Qmm-Assignment-6

Durga Prasad Gandi

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Loading required libraries

```
library(lpSolve)
library(lpSolveAPI)
```

Problem

AP is a package delivery service in the continental United States that guarantees overnight delivery. Throughout the country, the company has hubs in major cities and airports. Packages are received at hubs before being routed to intermediate or final destinations. The manager of the AP hub in Cleveland is concerned about labor costs and wants to know how to schedule employees in the most efficient way possible. The hub is open seven days a week, and the volume of packages handled varies by day. The table below estimates how many workers are needed on each day of the week.

```
Days_of_Week = c("Sunday", "Monday", "Tuesday", "Wednesday", "Thursday", "Friday", "Saturday")
Workers_required_perday = c(20,25,22, 28, 25,22, 20)
Workers_per_day = data.frame(Days_of_Week, Workers_required_perday)
Workers_per_day
```

```
##   Days_of_Week Workers_required_perday
## 1       Sunday                      20
## 2       Monday                      25
## 3      Tuesday                      22
## 4    Wednesday                      28
## 5     Thursday                      25
## 6       Friday                      22
## 7     Saturday                      20
```

Package handlers at AP are guaranteed a five-day work week with two consecutive days off. The base wage for the handlers is $750 per week. Workers working on Saturday or Sunday receive an additional $20 per day. The possible shifts and salaries for package handlers are:

```
Shifts_day = c(1,2,3,4,5,6,7)
Days_of_Week_off = c("Sunday & Monday", "Monday & Tuesday", "Tuesday & Wednesday", "Wednesday & Thursda
Wages_day = c(770, 790, 790, 790, 790, 770, 750)
shift_wages = data.frame(Shifts_day, Days_of_Week_off, Wages_day)
shift_wages
```

```
##   Shifts_day     Days_of_Week_off Wages_day
## 1          1      Sunday & Monday       770
## 2          2     Monday & Tuesday       790
## 3          3  Tuesday & Wednesday       790
## 4          4 Wednesday & Thursday       790
## 5          5    Thursday & Friday       790
## 6          6    Friday & Saturday       770
## 7          7    Saturday & Sunday       750
```

Questions

The manager wants to keep the total wage expenses as low as possible while ensuring that there are sufficient number of workers available each day.

1. Formulate the problem.
2. Solve the problem in R markdown.
3. Find the total cost and the number of workers available each day.

Hint: The number of available workers each day can exceed, but cannot be below the required amount.

1. Problem Formulation

Let us consider,

Number of people working in shift 1

$$x_1$$

Number of people working in shift 2

$$x_2$$

Number of people working in shift 3

$$x_3$$

Number of people working in shift 4

$$x_4$$

Number of people working in shift 5

$$x_5$$

Number of people working in shift 6

$$x_6$$

Number of people working in shift 7

$$x_7$$

People working on Sunday

$$x_2 + x_3 + x_4 + x_5 + x_6$$

People working on Monday

$$x_3 + x_4 + x_5 + x_6 + x_7$$

People working on Tuesday

$$x_4 + x_5 + x_6 + x_7 + x_1$$

People working on Wednesday

$$x_5 + x_6 + x_7 + x_1 + x_2$$

People working on Thursday

$$x_6 + x_7 + x_1 + x_2 + x_3$$

People working on Friday

$$x_7 + x_1 + x_2 + x_3 + x_4$$

People working on saturday

$$x_1 + x_2 + x_3 + x_4 + x_5$$

$$MinZ = 770x_1 + 790x_2 + 790x_3 + 790x_4 + 790x_5 + 770x_6 + 750x_7$$

Constraints:

Sunday workers may go over the required amount, but they may not go below it.

$$x_2 + x_3 + x_4 + x_5 + x_6 >= 20$$

Monday workers may go over the required amount, but they may not go below it.

$$x_3 + x_4 + x_5 + x_6 + x_7 >= 25$$

Tuesday workers may go over the required amount, but they may not go below it.

$$x_4 + x_5 + x_6 + x_7 + x_1 >= 22$$

Wednesday workers may go over the required amount, but they may not go below it.

$$x_5 + x_6 + x - 7 + x_1 + x_2 >= 28$$

Thursday workers may go over the required amount, but they may not go below it.

$$x_6 + x_7 + x_1 + x_2 + x_3 >= 25$$

Friday workers may go over the required amount, but they may not go below it.

$$x_7 + x_1 + x_2 + x_3 + x_4 >= 22$$

Saturdayday workers may go over the required amount, but they may not go below it.

$$x_1 + x_2 + x_3 + x_4 + x_5 >= 20$$

Non-negativity of constraints:

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 >= 0$$

2. Solve the problem in R markdown.

Creating a LP problem with 7 constraints and 7 decision variables.

```
solvelp = make.lp(7, 7)
```

The LP problem is represented by the object solvelp.

The objective function coefficients for each of the seven choice variables are defined in this line. Here is a vector with the coefficients in it. Ordering the variables consistently is important. Constraints and the objective function should be in the same sequence.

```
set.objfn(solvelp, c(770, 790, 790, 790, 790, 770, 750))
```

By setting the objective sense to "min" this line's aim is to minimize the objective function.

```
lp.control(solvelp, sense = 'min')
```

```
## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"       "dynamic"      "rcostfixing"
##
## $break.at.first
## [1] FALSE
##
## $break.at.value
## [1] -1e+30
##
## $epsilon
##       epsb       epsd      epsel     epsint epsperturb   epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##    1e-11    1e-11
##
## $negrange
```

```
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"    "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"   "equilibrate" "integers"
##
## $sense
## [1] "minimize"
##
## $simplextype
## [1] "dual"   "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"
```

Adding constraints to the LP problem.

```
set.row(solvelp,1,c(0,1,1,1,1,1,0))

set.row(solvelp,2,c(0,0,1,1,1,1,1))

set.row(solvelp,3,c(1,0,0,1,1,1,1))

set.row(solvelp,4,c(1,1,0,0,1,1,1))

set.row(solvelp,5,c(1,1,1,0,0,1,1))

set.row(solvelp,6,c(1,1,1,1,0,0,1))

set.row(solvelp,7,c(1,1,1,1,1,0,0))
```

Setting the right hand side values for the constraints in LP problem.

```
rights = c(20, 25, 22, 28, 25, 22, 20)
set.rhs(solvelp,rights)
```

Setting the directions(Types) and bounds to the LP problem.

```r
set.constr.type(solvelp,c(">=", ">=", ">=", ">=", ">=", ">=", ">="))
set.bounds(solvelp,lower = rep(0,7))
```

```r
set.type(solvelp, 1:7 ,type = c("integer"))
```

Subject the rownames to the constraints.

```r
lp.rownames = c("1", "2", "3", "4", "5", "6", "7")
```

Assigning names to the decision variables. p stands for plus and m for minus.

```r
lp.colnames = c("x1", "x2", "x3", "x4", "x5", "x6", "x7")
solvelp
```

```
## Model name:
##            C1   C2   C3   C4   C5   C6   C7
## Minimize  770  790  790  790  790  770  750
## R1          0    1    1    1    1    1    0  >=  20
## R2          0    0    1    1    1    1    1  >=  25
## R3          1    0    0    1    1    1    1  >=  22
## R4          1    1    0    0    1    1    1  >=  28
## R5          1    1    1    0    0    1    1  >=  25
## R6          1    1    1    1    0    0    1  >=  22
## R7          1    1    1    1    1    0    0  >=  20
## Kind      Std  Std  Std  Std  Std  Std  Std
## Type      Int  Int  Int  Int  Int  Int  Int
## Upper     Inf  Inf  Inf  Inf  Inf  Inf  Inf
## Lower       0    0    0    0    0    0    0
```

Solving the Lp problem using the constraints and objective function.

```r
solve(solvelp)
```

```
## [1] 0
```

Getting the value of objective function.

```r
get.objective(solvelp)
```

```
## [1] 25550
```

Getting the values of decision variables.

```r
get.variables(solvelp)
```

```
## [1]  2  6  4  0  8  2 11
```

To Get the values of constraints.

```r
get.constraints(solvelp)
```

```
## [1] 20 25 23 29 25 23 20
```

3.Find the total cost and the number of workers available each day

The optimal solution total cost is $25,550 .

The number of workers available on any given day is determined by the following decision variables:

During Sunday 20 people are working.

During Monday, there are 25 people working.

During Tuesday, there are 23 people working.

During Wednesday has a total of 29 employees.

During Thursday has a workforce of 25 people.

During Friday has a total of 23 employees.

During Saturday has a workforce 20 people.

In summary, the approach offers the best possible worker schedule that meets the needed number of workers per day while minimizing overall costs. Interpretation includes comprehending the formulated LP problem, solving it, and deriving relevant information from the outcomes.