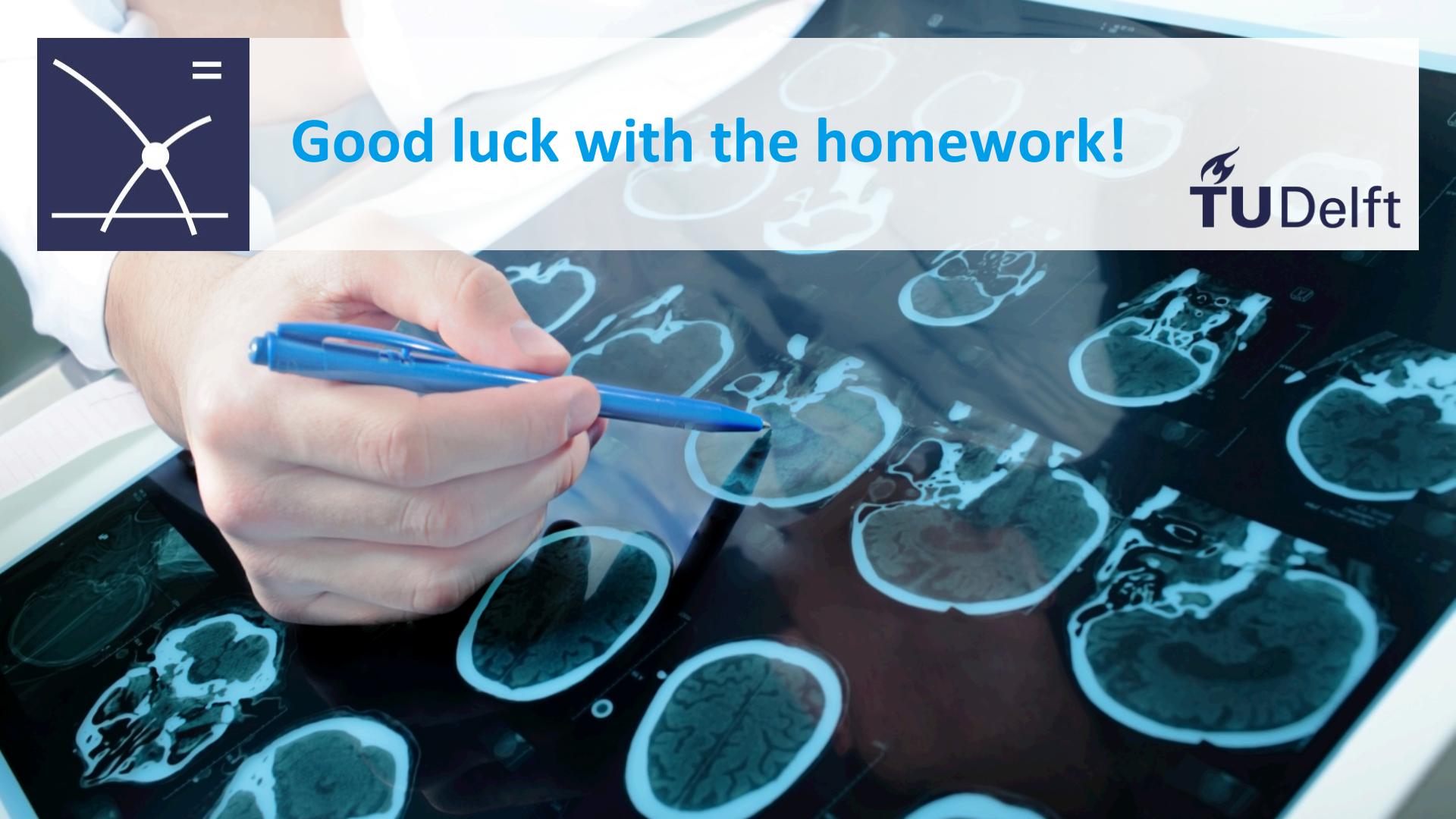
Next week…

Equations involving

- exponential functions
- logarithms
- trigonometric functions

Simplify ☑

Solve ☑

Check ☑

Good luck with the homework!