CS 481 - Dice Wars, Poker, and PTSD
Homework 1

Dennis George

4/9/2021

1 Russell & Norvig Problem 22

Suppose you are given a bag containing n unbiased coins. You are told that n - 1 of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

1. Suppose you reach into the bag, pick out a coin at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin?
   $P(fake \mid heads) = \frac{P(heads|fake)P(fake)}{P(heads)}$
   $= \frac{P(heads|fake)P(fake)}{P(heads|fake)P(fake)+P(heads|!fake)P(!fake)} = \frac{1(\frac{1}{n})}{1(\frac{1}{n})+0.5\frac{n-1}{n}} = \frac{2}{n+1}$

2. Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin?
   $P(fake \mid kheads) = \frac{P(kheads|fake)P(fake)}{P(kheads)}$
   $= \frac{P(kheads|fake)P(fake)}{P(kheads|fake)P(fake)+P(kheads|!fake)P(!fake)} = \frac{1(\frac{1}{n})}{1(\frac{1}{n})+2^{-k}\frac{n-1}{n}} = \frac{2^k}{2^k+n-1}$

3. Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns fake if all k flips come up heads; otherwise it returns normal. What is the (unconditional) probability that this procedure makes an error?
   $P(kheads, !fake) = P(kheads\ |!fake)P(!fake) = \frac{n-1}{n2^k}$

2 Right Handed Accuracy

A doctor says that an infant who turns the head to the right while lying on the back will be right-handed, and one who turns to the left will be left-handed. Isabella predominantly turned her head to the left. Given that 90% of the population is right-handed, what is Isabella's probability of being right-handed if the test is 90% accurate? If it is 80% accurate?

1. 90% accurate
   $P(!a \mid rh) = 0.1, P(!a\ |!rh) = 0.1, P(rh) = 0.9$
   $P(rh\ |!a) = \frac{P(!a|rh)P(rh)}{P(!a|rh)P(rh)+P(!a|!rh)P(!rh)} = \frac{0.1 \cdot 0.9}{0.1 \cdot 0.9+0.1 \cdot 0.9} = 0.5 = 50\%$

2. 80% accurate
   $P(!a \mid rh) = 0.2, P(!a\ |!rh) = 0.8, P(rh) = 0.9$
   $P(rh\ |!a) = \frac{P(!a|rh)P(rh)}{P(!a|rh)P(rh)+P(!a|!rh)P(!rh)} = \frac{0.2 \cdot 0.9}{0.2 \cdot 0.9+0.8 \cdot 0.1} = 0.69 = 69\%$

3 Poker Face

Poker Face. You are dealt 5 cards from a standard 52-card bridge deck. Determine the probabilities of the following events. Explain your reasoning and calculations!
Total number of distinct poker hands = $\binom{52}{5} = 2,598,960$

1. One pair
   $\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}^3 = 13 \cdot 6 \cdot 220 \cdot 64 = 1,098,240$
   $1,098,240/2,598,960 = 0.423 = 42.3\%$
   The $\binom{13}{1}$ comes from choosing the matching rank. There are 13 total options.
   The $\binom{4}{2}$ comes from picking the suits for the matching rank
   The $\binom{12}{3}$ comes from picking the remaining 3 non-matching ranks from the available options to complete the hand.
   The $\binom{4}{1}^3$ comes from picking the suits for the non-matching ranks.

2. Three of a kind
   $\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2 = 13 \cdot 4 \cdot 66 \cdot 16 = 54,912$
   $54,912/2,598,960 = 0.021 = 2.1\%$
   The $\binom{13}{1}$ comes from choosing the matching rank. There are 13 total options.
   The $\binom{4}{3}$ comes from picking the suits for the matching rank
   The $\binom{12}{2}$ comes from picking the remaining 2 non-matching ranks from the available options to complete the hand.
   The $\binom{4}{1}^2$ comes from picking the suits for the non-matching ranks.

3. Full House $\binom{13}{2}\binom{2}{1}\binom{4}{3}\binom{4}{2} = 78 \cdot 2 \cdot 4 \cdot 6 = 3,744$
   $3,744/2,598,960 = 0.00144 = 0.144\%$
   The $\binom{13}{2}$ comes from choosing the ranks for the cards in the full house.
   The $\binom{2}{1}$ comes from choosing one of the ranks to be the three-card combination
   The $\binom{4}{3}$ comes from picking suit for the three-card combination.
   The $\binom{4}{2}$ comes from picking the suits for the two-card combination.

4. Straight
   $\binom{10}{1}\binom{4}{1}^5 = 10 \cdot 1024 = 10,240$
   $10,240/2,598,960 = 0.00394 = 0.394\%$
   The $\binom{10}{1}$ comes from choosing the ranks for the lowest card in the hand. There are 10 ranks (A,2,3,4,5,6,7,8,9,10)
   The $\binom{4}{1}^5$ comes from choosing the suit for all of the cards in the hand.
   This calculation does not take into consideration all of the straight flushes and royal flushes. These values will need to be subtracted out.
   There are 4 total royal flushes ($\binom{4}{1}$).
   For straight flushes, we pick a suit, ($\binom{4}{1}$) and multiple by 9 for all the ways to get a normal straight.
   $9 \cdot \binom{4}{1} = 9 \cdot 4 = 36$
   $\frac{10,240-(36+4)}{2,598,960} = \frac{10,200}{2,598,960} = 0.00392 = 0.392\%$

4 Poker Faces

Recompute the probabilities in the previous problem, with the exception that instead of using 1 deck of cards, you have two decks shuffled together.
Total number of distinct poker hands $= \binom{104}{5} = 91,962,520$

1. One pair
   $\binom{13}{1}\binom{12}{3}\binom{8}{2}\binom{8}{1}^3 = 13 \cdot 220 \cdot 28 \cdot 512 = 41,000,960$
   The $\binom{13}{1}$ comes from picking a rank to pair.
   The $\binom{12}{3}$ comes from picking the ranks for the non-paired cards.
   The $\binom{8}{2}$ comes from picking the suits for the paired cards.
   The $\binom{8}{1}^3$ comes from picking the remaining suits for the non-paired cards.
   This does not account for all of the hands that have a pair and are a flush. These values will need to subtract out all of the flush hands that have a pair.
   $\binom{4}{1}\binom{13}{1}\binom{12}{3}\binom{2}{1}^3 = 4 \cdot 13 \cdot 220 \cdot 8 = 91,520$
   The $\binom{4}{1}$ comes from picking a suit to flush in.
   The $\binom{13}{1}$ comes from picking a rank to pair.
   The $\binom{12}{3}$ comes from picking ranks for the extra cards.
   The $\binom{2}{1}^3$ comes from getting each card in chosen rank and suit.
   $\frac{(41{,}000{,}960-91{,}520)}{91{,}962{,}520} = \frac{40{,}909{,}440}{91{,}962{,}520} = .445 = 44.5\%$

2. Three of a kind
   $\binom{13}{1}\binom{8}{3}\binom{12}{2}\binom{8}{1}^2 = 13 \cdot 66 \cdot 56 \cdot 64 = 3,075,072$
   The $\binom{13}{1}$ comes from picking a rank to have three of a kind.
   The $\binom{8}{3}$ comes from picking the suits for the three of a kind.
   The $\binom{12}{2}$ comes from picking the ranks for the extra cards.
   The $\binom{8}{1}^2$ comes from picking the suit for the remaining cards.
   $\frac{3{,}075{,}072}{91{,}962{,}520} = 0.0334 = 3.34\%$

3. Full House $\binom{13}{1}\binom{12}{1}\binom{8}{3}\binom{8}{2} = 13 \cdot 12 \cdot 56 \cdot 28 = 244,608$
   The $\binom{13}{1}$ comes from picking a rank to have three of a kind.
   The $\binom{12}{1}$ comes from picking a rank to have a pair in.
   The $\binom{8}{3}$ comes from picking a suit for the three of a kind.
   The $\binom{8}{2}$ comes from picking a suit for the pair.
   $\frac{244{,}608}{91{,}962{,}520} = 0.00266 = 0.266\%$

4. Straight

   $\binom{10}{1}\binom{8}{1}^5 = 10 \cdot 32,768 = 327,680$

   The $\binom{10}{1}$ comes from picking a possible high card for a straight.

   The $\binom{8}{5}$ comes from picking any of the suits for the straight.

   This calculation does not take into consideration all of the straight flushes and royal flushes. These values will need to be subtracted out.

   $\binom{4}{1}\binom{9}{1}\binom{2}{1}^5 = 4 * 9 * 2^5 = 1,172$

   The $\binom{4}{1}$ comes from picking a suit to flush in.

   The $\binom{9}{1}$ comes from picking a possible high card for a straight (Ace is excluded for a royal flush).

   The $\binom{2}{1}^5$ comes from picking the cards/suits to make it a straight flush.

   $\frac{(327,680-1,172)}{91,962,520} = \frac{326,508}{91,962,520} = 0.00355 = 0.355\%$

5 DiceWars

The game DiceWars is similar to Risk, but you don't age as much while you play it. Each player is assigned a selection of regions on a map. Each region contains a stack of 6-sided dice which equate to relative power. On a player's turn, the player may challenge any other player having dice in a neighboring region. The challenge is resolved when each player rolls their dice, with the higher total winning. Ties are settled in favor of the defender, not the attacker. The goal of the game, of course is total global domination. In your case, the questions are:

1. If you and an opponent each roll one die (not an option in DiceWars, but a good warmup), what is the probability of a tie?
   $P(x) = 6 \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{6}{36} = \frac{1}{6}$

2. If you and an opponent each roll one die, what is the probability that you roll higher than the opponent?
   $P(me > you) + P(you > me) + P(you = me) = 1$
   $P(me > you) = P(you > me)$ because of symmetry
   $P(me = you) = \frac{1}{6}$
   $P(me > you) = \frac{1 - P(me=you)}{P(you>me)} = \frac{1-\frac{1}{6}}{2} = \frac{5}{12}$

3. If you roll m dice, what is the probability that your total is n? Compute this for each value of n such that $m \leq n \leq 6m$, where $1 \leq m \leq 8$. You may express this as a recursive function.

   In order to determine the probability, we need to know two things.

   (a) The number of ways n can be divided by m where m is an integer between 1 and 6.

   (b) The total number of ways to roll k dice.

   For (b), we can just take 6^k.
   For (a), we can define a recursive function that will give use this answer.

   $$F_m(n) = \begin{cases} \sum_{i=1}^{m} F_{n-1}(n-i), & \text{if } n > 1, \\ 1, & \text{if } n = 1 \end{cases}$$

   Dividing (b) and (a) will give us the probability of rolling n with m dice.

   $$P_m(n) = \frac{F_m(n)}{6^k}$$

4. If you roll m dice and your opponent rolls k dice, what is the probability you and your opponent tie? Assume you know the answers from part (c).
   The probability to tie for any given k and m can be represented as:

   $$P_{m,k}(n) = P_m(n) \cdot P_k(n)$$

   If we wanted to calculate the probabilities for all $m \leq n \leq 6m$ and $k \leq n \leq 6k$ we would have to do a summation of all the probabilities over the intersections of the sets that are created by $m \leq n \leq 6m$ and $k \leq n \leq 6k$. $N_1 = \{n_1 \mid k \leq n_i \leq 6k\}$ $N_2 = \{n_2 \mid m \leq n_i \leq 6m\}$

   $$P(tie) = \sum_{n=N_1 \bigcap N_2} P_k(n) \cdot P_m(n)$$

5. If you roll m dice and your opponent rolls k dice, what is the probability you roll higher than the opponent?
   The probability to tie for any given k and m can be represented as:

$$P_m(i) \cdot P_k(j), \text{where } i > j$$

   If we wanted to calculate the probabilities for all $m \leq n \leq 6m$ and $k \leq n \leq 6k$ we would have to do a summation of all the probabilities over the intersections of the sets that are created by $m \leq n \leq 6m$ and $k \leq n \leq 6k$. Using N_1 and N_2 from the previous question:

$$P(win) = \sum_{i=N_2} P_k(i) \cdot \sum_{j \in N_1; j>i} P_m(j)$$

6. For parts (d) and (e), use your formulas to produce tables for each combination of m and k, $1 \leq m, k \leq 8$.
   I wrote some python code in order to generate these tables. Some values are rounded up to either 1 or 0. Values of m increase from 1 - 8 along the rows. Values of k increase from 1-8 along the columns.

```
Tie table:
    [[0.166667 0.069444 0.015432 0.001929 0.000129 0.000004 0.       0.      ]
     [0.069444 0.112654 0.069444 0.024884 0.005955 0.001017 0.000128 0.000012]
     [0.015432 0.069444 0.09285  0.065469 0.02994  0.009822 0.002447 0.000479]
     [0.001929 0.024884 0.065469 0.080944 0.061479 0.032624 0.013014 0.004086]
     [0.000129 0.005955 0.02994  0.061479 0.072693 0.057935 0.033997 0.015527]
     [0.000004 0.001017 0.009822 0.032624 0.057935 0.066539 0.054851 0.034619]
     [0.       0.000128 0.002447 0.013014 0.033997 0.054851 0.061722 0.052169]
     [0.       0.000012 0.000479 0.004086 0.015527 0.034619 0.052169 0.057819]]
```

```
Win table:
    [[0.416667 0.092593 0.011574 0.000772 0.000021 0.       0.       0.      ]
     [0.837963 0.443673 0.152006 0.03588  0.006105 0.000766 0.000071 0.000005]
     [0.972994 0.778549 0.453575 0.191701 0.060713 0.014879 0.00289  0.000452]
     [0.997299 0.939236 0.742831 0.459528 0.220442 0.083423 0.02545  0.006379]
     [0.99985  0.98794  0.909347 0.718078 0.463654 0.242449 0.103626 0.036742]
     [0.999996 0.998217 0.9753   0.883953 0.699616 0.466731 0.259984 0.121507]
     [1.       0.999801 0.994663 0.961536 0.862377 0.685165 0.469139 0.274376]
     [1.       0.999983 0.999069 0.989534 0.947731 0.843874 0.673456 0.471091]]
```

6 Current Events - Learning

This article highlights the new development in PTSD detection using an AI-assisted tool. Currently, patients are diagnosed with PTSD by an interview conducted by a clinician. The interview could be an inaccurate way of determining PTSD, as a lot of patients will hide their symptoms, in a way that becomes undetectable to a human. This new AI tool will identify 18 speech features that are most useful in determining if a person has PTSD. A study was done to determine the accuracy of this new tool. In order to make sure the tool was distinguishing PTSD from other mental illnesses, the researchers excluded participants with a diagnosis of major depression. The program was able to analyze a war verterans voice and detect PTSD with 89% accuracy. In the future, there is hopes to incorporate this technology in to an app, so the diagnosis process can be a lot quicker.

Bayes' rule allows us to describe the probability of an event based on prior knowledge of conditions that might be related to the event. In this case, a particular speech feature might be related to PTSD. The rule allows us to describe the probability that the candidate has PTSD, based on the prior knowledge that they have a particular speech feature (ex. lifeless, metallic tone). In the study, the researchers are using a combination of 18 particular speech features in order to improve the assessment of the AI tool. Since there is more available evidence (18 features), the degree of belief of the probability should rationally change.