

ABSTRACT

Title of dissertation: Measuring topology of BECs
in a synthetic dimensions lattice

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Measuring topology of BECs in a synthetic dimensions lattice

by

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Chapter 1: Introduction

1.1 Bose-Einstein condensation

1.1.1 Phase transition of a non-interacting Bose gas

Bose gases are characterized by the Bose-Einstein distribution giving the number of atoms $n(E_j)$ occupying each energy eigenstate E_j as

$$n(E_j) = \frac{1}{e^{(E_j - \mu)/k_B T} - 1}, \quad (1.1)$$

where k_B is the Boltzmann constant, T is the temperature in Kelvin, μ is the chemical potential. Assuming the total atom number N is fixed, the chemical potential $\mu(T, N)$ ensures that the total occupation of all $\sum_j n(E_j) = N$.

The Bose distribution leads to Bose-Einstein condensation, the collapse of a macroscopic fraction of the atoms into the ground state. This comes as a direct consequence of the Bose distribution's characteristic -1 in the denominator. Consider the occupation number $n(E_j)$ —it must remain positive, as a negative occupation number is unphysical. That means that the quantity $e^{(E_j - \mu)/k_B T}$ must remain greater than 1, or $(E_j - \mu)/k_B T < 0$ for all E_j . Therefore, $\mu \leq E_0$, where E_0 is the ground state energy.

Then, for a given temperature T , there is a maximum occupation number for each excited state given by $n(E_j) = \frac{1}{e^{E_j/k_B T} - 1}$. The only energy state whose occupation number is unbounded is the ground state, as $n(E_0)$ tends toward infinity as μ

tends towards 0. Therefore, as the temperature decreases, the maximum occupation of each excited state decreases until they can no longer support all of the atoms. The remaining atoms then have no choice but to collapse into the lowest energy level and Bose condense.

We will show this quantitatively for the case of a 3-D harmonically trapped gas of atoms, relevant to the experiments described in this thesis. It is convenient to define the fugacity $\zeta = e^{\mu/k_B T}$, and re-write the Bose-Einstein distribution as eigenstate E_j as

$$n(E_j) = \frac{\zeta}{e^{E_j/k_B T} - \zeta}. \quad (1.2)$$

The harmonic oscillator potential can be written as

$$V(r) = \frac{1}{2}m(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2), \quad (1.3)$$

where ω_x , ω_y and ω_z are the angular trapping frequencies along \mathbf{e}_x , \mathbf{e}_y , and \mathbf{e}_z . The eigenenergies with this potential are

$$E(j_x, j_y, j_z) = \left(\frac{1}{2} + j_x\right)\hbar\omega_x + \left(\frac{1}{2} + j_y\right)\hbar\omega_y + \left(\frac{1}{2} + j_z\right)\hbar\omega_z. \quad (1.4)$$

In order to find μ , we must find $\sum_{j_x, j_y, j_z} n(E(j_x, j_y, j_z))$ and set it equal to N . This task is greatly simplified by going to the continuum limit and finding the density of states. To do this, we neglect the zero-point energy (setting $E_0 = 0$, the effects of the zero-point energy are discussed in [?] section 2.5) and assume there is on average one state per volume element $\hbar^3\omega_x\omega_y\omega_z$. Then, the total number of states with energy less than or equal to some value ϵ is given by the volume of a prism made between points $(x, y, z) = (0, 0, 0)$, $(\epsilon, 0, 0)$, $(0, \epsilon, 0)$ and $(0, 0, \epsilon)$ in units of the volume element:

$$G(\epsilon) = \frac{\epsilon^3}{6\hbar^3\omega_x\omega_y\omega_z}. \quad (1.5)$$

The density of states is given by

$$g(\epsilon) = \frac{d}{d\epsilon}G(\epsilon) = \frac{\epsilon^2}{3\hbar^3\omega_x\omega_y\omega_z}. \quad (1.6)$$

Note that the occupation of the ground state is not included in this continuum picture. We can therefore use it only to calculate the total number of atoms in all of the excited states:

$$N_{\text{ex}} = \int_0^\infty d\epsilon g(\epsilon)n(\epsilon) = \int_0^\infty d\epsilon \frac{\epsilon^2}{3\hbar\omega_x\omega_y\omega_z} \frac{\zeta}{e^{\epsilon/k_B T} - \zeta} = \frac{(k_B T)^3}{\hbar^3\omega_x\omega_y\omega_z} \text{Li}_3(\zeta), \quad (1.7)$$

where $\text{Li}_3(\zeta)$ is the polylogarithm function¹. We define the mean trapping frequency $\bar{\omega} = (\omega_x\omega_y\omega_z)^{1/3}$ and the harmonic oscillator energy as $\hbar\bar{\omega}$, with the thermal energy in harmonic oscillator units $\tau = k_B T/\hbar\bar{\omega}$, giving

$$N_{\text{ex}} = \tau^3 \text{Li}_3(\zeta). \quad (1.8)$$

Finding the occupation number of the ground state from the Bose-Einstein distribution

$$N_0 = \frac{\zeta}{1 - \zeta}, \quad (1.9)$$

we can then find the chemical potential, or equivalently the fugacity ζ , to satisfy

$$N = N_0 + N_{\text{ex}}. \quad (1.10)$$

This is a transcendental equation that can only be solved numerically. We present an example of the solution in Figure ??.

Hard limit on excited state population due to $\mu \geq E_0$, which is because occupation $n(E_0) \geq 0$.

¹This calculation was done with Wolfram Alpha, not Russian algebra skills

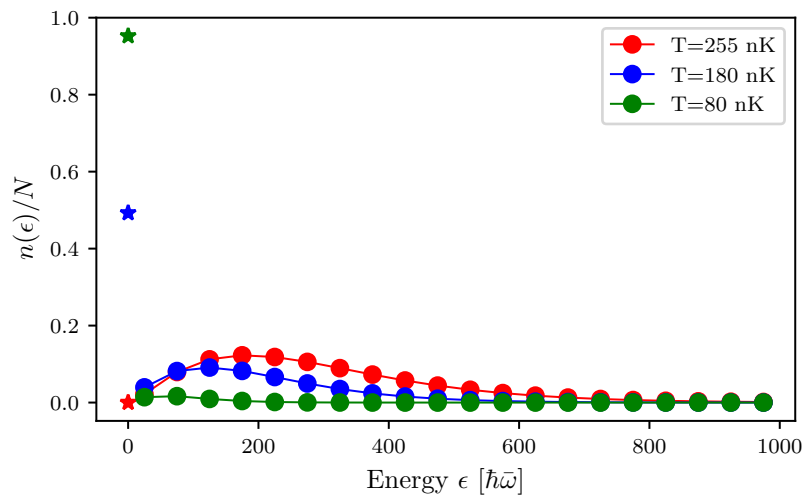


Figure 1: Occupation of energy states of a 3-D harmonic oscillator. Here $\omega_x = \omega_y = \omega_z$

Density of states - $\rho(\epsilon)$ excited state population

Critical temperature

Plot: occupation fraction as a function of energy for fixed N at different T?

Phase coherence

1.1.2 Interacting Bose gas

Add in interactions: full Hamiltonian

Mean-field approximation

Thomas-Fermi radii

Position space - pictures from wave lock?

Momentum space

Pictures of BEC condensing (steal from talk?)

1.2 Degenerate Fermi Gas

1.3 RbK apparatus

Bird's eye view picture

Brief description of BEC making procedure

Differences from Lauren's thesis:

Describe Hsin-I's new imaging path Describe extra lens for beam shaping the dipole trap

Brief description of DFG making procedure (of old)

Differences from Lauren's thesis: Describe 2D MOT'

Bibliography

- [1] Daniel Adam Steck. Rubidium 87 d line data. Available online, <http://steck.us/alkalidata>, April 2018. revision 2.1.5.
- [2] K. Jiménez-García, L. J. LeBlanc, R. A. Williams, M. C. Beeler, A. R. Perry, and I. B. Spielman. Peierls substitution in an engineered lattice potential. *Phys. Rev. Lett.*, 108:225303, May 2012.
- [3] H.J. Metcalf and P. van der Straten. *Laser Cooling and Trapping*. Graduate Texts in Contemporary Physics. Springer New York, 1999.
- [4] N.W. Ashcroft and N.D. Mermin. *Solid State Physics*. Saunders College, Philadelphia, 1976.
- [5] Nicola Marzari, Arash A. Mostofi, Jonathan R. Yates, Ivo Souza, and David Vanderbilt. Maximally localized wannier functions: Theory and applications. *Rev. Mod. Phys.*, 84:1419–1475, Oct 2012.
- [6] Karina Jimenez-Garcia. *Artificial Gauge Fields for Ultracold Neutral Atoms*. PhD thesis, Joint Quantum Institute, National Institute of Standards and Technology, and the University of Maryland, 2012.

- [7] Daniel Adam Steck. Quantum and atom optics. Available online at <http://steck.us/teaching>, January 2015. revision 0.12.2.
- [8] K. v. Klitzing, G. Dorda, and M. Pepper. New method for high-accuracy determination of the fine-structure constant based on quantized hall resistance. *Phys. Rev. Lett.*, 45:494–497, Aug 1980.
- [9] D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs. Quantized hall conductance in a two-dimensional periodic potential. *Phys. Rev. Lett.*, 49:405–408, Aug 1982.
- [10] Douglas R. Hofstadter. Energy levels and wave functions of bloch electrons in rational and irrational magnetic fields. *Phys. Rev. B*, 14:2239–2249, Sep 1976.
- [11] M. C. Geisler, J. H. Smet, V. Umansky, K. von Klitzing, B. Naundorf, R. Ketzmerick, and H. Schweizer. Detection of a landau band-coupling-induced rearrangement of the hofstadter butterfly. *Phys. Rev. Lett.*, 92:256801, Jun 2004.
- [12] B. Hunt, J. D. Sanchez-Yamagishi, A. F. Young, M. Yankowitz, B. J. LeRoy, K. Watanabe, T. Taniguchi, P. Moon, M. Koshino, P. Jarillo-Herrero, and R. C. Ashoori. Massive Dirac Fermions and Hofstadter Butterfly in a van der Waals Heterostructure. *Science*, 340:1427, 2013.
- [13] P. Zoller D. Jaksch. Creation of effective magnetic fields in optical lattices: the hofstadter butterfly for cold neutral atoms. *New Journal of Physics*, 5(1):56, 2003.
- [14] M. Aidelsburger, M. Atala, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch. Realization of the hofstadter hamiltonian with ultracold atoms in optical lattices. *Phys. Rev. Lett.*, 111(18):185301–, October 2013.

- [15] Hirokazu Miyake, Georgios A. Siviloglou, Colin J. Kennedy, William Cody Burton, and Wolfgang Ketterle. Realizing the harper hamiltonian with laser-assisted tunneling in optical lattices. *Phys. Rev. Lett.*, 111:185302, Oct 2013.
- [16] Gregor Jotzu, Michael Messer, Remi Desbuquois, Martin Lebrat, Thomas Uehlinger, Daniel Greif, and Tilman Esslinger. Experimental realization of the topological haldane model with ultracold fermions. *Nature*, 515(7526):237–240, Nov 2014.
- [17] M Aidelsburger, M Lohse, C Schweizer, M Atala, J T Barreiro, S Nascimbène, N. R. Cooper, I. Bloch, and N. Goldman. Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms. *Nature Physics*, 11(2):162–166, December 2014.
- [18] M. Mancini, G. Pagano, G. Cappellini, L. Livi, M. Rider, J. Catani, C. Sias, P. Zoller, M. Inguscio, M. Dalmonte, and L. Fallani. Observation of chiral edge states with neutral fermions in synthetic hall ribbons. *Science*, 349(6255):1510–, Sep 2015.
- [19] M Hafezi, S Mittal, J Fan, A Migdall, and J M Taylor. Imaging topological edge states in silicon photonics. *Nat. Photon.*, 7(12):1001–1005, October 2013.
- [20] A. Celi, P. Massignan, J. Ruseckas, N. Goldman, I.B. Spielman, G. Juzeliunas, and M. Lewenstein. Synthetic gauge fields in synthetic dimensions. *Phys. Rev. Lett.*, 112(4):043001–, Jan 2014.
- [21] B. K. Stuhl, H.-I. Lu, L. M. Ayccock, D. Genkina, and I. B. Spielman. Visualizing edge states with an atomic bose gas in the quantum hall regime. *Science*, 349(6255):1514–, Sep 2015.
- [22] Y. Aharonov and D. Bohm. Significance of electromagnetic potentials in quantum theory. *Phys. Rev.*, 115:485, 1959.

- [23] Yakir Aharonov and Ady Stern. Origin of the geometric forces accompanying berry's geometric potentials. *Phys. Rev. Lett.*, 69(25):3593–3597, 1992.
- [24] Dario Hgel and Beln Paredes. Chiral ladders and the edges of quantum Hall insulators. *Phys. Rev. A*, 89(2):023619, 2014.