1 Bisection algorithm for simplical meshes

- Serial algorithm
- Longest edge splitting with propagation front
- For 2D meshes with triangles isosceles the number of congruence classes is 1.
- The algorithm works directly for 3D and 4D.

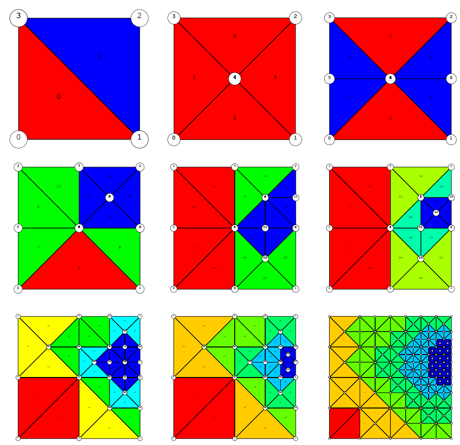

Figure 1: Steps of the bisection algorithm.

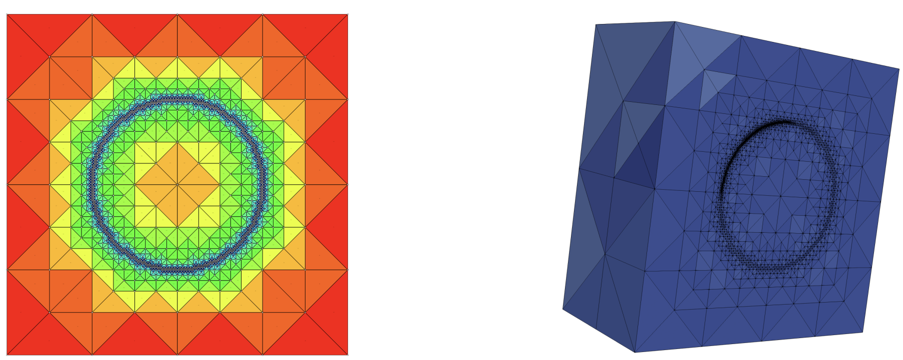

Figure 2: Result for sphere refinement pattern. Left: 2D. Right: 3D.

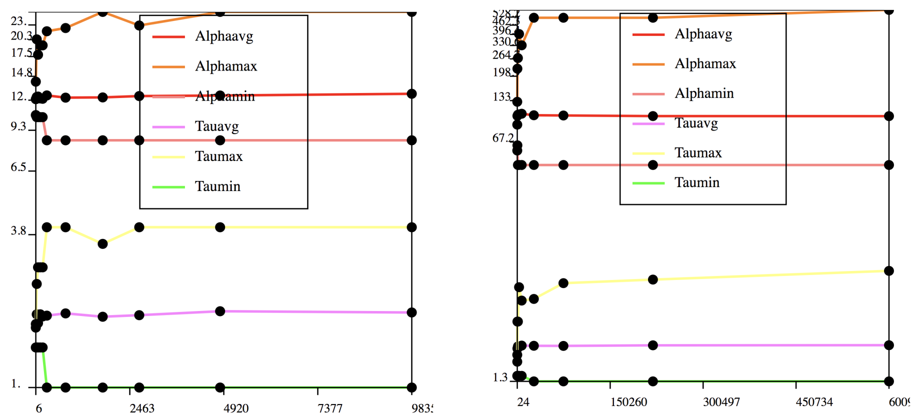

Figure 3: Quality of the elements for several iterations of the adaptive refinement shown in Figure 1. The x axis represent the total number of elements in the mesh. The y axis the quality criterion (lower is better). Left: 3D. Right: 4D.

Table 1
A list of tetrahedron shape measures used in literature

Aspect ratio measure	Value for a equilateral tetrahedron	Used in
$\beta = \frac{CR}{IR}$	$\beta^* = 3.0$	[1]
$\sigma = \frac{S_{\max}}{IR}$	$\sigma^* = 4.898979$	[2]
$\omega = \frac{CR}{S_{\max}}$	$\omega^* = 0.612507$	[2]
$\tau = \frac{S_{\max}}{S_{\min}}$	$\tau^* = 1.0$	[2]
$\kappa = \frac{V^4}{\left[\sum_{i=1}^{i=4} SA^2\right]^3}$	$\kappa^* = 4.58457e-04$	[3]
$\alpha = \frac{S_{\mathrm{avg}}^3}{V}$	$\alpha^* = 8.479670$	[4]
$\gamma = \frac{S_{\mathrm{rms}}^3}{V}$	$\gamma^* = 8.479670$	[5]

Nomenclature:
CR = radius of the circumscribed sphere, IR = radius of the inscribed sphere, S_i = length of any edge i, $S_{\max} = \max(S_i)$ $(i = 1\ldots6)$, $S_{\min} = \min(S_i)$ $(i = 1\ldots6)$, SA = surface area of a triangular facet, S_{avg} = average (S_i) $(i = 1\ldots6)$, S_{rms} = root mean square (S_i) $(i = 1\ldots6)$, V = volume of the tetrahedron, and

$$V = \tfrac{1}{6} * \begin{vmatrix} 1 & x_1 & y_1 & z_1 \\ 1 & x_2 & y_2 & z_2 \\ 1 & x_3 & y_3 & z_3 \\ 1 & x_4 & y_4 & z_4 \end{vmatrix}, \qquad IR = \frac{4V}{\left[\sum_{i=1}^{i=4} SA\right]}, \qquad S_{\mathrm{rms}} = \sqrt{\tfrac{1}{6} \sum_{i=1}^{i=6} S_i^2}\,.$$

Figure 4: Quality metics. We used “Alpha” and “Tau”.