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NECESSARY CONDITIONS FOR AGGREGATION IN SECURITIES MARKETS

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I. Introduction

An important aggregation problem is the derivation of equilibrium security prices which are independent of the allocation of initial wealth among investors. The problem is of interest because, if investors are conceived as being endowed with initial holdings of securities, it is clear that the initial wealth allocation which depends on security prices is endogenous to the model. Although he addresses a differently defined objective, Rubinstein [8] has shown that sufficient conditions for the solution of the problem described above are conditions that permit construction of "composite" (representative) investors whose resources, beliefs, and tastes depend on the exogenous specifications of the economy (viz., the beliefs and tastes of all investors and production conditions) but not on the initial allocation of securities.[1]

Other writers[2] have considered the distinct but related problem of the effect of changes in the number of investors on the market price of risk in a mean-variance context. In this paper, we hold constant the number of investors and the aggregate endowment of the economy and consider the necessary conditions for a reallocation of the aggregate endowment across investors to leave equilibrium security prices unchanged.[3]

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[1] Rubinstein addresses the problem of deriving equilibrium prices as closed-form functions of the exogenous specifications of the economy and remarks that "generally, these exogenous specifications should also include the initial endowment of each individual's claims" [8, fn.4]. However, as Rubinstein notes, the conditions he finds sufficient for the result he seeks are also conditions under which equilibrium prices are independent of the allocation of initial wealth.

[2] e.g., Lintner [4]; Litzenberger and Budd [5].

[3] This question has also been considered independently by Milne [6].

After setting up the basic problem in Part II, necessary conditions for aggregation are derived in Part III. The necessary conditions are that the Engel curves of all investors are parallel straight lines. These turn out, on examination, to be the conditions Rubinstein found to be sufficient for aggregation when investors are not necessarily identical, nor necessarily possess identical resources. Part IV shows that, when the conditions for aggregation are met, explicit expressions for primitive security prices may be derived, and these expressions may be used to value complex securities in incomplete markets under certain conditions.

II. The Investor's Decision Problem

Consider an economy with n investors, each of whom is concerned with maximizing the expected value of a monotone, increasing, strictly concave, von Neumann-Morgenstern utility function defined on wealth at the end of one period.[4] If the investors are faced with a complete securities market in which there are as many independent securities as there are possible states of nature, we may, without loss of generality, consider these securities to be primitive Arrow-Debreu securities each of which pays off in only a single state of nature. Then the decision problem of a typical investor i(i=1,...,n) may be represented by

$$\text{(1)} \qquad \underset{\{W_{is}\}}{\text{Maximize}} \sum_{s=1}^{m} \pi_{is} U_i(W_{is}),$$

subject to the budget constraint

$$\text{(2)} \qquad \sum_{s=1}^{m} P_s W_{is} = A_i \equiv \sum_{s=1}^{m} P_s \bar{W}_{is},$$

where

W_{is} is the end of period wealth of the i^{th} investor if state s occurs (s = 1,...,m),

$\bar{W}_{is}$ is the endowment of end-of-period wealth in state s for the i^{th} investor,

$A_i \equiv \sum_{s=1}^{m} P_s \bar{W}_{is}$ is the endowed wealth of the i^{th} investor, obtained by valuing his endowments of state contingent wealth $\bar{W}_{is}$(s = 1,...,m) at the equilibrium prices P_s(s = 1,...,m),

$U_i(\cdot)$ is the utility function of the i^{th} investor, defined over end-of-period wealth,

π_{is} is the assessment by the i^{th} investor of the probability that state s will occur, and

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[5]The analysis may be generalized in a straightforward manner to include consumption at the beginning of the period, assuming a separable utility function. While this is the approach taken in Rubinstein [8], we neglect initial consumption for simplicity.

P_s is the current price of a primitive Arrow-Debreu security which pays \$1 if state s occurs, and nothing otherwise.

Thus each investor is endowed with an initial bundle of claims to state contingent wealth, $\bar{W}_{is}(s = 1,\ldots,m)$, and by trading with other investors at the equilibrium prices, $P_s(s = 1,\ldots,m)$, attains an optimal allocation of state-contingent wealth $W_{is}(s = 1,\ldots m)$. This optimal allocation for a given set of prices is yielded by the solution to the above decision problem. The first-order conditions for a maximum in the investor's decision problem are the budget constraint, (2), and

$$\pi_{is}\, U_i'(W_{is}) = \theta_i\, P_s \qquad (s = 1,\ldots,m) \tag{3}$$

where θ_i is the Lagrangean multiplier associated with the budget constraint.

III. Necessary Conditions for Aggregation

We show first, in Theorem 1, that aggregation requires that the Engel curves of all investors be parallel straight lines. Then, in Theorem 2, it is established that a necessary and sufficient condition for an investor's Engel curves to be (locally) linear is that the risk tolerance of his utility function (defined as the reciprocal of the Pratt-Arrow coefficient of absolute risk aversion) is linear in wealth. Finally, Theorem 3 establishes that for all investors to have parallel Engel lines, for any given set of primitive security prices, either (a) all investors must have power or logarithmic utility functions and share a common degree of cautiousness and common beliefs, or (b) all investors must have constant absolute risk aversion (exponential utility).

Theorem 1: A necessary condition for equilibrium security prices to be expressible solely in terms of the exogenous specifications of the economy (i.e., aggregation) is that the Engel curves relating demands for contingent wealth in different states as initial wealth varies, for a given set of primitive security prices, are parallel straight lines for all investors.

Proof: For a given set of probability beliefs, the demand of the i^{th} investor for contingent wealth in state s is a function of primitive security prices $(P_1,\ldots,P_m)$ and initial wealth, A_i, which we may write as $f_{is}(P_1,\ldots,P_m; A_i)$. Equilibrium primitive security prices are obtained by summing demands for state-contingent wealth over all investors and equating the result to the exogenous aggregate endowment of contingent wealth in each state, $\bar{W}_s \equiv \sum_{i=1}^{n} \bar{W}_{is}(s = 1,\ldots,m)$, so that at equilibrium:

$$\sum_{i=1}^{n} f_{is}(P_1,\ldots,P_m;\ A_i) = \bar{W}_s \qquad (s = 1,\ldots,m). \tag{4}$$

Aggregation requires that the equilibrium prices be independent of the allocation across individual investors of the given aggregate endowment of state contingent wealth, $\bar{W}_s$ $(s = 1,\ldots,m)$. Thus, taking the differential of (4) as $\bar{W}_{is}$ changes, holding constant $\bar{W}_s$, and recalling that $A_i = \sum_{s=1}^{m} P_s \bar{W}_{is}$, aggregation requires that

$$\sum_{i=1}^{n} \frac{\partial f_{it}}{\partial A_i} P_s\, d\bar{W}_{is} = 0, \text{ when } \sum_{i=1}^{n} d\bar{W}_{is} = 0 \qquad (t = 1,\ldots,m). \tag{5}$$

Consider a reallocation of endowed wealth in some state s between individuals i and j, holding constant the aggregate endowment of contingent wealth in state s, so that $d\bar{W}_{js} = -d\bar{W}_{is}$, and $d\bar{W}_{ks} = 0$ $(k \neq i,j)$. Then using (5), we have that for any two states q and r,

$$\begin{aligned} \frac{\partial f_{iq}}{\partial A_i} P_s\, d\bar{W}_{is} - \frac{\partial f_{iq}}{\partial A_j} P_s\, d\bar{W}_{is} &= 0, \\ \frac{\partial f_{ir}}{\partial A_i} P_s\, d\bar{W}_{is} - \frac{\partial f_{jr}}{\partial A_j} P_s\, d\bar{W}_{is} &= 0. \end{aligned} \tag{6}$$

But (6) implies that the slope of the projection of the Engel curve of the i^{th} investor in the (q,r) plane at the point where it is intersected by the budget plane corresponding to initial wealth A_i, $\frac{\partial f_{iq}}{\partial A_i} / \frac{\partial f_{ir}}{\partial A_i}$, is equal to the corresponding slope for the j^{th} investor. Since this must hold for all levels of initial wealth A_i and A_j, the Engel curve projections of these investors must be linear and parallel. Finally, since this must hold for all investors i and j, and for all states q and r, the Engel curves of all investors must be parallel straight lines, which completes the proof of the theorem.

Theorem 1 merely reestablishes a well-known result that "a given system of personal indifference maps yields a unique community indifference map if, and only if, the personal Engel curves are parallel straight lines for different individuals at the same prices" (Gorman [2]). As Samuelson [10] has pointed out, "community indifference curves between the totals of two goods X and Y--where $X = X^1 + X^2$, $Y = Y^1 + Y^2$--give us a 'demand relationship' between prices and quantities of the following form, $Px/Py = F(X^1 + X^2, Y^1 + Y^2)$, where the latter can be called the marginal-rate-of-substitution function of the group.

They provide this and essentially nothing more." Thus the problem of aggregation is simply the problem of constructing community indifference curves. Moreover, Samuelson also points out that if the Engel lines are to be everywhere linear then they must pass through the origin. This behavior is not exhibited by all members of the class of utility functions admitted by Rubinstein as sufficient for aggregation. Therefore, both his proof of sufficiency and our proof of necessity apply to "local" (rather than global) aggregation, *i.e.*, where extra conditions are assumed to be present to ensure that all equilibrium demands lie in the positive orthant.[5]

We turn next to the conditions for linearity of the Engel curves and derive the equations of the Engel lines in terms of primitive security prices, tastes and beliefs.

Theorem 2: A necessary (and sufficient) condition for an investor to have linear Engel curves is that his utility function, $U_i(\cdot)$, exhibit linear risk tolerance.[6]

Proof: The projection in the (s,t) plane of an Engel curve of the i^{th} investor is the locus of points such that the marginal rate of substitution between W_{is} and W_{it} is equal to the price ratio. From (1), this is the locus of points satisfying

$$\frac{\pi_{is}U_i'(W_{is})}{\pi_{it}U_i'(W_{it})} = \frac{P_s}{P_t}\,. \tag{7}$$

Setting the differential of the left-hand side of (7) equal to zero, the slope of the Engel curve projection is

$$\frac{dW_{is}}{dW_{it}} = \frac{T_i(W_{is})}{T_i(W_{it})} \tag{8}$$

where $T_i(\cdot) \equiv -U_i'(\cdot)/U_i''(\cdot)$ is the risk tolerance of the utility funtion, *i.e.*, the reciprocal of the Pratt-Arrow coefficient of absolute risk aversion.

Suppose the Engel curve projection is a straight line:

$$W_{is} = h_{ist}W_{it} + g_{ist}. \tag{9}$$

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[5]For conditions assuring a solution in the positive orthant under generalized logarithmic utility, see Rubinstein [9].

[6]A similar theorem is proved by Pollack [7].

For (9) to hold, considering (8), requires

$$T_i(h_{ist}W_{it} + g_{ist}) = h_{ist}T_i(W_{it}). \tag{10}$$

Differentiating both sides of (10) with respect to W_{it} and substituting from (9) yields $T_i'(W_{is}) = T_i'(W_{it})$. However, since (2) and (3) imply that it is possible to change A_i and $P_1,\ldots,P_m$ so that W_{is} is changed arbitrarily while W_{it} remains constant, it must be the case that $T_i'(\cdot)$ is a constant, λ_i, referred to as the cautiousness of the utility function. This establishes the necessity of linear risk tolerance for linear Engel curves.

To prove sufficiency, suppose risk tolerance is linear:

$$T_i(W) = \lambda_i W + \mu_i. \tag{11}$$

Consider the following linear relation between W_{is} and W_{it}.

$$W_{is} = \begin{cases} h_{ist}\left(W_{it} + \dfrac{\mu_i}{\lambda_i}\right) - \dfrac{\mu_i}{\lambda_i} & \text{if } \lambda_i \neq 0 \\ W_{it} + g_{ist} & \text{if } \lambda_i = 0. \end{cases} \tag{12}$$

Since (11) and (12) satisfy (8), the straight line (12) is indeed an Engel curve projection when risk tolerance is linear as in (11). This completes the proof of the theorem.

Linearity of the budget constraint (2) implies that linearity of the Engel curves is obviously equivalent to investment in any security being a linear function of initial wealth. Therefore, the proof of Theorem 2 above may be viewed as a simpler and more direct proof of the Cass and Stiglitz [1] proposition that investment in any security will be linear in initial wealth if and only if the utility function belongs to the class exhibiting risk tolerance that is linear in wealth.

Theorem 3: A necessary condition for the Engel curves of all investors to be *parallel* straight lines (and therefore, by Theorem 1, a necessary condition for aggregation) is that either

(i) all investors have the same cautiousness ($\lambda_i = \lambda$ for all i) and the same beliefs ($\pi_{is} = \pi_s$ for all i, for s = 1,...,m)

or

(ii) all investors have exponential utility ($\lambda_i = 0$ for all i).

Proof: By Theorem 2, an investor with linear Engel curves must have linear risk tolerance, as in (11), with the equation for the projection in the (s,t) plane of his Engel curve being given by (12). Integrating (11), his marginal utility must be of the following form.

$$U_i'(W) = \begin{cases} (\lambda_i W + \mu_i)^{-1/\lambda_i} & \text{if } \lambda_i \neq 0 \\ e^{-W/\mu_i} & \text{if } \lambda_i = 0. \end{cases} \tag{13}$$

Substituting in (7) from (12) for W_{is}, and from (13) for $U_i'(\cdot)$ and simplifying, yields

$$\begin{aligned} (\pi_{is}/\pi_{it})\; h_{ist}^{-1/\lambda_i} &= P_s/P_t & \text{if } \lambda_i \neq 0 \\ (\pi_{is}/\pi_{it})\; e^{-g_{ist}/\mu_i} &= P_s/P_t & \text{if } \lambda_i = 0. \end{aligned} \tag{14}$$

Therefore,

$$\begin{aligned} h_{ist} &= \left(\frac{P_s/P_t}{\pi_{is}/\pi_{it}}\right)^{-\lambda_i} & \text{if } \lambda_i \neq 0 \\ g_{ist} &= -\mu_i \log \left(\frac{P_s/P_t}{\pi_{is}/\pi_{it}}\right) & \text{if } \lambda_i = 0. \end{aligned} \tag{15}$$

For aggregation to obtain, each investor's Engel curve projection must have the same slope, so that $h_{ist} = h_{st}$ in (9) for all i. If all investors have exponential utility, as in case (ii) of the theorem, then $h_{ist} = 1$ for all i, as shown in (12), so that aggregation obtains.

Suppose now that aggregation obtains, so that $h_{ist} = h_{st}$ for all i, but that all investors do not have exponential utility and furthermore that, in contradiction of case (i), $0 \neq \lambda_i \neq \lambda_j \neq 0$ for two investors i and j. Since it is assumed that $h_{ist} = h_{jst} = h_{st}$, (15) implies in this case

$$\left(\frac{P_s/P_t}{\pi_{is}/\pi_{it}}\right)^{-\lambda_i} = h_{st} = \left(\frac{P_s/P_t}{\pi_{js}/\pi_{jt}}\right)^{-\lambda_j} . \tag{16}$$

If $\lambda_i \neq \lambda_j$, as assumed, then (16) may be solved for P_s/P_t, yielding

$$\frac{P_s}{P_t} = \left[\frac{(\pi_{is}/\pi_{it})^{-\lambda_i}}{(\pi_{js}/\pi_{jt})^{-\lambda_j}}\right]^{\frac{1}{\lambda_j - \lambda_i}} . \tag{17}$$

Relation (17) implies that relative security prices, in equilibrium, are determined completely independently of relative aggregate supplies of state-contingent wealth. Since such a condition is impossible, aggregation requires $\lambda_i = \lambda_j$. In general, therefore, aggregation requires $\lambda_i = \lambda$ for all i. Furthermore, $\lambda_i = \lambda_j = \lambda \neq 0$ in (16) implies $\pi_{is}/\pi_{it} = \pi_{js}/\pi_{jt}$. Since this holds for arbitrary s and t and, obviously, $\Sigma_{s=1}^{m}\pi_{is} = \Sigma_{s=1}^{m}\pi_{js} = 1$, the assumption of aggregation with $\lambda_i = \lambda_j = \lambda \neq 0$ requires $\pi_{is} = \pi_{js}$ for all s. Therefore, if case (ii) does not hold, aggregation requires that case (i) be satisfied, which establishes the theorem.

Theorem 3 shows that aggregation requires either that all investors possess utility functions exhibiting constant absolute risk aversion (i.e., $\lambda_i = 0$ for all i), but do not necessarily share common beliefs, or that all investors possess utility functions with identical cautiousness (i.e., $\lambda_i = \lambda$ for all i) and have the same beliefs. In the latter case, investors may differ only in the parameter μ_i (see equation (11)) of their risk tolerance functions.

The results of this section betray their lineage in Wilson's [11] paper on syndicates which is also the basis for part of Rubinstein's [8] paper. Wilson dealt with the conditions under which a surrogate utility function, which depends only on payoffs, and a surrogate group probability assessment, which is independent of state payoffs, could be constructed for a group when sharing rules were Pareto optimal. Since sharing rules in a complete capital market are always Pareto optimal, his results apply in our context also. However, Wilson's surrogate functions in general depend upon the weights assigned to the utility of individual members of the group, which correspond to the allocation of initial wealth in our context. Hence, while the existence of surrogate functions is a necessary condition for aggregation and the construction of community indifference curves (since the representative investor must satisfy the Savage axioms), it is not sufficient. Aggregation requires, in addition, that the surrogate functions be independent of the allocation of initial wealth.

IV. Aggregation and Equilibrium Security Prices

When the conditions for aggregation are met, straightforward derivations produce closed-form expressions for equilibrium relative prices of primitive securities in a complete market.[7] Suppose, first, that case (i) of Theorem 3

[7] Since we have omitted initial consumption for simplicity, the equilibrium relationships in our model are capable of determining only relative prices. As noted previously, the extension of the model to incorporate consumption at the beginning of the period, as in Rubinstein [8], is straightforward.

holds: investors have identical cautiousness and beliefs. Then, substituting from (15) for h_{ist} in (12), for the i^{th} investor,

$$W_{is} = \left(\frac{P_s/P_t}{\pi_s/\pi_t}\right)^{-\lambda}\left(W_{it} + \frac{\mu_i}{\lambda}\right) - \frac{\mu_i}{\lambda}. \tag{18}$$

Summing (18) over all investors, equating total demands for state-contingent wealth to exogenous total supplies, and solving for relative primitive security prices,

$$\frac{P_s}{P_t} = \frac{\pi_s}{\pi_t}\left(\frac{W_s + \Sigma_{i=1}^{n}\mu_i/\lambda}{W_t + \Sigma_{i=1}^{n}\mu_i/\lambda}\right)^{-1/\lambda} \tag{19}$$

It is clear from (19) that the same market equilibrium would result if the market were populated by "representative" investors, all with the same beliefs, cautiousness, and rate of patience, provided that the value of the parameter μ for the representative investors were equal to the average value of this parameter for all actual investors. Moreover, since the conditions under which (19) was derived yield universal portfolio separation (see Rubinstein [8, p. 228]), all investors hold the same risky asset portfolio, and rates of return determined in an incomplete market are the same as those determined in an otherwise identical complete market. Thus, (19) is a very general valuation equation which, under the stated assumptions, may be employed to value complex securities in an incomplete market as well.

To apply case (ii) of Theorem 3, suppose $\lambda_i = 0$ for all i. Substituting from (15) for g_{ist} in (12),

$$W_{is} = W_{it} - \mu_i \log\left(\frac{P_s/P_t}{\pi_{is}/\pi_{it}}\right). \tag{20}$$

Proceeding as before, market equilibrium in this case is described by the following relation for relative prices of primitive securities.

$$\frac{P_s}{P_t} = e^{[W_t - W_s + \Sigma_{i=1}^{n}\mu_i \log(\pi_{is}/\pi_{it})]/\Sigma_{i=1}^{n}\mu_i}. \tag{21}$$

Thus, (21) is the expression for equilibrium relative prices when all investors have constant absolute risk aversion but do not necessarily share common beliefs. The same equilibrium relative prices would prevail if the market consisted only of representative investors, each having constant absolute risk aversion with parameter $\mu = \Sigma_{i=1}^{n}\mu_i/n$ and sharing common beliefs, π_s (s=1,...,m) satisfying,

$$\frac{\pi_s}{\pi_t} = \prod_{i=1}^{n} \left(\frac{\pi_{is}}{\pi_{it}} \right)^{\mu_i/\Sigma_{j=1}^n \mu_j} . \tag{22}$$

Besides defining the ratios of probabilities constituting the beliefs of the representative investor, it is obviously desirable to require, in addition, that these probabilities sum to unity. This is achieved by normalizing the representative investor's probability beliefs by requiring them to satisfy[8]

$$\Pi_s = \frac{\prod_{i=1}^{n} \pi_{is}^{\mu_i/\Sigma_{j=1}^n \mu_j}}{\Sigma_{t=1}^m \prod_{i=1}^{n} \pi_{it}^{\mu_i/\Sigma_{j=1}^n \mu_j}} \qquad (s = 1,\ldots,m). \tag{23}$$

Clearly, (23) implies both that the ratios π_s/π_t satisfy (22) and also that $\Sigma_{s=1}^m \pi_s = 1$.

The equilibrium relation (21), unlike (19), is valid only for a complete securities market. On the other hand, (19) incorporates the assumption of common probability beliefs. If it is assumed in the case of (21) that all investors share common probability beliefs, $\pi_{is} = \pi_s$ $(i=1,\ldots,n;\ s=1,\ldots,m)$, then universal portfolio separation obtains and the resulting equilibrium price relation (i.e., (21) with the substitution of π_s and π_t for π_{is} and π_{it}) may be employed for valuing complex securities in arbitrary incomplete markets under exponential utility with heterogeneous risk tolerance.

As an intermediate case, suppose that all investors agree on the ratio π_{is}/π_{it} in (21) for two states, s and t, and that aggregate supplies of state-contingent wealth in states s and t are equal (*i.e.*, $W_s = W_t$). Under these conditions, the ratio W_{is}/W_{it} is the same for all investors, so that the equilibrium would be unchanged if the primitive securities for states s and t were

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[8] In a model that includes the choice of initial consumption as a decision variable and an impatience factor as a parameter, as in Rubinstein [8], the condition that the representative probabilities sum to unity would require them to depend on both the probability beliefs and impatience factors of the actual investors in the market. These representative probabilities would not meet Rubinstein's requirements for a representative individual that his beliefs should be independent of his impatience factor and vice versa. However, the only way these requirements could be satisfied in general for the exponential utility case would be at the cost of having "probabilities" for the representative individual not necessarily sum to unity.

combined into a single, composite security.[9] This case shows that even when completely homogenous beliefs are not assumed, equilibrium price relation (21) does not necessarily require a complete market to be applicable.

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[9] See Hakansson [3], especially his Theorem 4.

REFERENCES

[1] Cass, D., and J. Stiglitz. "The Structure of Investor Preferences and Asset Returns, and Separability in Portfolio Allocation: A Contribution to the Pure Theory of Mutual Funds." *Journal of Economic Theory*, Vol. 2 (1970), pp. 122-160.

[2] Gorman, W. "Community Preference Fields." *Econometrica*, Vol. 21 (1953), pp. 63-80.

[3] Hakansson, N. "The Superfund: Efficient Paths toward a Complete Financial Market." Working Paper No. 25, Institute of Business and Economic Research, University of California, Berkeley (1974).

[4] Lintner, J. "The Market Price of Risk, Size of Market and Investor's Risk Aversion." *Review of Economic Statistics*, Vol. 52 (1970), pp. 87-99.

[5] Litzenberger, R., and A. Budd. "Secular Trends in Risk Premiums." *Journal of Finance*, Vol. 27 (1972), pp. 857-864.

[6] Milne, F. "Consumer Preferences, Linear Demand Functions and Aggregation in Competitive Asset Markets." Unpublished manuscript, University of Rochester (1976).

[7] Pollack, R. "Additive Utility Functions and Linear Engel Curves." *Review of Economic Studies*, Vol. 38 (1971), pp. 401-414.

[8] Rubinstein, M. "An Aggregation Theorem for Securities Markets." *Journal of Financial Economics*, Vol. 1 (1974), pp. 225-244.

[9] __________. "The Strong Case for the Generalized Logarithmic Utility Model as the Premier Model of Financial Markets." *Journal of Finance*, Vol. 31 (1976), pp. 551-571.

[10] Samuelson, P. "Social Indifference Curves." *Quarterly Journal of Economics*, Vol. 70 (1956), pp. 1-22.

[11] Wilson, R. "The Theory of Syndicates." *Econometrica*, Vol. 36 (1968), pp. 119-132.