Gaussian Process

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Jupyter Notebook

1 Introduction

A Gaussian Process (GP) define a prior over functions f, which can be converted into a posterior over functions once we have seen some data. Let $x_1, \ldots, x_N$ a set of point, a GP assumes that $p(f(x_1, \ldots, f(x_N)))$ is jointly Gaussian, with some mean $\mu(x)$ and covariance $\Sigma(x)$ given by $\Sigma_{ij} = \kappa(x_i, x_j)$, where κ is a kernel function. In this work we sample from a multivariate gaussian (MVN) with different kernels, using this general setting: N=100, $x_1, \ldots, x_N = 0, \ldots, 99$ and $\mu(x) = 0$.

2 Square Exponential Kernel

The squared exponential kernel is the more common kernel used. In Figure 1 we observe that to lower length scale (ℓ) the function look more "wiggle".

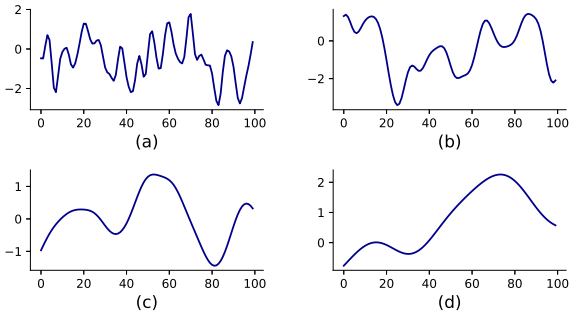

Figure 1: Samples from a MVN with square exponential kernel with $\mu = 0$ and $\sigma = 1$ for differents length scale (ℓ). Axis x represent the support and axis y its values. (a) Plot with $\ell = 1$, (b) $\ell = 4$, (c) $\ell = 8$ and (d) $\ell = 16$.

3 Locally Periodic Kernel

This kernel is the product between the periodic kernel and the square exponential kernel. In Figure 2 we observe that to higher periodicity (p) lower is the distance between repetitions. Also, to difference of periodic kernel, this kernel allows periodicity without repeating exactly the same pattern, allowing variations over time that are smother to higher ℓ.

4 Linear Plus Periodic Kernel

This kernel is the sum between periodic kernel and linear kernel. In Figure 3 we have some samples from this kernel, we observe that this kernel result in a periodic function with increasing mean. The parameter σ_v control the mean increment, that is higher to higher σ_v, on the other hand, the σ_b parameter has an offset behavior, it determine how far from 0 the height of the function will be at zero.

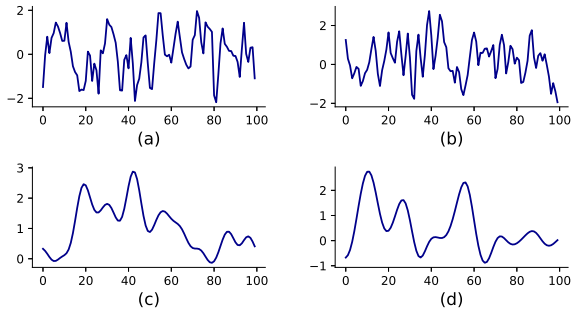

Figure 2: Samples from a MVN with locally periodic kernel with $\mu = 0$ and $\sigma = 1$ for differents length scale (ℓ) and periodicity (p). Axis x represent the support and axis y its values. (a) Plot with ($\ell = 1, p = 0.1$), (b) ($\ell = 1, p = 1$), (c) ($\ell = 5, p = 0.1$) and (d) ($\ell = 5, p = 1$).

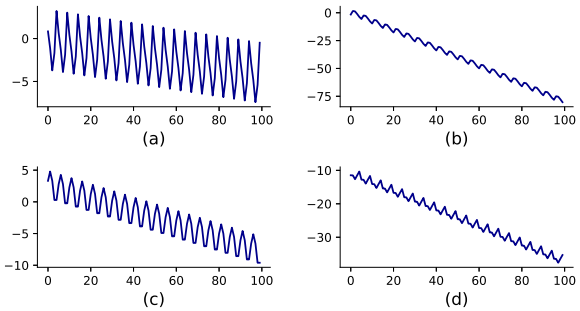

Figure 3: Samples from a MVN with linear plus periodic kernel with ($\mu = 0, \sigma = 2, c = 0, \ell = 1, p = 5$) for differents σ_b and σ_v. Axis x represent the support and axis y its values. (a) Plot with ($\sigma_b = 0, \sigma_v = 0.01$), (b) ($\sigma_b = 0, \sigma_v = 0.1$), (c) ($\sigma_b = 4, \sigma_v = 0.01$) and (d) ($\sigma_b = 4, \sigma_v = 0.1$).