1 Bisection algorithm for simplical meshes

- Serial algorithm
- Longest edge splitting with propagation front
- For 2D meshes with triangles isosceles the number of congruence classes is 1.
- The algorithm works directly for 3D and 4D.

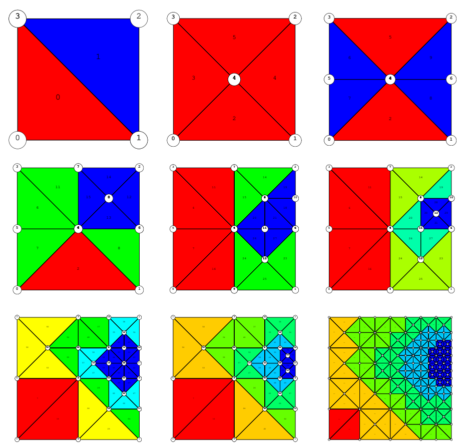

Figure 1: Steps of the bisection algorithm.

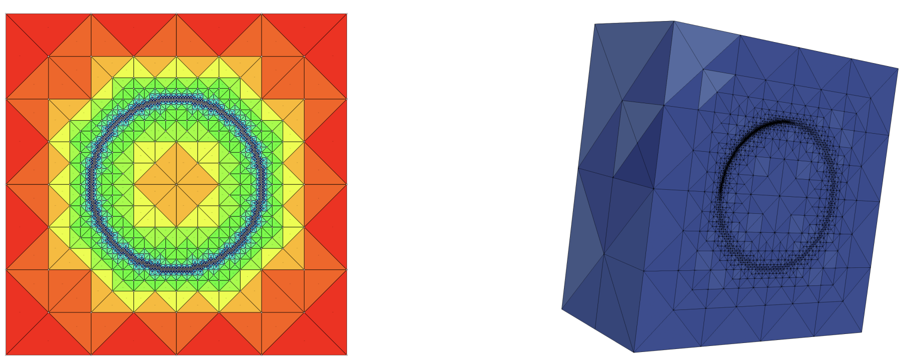

Figure 2: Result for sphere refinement pattern. Left: 2D. Right: 3D.

2 Quality for recursive longest-edge

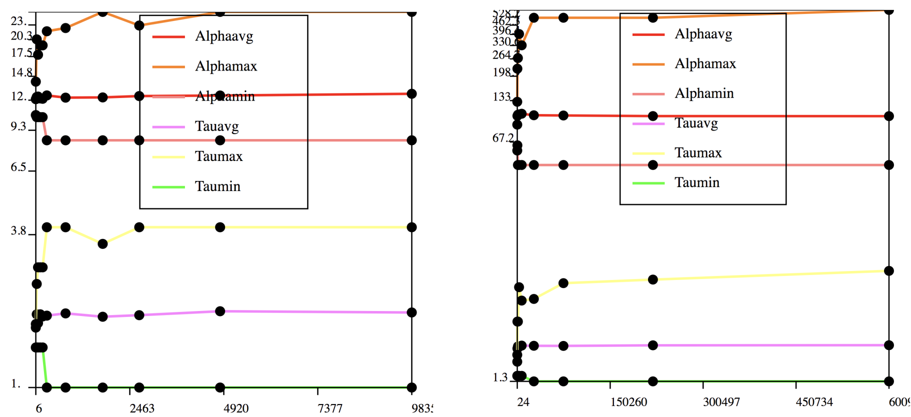

Figure 3: Quality of the elements for several iterations of the adaptive refinement shown in Figure 2. The x axis represent the total number of elements in the mesh. The y axis the quality criterion (lower is better). Left: 3D. Right: 4D.

Table 1
A list of tetrahedron shape measures used in literature

Aspect ratio measure	Value for a equilateral tetrahedron	Used in
$\beta = \frac{CR}{IR}$	$\beta^* = 3.0$	[1]
$\sigma = \frac{S_{\max}}{IR}$	$\sigma^* = 4.898979$	[2]
$\omega = \frac{CR}{S_{\max}}$	$\omega^* = 0.612507$	[2]
$\tau = \frac{S_{\max}}{S_{\min}}$	$\tau^* = 1.0$	[2]
$\kappa = \frac{V^4}{\left[\sum_{i=1}^{i=4} SA^2\right]^3}$	$\kappa^* = 4.58457e-04$	[3]
$\alpha = \frac{S_{\text{avg}}^3}{V}$	$\alpha^* = 8.479670$	[4]
$\gamma = \frac{S_{\text{rms}}^3}{V}$	$\gamma^* = 8.479670$	[5]

Nomenclature:
CR = radius of the circumscribed sphere, IR = radius of the inscribed sphere, S_i = length of any edge i, $S_{\max} = \max(S_i)$ $(i = 1 \ldots 6)$, $S_{\min} = \min(S_i)$ $(i = 1 \ldots 6)$, SA = surface area of a triangular facet, S_{avg} = average (S_i) $(i = 1 \ldots 6)$, S_{rms} = root mean square (S_i) $(i = 1 \ldots 6)$, V = volume of the tetrahedron, and

$$V = \tfrac{1}{6} * \begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix}, \quad IR = \frac{4V}{\left[\sum_{i=1}^{i=4} SA\right]}, \quad S_{\text{rms}} = \sqrt{\tfrac{1}{6} \sum_{i=1}^{i=6} S_i^2}.$$

Figure 4: Quality metics. We used "Alpha" and "Tau".

3 Newest-vertex with longest edge for $D > 2$

Without a recurisve closure of the refined edge the overall element quality degrades very fast.

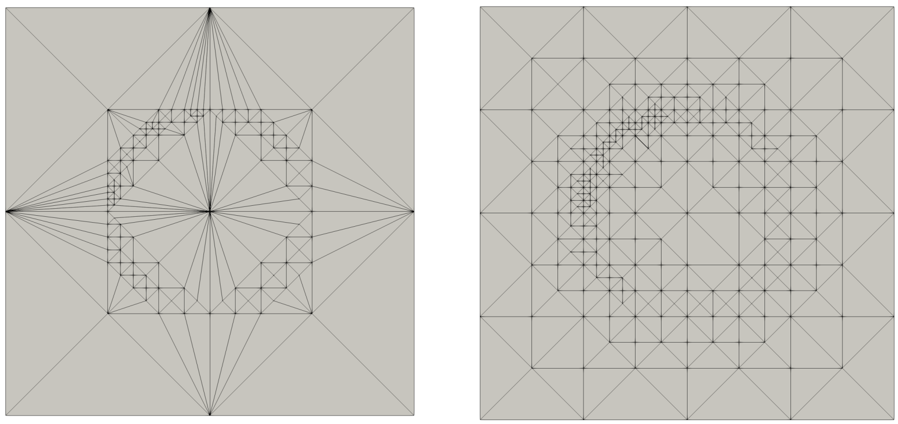

Figure 5: Non-recursive vs. recursive (3D).

4 Quality of newest vertex with longest edge.

While the longest-edge criterion is superior, the hybrid approach (newest vertex + longest edge) stays in the same order of magnitude for the degrading element quality. With further experiments it appears that the hybrid approach does not consistently generates conforming meshes.

5 Parallelization

The parallel algorithm in 2D works by refinining independently the subdomains, then conforming the interfacing edges (see Figure 7).

In higher dimensions the only local refinement strategy that seems to work well is newest-vertex bisection.

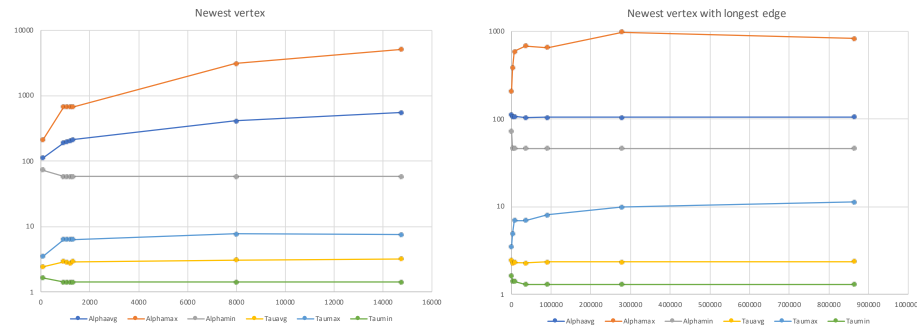

Figure 6: Quality of the elements for several iterations of the adaptive refinement shown in Figure 2. The x axis represent the total number of elements in the 4D mesh. The y axis the quality criterion (lower is better). The influence of longest edge shows on both the better quality and the larger number of elements generated.

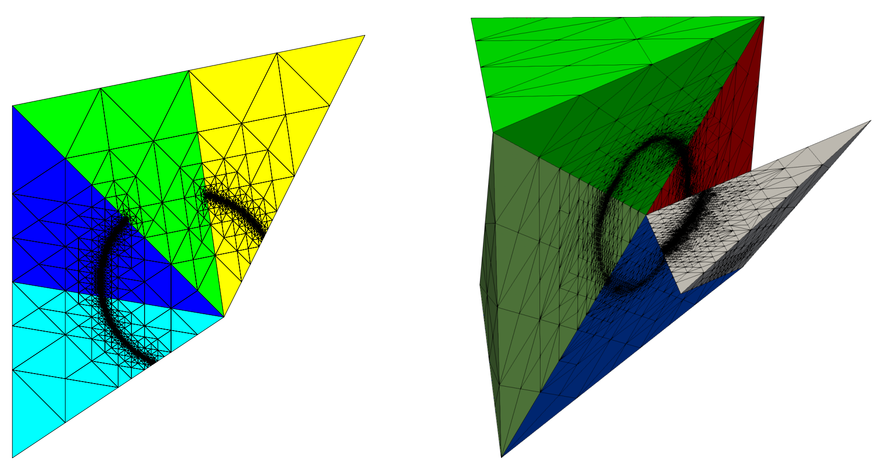

Figure 7: Partitioning with one part without refinement. Color represent partitions.