

## 1 Bisection algorithm for simplicial meshes

- Serial algorithm
- Longest edge splitting with propagation front
- For 2D meshes with triangles isosceles the number of congruence classes is 1.
- The algorithm works directly for 3D and 4D.

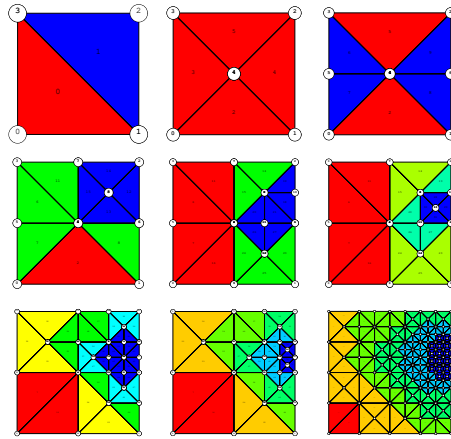


Figure 1: Steps of the bisection algorithm.

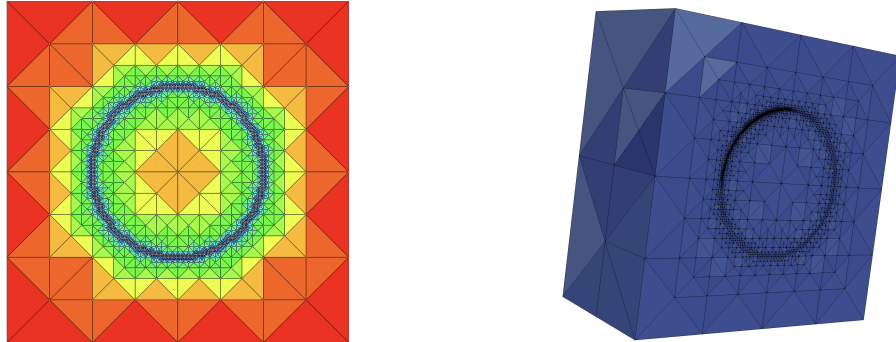


Figure 2: Result for sphere refinement pattern. Left: 2D. Right: 3D.

## 2 Quality for recursive longest-edge

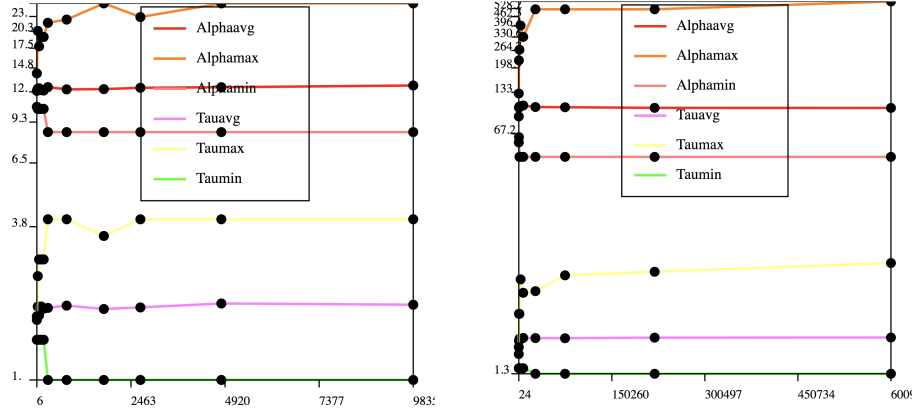


Figure 3: Quality of the elements for several iterations of the adaptive refinement shown in Figure 2. The  $x$  axis represent the total number of elements in the mesh. The  $y$  axis the quality criterion (lower is better). Left: 3D. Right: 4D.

Table 1  
A list of tetrahedron shape measures used in literature

Aspect ratio measure	Value for a equilateral tetrahedron	Used in
$\beta = \frac{CR}{IR}$	$\beta^* = 3.0$	[1]
$\sigma = \frac{S_{\max}}{IR}$	$\sigma^* = 4.898979$	[2]
$\omega = \frac{CR}{S_{\max}}$	$\omega^* = 0.612507$	[2]
$\tau = \frac{S_{\max}}{S_{\min}}$	$\tau^* = 1.0$	[2]
$\kappa = \frac{V^4}{\left[ \sum_{i=1}^4 S_i^2 \right]^3}$	$\kappa^* = 4.58457e-04$	[3]
$\alpha = \frac{S_{\text{avg}}^3}{V}$	$\alpha^* = 8.479670$	[4]
$\gamma = \frac{S_{\text{rms}}^3}{V}$	$\gamma^* = 8.479670$	[5]

Nomenclature:  
 $CR$  = radius of the circumscribed sphere,  $IR$  = radius of the inscribed sphere,  $S_i$  = length of any edge  $i$ ,  $S_{\max} = \max(S_i)$  ( $i = 1 \dots 6$ ),  $S_{\min} = \min(S_i)$  ( $i = 1 \dots 6$ ),  $SA$  = surface area of a triangular facet,  $S_{\text{avg}}$  = average ( $S_i$ ) ( $i = 1 \dots 6$ ),  $S_{\text{rms}}$  = root mean square ( $S_i$ ) ( $i = 1 \dots 6$ ),  $V$  = volume of the tetrahedron, and

$$V = \frac{1}{6} \begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix}, \quad IR = \frac{4V}{\left[ \sum_{i=1}^4 SA \right]}, \quad S_{\text{rms}} = \sqrt{\frac{1}{6} \sum_{i=1}^6 S_i^2}.$$

Figure 4: Quality metrics. We used “Alpha” and “Tau”.

### 3 Newest-vertex with longest edge for $D > 2$

Without a recursive closure of the refined edge the overall element quality degrades very fast.

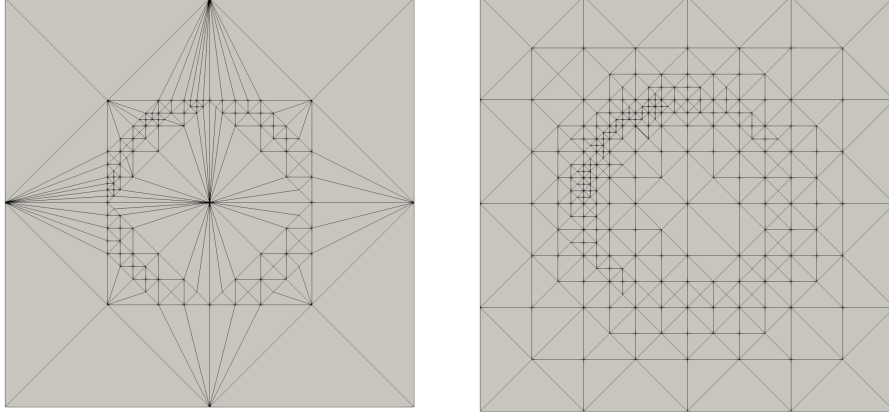


Figure 5: Non-recursive vs. recursive (3D).

### 4 Quality of newest vertex with longest edge.

While the longest-edge criterion is superior the hybrid approach (newest vertex + longest edge) stays in the same order of magnitude for the degrading element quality.

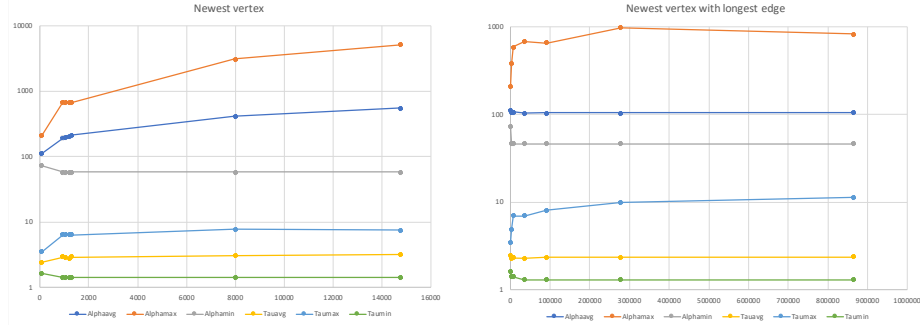


Figure 6: Quality of the elements for several iterations of the adaptive refinement shown in Figure 2. The  $x$  axis represent the total number of elements in the 4D mesh. The  $y$  axis the quality criterion (lower is better). The influence of longest edge shows on both the better quality and the larger number of elements generated.

## 5 Parallelization

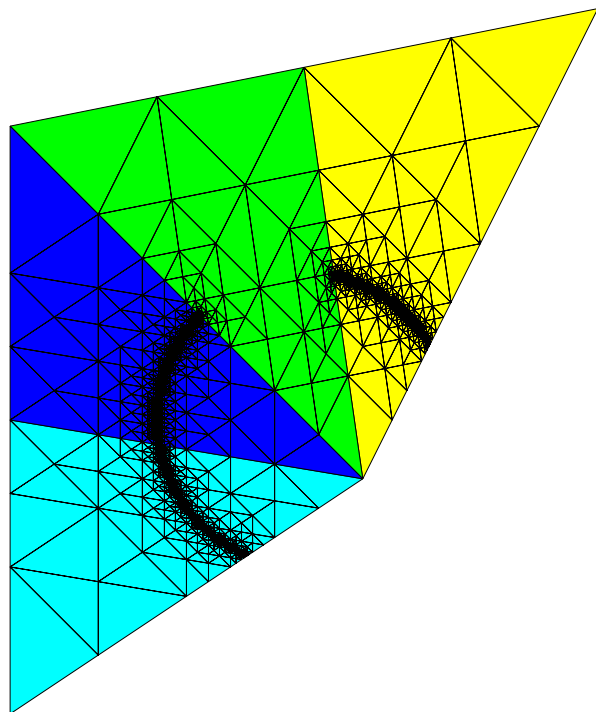


Figure 7: Partitioning with one part without refinement.