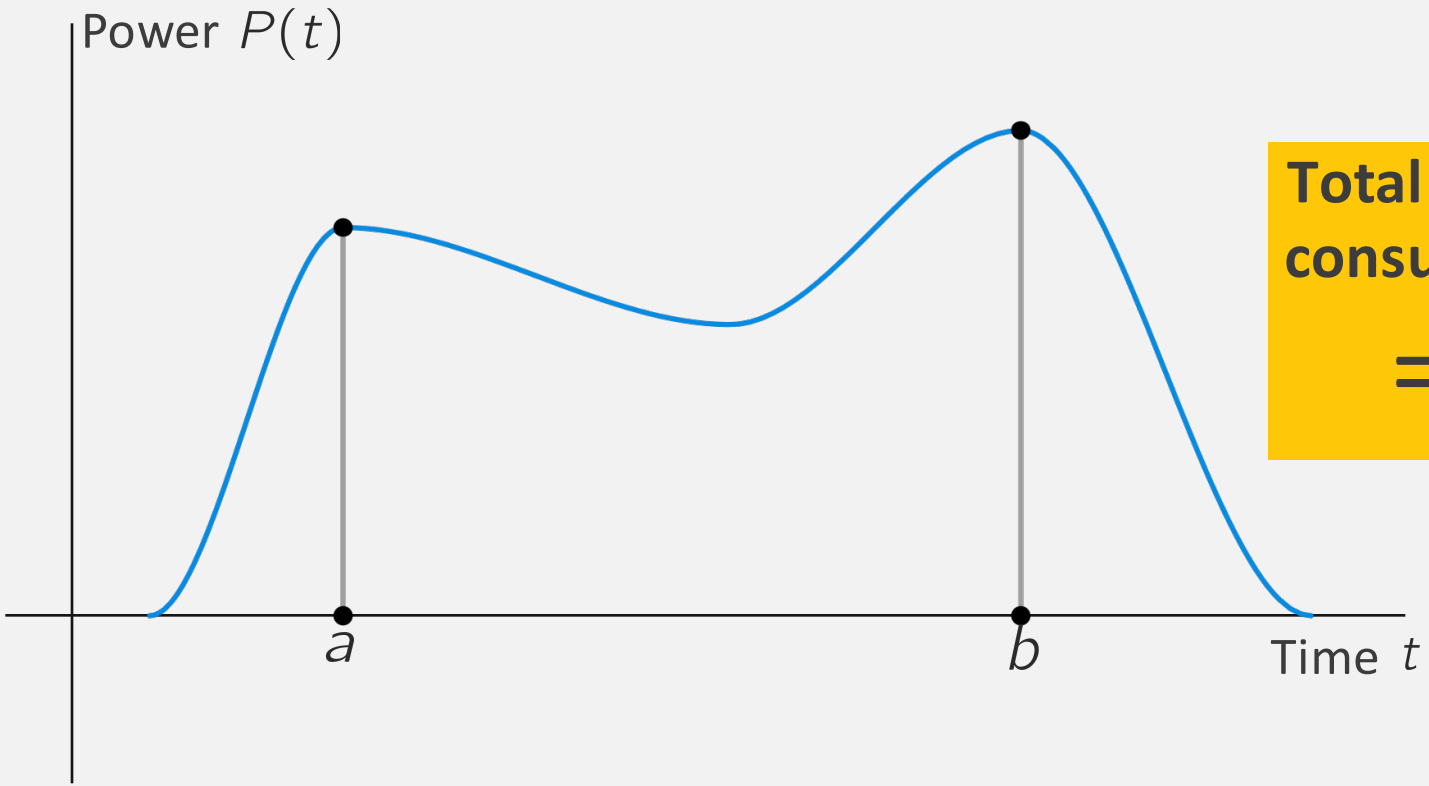
Integration

Summary

Pascal de Koster

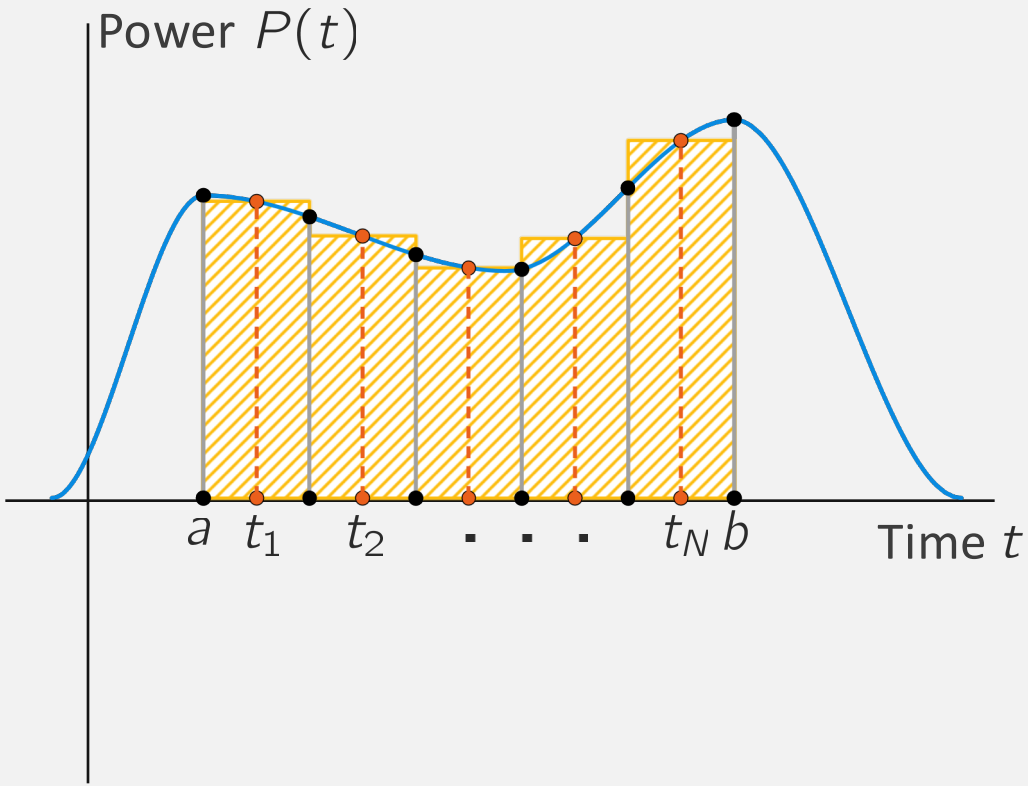
Determine total energy consumption



Total energy consumption

= ?

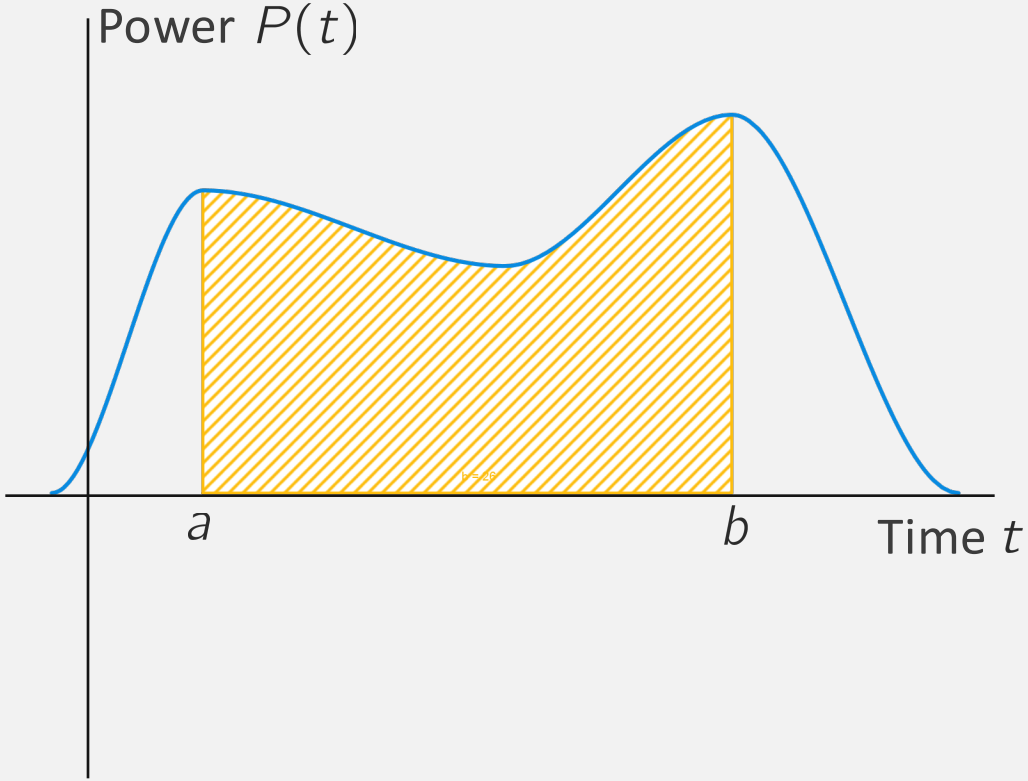
Determine total energy consumption



- Subintervals of size Δt
- Sample points $t_1, \ldots, t_N$
- Riemann sum:

$$\sum_{i=1}^{N} P(t_i) \cdot \Delta t$$

Determine total energy consumption

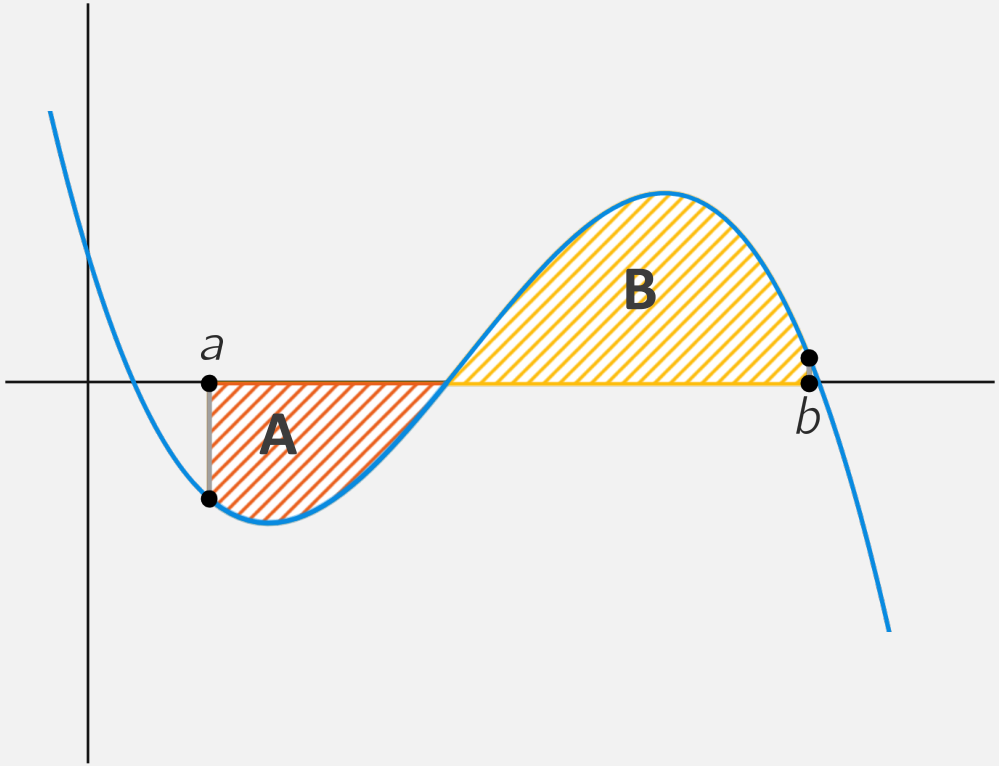


Limit: $\begin{cases} \Delta t \to 0 \\ N \to \infty \end{cases}$

Integral $\displaystyle\int_a^b P(t)\,dt$

= exact energy consumption

The integral – what does it mean?



$$\int_a^b f(x)\,dx$$

= **signed area**

= area B – area A

The Fundamental Theorem of Calculus

- f continuous function on $[a, b]$;
- F function such that $F' = f$;

then $$\int_a^b f(x)\,dx = F(b) - F(a)$$

- F is a **primitive function** of f, *so is* $F + C$

Standard Integrals

$$\int x^n = \frac{1}{n+1}x^{n+1} + C \quad \text{whenever} \quad n \neq -1$$

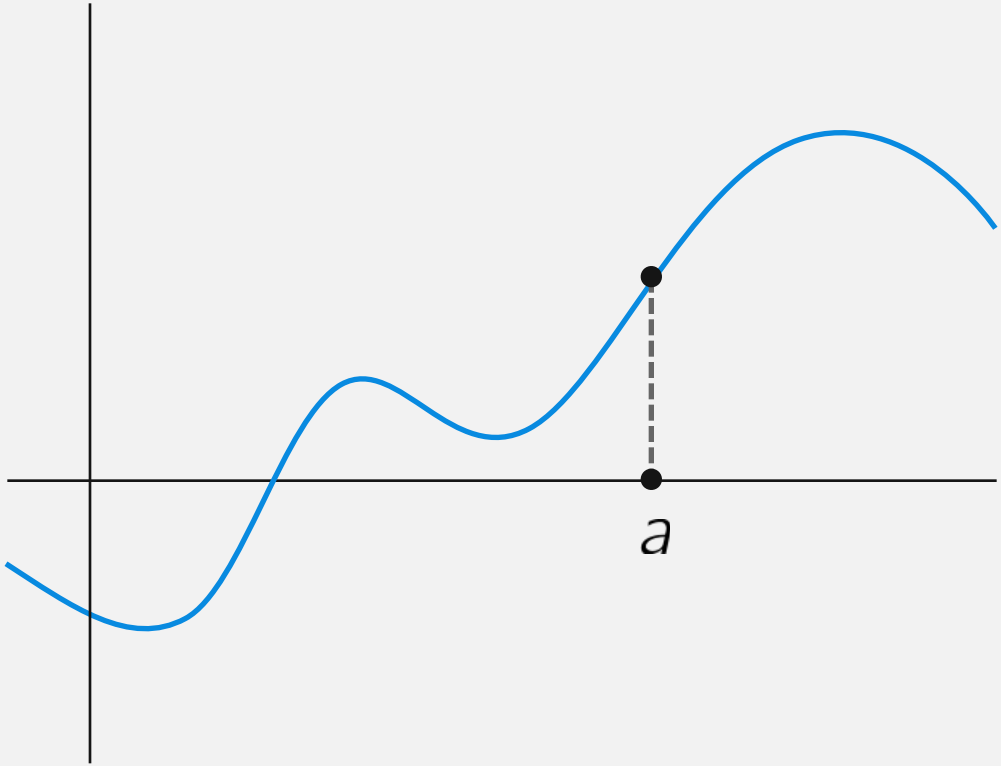
$$\int \frac{1}{x} = \ln(x) + C$$

$$\int a^x dx = \frac{1}{\ln(a)}a^x + C$$

$$\int \sin(x)dx = -\cos(x) + C$$

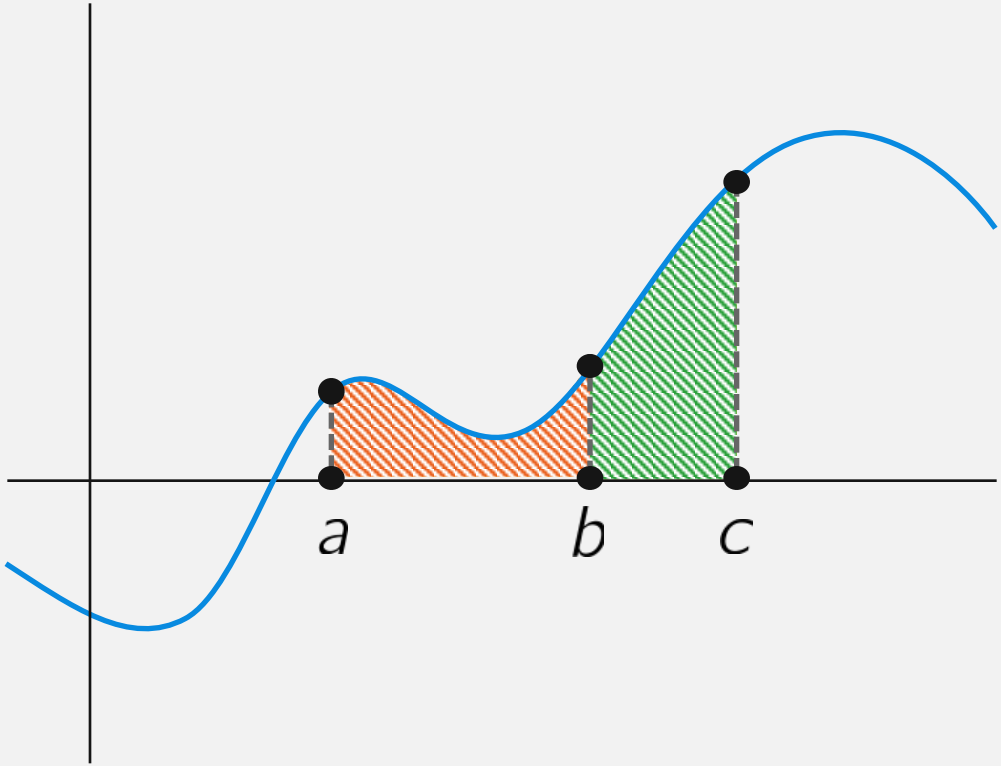
$$\int \cos(x)dx = \sin(x) + C$$

Integral with same upper and lower bound



$$\int_a^a f(x)dx = 0$$

Splitting an integral



$$\int_a^c f(x)dx$$
$$= \int_a^b f(x)dx + \int_b^c f(x)dx$$

Interval rules

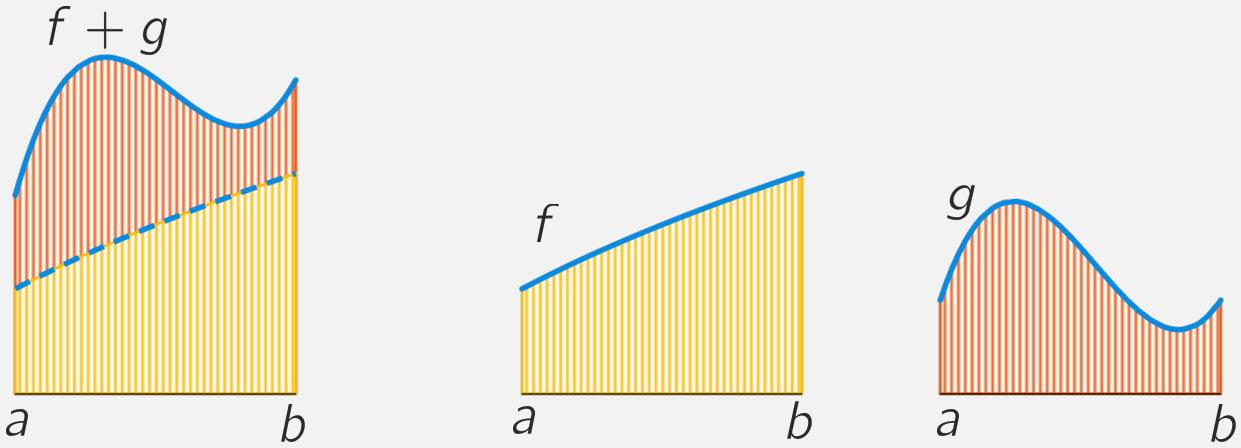
$$\int_a^a f(x)dx = 0$$

$$\int_a^c f(x)dx = \int_a^b f(x)dx + \int_b^c f(x)dx$$

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

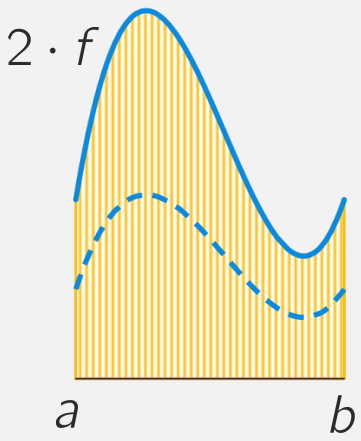
Function rules: addition

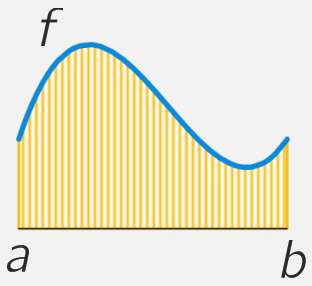
$$\int_a^b f(x) + g(x)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$



Function rules: scaling

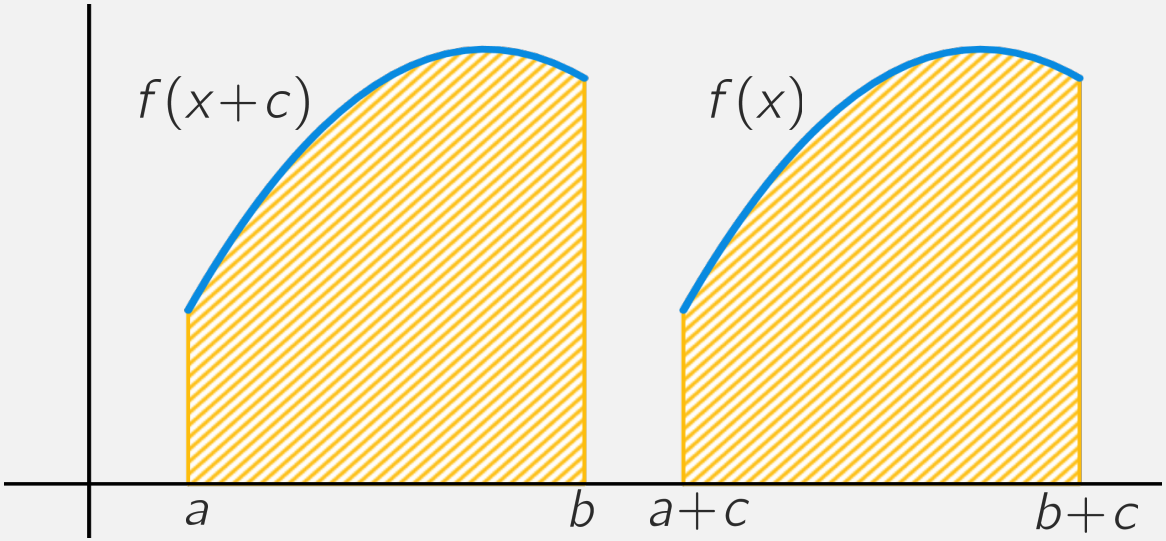
$$\int_a^b c \cdot f(x)dx = c \cdot \int_a^b f(x)dx$$





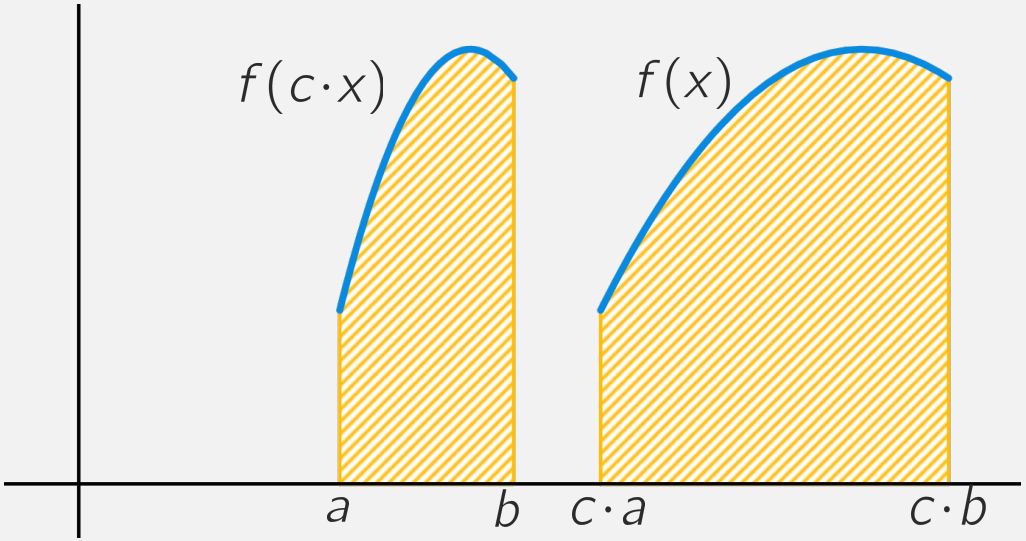
Linear substitution: translation

$$\int_a^b f(x+c)dx = \int_{a+c}^{b+c} f(x)dx$$



Linear substitution: scaling

$$\int_a^b f(c \cdot x)dx = \int_{c \cdot a}^{c \cdot b} \frac{1}{c} f(x)dx$$



The Substitution Method

1. substitution

$$u = x^3 + 5x$$

2. relation *du* and *dx*

$$\begin{cases} \dfrac{du}{dx} = 3x^2 + 5 \\ du = (3x^2 + 5)\,dx \end{cases}$$

$$\int \cos(x^3 + 5x) \cdot (3x^2 + 5)\,dx$$

3. express integral in *u* and *du*

$$= \int \cos(u)\,du = \sin(u) + C$$

4. evaluate

$$= \sin(x^3 + 5x) + C$$

5. re-express in *x*

Integration by parts

$$\int fg\,dx = Fg - \int Fg'\,dx$$

$$f = F'$$

Integration

Good luck with your homework!