Tax Avoidance and Evasion in a Dynamic Setting*

Duccio Gamannossi degl'Innocenti[†] Rosella Levaggi[‡]
Francesco Menoncin[§]

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Abstract

We study an intertemporal utility maximization problem where taxpayers can engage in both tax avoidance and tax evasion. Evasion is costless but is fined if discovered, while avoidance is costly but might be successful (i.e. deemed legitimate) with a given probability (β) upon audit. We find that traditional deterrence instruments (fine and frequency of audit) reduce optimal evasion but, in contrast with results in a static framework, they have no impact on optimal avoidance. In fact, tax avoidance depends negatively on its marginal cost and positively on both its probability of success (β) and the tax rate. We show that non-compliance behavior may result in a Laffer curve for fiscal revenues and that the revenue maximizing tax rate is lower the higher β. We characterize the optimal level of β by taking into account different government objectives: minimizing evasion, minimizing non-compliance (evasion plus avoidance), or maximizing revenues. Our results suggest that specific policies (e.g., tax simplification) need to be implemented to deter avoidance and we illustrate their impact on evasion.

Keywords: Tax Avoidance; Tax Evasion; Dynamic Programming; Tax Simplification

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[†]Corresponding author at: Department of Economics and Finance, Università Cattolica del Sacro Cuore, Via Necchi 5, 20123, Milan, Italy. E-mail: duccio.gamannossi@unicatt.it.

[‡]Department of Economics and Management, University of Brescia, Via San Faustino 74b, 25122, Brescia, Italy.

[§]Department of Economics and Management, University of Brescia Via San Faustino 74b, 25122, Brescia, Italy.

1 Introduction

In Europe, income under-reporting is about 20% of GDP, accounting for a potential loss of about 750-900 billion Euros each year (Buehn and Schneider, 2012; Murphy, 2019), i.e. about 13.2% of total revenue (Albarea et al., 2020). Intentional underreporting of income is about 18-19% of the total reported income in the US, leading to a tax gap (Feige and Cebula, 2011; IRS, 2019) that, according to some estimates, may have reached 630 billion dollars in 2020 (Sarin and Summers, 2019) but the latter may be even higher when tax avoidance is taken into account. Since the revenue loss is only the tip of the iceberg for what concerns the effect of tax evasion (Slemrod, 2007; Alm, 2012; Dzhumashev and Gahramanov, 2011; Markellos et al., 2016), reducing non-compliance is a priority for many governments, both in developed and developing countries.[1]

In order to reduce their tax liabilities, taxpayers may take three kinds of actions: (i) illegal ones (tax evasion), (ii) those that use tax law to gain an advantage that lawmakers never intended (tax avoidance), and (iii) those that use tax allowances for the purposes intended by lawmakers (tax planning).[2] Since (iii) is legit, here we focus on (i) and (ii). In contrast to most of the literature that disentangles the study of avoidance and evasion, we study them jointly (similar to Gamannossi degl'Innocenti and Rablen, 2017) as the taxpayer optimally chooses them at the same time.

We assume evasion to be costless since it can be performed quite easily just by concealing a part of the revenue. Conversely, tax avoidance schemes are usually sophisticated[3] and require considerable expertise to be devised.[4] Our model is cast in a dynamic framework and we study the dynamic programming problem of a representative taxpayer who maximizes the expected utility of her/his inter-temporal consumption and decides the optimal percentage of evasion and avoidance. The taxpayer receives utility from the consumption that exceeds a minimum (subsistence) amount in each period and her/his utility increases with the consumption of both a private and a public produced good. The taxpayer is endowed with a linear Ak technology and a constant tax rate is levied on the yield produced.

The model has two sources of uncertainty: the occurrence of an audit and the success of avoidance.

In particular, the taxpayer knows the frequency of audits but does not know when they occur. When an audit happens, we assume that evasion is detected

[1] The literature is not conclusive on its exact amount. For Davison (2021) it may also reach 1 trillion dollars, while other estimates are more in line with the tax gap.

[2] Despite a marked heterogeneity in the details, in most tax systems (e.g. IRS, 2014; European Parliament, 2017; HMRC, 2019; UN, 2019) adherence to the letter of the law does not imply legality. Courts decide whether a given tax liability reduction is admissible based on its agreement with the purposes of the tax legislation.

[3] Stiglitz (1985) provides several examples and identifies three basic principles of these activities: postponement of taxes, arbitrage across individuals subject to different marginal tax rates, and manipulation of different types of income that are taxed.

[4] Empirical evidence shows that people's understanding of tax law, tax rates, and basic concepts of taxation is limited, i.e. Blaufus et al. (2015); Gideon (2017); Stantcheva (2021).

and a fine must be paid on the evaded tax. Conversely, upon audit, there is some positive probability that avoidance will be successful, because: (i) it goes undetected, (ii) it is not challenged by the tax authority, or (iii) it is not recognized as illegitimate by courts. If avoidance is unsuccessful, then in our model it is considered akin to evasion and fined accordingly. The success probability of avoidance (β) represents, in some way, a broad measure of the vulnerability of the tax system to avoidance and we assume it to be constant over time.

Our analysis shows that optimal avoidance is constant over time, it does not depend on the preference parameters, it is independent of the form of the utility function and is not directly affected by traditional deterrence instruments, like the fine and the frequency of audits. Instead, avoidance depends only on (i) the tax rate (positively), (ii) the avoidance probability of success β (positively), and (iii) the cost of avoidance (negatively). However, avoidance has a negative impact on tax evasion and, thus, total unreported income crucially depends on both traditional fiscal parameters and tax avoidance determinants.

As in Gamannossi degl'Innocenti and Rablen (2017) (i) the taxpayer optimally decides both evasion and avoidance, (ii) the audit is performed with an exogenous frequency and probability, and (iii) when audited, evasion is immediately detected, while avoidance may be successful with a given probability. However, in Gamannossi degl'Innocenti and Rablen (2017) the evasion and avoidance decisions are taken in two separate steps, while in our model we assume them to happen simultaneously. As a consequence, it is possible to decide evasion and avoidance by managing two different risks. The risk of avoidance to be unsuccessful is managed just by choosing an optimal amount of avoidance, while the risk to be audited is managed inside the optimal decision of evasion. Instead, if these two decisions are taken separately, the same parameters should affect both of them, as demonstrated in Gamannossi degl'Innocenti and Rablen (2017).

We find that the impact of the tax rate on total unreported income share is non-monotonic, and depends on the assumptions about the taxpayer's preferences. In particular, we investigate two cases, with and without a subsistence level of consumption for the taxpayer. While the presence of a subsistence level does not affect the optimal avoidance, it makes the optimal evasion (as well as the optimal consumption) a dynamic variable which converges towards a long term value.

We show that, for low levels of either avoidance or taxation, an increase in the tax rate leads to a net improvement in compliance, while the opposite holds true when the level of either avoidance or tax rate is high. As a result, in our model non-compliance behavior may result in a Laffer curve for fiscal revenues, providing a theoretical explanation for a phenomenon documented by policymakers (Papp and Takáts, 2008; Vogel, 2012). We also show the revenue maximizing tax rate to be lower the higher the probability β, so that the ability to raise revenues from taxes is directly affected by the vulnerability of the economic system to avoidance.

Interestingly, an increase in evasion is more likely when minimum consumption is relatively high (which happens, in general, in more developed economies,see Chetty and Szeidl, 2016; Havranek et al., 2017). Notice, however, that a reduction in β always increases declared income (and expected Government revenue) since the reduction induced in avoidance more than offsets any positive effect on evasion.

The first studies on tax compliance (e.g. Allingham and Sandmo, 1972, Yitzhaki, 1974) adapted Becker (1968)'s model of crime to tax evasion. Since then, the economic literature has been mostly focusing on tax evasion (e.g. Gamannossi degl'Innocenti and Rablen, 2017), either neglecting tax avoidance or considering it independently of other non-compliance opportunities.[5] However, already Cross and Shaw (1981, 1982) point out the importance of a joint study of evasion and avoidance, since taxpayers may consider them either substitutes or complements, and tax authorities should take into account both channels of response to their deterrence activities.

Furthermore, most of the economic literature —with few recent exceptions— relies on a static (timeless) framework (Dzhumashev and Gahramanov, 2011), even though the properties of the optimal solutions in a static and a dynamic framework may be quite different (Wen-Zhung and Yang, 2001).

Actually, the most recent models are cast into a dynamic framework (Dzhumashev and Gahramanov, 2011; Levaggi and Menoncin, 2012).

Finally, some recent extensions take into account the impact of uncertainty over fiscal parameters on evasion and growth (Bernasconi et al., 2015), the relationship between evasion and investment choices (Levaggi and Menoncin, 2013), and the role of habit in consumption (Bernasconi et al., 2019).

In this paper, we aim at merging most of the cited approaches, by allowing for both avoidance and evasion (as in Gamannossi degl'Innocenti and Rablen, 2017) in a dynamic setting (as in Levaggi and Menoncin, 2013).

In both Gamannossi degl'Innocenti and Rablen (2017) and Levaggi and Menoncin (2013), an increase in tax rate reduces non-compliance because of an income effect. In our setting, instead, an increase in the tax rate may have a non-monotonic effect on Government revenues as in Li et al. (2021) who, however, model the sole avoidance.

Our model is also the first one to analytically study the impact of the probability of success of avoidance.[6]

The article is structured as follows: Section 2 outlines the model. The optimization problem for the taxpayer and some possible government goals are analyzed in Sections 3 and 4. Section 5 discusses the results and their policy implications, while Section 6 draws some conclusions. The proofs are provided in the appendices.

[5]Notable exceptions considering both avoidance and evasion are Alm (1988a), Alm et al. (1990), Alm and McCallin (1990), and Gamannossi degl'Innocenti and Rablen (2017).

[6]In Gamannossi degl'Innocenti and Rablen (2017), an analogous investigation was performed only numerically.

2 The model

We model the choices of a representative taxpayer who maximizes her/his intertemporal utility that depends on the consumption of a private good (c_t) and a merit good (g_t) which is financed by a linear income tax. Our assumption is justified because, starting from the inception of the welfare state, the supply of goods like health care, education, and other personal services, has been increasing over time to become one of the biggest shares of public expenditure in western countries (OECD, 2021). We assume that the taxpayer is affected by fiscal illusion, which means that s/he does not perceive the link between public good provision and income tax so that s/he may engage in tax evasion and tax avoidance without internalizing the consequences of such behavior in the future supply of g_t.

2.1 Capital accumulation

The taxpayer is endowed with an initial capital k_{t_0} that is used over the period $[t_0, \infty[$ to produce an income y_t through the deterministic production function

$$y_t = Ak_t, \tag{1}$$

where A measures total factor productivity. Since the stock of capital cannot produce an aggregate income greater than the capital itself, it is reasonable to assume that $0 < A < 1$. Although A is exogenous and deterministic, the process of capital accumulation is endogenous because of the taxpayer's consumption choice (c_t), and it is also stochastic due to the choices related to tax evasion (e_t) and tax avoidance (a_t).

Government levies a linear tax $0 \leq \tau \leq 1$ on income, which is used to finance the provision of the merit good (g_t). With perfect compliance, the net change in capital is

$$dk_t = ((1-\tau)\, y_t - c_t)\, dt. \tag{2}$$

The taxpayer assumes that g_t does not depend on the income tax s/he pays. For this reason, s/he may try to reduce her/his tax burden by either evading a percentage (e_t) of the yield or by eroding her/his tax base through avoidance (a_t) whose effectiveness depends on the vulnerability of the tax system.

Avoidance is always audited together with evasion, and, in the case of an audit, two scenarios may happen: either avoidance is considered to be legal, or not. We define as $\mathbb{I}_{a \neq e}$ the indicator function whose value is either 1 if avoidance is "successful", i.e. it is assumed to be different from evasion, or 0 otherwise, i.e. it is considered to be evasion.

When the taxpayer is caught reducing the tax base, s/he has to pay a fine η that is proportional to the hidden part of the total tax, given by evasion and avoidance if it is illegal. Accordingly, the total fine that must be paid in case of an audit is

$$\eta\, (e_t + (1 - \mathbb{I}_{a \neq e})\, a_t)\, \tau A k_t,$$

whose expected value at time t is

$$\mathbb{E}_t\left[\eta\left(e_t+\left(1-\mathbb{I}_{a\neq e}\right)a_t\right)\tau Ak_t\right]=\eta\left(e_t+\left(1-\mathbb{E}_t\left[\mathbb{I}_{a\neq e}\right]\right)a_t\right)\tau Ak_t.$$

We recall that the expected value of the indicator function of an event coincides with the probability of the event. In particular, we set

$$\mathbb{E}\left[\mathbb{I}_{a\neq e}\right]:=\beta,$$

which measures the probability that avoidance is "successful", i.e. is considered to be legal. This parameter provides a measure of the vulnerability of the tax system to tax avoidance and is lower in economies in which: (i) tax codes are simpler and less ambiguous, (ii) tax authorities are endowed with relatively sizable operational[7] and litigation resources, and (iii) courts have higher effectiveness. If $\beta=1$ avoidance is riskless (like in Alm, 1988b; Alm et al., 1990), and deterrence is completely ineffective against avoidance. Instead, for $\beta=\frac{\eta-1}{\eta}$ the taxpayer's payment conditional on audit is the same as for true income reporting. This implies that, for any level above this threshold, the taxpayer gets an actual discount on her/his tax bill even in the case of an audit, while if β is below the threshold the taxpayer reduces the fines s/he has to pay if caught.

In line with other works in both the static and the dynamic literature (Wen-Zhung and Yang, 2001; Levaggi and Menoncin, 2013; Gamannossi degl'Innocenti and Rablen, 2017), we assume tax evasion to be a costless activity. Conversely, avoidance is assumed to be expensive since a considerable effort (or expertise) is needed to reduce the tax burden while not violating the law.

To keep our results as general as possible, the costs of avoidance are represented through any increasing and convex function $f\left(a_t\right)$.[8] Notably, this formulation allows accounting for the (likely) occurrence of fixed costs $f(0)\geq 0$ (setup costs, e.g. creation of legal entities) and has the flexibility to represent any mix of avoidance instruments.

In line with Levaggi and Menoncin (2012, 2013); Bernasconi et al. (2015); Levaggi and Menoncin (2016a,b), we model the audit process through a Poisson jump process $d\Pi_t$ whose frequency is λdt which coincides with the two first moments of the jump

$$\mathbb{E}_t\left[d\Pi_t\right]=\mathbb{V}\left[d\Pi_t\right]=\lambda dt.$$

Thus, the final dynamics of capital k_t is

$$\begin{aligned} dk_t = & \left(Ak_t-\tau\left(1-e_t-a_t\right)Ak_t-c_t-f\left(a_t\right)Ak_t\right)dt \\ & -\eta\left(e_t+\left(1-\mathbb{I}_{a\neq e}\right)a_t\right)\tau Ak_t d\Pi_t, \end{aligned} \tag{3}$$

[7] In a recent paper, Guyton et al. (2021) show that more detailed/thorough audits are able to uncover avoidance activities that are mostly missed by standard random audits.

[8] Evidence on the contractual terms upon which avoidance schemes are typically sold is scarce. However, Committee of Public Accounts (2013) reports that the majority of schemes entail a fee related to the reduction in the annual theoretical tax liability of the user and Kantar Public UK (2015) shows that fees vary with the value of the amount of the investment realized by the scheme.

where the tax τ is paid only on the income that is not hidden $(1 - e_t - a_t)$, the avoidance cost $f(a_t)$ is proportional to income, and we assume that the probability that avoidance is successful is independent of the probability to be audited.

The expected value of dk_t is

$$\mathbb{E}_t [dk_t] = \left((1-\tau) Ak_t + (1-\eta\lambda) e_t \tau Ak_t + \left(1 - \eta\lambda (1-\beta) - \frac{f(a_t)}{a_t \tau}\right) a_t \tau Ak_t - c_t \right) dt,$$

from which we see that evasion is expedient on average if

$$\mathbb{E}_t [dk_t] > \mathbb{E}_t [dk_t]_{e_t=0},$$

which becomes

$$1 - \eta\lambda > 0, \tag{4}$$

and, accordingly, we will assume that the product between the frequency of audit (λ) and the fine (η) is lower than 1. Instead, avoidance is expedient on average if

$$\mathbb{E}_t [dk_t] > \mathbb{E}_t [dk_t]_{a_t=0},$$

which becomes

$$\frac{f(a_t)}{a_t} < \frac{f(0)}{a_t} + (1 - \eta\lambda (1-\beta)) \tau.$$

Hence, the taxpayer will engage in avoidance if its costs are lower than a threshold dependent on both fixed avoidance cost, and fiscal and enforcement parameters.

Since the product $\eta\lambda$ is higher than 1 for an expedient evasion, then the minorant of the right hand side is $\beta\tau$, and so we can impose that

$$f(a_t) < f(0) + a_t \beta\tau, \tag{5}$$

in which we further assume that $f(0) < 1 - \tau$.

2.2 Taxpayer's preferences

The representative taxpayer receives utility from consuming both a private produced good (c_t) and a public produced good (g_t), and we assume that such a utility is additive in these two goods.

The taxpayer's behavior is described by a Hyperbolic Absolute Risk Aversion utility (see, for instance, Gollier, 2001) written on the instantaneous consumption as

$$U(c_t) = \frac{(c_t - c_m)^{1-\delta}}{1-\delta} + v(g_t), \tag{6}$$

where c_m is a minimum (subsistence) amount of consumption that the taxpayer needs to consume, the parameter $\delta > 0$ measures the risk aversion, and $v(\bullet)$ is an increasing and concave function. The existence of a strictly positive subsistence consumption level allows us to solve some puzzles and reconcile theoretical findings with empirical evidence (see, for instance, Sethi et al., 1992; Weinbaum, 2005; Achury et al., 2012 for the role of subsistence consumption in portfolio choice and Strulik, 2010 for its role in modeling economic growth). The Arrow Pratt absolute risk aversion index is

$$-\frac{\frac{\partial^2 U(c_t)}{\partial c_t^2}}{\frac{\partial U(c_t)}{\partial c_t}} = \frac{\delta}{c_t - c_m}, \tag{7}$$

which increases when either δ or c_m increase. In other words, a taxpayer with a low δ but whose consumption is closer to c_m behaves exactly as a taxpayer with a higher δ but with a consumption level farther from c_m.

3 The Problem

If the taxpayer discounts future utility at a constant rate ρ, the optimization problem can be written as

$$\max_{\{c_t, e_t, a_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} \frac{(c_t - c_m)^{1-\delta}}{1-\delta} e^{-\rho(t-t_0)} dt\right], \tag{8}$$

under the capital dynamics (3).

Proposition 1. *The optimal solution to Problem (8), given the capital dynamics (3), is*

$$a^* = (f')^{-1}(\tau\beta), \tag{9}$$

$$e_t^* = \frac{k_t - H}{\tau\eta A k_t}\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) - (1-\beta)\, a^*, \tag{10}$$

$$c_t^* = c_m + (k_t - H)\left(\frac{\rho+\lambda}{\delta} + \frac{\delta-1}{\delta}\frac{1}{\eta} + \frac{\delta-1}{\delta}(1-\tau)A - \frac{1}{\eta}(\lambda\eta)^{\frac{1}{\delta}} + \frac{\delta-1}{\delta}(\tau\beta a^* - f(a^*))\, A\right), \tag{11}$$

in which $(f')^{-1}$ is the inverse of the first derivative of the function f, and

$$H := \frac{c_m}{A(\tau\beta a^* - f(a^*) + (1-\tau))}.$$

Proof. See Appendix A. □

Equation (10) shows that optimal evasion is affected by all the model parameters. The same is not true for (9), even if both variables are subject to the same risk of being audited.

In the proposition above, H is a constant whose value coincides with the present value of a perpetuity. In fact, we can write

$$H = \int_t^{\infty} c_m e^{-A(\tau\beta a^* - f(a^*) + (1-\tau))(s-t)} ds,$$

which is always positive because of condition (5). Thus, we can conclude that H represents the total present value of the future subsistence consumption c_m, discounted at a rate given by the total factor productivity corrected by both the tax rate and a function of avoidance. Accordingly, $k_t - H$ can be considered as the disposable capital that remains after saving enough for financing the future streams of subsistence consumption.

We note that when avoidance is not expedient (i.e. $a^* = 0$), the discount rate is given by the total factor productivity net of tax and fixed avoidance costs: $A((1-\tau) - f(0))$. We can immediately check that, under condition (5), the presence of avoidance (i.e $a^* > 0$) increases optimal consumption.

Optimal tax avoidance is constant across time and does not depend on the audit parameters η (the fine) and λ (the frequency of controls), while it simply depends on its cost (the shape of the function $f(\bullet)$), the vulnerability of the tax system to avoidance β, and the tax rate τ. In particular, from Eq. (9), we see that the representative taxpayer balances the marginal costs of avoidance (the derivative $f'(\bullet)$) with the marginal benefits from avoidance ($\tau\beta$).

Intuitively, avoidance exploits a loophole in the law to attain a lower expected penalty relative to evasion. Accordingly, optimal avoidance depends only on the parameters that affect its net marginal return relative to evasion in case of an audit. The lack of effect of classic deterrence instruments on avoidance is in sharp contrast with the rest of the static literature (Gamannossi degl'Innocenti and Rablen, 2017; Alm, 1988b; Alm et al., 1990).[9] Interestingly, we show that not only is avoidance not affected by the preference parameters (taxpayer's risk aversion), but it is even independent of the functional form chosen for utility (see Appendix A, Remark 1, where first order conditions are computed). Given that avoidance does not depend on preference parameters, our model implies that all the taxpayers will avoid the same share of income if the system is vulnerable to avoidance ($\beta > 0$).

Tax evasion, on the contrary, is used as a "top up" to tax avoidance even if the "substitution rate" is not one. The share of evaded income depends on the fiscal parameters and is similar to the optimal tax evasion of other dynamic models (e.g., Levaggi and Menoncin, 2013), but it is lower in magnitude given that taxpayers substitute it in part with avoidance.

In our setting, we show that tax avoidance, while reducing evasion, increases total unreported income share, i.e., the sum of optimal evasion and avoidance:

$$E_t^* = e_t^* + a^* = \frac{k_t - H}{\tau\eta A k_t}\left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) + \beta\left(f'\right)^{-1}(\tau\beta). \tag{12}$$

The existence of a subsistence level of consumption implies that optimal evasion is time dependent as shown in Proposition 1. In the following corollary, we show that with $c_m = 0$ evasion is constant over time and so is consumption share.

[9] One exception is Li et al. (2021), where the effect of increased deterrence on avoidance is entirely offset by the endogenous adjustment of price by suppliers.

Corollary 1. *The optimal solution to Problem (8) for a CRRA taxpayer (i.e. $c_m = 0$), given the capital dynamics (3), is*

$$a^* = (f')^{-1}(\tau\beta),$$
$$e^* = \frac{1}{\tau\eta A}\left(1-(\lambda\eta)^{\frac{1}{\delta}}\right)-(1-\beta)\,a^*,$$
$$\frac{c_t^*}{k_t} = \frac{\rho+\lambda}{\delta}+\frac{\delta-1}{\delta}\frac{1}{\eta}+\frac{\delta-1}{\delta}(1-\tau)\,A-\frac{1}{\eta}(\lambda\eta)^{\frac{1}{\delta}}+\frac{\delta-1}{\delta}(\tau\beta a^*-f(a^*))\,A.$$

Proof. It is sufficient to set $c_m = 0$ in Proposition 1. □

Corollary 1 highlights that, with $c_m = 0$, all the optimal strategies are time independent. Instead, with $c_m > 0$, the dynamics of optimal solutions arise because of a habit effect that leads to a gradual convergence towards long term levels coinciding with the case $c_m = 0$.

The results in Proposition 1 and Corollary 1 allow drawing some interesting conclusions on the dynamic path of the choice variables of the taxpayer. While the optimal share of avoided income is fixed, the dynamics of consumption, capital, and tax evasion are more nuanced. In Figure 1, black lines show the case with $\beta > 0$, so that the optimal avoidance share is positive, while gray lines show the case with $\beta = 0$ which implies zero optimal avoidance. Thus, the figure makes it easier to compare our result with the previous literature (i.e. Levaggi and Menoncin, 2013). When $c_m > 0$, these variables are affected by the (random) audits, so we perform $N = 1000$ replications and report the average (solid line) along with the zero and one quantile (dashed lines).

Panel a) shows that, for $c_m = 0$ (dot-dashed line), the evaded share of income is fixed in time due to the constant relative risk aversion. When $c_m > 0$, the evasion is increasing in time and in the long run it tends to its optimal level with $c_m = 0$. This dynamics of evasion is driven by the growth of taxpayer's consumption c_t^*, that reduces risk aversion $\frac{\delta}{c_t^*-c_m}$. Conversely, when an audit occurs, we observe a sharp drop in quantile lines driven by the fall in the taxpayer's consumption. While the level of optimal evasion when $a^* > 0$ (in black) is lower than the one when $a^* = 0$ (in gray), the graph highlights the identical behavior of the two, given that optimal avoidance is constant.

Panel b) illustrates the evolution of consumption as a ratio of capital. The dynamics of c_t^*/k_t when $c_m = 0$ is shown to be constant over time (Corollary 1) and lower than the case with $c_m > 0$. When $c_m > 0$ (Proposition 1), c_t^*/k_t is decreasing over time due to a more sustained growth of the denominator. Given that optimal consumption is an affine transformation of capital, quantile lines in this panel experience a jump upon audit (since c_t^* decreases less than k_t). Finally, the plot shows that allowing for avoidance leads to a higher $\frac{c_t^*}{k_t}$ since the last term in (11) is positive when $a^* > 0$ by condition (5).

— Insert Figure 1 about here —

3.1 Comparative statics

Here, we compute how the three choice variables a^*, e_t^*, and E_t^* respond to a change in the model parameters.

Optimal avoidance a^* increases with respect to both β and τ as expected. This result matches evidence in the empirical literature[10] and shows that the Yitzhaki puzzle (Yitzhaki, 1974) does not hold for tax avoidance in a dynamic setting. The positive relationship between a^* and τ is in contrast with Gamannossi degl'Innocenti and Rablen (2017) and follows from optimal avoidance being driven only by its marginal net return relative to evasion in the case of an audit. In the static framework, a similar result is observed only in Alm and McCallin (1990), where the joint avoidance and evasion decision is analyzed using a portfolio approach.

From (10), optimal evasion decreases if τ increases, thus confirming the Yitzhaki puzzle. In our model, the presence of avoidance reinforces the dampening effect already observed when the fine is proportional to evaded taxes. The same effect can be observed for an increase in λ and η. Instead, the reaction of e_t^* to changes in β, measured by

$$\frac{\partial e_t^*}{\partial \beta} = \frac{\tau a^* A H^2}{c_m} \frac{e_t^* + (1-\beta)\, a^*}{k_t - H} + a^* - (1-\beta)\, \frac{\partial a^*}{\partial \beta},$$

is not trivial to compute. In fact, the elements of its latter term, $(1-\beta)\,\frac{\partial a^*}{\partial \beta}$, are impacted in opposite directions by a change in β. Indeed, an increase in β increases avoidance (hence reducing evasion) but this (negative) effect is multiplied by $1-\beta$ which is decreasing in β.

For $c_m = 0$ (i.e. $H = 0$), the optimal tax evasion may be either increasing or decreasing w.r.t. β:

$$\left.\frac{\partial e_t^*}{\partial \beta}\right|_{c_m=0} = a^* - (1-\beta)\, \frac{\partial a^*}{\partial \beta} \gtreqless 0 \iff \frac{\partial a^*}{\partial \beta} \frac{1}{a^*} \lesseqgtr \frac{1}{1-\beta},$$

i.e. evasion is increasing in β only if the elasticity of a^* w.r.t. β is lower than a given threshold. This result has an interesting interpretation from a policy point of view because of the twofold interpretation of the parameter β: it is the probability of avoidance to be successful, but it can also be interpreted as the vulnerability of the fiscal system to avoidance. Thus, decreasing the tax system vulnerability to avoidance (lowering β) reduces (increases) tax evasion if the probability of success in avoidance (β) is low (high). Hence, for a system rather vulnerable to tax avoidance, a marginal decrease in the probability of success of avoidance may worsen tax evasion statistics. Other things being equal, $\frac{\partial e_t^*}{\partial \beta}$ is

[10] Long and Gwartney (1987), Alm et al. (1990), and Lang et al. (1997) show that tax avoidance increases with the tax rate for US, Jamaican, and German households. As reviewed in Riedel (2018), the scientific literature unanimously reports evidence of substantial tax motivated profit shifting. Also Beer et al. (2020) perform a comprehensive meta-analysis of existing studies suggesting an elasticity of before tax income to corporate tax rate of minus one.

higher when $c_m > 0$, meaning that the effectiveness of β in preventing evasion is higher if there is a positive subsistence consumption level.

It is interesting to observe that if $c_m = 0$, the value of β which minimizes the evasion must satisfy the condition

$$\frac{\partial a^*}{\partial \beta}\frac{\beta}{a^*} = \frac{\beta}{1-\beta},$$

that coincides with the odds of the event that avoidance is successful.

The total unreported income share (12) reacts to changes in τ in an ambiguous way because of the reduction in tax evasion and the increase in tax avoidance. The derivative can be written as:

$$\frac{\partial E_t^*}{\partial \tau} = \underbrace{-\frac{1}{\tau\eta A k_t}\left(1-(\lambda\eta)^{\frac{1}{\delta}}\right)\left(H\frac{1-\beta a^*}{\tau\beta a^* - f(a^*) + (1-\tau)} + \frac{k_t - H}{\tau}\right)}_{<0} + \beta\underbrace{\frac{\partial a^*}{\partial \tau}}_{>0},$$

and even for the simpler case with $c_m = 0$ the sign remains ambiguous:

$$\left.\frac{\partial E_t^*}{\partial \tau}\right|_{c_m=0} = \underbrace{-\frac{1}{\tau^2\eta A}\left(1-(\lambda\eta)^{\frac{1}{\delta}}\right)}_{<0} + \beta\underbrace{\frac{\partial a^*}{\partial \tau}}_{>0}.$$

Notably, when τ is relatively high, the negative term is smaller (in absolute value) and the positive one is bigger. This implies that an increase in tax rate reduces total reported income share when taxation is sufficiently high, i.e., increasing fiscal pressure improves compliance in economic systems with a relatively high tax burden and vice versa. Our result is in contrast with both Gamannossi degl'Innocenti and Rablen (2017) and Levaggi and Menoncin (2013), where a tax raise only induces an income effect (on both avoidance and evasion or evasion only) that lowers non-compliance.

A result analogous to ours is reported by Alm (1988b) in a static framework, but, in that case, the ambiguity is made possible by the very general specification of the fine, tax and avoidance cost functions.

Finally, the derivative of E_t^* with respect to β can be written as:

$$\frac{\partial E_t}{\partial \beta} = \frac{1}{\tau\eta A k_t}\left(1-(\lambda\eta)^{\frac{1}{\delta}}\right)\frac{H\tau a^*}{(\tau\beta a^* - f(a^*) + (1-\tau))} + a^* + \beta\frac{\partial a^*}{\partial \beta} > 0,$$

so that reducing the system's vulnerability to avoidance, despite possibly worsening tax evasion, always increases declared revenue. The comparative statics results derived in this section, along with the results on government revenue in the next section, are summarized in Table 1.

Table 1: Effect of enforcement/fiscal parameters on avoidance, evasion, and tax revenue. The table presents the sign of derivatives (null, positive, negative, or undetermined) of the function in the column with respect to the parameter in the row: sign $\left(\frac{\partial \mathrm{Col}}{\partial \mathrm{Row}}\right)$

	a^*	e_t^*	$E_t^* = a^* + e_t^*$	$\mathbb{E}_t\left[dT_t\right]$
λ	0	$-$	$-$	$+$
η	0	$-$	$-$	$+$
β	$+$	und.	$+$	$-$
τ	$+$	$-$	und.	und.

4 The optimal capital dynamics and government revenue

We first consider the optimal capital dynamics and its relation with β.

Proposition 2. *The expected growth rate of optimal modified capital $k_t^* - H$ is*

$$\gamma^* := \frac{1}{dt}\mathbb{E}_t\left[\frac{d\left(k_t^* - H\right)}{\left(k_t^* - H\right)}\right] = \frac{1}{\delta}\left(\left(1-\tau\right)A - \left(\rho+\lambda\right) + \frac{1}{\eta} + \left(\tau\beta a_t^* - f\left(a_t^*\right)\right)A\right) - \left(1 - \left(\lambda\eta\right)^{\frac{1}{\delta}}\right)\lambda, \tag{13}$$

whose first derivative with respect to β is

$$\frac{\partial \gamma^*}{\partial \beta} = \frac{1}{\delta}\frac{\tau}{\eta}a_t^* A > 0. \tag{14}$$

Proof. See Appendix B. □

Equation (13) follows from Equation (11) and describes the dynamics of the capital in excess of H,[11] which measures the discounted present value of the future subsistence consumption levels c_m.

We stress that the solution to the process $k_t^* - H$ is exponential, and is always positive if the initial value $k_{t_0}^* - H$ is positive. Thus, we can conclude that the optimal capital will never fall below the value H if $k_{t_0}^* > H$, i.e., the taxpayer behaves in such a way to guarantee that her/his capital is always able to finance the future flow of subsistence consumption.

Given condition (14), the capital growth is maximized by choosing the highest value for β (i.e. $\beta^* = 1$). This result is due to the nature of the good produced by the Government: by assuming a merit good that does not increase private capital productivity, growth is maximized with minimum tax revenue.

We now characterize government revenue and its relationship with the success probability of avoidance, the fine, and the tax rate.

[11] Since H is constant, we get $\frac{1}{dt}\mathbb{E}_t\left[\frac{d\left(k_t^* - H\right)}{\left(k_t^* - H\right)}\right] = \frac{1}{dt}\mathbb{E}_t\left[\frac{d\left(k_t^*\right)}{\left(k_t^*\right)}\right]$.

Proposition 3. *The expected dynamics of government revenue dT_t is*

$$\mathbb{E}_t\left[dT_t\right] = \left[\tau\left(1-\beta a_t^*\right)Ak_t - \frac{k_t-H}{\eta}\left(1-\lambda\eta\right)\left(1-\left(\lambda\eta\right)^{\frac{1}{\delta}}\right)\right]dt,$$

whose derivatives are

$$\frac{\partial}{\partial\beta}\left(\frac{1}{dt}\mathbb{E}_t\left[dT_t\right]\right) > 0,$$

$$\frac{\partial}{\partial\eta}\left(\frac{1}{dt}\mathbb{E}_t\left[dT_t\right]\right) > 0,$$

$$\frac{1}{dt}\frac{\partial\mathbb{E}_t\left[dT_t\right]}{\partial\tau} = \left(1-\beta a_t^*\right)Ak_t - \tau\beta\frac{\partial a_t^*}{\partial\tau}Ak_t + \frac{1}{\eta}\left(1-\lambda\eta\right)\left(1-\left(\lambda\eta\right)^{\frac{1}{\delta}}\right)H\frac{1-\beta a^*}{\left(\tau\beta a^* - f\left(a^*\right)+\left(1-\tau\right)\right)}, \tag{15}$$

$$\left.\frac{1}{dt}\frac{\partial\mathbb{E}_t\left[dT_t\right]}{\partial\tau}\right|_{c_m=0} \gtreqless 0 \iff \beta a_t^*\left(\tau\frac{\partial a_t^*}{\partial\tau}\frac{1}{a_t^*}+1\right) \lesseqgtr 1. \tag{16}$$

Proof. See Appendix C. □

Proposition 3 shows that increases in β lead to higher expected revenues through a reduction in total unreported income share. A positive relationship also holds between η and expected revenues despite a reduction in expected fines caused by the lower evasion.

Equation (16) in Proposition 3 reveals that there is an ambiguity about how Government tax revenue reacts to changes in the tax rate. A relevant result is obtained for a particular case.

Corollary 2. *If the elasticity of the optimal avoidance w.r.t the tax rate is constant (i.e. $\exists\kappa,\chi>0 : a^* = \kappa\tau^\chi$), then*

$$\left.\frac{1}{dt}\frac{\partial\mathbb{E}_t\left[dT_t\right]}{\partial\tau}\right|_{c_m=0} \gtreqless 0 \iff \tau \lesseqgtr \left(\left(\gamma+1\right)\beta\kappa\right)^{-\frac{1}{\chi}}.$$

Proof. It is sufficient to substitute $a^* = \kappa\tau^\chi$ in (16) with κ and χ positive. □

Corollary 2 shows the condition under which our model entails a Laffer curve behavior. For τ sufficiently low, the revenue increases as τ increases because the rise in the tax rate and the reduction of tax evasion more than offset the increase in tax avoidance. However, as τ increases, the latter effect becomes prevalent and the revenue starts decreasing. Notably, the level of the revenue maximizing tax rate is inversely related to β. The analogous result in Li et al. (2021) arises due to a non-monotonic impact of the tax rate on the minimum income to engage in avoidance.

Figure 2 shows a graphical representation of the Laffer curve in our setting. On the horizontal axis, we measure the tax rate while on the vertical axis we measure the variation in expected revenues relative to capital. In the figure, we consider three different constants of variation ω in a power cost function of the form $f(a) = \omega a^{\gamma}$. The plot shows the relationship between tax rate and expected revenues becoming negative at lower tax rates when ω is smaller. This behaviour is expected, as the increase in avoidance following a rise in the tax rate is higher when the marginal cost f' is lower.

— Insert Figure 2 about here —

5 Discussion and policy implications

The results and the comparative statics presented in the previous sections highlight the importance of studying tax evasion and tax avoidance as a joint decision. The results of our model indeed show that several interesting policy implications can be derived from this analysis and that the institutional setting (especially β) may change the outcome of policies aimed at reducing non compliance. In what follows we summarize and discuss the most important results of our model.

1. Tax avoidance depends neither on taxpayer's risk attitude, nor her/his utility function, nor on traditional audit parameters (frequency and fines). This implies that government cannot alter the avoidance decision using ordinary tax enforcement tools. Instead, this result can be obtained through an increase in the quality (litigation resources and thoroughness) of the audits or fiscal/legal reforms (which in our model would affect β, the vulnerability of the tax system to avoidance). However, avoidance deterrence might entail unintended consequences:

    (a) Even if evasion is decreasing in the tax rate, there are limits to the use of the tax rate as an instrument to improve compliance due to the presence of a Laffer curve on total Government revenue, which provides a theoretical explanation for a phenomenon documented by policymakers (Papp and Takáts, 2008; Vogel, 2012). This finding follows from the three effects induced by a rise in the tax rate: (i) a mechanical increase of revenues due to the higher marginal tax rate, (ii) a reduction in evasion, and (iii) an increase in avoidance.

    (b) Policies aimed at increasing avoidance costs, while theoretically identifiable, seem to have limited practical relevance. The costs to engage in avoidance are related to the effort (or expertise) required to have a deep understanding of the "loopholes" in the tax law. An increase in these costs entails a trade-off, as these costs also apply to "intended" economic activities. A more effective way to reduce tax avoidance is to lower the probability of a successful avoidance (i.e. reduce β), through a simplification of the tax system.[12] Invest-

[12] On specific anti avoidance reforms of the tax system, see Gravelle (2014).

ing in tax simplification, intended as the reduction of the extent of variation in possible tax treatments of economic activities (number of deductions, exemptions and instances of preferential treatment of income), has also been recommended in the literature (e.g., Skinner and Slemrod, 1985; Mccaffery, 1990; Kopczuk, 2006) for its several desirable outcomes.

2. Our analysis shows that tax avoidance deterrence performed by lowering either the tax rate or the probability that the audited avoidance is successful, might entail an unwanted increase in tax evasion, which can however be offset by raising either the frequency of audit or the fine.

3. The opposite impacts of the tax rate on avoidance and evasion may provide an alternative interpretation for the Yitzhaki puzzle. While, from a theoretical point of view, it is possible to disentangle evasion from avoidance, the distinction is much more complex in an empirical setting. An imperfect measure of tax evasion (which may also include a part of tax avoidance) would lead to a spurious estimation, as the recent estimates on the tax gap show (Sarin and Summers, 2019).

Over the last few decades, the most striking worldwide trend in tax policy has been the decline in corporate income tax rates. Some argue (e.g. Tørsløv et al., 2020) that this is an effect of the tax reduction performed in many countries to face the competition of tax havens. We show that a similar mechanism might also be at work for individual income tax: when avoidance is more profitable (higher β), the tax rate that maximizes government revenue and the revenues themselves are lower. Our results suggest that anti-avoidance efforts of tax authorities/governments/international organizations should be extended to personal income to prevent a deterioration of Government revenue.

6 Conclusion

In this paper we developed what can be considered, to the best of our knowledge, the first dynamic model studying taxpayer's avoidance and evasion. Evasion is costless, but entails the payment of a fine if detected. Instead, avoidance is costly, but there is a probability that it will be considered legitimate upon audit.

Contrary to previous studies in a static framework, our results show that optimal avoidance does not depend on audit parameters (frequency of the audits and fine to be paid when caught evading) in an intertemporal setting. Tax avoidance, unlike evasion, is also not affected by the preferences of the taxpayer. The share of avoided income results from a cost-benefit analysis: the (certain) avoidance cost measures the (money equivalent) effort (or hired expertise cost) needed to engage in avoidance, while the (uncertain) benefit is the potential reduction of the probability of being fined when audited.

From a policy point of view, our model shows that reducing tax evasion may be a government objective that is rather different from maximizing revenue, especially in the presence of tax avoidance. Given the opposite impact of the tax rate on avoidance and evasion, we find that a Laffer curve exists between the tax rate and fiscal revenue. Our analysis also shows the importance of the probability that the audited avoidance is successful β and highlights its possible detrimental impact on evasion. In particular, a reduction in β leads to an increase of collected revenues but might entail a rise of tax evasion for economies more vulnerable to avoidance.

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A Proof of Proposition 1

Given Problem (8), we can define the value function at any time $t \in [t_0, \infty)$ as

$$J(t, k_t) = \max_{\{c_s, e_s, a_s\}_{s\in[t,\infty[}} \mathbb{E}_{t_0} \left[\int_t^\infty \frac{(c_s - c_m)^{1-\delta}}{1-\delta} e^{-\rho(s-t)} ds \right],$$

and so the Hamilton-Jacobi-Bellman (HJB) equation is

$$0 = \frac{\partial J(t, k_t)}{\partial t} - (\rho + \lambda) J + \frac{\partial J(t, k_t)}{\partial k_t} (1 - \tau) A k_t + \max_{c_t} \left[\frac{(c_t - c_m)^{1-\delta}}{1-\delta} - \frac{\partial J(t, k_t)}{\partial k_t} c_t \right]$$
$$+ \max_{e_t, a_t} \left[\frac{\partial J(t, k_t)}{\partial k_t} (\tau (e_t + a_t) A k_t - f(a_t) A k_t) + \lambda J(t, k_t - \eta (e_t + (1-\beta) a_t) \tau A k_t) \right].$$

The First Order Condition (FOC) with respect to c_t is

$$c_t^* = c_m + \left(\frac{\partial J(t, k_t)}{\partial k_t} \right)^{-\frac{1}{\delta}}.$$

The FOC with respect to a_t is

$$\frac{\partial J(t, k_t)}{\partial k_t} \left(\tau - \frac{\partial f(a_t^*)}{\partial a_t^*} \right) - \lambda \frac{\partial J(t, k_t - \eta (e_t + (1-\beta) a_t) \tau A k_t)}{\partial (k_t - \eta (e_t + (1-\beta) a_t) \tau A k_t)} \eta (1 - \beta) \tau = 0$$

and the FOC with respect to e_t is

$$\frac{\partial J(t, k_t)}{\partial k_t} - \lambda \frac{\partial J(t, k_t - \eta (e_t + (1-\beta) a_t) \tau A k_t)}{\partial (k_t - \eta (e_t + (1-\beta) a_t) \tau A k_t)} \eta = 0.$$

The comparison between the two last FOCs gives

$$a^* = (f')^{-1} (\tau \beta),$$

in which $(f')^{-1}$ is the inverse of the derivative of the function f.

Remark 1. We immediately see that the optimal avoidance does not depend on the value function $J(t, k_t)$, which means that it neither depends on the utility function.

For computing the other two variables, instead, we must know the functional form of the value function. The guess function is

$$J = F^\delta \frac{(k_t - H)^{1-\delta}}{1-\delta},$$

in which F and H are constant that will be obtained from the HJB equation. Given this function, the optimal values for evasion and consumption are

$$c_t^* = c_m + \frac{k_t - H}{F},$$

$$e_t^* = \frac{k_t - H}{\tau \eta A k_t} \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) - (1-\beta) a_t^*.$$

Once a_t^* and e_t^* are substituted into the HJB we get:

$$\begin{aligned} 0 =& F^{\delta} (k_t - H)^{1-\delta} \frac{\delta}{\delta - 1} \left(\frac{\rho + \lambda}{\delta} + \frac{\delta - 1}{\delta} \frac{1}{\eta} + \frac{\delta - 1}{\delta} (1-\tau) A - \frac{1}{\eta} (\lambda\eta)^{\frac{1}{\delta}} + \frac{\delta - 1}{\delta} (\tau\beta a^* - f(a^*)) A - F^{-1} \right) \\ &+ F^{\delta} (k_t - H)^{-\delta} (-c_m + (\tau\beta a^* - f(a^*) + (1-\tau)) AH). \end{aligned}$$

This function can be split into two equations: one which contains the terms with $(k_t - H)^{1-\delta}$ and one which contains the terms with $(k_t - H)^{-\delta}$. Thus, we get

$$\begin{aligned} F^{-1} =& \frac{\rho + \lambda}{\delta} + \frac{\delta - 1}{\delta} \frac{1}{\eta} + \frac{\delta - 1}{\delta} (1-\tau) A - \frac{1}{\eta} (\lambda\eta)^{\frac{1}{\delta}} + \frac{\delta - 1}{\delta} (\tau\beta a^* - f(a^*)) A, \\ H =& \frac{c_m}{A (\tau\beta a^* - f(a^*) + (1-\tau))}. \end{aligned}$$

B Proof of Proposition 2

Equation (13) follows by taking the derivative with respect to time of the optimal capital dynamics:

$$\frac{d(k_t^* - H)}{(k_t^* - H)} = \frac{1}{\delta} \left((1-\tau) A - (\rho + \lambda) + \frac{1}{\eta} + (\tau a_t^* \mathbb{I}_{a \neq e} - f(a_t^*)) A \right) dt - \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) d\Pi_t.$$

If we compute the first derivative of γ^* with respect to β we get

$$\frac{\partial \gamma^*}{\partial \beta} = \frac{1}{\delta} \left(\frac{\tau}{\eta} a_t^* + \tau\beta \frac{\partial a_t^*}{\partial \beta} - f'(a_t^*) \frac{\partial a_t^*}{\partial \beta} \right) A,$$

and because of the first order condition $f'(a_t^*) = \tau\beta$, we can write

$$\frac{\partial \gamma^*}{\partial \beta} = \frac{1}{\delta} \frac{\tau}{\eta} a_t^* A > 0.$$

From direct differentiation of equation (15) it is

$$\frac{\partial}{\partial \beta} \left(\frac{1}{dt} \mathbb{E}_t [dT_t] \right) = -\tau a_t^* A k_t - \tau\beta \frac{\partial a_t^*}{\partial \beta} A k_t - \frac{1}{\eta} (1 - \lambda\eta) \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) \frac{1}{A} \frac{c_m \tau a^*}{(\tau\beta a^* - f(a^*) + (1-\tau))^2} > 0,$$

$$\frac{\partial}{\partial \eta} \left(\frac{1}{dt} \mathbb{E}_t [dT_t] \right) > 0 \iff \left(1 - (\lambda\eta)^{\frac{1}{\delta}}\right) + (1 - \lambda\eta) \frac{1}{\delta} (\lambda\eta)^{\frac{1}{\delta}} > 0.$$

C Proof of Proposition 3

The dynamics of government revenues can be written as

$$dT_t = \left(\tau\left(1-\mathbb{I}_{a\neq e}a_t^*\right)Ak_t - \frac{k_t-H}{\eta}\left(1-(\lambda\eta)^{\frac{1}{\delta}}\right)\right)dt + (k_t-H)\left(1-(\lambda\eta)^{\frac{1}{\delta}}\right)d\Pi_t,$$

whose expected value is

$$\mathbb{E}_t\left[dT_t\right] = \left[\tau\left(1-\beta a_t^*\right)Ak_t - \frac{k_t-H}{\eta}\left(1-\lambda\eta\right)\left(1-(\lambda\eta)^{\frac{1}{\delta}}\right)\right]dt.$$

When $c_m = 0$ the derivative of government income w.r.t. tax is

$$\left.\frac{1}{dt}\frac{\partial\mathbb{E}_t\left[dT_t\right]}{\partial\tau}\right|_{c_m=0} = \left(1-\beta a_t^*\right)Ak_t - \tau\beta\frac{\partial a_t^*}{\partial\tau}Ak_t,$$

whose second derivative is

$$\left.\frac{1}{dt}\frac{\partial^2\mathbb{E}_t\left[dT_t\right]}{\partial\tau^2}\right|_{c_m=0} = \beta\left(-2\frac{\partial a_t^*}{\partial\tau} - \tau\frac{\partial^2 a_t^*}{\partial\tau^2}\right)Ak_t.$$

If this derivative is always negative, then the curve has a unique maximum. Since $f(a)$ is increasing and convex, then $(f')^{-1}$ is increasing, which means that a^* is increasing in τ. The second derivative of a^* w.r.t. τ depends on the sign of the third derivative of $f(a)$, which has not been defined.

In the case of a power function

$$f(a) = f(0) + \omega a^\gamma,$$

with positive ω and $\gamma > 1$, the second derivative is always negative. In this case, in fact

$$a^* = \left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}},$$

from which

$$\frac{\partial a_t^*}{\partial\tau} = \frac{1}{\gamma-1}\frac{1}{\tau}\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{\partial^2 a_t^*}{\partial\tau^2} = \frac{1}{\gamma-1}\left(\frac{1}{\gamma-1}-1\right)\frac{1}{\tau^2}\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}}$$

and so

$$\left.\frac{1}{dt}\frac{\partial\mathbb{E}_t\left[dT_t\right]}{\partial\tau}\right|_{c_m=0} = \left(1-\frac{\gamma}{\gamma-1}\beta\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}}\right)Ak_t$$

$$\left.\frac{1}{dt}\frac{\partial^2\mathbb{E}_t\left[dT_t\right]}{\partial\tau^2}\right|_{c_m=0} = -\beta\frac{1}{\tau}\frac{\gamma}{(\gamma-1)^2}\left(\frac{\tau\beta}{\omega\gamma}\right)^{\frac{1}{\gamma-1}}Ak_t < 0.$$

So, if the function $f(a)$ is a power function, there is only one maximum.

Figure 1: a) Evasion dynamics b) Ratio of consumption to capital dynamics. Black lines are obtained with $\beta = .5$ ($a^* > 0$) while gray ones with $\beta = 0$ (which implies $a^* = 0$). Parameters specification: $A = 0.3$, $c_m = 10$, $\delta = 2.5$, $\eta = 2.5$, $k_0 = 100$, $\lambda = 0.3$, $\rho = 0.05$, $\tau = .3$, $f(a_t) = \omega a_t^{\gamma}$, $\gamma = 2$, $\omega = 1$.

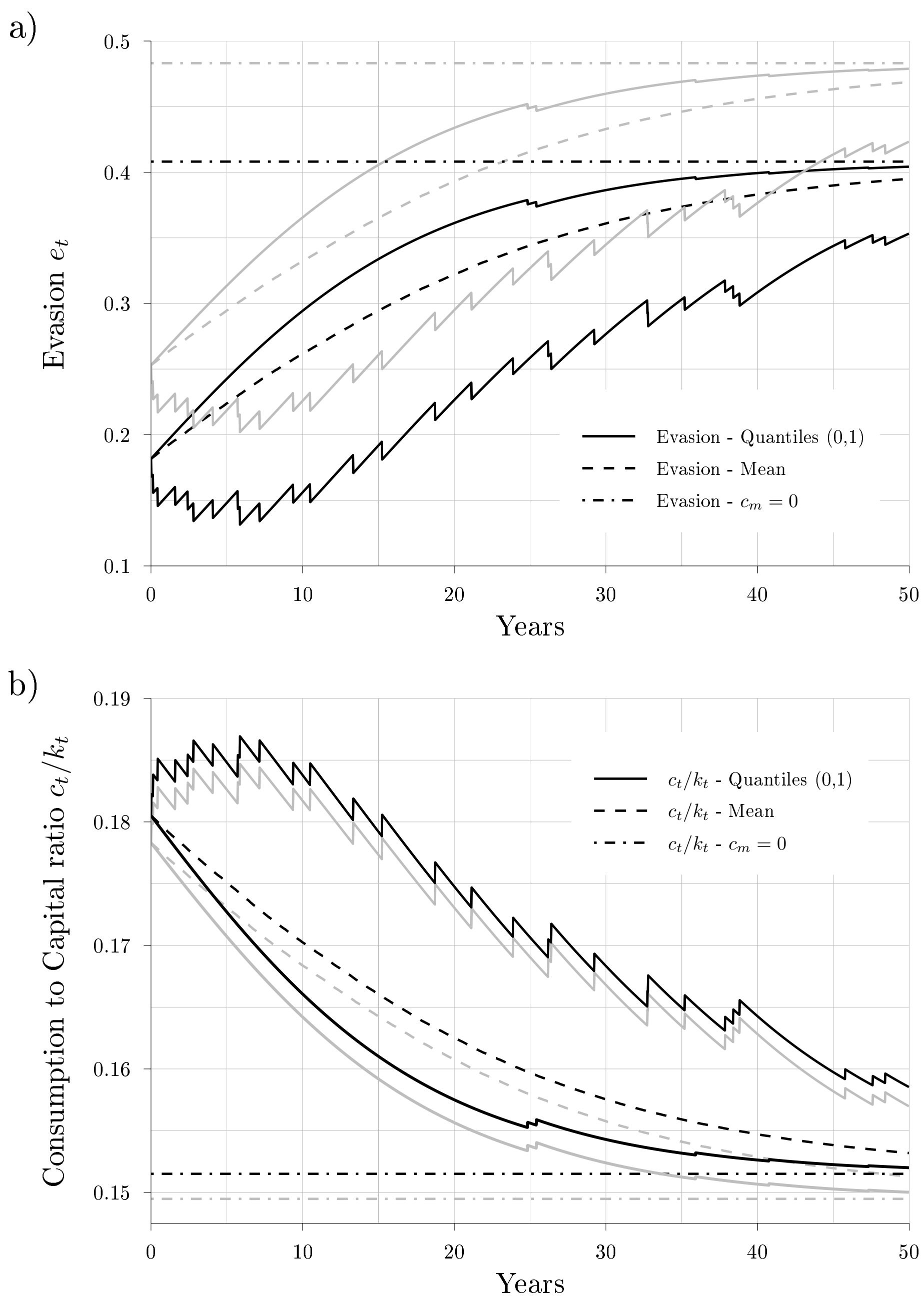

Figure 2: Ratio of expected revenues to capital as a function of the tax rate (τ) for different constants of variation (ω) of the cost function. Parameters are the same of Figure 1 except for $c_m = 0$ and $\gamma = 2.5$.