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CS360 HW 3

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Problem 1. Formulate expressions of predicate logic, making use of basic integer arithmetic operation and relations, capturing the following meaning.
(i) Predicates: p(X), X is a prime number, t(X,Y), X,Y are a pair of twin primes.
(ii) Statements: there are infinitely many prime numbers, there are infinitely many pairs of twin primes.

Provide justifications. Hint: Express infinitely many as arbitrary large.

Assume all variables for the expressions below are integers.
(i). $p(X) \equiv (X > 1) \wedge (\forall Y)((\forall Z)(((Y > 1) \wedge (Z > 1)) \rightarrow ((Y * Z) \neq X)))$
X must be greater than 1 since the first prime number is technically 2. If X is prime, then the only factors of X are 1 and X. There are no other factors which, when multiplied together, produce X. Therefore, for all pairs of possible integers Y and Z, if both integers are greater than 1, then their product cannot produce X. If this holds for all pairs, then X is prime. If this fails for any single pair, then X can be factored, so X is not prime.

$t(X, Y) \equiv p(X) \wedge p(Y) \wedge (Y - X = 2)$
Given two numbers, if both are prime and have a relative difference of 2, then the two numbers are twin primes. This definition assumes that Y is the larger prime of the pair.

(ii).
For infinitely prime numbers:
$(\forall X)(p(X) \rightarrow (\exists Y)((Y > X) \wedge p(Y)))$
Given any prime number, there exists another number which is greater than the prime number and is also prime. This property holds for all numbers that are prime. Hence for every prime, there is always a greater prime, so therefore there are infinitely many prime numbers.

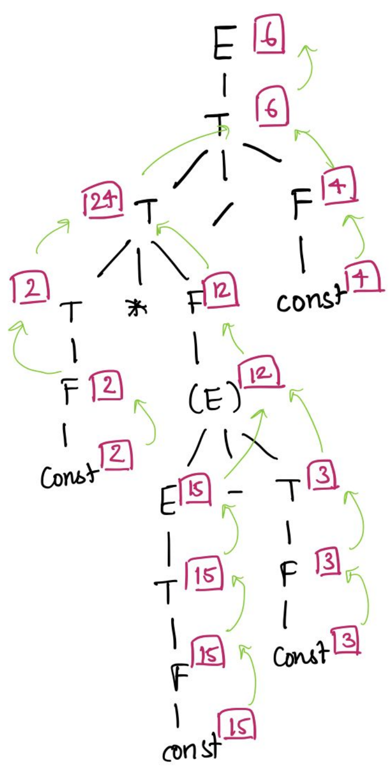
For infinitely many pairs of twin primes:
$(\forall X)((\forall Y)(t(X, Y) \rightarrow (\exists A)((\exists B)((A > X) \wedge (B > Y) \wedge t(A, B))))$
Given any pair of twin primes, there exists two numbers A and B such that A is greater than X and B is greater than Y (meaning the pair A, B is greater) and A and B form a new twin prime. This property holds for all twin primes. Hence, for every twin prime pair, there is always a greater twin prime pair, so therefore there are infinitely many pairs of twin primes.

Problem 2. Illustrate, with representative examples of decorated parse trees, solutions of PLP Exercises 4.7, 4.8 (Quiz 5 practice problems) explaining the attribute flow in the style of PLP Figures 4.2, 4.4.

4.7:
CFG:

$E_1 \longrightarrow E_2 \texttt{ + } T$	▷ E_1.seq := cat(E_2.seq, T.seq, "+")
$E_1 \longrightarrow E_2 \texttt{ - } T$	▷ E_1.seq := cat(E_2.seq, T.seq, "−")
$E \longrightarrow T$	▷ E.seq := T.seq
$T_1 \longrightarrow T_2 \texttt{ * } F$	▷ T_1.seq := cat(T_2.seq, F.seq, "×")
$T_1 \longrightarrow T_2 \texttt{ / } F$	▷ T_1.seq := cat(T_2.seq, F.seq, "÷")
$T \longrightarrow F$	▷ T.seq := F.seq
$F_1 \longrightarrow \texttt{ - } F_2$	▷ F_1.seq := cat(F_2.seq, "+/_")
$F \longrightarrow \texttt{ (} E \texttt{) }$	▷ F.seq := E.seq
$F \longrightarrow \textbf{const}$	▷ F.seq := buttons(const.val)

Parse tree for 2 x (15 - 3) / 4



4.8
CFG:

Answer:

$E \longrightarrow T\ TT$

▷ TT.st := T.seq ▷ E.seq := TT.seq

$TT_1 \longrightarrow +\ T\ TT_2$

▷ TT_2.st := cat(TT_1.st, T.seq, "+") ▷ TT_1.seq := TT_2.seq

$TT_1 \longrightarrow -\ T\ TT_2$

▷ TT_2.st := cat(TT_1.st, T.seq, "−") ▷ TT_1.seq := TT_2.seq

$TT \longrightarrow \epsilon$

▷ TT.seq := TT.st

$T \longrightarrow F\ FT$

▷ FT.st := F.seq ▷ T.seq := FT.seq

$FT_1 \longrightarrow *\ F\ FT_2$

▷ FT_2.st := cat(FT_1.st, F.seq, "×") ▷ FT_1.seq := FT_2.seq

$FT_1 \longrightarrow /\ F\ FT_2$

▷ FT_2.st := cat(FT_1.st, F.seq, "÷") ▷ FT_1.seq := FT_2.seq

$FT \longrightarrow \epsilon$

▷ FT.seq := FT.st

$F_1 \longrightarrow -\ F_2$

▷ F_1.seq := cat(F_2.seq, "+/_")

$F \longrightarrow (\ E\ )$

▷ F.seq := E.seq

$F \longrightarrow$ **const**

▷ F.seq := const.val

Parse tree for 2 x (15 - 3) / 4: