The Circlette Lattice
Background Independence, Fermion Doubling, and the Topological Structure of the Holographic Vacuum

Sequel to: “The Holographic Circlette” (Part I)

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The lattice does not simulate quantum mechanics. It is quantum mechanics.

Abstract

In the companion paper (Part I) **?**, we showed that the Standard Model fermion spectrum emerges as the set of valid codewords of an 8-bit error-correcting code (the circlette) on a 2D holographic lattice, and derived the Dirac and Schrödinger equations as the continuum limit of a CNOT quantum walk. Part I (v2) further derived the charged lepton mass ratios to 0.007% precision from a single geometric parameter $\delta = 2/9$ - the ratio of a 2-bit topological defect to a 9-qubit plaquette - together with the weak mixing angle $\sin^2 \theta_W = 2/9$ and the flavour mixing angles.

Here we address the foundational question: *what is the lattice?*

We propose that the circlettes are not entities on a lattice but rather *constitute* the lattice itself - a background-independent quantum cellular automaton in which spacetime is an emergent property of informational adjacency. We identify the 4.8.8 truncated square tiling as the natural topological graph, with octagons as self-contained 8-bit registers (matter) and interstitial squares as communication channels (gauge links).

The 9-qubit plaquette - 8 boundary bits on the octagon ring plus 1 bulk parity bit at the centre - provides the geometric origin of the key parameter $\delta = d/N = 2/9$, where $d = 2$ is the support of the ν_R topological defect and $N = 9$ is the plaquette size. This integer ratio determines the charged lepton mass spectrum (via the Koide formula **?**), the weak mixing angle, and the flavour mixing angles. The colour dilution pattern $\delta_u = 2/27$, $\delta_d = 1/9$ for the quark sectors follows from the interaction of the defect with the $N_c = 3$ colour multiplicity.

We investigate whether the 4.8.8 topology resolves the Nielsen-Ninomiya fermion doubling problem **??**. By explicit construction of the tight-binding Dirac operator, we prove that the physical next-nearest-neighbour coupling through interstitial squares generates a term proportional to $\alpha_1\alpha_2 \times \sin k_x \sin k_y$, which vanishes at all

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doubler locations and cannot gap the unphysical species. However, the circlette's CNOT coin operator - the discrete Z_2 chiral update - provides a dynamical resolution: it breaks the continuous $\mathrm{U}(1)_A$ symmetry that the Nielsen-Ninomiya theorem requires, rendering the theorem inapplicable at the axiomatic level.

This establishes a clean separation of concerns: the *hardware* (4.8.8 topology) provides spatial structure, bandwidth matching, gauge plaquettes, and the 9-qubit geometry from which $\delta = 2/9$ emerges; the *software* (CNOT dynamics) provides particle physics, chiral mixing, mass generation, and doubler resolution.

Keywords: background independence, quantum cellular automaton, fermion doubling, Nielsen-Ninomiya theorem, truncated square tiling, lattice gauge theory, CNOT gate, discrete chirality, 9-qubit plaquette

Contents

1 Introduction

In Part I of this series **?**, we established that 45 Standard Model fermion states emerge as the valid codewords of an 8-bit ring code on a holographic lattice, and that the Dirac equation arises as the continuum limit of a CNOT quantum walk on the chirality–isospin bits. Part I (v2) extended the framework to derive, from the single geometric parameter $\delta = 2/9$:

- the charged lepton mass ratios to 0.007% precision,
- the weak mixing angle $\sin^2 \theta_W = 2/9$ (0.5% error),
- the W/Z boson mass ratio $M_W/M_Z = \sqrt{7/9}$ (0.06% error),
- the Cabibbo angle and PMNS neutrino mixing angles (2–5% errors), and
- the colour dilution pattern for quark masses ($\delta_u = 2/27$, $\delta_d = 1/9$).

But a foundational question was deferred: *what is the lattice itself?* In Part I, the lattice was treated as a given substrate - a background graph on which the circlettes reside. This is unsatisfactory for two reasons. First, it introduces an unexplained scaffold; second, it fails to address a standard objection from lattice gauge theory: the Nielsen-Ninomiya fermion doubling problem **??**.

In this paper we address both issues. Section 2 establishes the ontological framework. Section 3 identifies the 4.8.8 tiling and derives the bandwidth-matching principle. Section 4 shows how the 9-qubit plaquette geometry gives rise to $\delta = 2/9$ and the mass spectrum. Section 5 establishes the correspondence with lattice gauge theory. Section 6 contains the central technical result on fermion doubling. Section 7 discusses the hardware/software separation. Section 8 sketches cosmological implications.

2 Ontology: Background Independence and the Vacuum

2.1 Three Pictures of the Lattice

Consider three possible relationships between circlettes and the lattice:

Picture A (Tenant Model): The lattice is a separate graph; circlettes are states at its nodes. This is the working assumption of Part I.

Picture B (Identity Model): The circlettes *are* the nodes. "Space" is the graph of who-connects-to-whom. No separate lattice exists.

Picture C (Woven Model): The circlettes share edges with their neighbours; the lattice is woven from overlapping rings.

We adopt Picture B as the ontological foundation. Picture A requires explaining what the lattice is made of (substrate regress). Picture C introduces shared degrees of freedom that conflict with the self-contained bit ownership of the code. Picture B is background-independent: it defines spatial relationships purely through informational adjacency, with no embedding space required.

2.2 Architecture

The fundamental objects are:

Circlette: An isolated 8-bit register. Fully self-contained; owns all its bits absolutely. Has an internal state that is either a valid codeword (one of 45) or an error state.

Link: A communication channel between two circlettes. Carries a U(1) phase (the gauge field).

Lattice: The graph of circlettes connected by links. Not a separate entity - defined entirely by which circlettes are linked to which.

This is precisely the architecture of Hamiltonian lattice gauge theory in the Kogut-Susskind formulation **?**: matter fields live at sites, gauge fields live on links.

2.3 The Vacuum Is Not Empty

Every node in the lattice is occupied by a circlette in its ground state. "Empty space" is a lattice of circlettes all in the vacuum configuration. Removing the circlettes does not produce empty space - it produces nothing: no distance, no dimension, no geometry. The lattice is load-bearing.

2.4 Particles as Information Gliders

Particle propagation is information flow across a stationary substrate. The circlettes do not move; the pattern moves. At each tick, the CNOT quantum walk transfers the excitation from one circlette to its neighbour through the communication channel, evaluating the U(1) link phase in transit **?**.

3 The 4.8.8 Topology

3.1 The Truncated Square Tiling

We identify the topological graph of the circlette lattice with the 4.8.8 Archimedean tiling (the truncated square tiling), in which regular octagons and squares tile the plane with vertex configuration $(4 \cdot 8^2)$.

3.2 The Bandwidth-Matching Principle

Each octagon in the 4.8.8 tiling has exactly 8 edges - the same as the number of bits in the circlette. These partition into two geometric sectors:

1. **4 orthogonal channels** connecting to nearest-neighbour octagons (N, S, E, W). These carry the Dirac kinetic term.
2. **4 diagonal channels** connecting through interstitial squares to next-nearest-neighbour octagons (NE, NW, SE, SW). These provide the gauge plaquette structure.

The total channel count is:

$$n_{\text{channels}} = 4_{\text{kinematic}} + 4_{\text{gauge}} = 8 = n_{\text{bits}}. \tag{1}$$

Conjecture 1 (Bandwidth Matching). *The 4.8.8 truncated square tiling is the unique Archimedean tiling of the plane for which the polygon coordination number equals the minimum code length required to support the Standard Model fermion spectrum.*

3.3 Orthogonal Gauge Separation

On a standard square lattice, the kinematic hops and gauge plaquettes share the same edges. On the 4.8.8 lattice, they are topologically distinct: fermion hops use octagon-to-octagon edges, while gauge loops traverse the interstitial square boundaries. This provides a natural separation of data from forces.

Conjecture 2 (Single-Tick Consistency). *The 4.8.8 topology is the minimal planar graph that permits both the CNOT kinematic update and the U(1) gauge evaluation within a single computational tick without channel conflict.*

4 The 9-Qubit Plaquette and the Origin of $\delta = 2/9$

This section, new to v2, establishes the connection between the 4.8.8 topology and the quantitative predictions of Part I.

4.1 The Plaquette as Unit Cell

Each octagon of the 4.8.8 tiling is an 8-bit ring: the circlette. But the *unit cell* of the tiling includes one additional degree of freedom: the bulk parity (or syndrome) bit at the centre of the plaquette, shared among the stabiliser constraints. The total unit cell is therefore a **9-qubit plaquette**:

- 8 boundary sites: the ring bits $G_0, G_1, C_0, C_1, \mathrm{LQ}, I_3, \chi, W$,
- 1 centre site: the bulk parity.

The vacuum state (ground state of the stabiliser Hamiltonian) is delocalised across all 9 sites.

4.2 The ν_R Topological Defect

Three pseudocodewords - one per generation - satisfy constraints R1–R3 but violate only R4 (no right-handed neutrino). These ν_R states are localised at the 2 boundary sites where the constraint is violated.

The defect has three key properties:

1. **Localisation:** It is pinned to $d = 2$ sites and cannot spread without additional energy cost.
2. **Three-fold degeneracy:** The Z_3 symmetry of the generation ring admits three ν_R states.
3. **Boundary character:** It lives on the boundary of the plaquette, not in the bulk.

4.3 The Geometric Origin of δ

The vacuum translation amplitude scales with the full plaquette: $T_{\text{vac}} \propto Nt$, where $N = 9$. The defect, pinned to its 2-site support, has $T_{\text{def}} \propto dt$, where $d = 2$. The geometric (Berry) phase is:

$$\delta = \frac{d}{N} = \frac{2}{9}\ \text{rad} \tag{2}$$

This single ratio determines:

- The Koide mass formula phase: $m_n = \mu(1 + \sqrt{2}\cos(\delta + 2\pi n/3))^2$??.
- The weak mixing angle: $\sin^2\theta_W = d/N = 2/9$.
- The W/Z boson mass ratio: $M_W/M_Z = \sqrt{(N-d)/N} = \sqrt{7/9}$.
- The Cabibbo angle: $\theta_C \approx \delta$.

The 4.8.8 topology is no longer merely a convenient graph. It is the *source* of the integer ratio 2/9 that parameterises the Standard Model.

4.4 Colour Dilution and the Quark Sector

Part I (v2) showed that the quark mass sectors follow the same Koide functional form but with modified geometric parameters. Fitting to experimental quark masses ? reveals:

Sector	δ	R	Source	Status
Leptons	2/9	$\sqrt{2}$	Base geometry	Rigorous
Up quarks	2/27	$\approx\sqrt{3}$	Twist/N_c	Structural
Down quarks	1/9	$\approx$1.55	Twist/2	Phenomenological

Table 1: Geometric parameters for each charge sector. Colour introduces a dilution factor in the twist: the 2-bit defect is shared across $N_c = 3$ colour sheets for up quarks, and across 2 isospin channels for down quarks.

The colour dilution hypothesis $\delta_u = \delta_\ell/N_c = 2/27$ is accurate to 0.6% against the fitted value. The down quark twist $\delta_d = \delta_\ell/2 = 1/9$ is accurate to 1.1%. The down sector reproduces m_d and m_s within experimental uncertainties (3.6% and 1.0% respectively). The up sector correctly predicts the heavy quark hierarchy (m_c/m_t) but the lightest mass m_u sits near a node of the cosine function and requires sub-leading corrections.

In the lattice picture, the physical mechanism is clear: the defect's Berry phase is diluted because the colour degree of freedom provides N_c parallel sheets through which the defect can propagate, each carrying a fraction $1/N_c$ of the total phase.

5 Correspondence with Lattice Gauge Theory

5.1 Kogut-Susskind Architecture

The circlette lattice maps directly onto the Kogut-Susskind formulation of Hamiltonian lattice gauge theory ?:

LGT concept	Circlette realisation	4.8.8 element
Site (matter field)	8-bit register, 45-state code	Octagon
Link (gauge field)	U(1) phase $U(x,y) = e^{ieA\Delta x}$	Oct–oct edge
Plaquette (field strength)	Wilson loop $\prod_\square U$	Interstitial square
Hopping term	CNOT quantum walk	NN channel
Wilson mass	CNOT coin operator	Built into dynamics

Table 2: Mapping between lattice gauge theory and the circlette lattice.

5.2 Wilson Loops and Plaquettes

The interstitial squares of the 4.8.8 tiling are 4-cycles. In lattice gauge theory, the phase accumulated around the shortest closed loop is the plaquette variable **?**:

$$U_\square = U_{12}\, U_{23}\, U_{34}\, U_{41}, \tag{3}$$

which gives the discretised field strength tensor $F_{\mu\nu}$ in the continuum limit.

5.3 Anomaly Structure from the Code

As shown in Part I, the 45-state code automatically satisfies:

$$\sum_f Q_f = 0 \quad \text{(gravitational anomaly cancellation)}, \tag{4}$$

$$\sum_f Q_f^2 = 16 \quad \text{(exact Standard Model } \beta\text{-function coefficient)}. \tag{5}$$

6 The Fermion Doubling Problem

6.1 Background: the Nielsen-Ninomiya Theorem

The Nielsen-Ninomiya no-go theorem **??** states that on any regular lattice, a Dirac operator satisfying locality, translational invariance, Hermiticity, and exact continuous $U(1)_A$ chiral symmetry necessarily has an equal number of left-handed and right-handed fermion species. In 2D, this produces 4 Dirac points (1 physical, 3 doublers).

The standard remedy is the Wilson mass **?**: a momentum-dependent term $M(k) = r(2 - \cos k_x - \cos k_y)$ proportional to β that gaps the doublers while leaving the physical fermion massless. This is added by hand and explicitly breaks chiral symmetry.

6.2 The Dirac Operator on the 4.8.8 Graph

In the circlette framework, the coin space is spanned by the chirality bit χ and isospin bit I_3, giving 4-component spinors. The Dirac matrices are:

$$\alpha_1 = \sigma_x^\chi \otimes \sigma_x^{I_3}, \quad \alpha_2 = \sigma_x^\chi \otimes \sigma_y^{I_3}, \quad \beta = \sigma_z^\chi \otimes I^{I_3}. \tag{6}$$

On the square lattice with nearest-neighbour (NN) hopping only, the naive Bloch Hamiltonian is:

$$H_{\text{naive}}(k) = \alpha_1 \sin k_x + \alpha_2 \sin k_y, \tag{7}$$

which has zeros at $(0,0)$, $(\pi,0)$, $(0,\pi)$, and (π,π) - the standard doubling problem.

6.3 The Geometric Temptation: NNN Hopping

The 4.8.8 topology provides next-nearest-neighbour (NNN) connections through the interstitial squares. A natural hypothesis is that these diagonal channels act as a native Wilson mass.

The NNN coupling matrix from the two-hop process gives:

$$H_{\text{NNN}}(k) = 4t_2 \sin k_x \sin k_y \cdot \alpha_1\alpha_2, \tag{8}$$

where:

$$\alpha_1\alpha_2 = (\sigma_x \otimes \sigma_x)(\sigma_x \otimes \sigma_y) = I \otimes (i\sigma_z) = i(I^\chi \otimes \sigma_z^{I_3}). \tag{9}$$

This is a rotation in the isospin sector, *not* the mass matrix β which acts on chirality.

6.4 The Rigorous Disproof

The full 4.8.8 Bloch Hamiltonian is:

$$H_{4.8.8}(k) = \underbrace{\alpha_1 \sin k_x + \alpha_2 \sin k_y}_{\text{NN (Dirac)}} + \underbrace{4t_2 \sin k_x \sin k_y \, \alpha_1\alpha_2}_{\text{NNN (diagonal)}}. \tag{10}$$

At all three doubler locations:

$$H_{4.8.8}(\pi, 0) : \sin\pi \cdot \sin 0 = 0 \Rightarrow \text{NNN term vanishes}, \tag{11}$$
$$H_{4.8.8}(0, \pi) : \sin 0 \cdot \sin\pi = 0 \Rightarrow \text{NNN term vanishes}, \tag{12}$$
$$H_{4.8.8}(\pi, \pi) : \sin\pi \cdot \sin\pi = 0 \Rightarrow \text{NNN term vanishes}. \tag{13}$$

Result 1. *The physical NNN coupling generated by the 4.8.8 topology vanishes at all three doubler locations. The 4.8.8 geometry alone cannot resolve the fermion doubling problem, regardless of the NNN coupling strength t_2.*

This was confirmed by numerical diagonalisation across the full Brillouin zone **?**. Furthermore, exhaustive testing of all possible NNN coupling matrices with a Wilson-like $\cos k_x \cos k_y$ dependence showed that the (π, π) doubler cannot be distinguished from the physical fermion at $(0, 0)$ because $\cos\pi\cos\pi = \cos 0 \cos 0 = 1$.

6.5 The Algorithmic Resolution: the CNOT Coin

The resolution comes not from the spatial topology but from the dynamics.

In the theory of discrete-time quantum walks **???**, the coin operator is the sole generator of the effective mass in the continuum limit. In the circlette framework, the coin operator is the CNOT gate:

$$\text{CNOT} : \quad \chi \to \chi, \quad W \to W \oplus \chi. \tag{14}$$

This copies the chirality bit into the weak bit, enforcing constraint R2 at each tick - an explicit, dynamical mixing of left- and right-handed chiral sectors at every computational step.

The Nielsen-Ninomiya theorem is a topological winding theorem: it requires that the continuous phase $\mathrm{U}(1)_A$ accumulated around the Brillouin zone torus sums to zero. But in the circlette framework:

1. Chirality is a discrete Boolean variable $\chi \in \{0, 1\}$, not a continuous phase.

2. The "chiral symmetry" is Z_2 (bit flip), not $\mathrm{U}(1)_A$.

3. A discrete Z_2 space does not support topological winding. Formally, $\pi_1(\mathrm{U}(1)) = \mathbb{Z}$ permits integer winding numbers; a discrete Z_2 space has no continuous manifold, and the homotopy prerequisites are absent.

Result 2. *The Nielsen-Ninomiya theorem is inapplicable to the circlette lattice because its foundational premise - exact continuous $U(1)_A$ chiral symmetry - is violated by the discrete Z_2 structure of the chirality bit and its CNOT update rule.*

The doublers are not gapped by a penalty term; they never arise because the symmetry that would protect them does not exist in the discrete theory.

	Wilson fermions	Circlette (CNOT coin)
Chiral symmetry	$\mathrm{U}(1)_A$ broken explicitly	Z_2 (never continuous)
Mechanism	Momentum-dependent mass	Discrete chiral mixing
Origin	Added by hand	Fundamental dynamics
Level	Hamiltonian (spatial)	Coin operator (temporal)
Nielsen-Ninomiya	Evaded (symmetry broken)	Inapplicable (wrong axiom)
Physical mass	From Wilson parameter r	From CNOT frequency

Table 3: Comparison of doubler resolution mechanisms.

7 Hardware and Software: A Separation of Concerns

7.1 The Architecture

The results of the preceding sections establish a clean decoupling:

Hardware (the 4.8.8 topology):

- Establishes the $n = 8$ bandwidth matching (code length = coordination capacity).
- Separates kinematic channels from gauge channels.
- Provides native Wilson-loop plaquettes as interstitial squares.
- Defines the 9-qubit plaquette geometry from which $\delta = 2/9$ emerges.
- Sets the spatial adjacency structure from which distances emerge.

Software (the CNOT dynamics):

- Generates the Dirac equation as the continuum limit of the quantum walk.
- Provides dynamical chiral mixing, resolving fermion doubling.
- Produces rest mass as the CNOT execution frequency.

- Enforces the R1–R4 constraints that define the particle spectrum.
- Generates the mass hierarchy through Feshbach resonance via the ν_R defect.

The hardware is the routing network; the software is the physics. The spatial topology determines who can talk to whom; the update rule determines what they say.

7.2 The Complete Causal Chain

From the 4.8.8 topology to the Standard Model:

4.8.8 tiling	$\to$	$n_{\text{bits}} = 8$ (bandwidth matching)
8-bit ring + R1–R4	$\to$	45 fermion states
Octagon + centre	$\to$	9-qubit plaquette
ν_R defect on 2 sites	$\to$	$\delta = d/N = 2/9$
Z_3 circulant + quadrature	$\to$	$m_n = \mu(1 + \sqrt{2}\cos(\delta + 2\pi n/3))^2$
Bulk/boundary partition	$\to$	$\sin^2\theta_W = 2/9$, $M_W/M_Z = \sqrt{7/9}$
Colour sheets	$\to$	$\delta_u = 2/27$, $\delta_d = 1/9$
CNOT coin	$\to$	Dirac equation, mass, doubler resolution

Every quantitative prediction of the framework traces back to the topological properties of the 4.8.8 graph.

7.3 Distance and Geometry

In the background-independent picture, distance between two circlettes is the number of hops (graph distance). The link variables $U(x, y)$ modulate this: regions with large phase fluctuations have shorter effective distances. The metric tensor emerges from the Fisher information geometry of the link variables. Curved spacetime is a region where the link variables vary systematically.

8 Cosmological Implications

8.1 The Past Hypothesis and the Origin of Time

Time is emergent from the sequential execution of the CNOT update rule. There is no external clock; the tick $t \to t + 1$ *is* time. Asking "what happened before the lattice?" is a category error **?**.

The Big Bang is a symmetry-breaking phase transition of a pre-geometric lattice. The initial state is maximally symmetric (every node in the same ground state), giving Shannon entropy $S = 0$: the universe began at minimum entropy not because it was empty, but because it was perfectly uniform **?**.

8.2 Holographic Expansion

The holographic principle **??** and the Bekenstein-Hawking bound **?** require $S_{\max} \propto A/4\ell_P^2$. A fixed graph would cap total entropy, violating the Second Law. This necessitates a growing lattice: cosmic expansion corresponds to the continuous addition of new nodes to the 4.8.8 boundary graph.

The vacuum occupation fraction $\Phi = 45/256$ becomes the energy density per node. As the graph grows, the total vacuum energy increases proportionally with the node count (hence with the volume), naturally producing the constant energy density ρ_Λ that drives accelerated expansion.

8.3 Algorithmic Baryogenesis

The CNOT gate is an inherently asymmetric, directed operation: the chirality bit χ controls the update of the weak bit W, but W does not alter χ. This control-target hierarchy is not invariant under the combined operation of CP. Whether this structural non-commutativity, propagated through the Z_3 generation sector, natively generates the CKM complex phase remains an open question.

9 Open Questions

9.1 Is the 4.8.8 Tiling Unique?

The bandwidth-matching argument is currently heuristic. A proof would require showing that the 4.8.8 tiling is the unique Archimedean tiling with coordination number 8 and bipartite gauge structure. This is a well-posed mathematical question: there are only 11 Archimedean tilings.

9.2 Can the CNOT Resolution Be Made Rigorous?

A rigorous proof would require explicit construction of the DTQW propagator with the CNOT coin, computation of the effective continuum Lagrangian, and demonstration that no doubler poles appear in the fermion propagator **???**.

9.3 The Brillouin Zone Folding Question

The bipartite structure of the 4.8.8 tiling doubles the real-space unit cell and halves the Brillouin zone. Whether the resulting folding absorbs the final doubler into internal spinor degrees of freedom (as in Kogut-Susskind staggered fermions **?**) remains an open question.

9.4 The Down Quark Twist Factor

The colour dilution pattern gives $\delta_u = \delta_\ell/N_c$ for up quarks (0.6% from the fit) and $\delta_d = \delta_\ell/2$ for down quarks (1.1% from the fit). The factor of 2 for the down sector may relate to hypercharge or the isospin-doublet structure. A first-principles derivation from the (C_0, C_1) colour bits is needed.

9.5 The Down Quark Structure Factor

The down quark structure factor $R_d \approx 1.546$ does not correspond to a simple integer root ($R_d^2 \approx 2.39$). The closest candidate is $\sqrt{12/5}$ (0.2% error), but the geometric interpretation remains unclear.

10 Conclusions

We have established the ontological and topological foundations of the circlette lattice:

1. **Background independence:** The circlettes *are* the lattice. Spacetime is an emergent property of informational adjacency.

2. **The 4.8.8 topology:** Provides the natural graph with $n_{\text{bits}} = n_{\text{channels}} = 8$ (bandwidth matching), separated kinematic and gauge channels, and native Wilson-loop plaquettes.

3. **The 9-qubit plaquette:** The unit cell of the 4.8.8 tiling (8 boundary + 1 centre) gives $\delta = d/N = 2/9$ - the single integer ratio that parameterises the charged lepton mass spectrum, the weak mixing angle, and the flavour mixing angles.

4. **Colour dilution:** The defect Berry phase is divided by the colour multiplicity for quarks: $\delta_u = 2/27$ (twist/N_c), $\delta_d = 1/9$ (twist/2).

5. **Fermion doubling:** The NNN coupling from the 4.8.8 topology vanishes at all doubler locations (rigorously verified). The CNOT coin operator's Z_2 chiral mixing makes the Nielsen-Ninomiya theorem inapplicable.

6. **Hardware/software separation:** The topology provides spatial structure and the geometric ratio 2/9; the dynamics provides particle physics and doubler resolution.

The deepest result of Parts I and II combined is that every quantitative prediction of the framework - the mass ratios, the mixing angles, the gauge couplings - traces back through the CNOT dynamics to the integer topology of the 4.8.8 graph. The lattice does not simulate quantum mechanics. It is quantum mechanics.

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