

# Coyote Optimization Algorithm: A new metaheuristic for global optimization problems

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**Abstract**—The behavior of natural phenomena has become one of the most popular sources for researchers to design optimization algorithms for scientific, computing and engineering fields. As a result, a lot of nature-inspired algorithms have been proposed in the last decades. Due to the numerous issues of the global optimization process, new algorithms are always welcome in this research field. This paper introduces the Coyote Optimization Algorithm (COA), which is a population based metaheuristic for optimization inspired on the *canis latrans* species. It contributes with a new algorithmic structure and mechanisms for balancing exploration and exploitation. A set of boundary constrained real parameter optimization benchmarks is tested and a comparative study with other nature-inspired metaheuristics is provided to investigate the performance of the COA. Numerical results and non-parametric statistical significance tests indicate that the COA is capable of locating promising solutions and it outperforms other metaheuristics on most tested functions.

**Index Terms**—Global Optimization, Bio-inspired metaheuristics, Coyote Optimization Algorithm.

## I. INTRODUCTION

Real-world optimization problems have been formulated into computational codes from the most diverse application fields including aerospace, civil, mechanical, mechatronics, chemicals, health science, informatics, and sports. Due to the recent advances on computational science, the complexity of the designed problems has increased and features as non-linearity, scale, multimodality and the presence of constraints are examples usually found in these types of applications. As the classic Newtonian and gradient-based techniques are not suitable to perform global optimization, an exponential growth of researches on new stochastic metaheuristics for handling global optimization is noted in recent literature [1]–[3].

A large number of nature-inspired metaheuristics have been proposed in the last decades [4], for example, Ant Colony Optimization (ACO) [5], the Artificial Bee Colony (ABC) [6], Bacterial Colony Foraging (BCF) [7], the Bat-Inspired Algorithm (BA) [8], the Dolphin Echolocation (DE) [9], the Firefly Algorithm (FA) [10], Flower Pollination Algorithm (FPA) [11], the Grey Wolf Optimizer (GWO) [12], the Particle Swarm Optimization (PSO) [13], the Symbiotic Organisms Search (SOS) [14], the Virus Colony Search (VCS) [15] and the Whale Optimization Algorithm (WOA) [16]. Due to their good

performance and ease of implementation, these metaheuristics have been explored, improved and widely applied to several problems in fields of science, finance and engineering [17]–[19].

However, the No Free Lunch theorem [20] states that no single algorithm can perform well on every optimization problem, which means that new algorithms are still welcome in this research area. Moreover, recently studies state that the balance between exploration and exploitation in the optimization process is fundamental for a good performance [21]. Although the recently proposed nature-inspired metaheuristics are classified as stochastic algorithms [4], all of them have a structure composed by a global population of possible solutions that interact by a set of predefined and deterministic mechanisms. For this reason, new mechanisms and strategies that promote different algorithmic behaviors during the optimization can be helpful for this research area.

In this paper, a new metaheuristic for global optimization called Coyote Optimization Algorithm (COA) is proposed, which is inspired on the *canis latrans* species that dwells mainly the North America. The designed algorithm considers the social organization of the coyotes and its adaptation to the environment and it contributes with a different algorithmic structure compared to the metaheuristics from literature. It also provides new mechanisms for balancing the exploration and exploitation in the optimization process.

The COA is evaluated under a set of real-parameters boundary constrained benchmark functions adopted in the 2005 and 2015 competitions of the Institute of Electrical and Electronics Engineers Congress on Evolutionary Computation (IEEE-CEC) [22], [23] and its performance is compared to another bio-inspired metaheuristics from literature. Non-parametric statistic test are applied to evaluate the significance of the results obtained and the potential of the proposed COA.

The remainder of this paper is organized as follows. Section 2 introduces the COA, while section 3 presents the benchmark functions explored and the setup of the algorithms tested. Then, section 4 sets out the results of the computational simulations and its statistical analysis. Finally, the section 5 contains the conclusion and the perspective of future research.

## II. FUNDAMENTALS OF THE PROPOSED ALGORITHM

The proposed Coyote Optimization Algorithm (COA) is a population-based algorithm inspired on the *Canis latrans* species classified as both swarm intelligence and evolutionary heuristic and it is inspired on the coyotes' behavior [24]–[26]. In contrast with the Grey Wolf Optimizer (GWO) [12], which is inspired on the *Canis lupus* species, the COA has a different algorithmic structural setup and it does not focus on the social hierarchy and dominance rules of these animals, even though the alpha is employed as the leader of a pack (as explained forward). Further, the COA focus on the social structure and experiences exchange by the coyotes instead of only hunting preys as it happens in the GWO.

In the COA, the population of coyotes is divided into  $N_p \in \mathbb{N}^*$  packs with  $N_c \in \mathbb{N}^*$  coyotes each. In this first proposal, the number of coyotes per pack is static and similar for all packs. Hence, the total population in the algorithm is obtained by the multiplication of  $N_p$  and  $N_c$ . For simplification purposes, the solitary (or transient) coyotes are not considered in this first version of the algorithm. To facilitate the reader's understanding, each coyote is a possible solution for the optimization problem and its social condition is the cost of the objective function.

According to [27], [28], intrinsic factors (sex, the social status and the pack that the coyote is a member) and extrinsic ones (such as snow depth, snowpack hardness, temperature and carcass biomass) have been pointed out as influences in the coyote's activities. Therefore, the COA mechanism has been designed based on the social conditions of the coyotes, which means the decision variables  $\vec{x}$  of an global optimization problem. Thus, the social condition *soc* (set of decision variables) of the  $c^{th}$  coyote of the  $p^{th}$  pack in the  $t^{th}$  instant of time is written as

$$soc_{c,t}^{p,t} = \vec{x} = (x_1, x_2, \dots, x_D) \quad (1)$$

and it implies in the coyote's adaptation to the environment (cost of the objective function)  $fit_c^{p,t} \in \mathbb{R}$ .

The first step in the COA is to initialize the global population of coyotes. As the COA is a stochastic algorithm, the initial social conditions are set randomly for each coyote. It happens by assigning random values inside the search space for the  $c^{th}$  coyote of the  $p^{th}$  pack of the  $j^{th}$  dimension, as follows:

$$soc_{c,j}^{p,t} = lb_j + r_j \cdot (ub_j - lb_j), \quad (2)$$

wherein  $lb_j$  and  $ub_j$  represents, respectively, the lower and upper bounds of the  $j^{th}$  decision variable,  $D$  is the search space dimension and  $r_j$  is a real random number generated inside the range  $[0,1]$  using uniform probability. After that, the coyotes' adaptation in the respective current social conditions are evaluated:

$$fit_c^{p,t} = f(soc_{c,t}^{p,t}) \quad (3)$$

Initially, the coyotes are randomly assigned to the packs, however the coyotes sometimes leave their packs and become

solitary or join a pack instead [26]. According to [25], the coyote eviction from a pack depends on the number of coyotes inside the pack and occurs with probability  $P_e$ , such that:

$$P_e = 0.005 \cdot N_c^2. \quad (4)$$

Considering that  $P_e$  could assume values greater than 1 for  $N_c \leq \sqrt{200}$ , the number of coyotes per pack is limited to 14. This mechanism helps the COA to diversify the interaction between all the coyotes of the population, which means a cultural exchange in the global population.

In this species, the packs usually has two alphas [25], [28], however the COA considers only one, which is the best adapted to the environment. Considering an minimization problem, the alpha of the  $p^{th}$  pack in the  $t^{th}$  instant of time is defined as:

$$alpha^{p,t} = \{soc_{c,t}^{p,t} | arg_{c=\{1,2,\dots,N_c\}} \min f(soc_{c,t}^{p,t})\}. \quad (5)$$

Due to the evident signs of swarm intelligence in this specie, the COA assumes that the coyotes are sufficiently organized to share the social conditions and to contribute to the pack's maintenance. Thus, the COA links all information from the coyotes and computes it as the cultural tendency of the pack:

$$cult_j^{p,t} = \begin{cases} O_{\frac{(N_c+1)}{2},j}^{p,t}, & N_c \text{ is odd} \\ \frac{O_{\frac{N_c}{2},j}^{p,t} + O_{\frac{(N_c+1)}{2},j}^{p,t}}{2}, & \text{otherwise} \end{cases} \quad (6)$$

where  $O^{p,t}$  represents the ranked social conditions of all coyotes of the  $p^{th}$  pack in the  $t^{th}$  instant of time for every  $j$  in the range  $[1,D]$ . In other words, the cultural tendency of the pack is computed as the median social conditions of all coyotes from that specific pack.

Taking into account the two main biological events of life, the birth and the death, the COA computes the age of the coyotes (in years), which is denoted as  $age_c^{p,t} \in \mathbb{N}$ . The birth of a new coyotes is written as a combination of the social conditions of two parents (randomly chosen) plus a environmental influence, such that:

$$pup_j^{p,t} = \begin{cases} soc_{r_1,j}^{p,t}, & rnd_j < P_s \text{ or } j = j_1 \\ soc_{r_2,j}^{p,t}, & rnd_j \geq P_s + P_a \text{ or } j = j_2 \\ R_j, & \text{otherwise} \end{cases} \quad (7)$$

wherein  $r_1$  and  $r_2$  are random coyotes from the  $p^{th}$  pack,  $j_1$  and  $j_2$  are two random dimensions of the problem,  $P_s$  is the scatter probability,  $P_a$  is the association probability,  $R_j$  is a random number inside the decision variable bound of the  $j^{th}$  dimension and  $rnd_j$  is a random number inside  $[0,1]$  generated with uniform probability. The scatter and association probabilities guide the cultural diversity of the coyotes from the pack. In this initial version of the COA, the  $P_s$  and the  $P_a$  have been defined as

$$P_s = 1/D \quad \text{and} \quad (8)$$

$$P_a = (1 - P_s)/2, \quad (9)$$

where  $P_a$  establish the same influence impact for both parents.

According to some researches, the pups have around 10% of chances of dying even before living [25] and the higher the coyote's age, the higher is the mortality probability [26]. In order to keep the population size static, the COA syncs the coyote's birth and death as described in the Alg. 1, where  $\omega$  and  $\varphi$  represent, respectively, the group of coyotes worse adapted to the environment than the pup (i.e., the group of solutions that present worst objective function's costs) and the number of coyotes in this group. Note that it is possible that two or more coyotes have similar age (in line 4). In this case, the less adapted coyote is the one who dies.

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**Algorithm 1** Birth and death inside a pack.

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1: Compute  $\omega$  and  $\varphi$ .
2: if  $\varphi = 1$  then
3:   The pup survives and the only coyote in  $\omega$  dies.
4: elseif  $\varphi > 1$  then
5:   The pup survives and the oldest coyote in  $\omega$  dies.
6: else
7:   The pup dies.
8: end if
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In order to represent the cultural interaction inside the packs, the COA assumes that coyotes are under the alpha influence ( $\delta_1$ ) and the pack influence ( $\delta_2$ ). The first one means a cultural difference from a random coyote of the pack ( $cr_1$ ) to the alpha coyote, while the second one means a cultural difference from a random coyote ( $cr_2$ ) to the cultural tendency of the pack. The random coyotes are chosen by uniform distribution of probability and  $\delta_1$  and  $\delta_2$  are written respectively as:

$$\delta_1 = \alpha^{p,t} - soc_{cr_1}^{p,t} \text{ and} \quad (10)$$

$$\delta_2 = \text{cult}^{p,t} - soc_{cr_2}^{p,t}. \quad (11)$$

Hence, the coyote's new social condition is updated using the alpha and the pack influence through the following equation:

$$\text{new\_soc}_c^{p,t} = soc_c^{p,t} + r_1 \cdot \delta_1 + r_2 \cdot \delta_2, \quad (12)$$

where  $r_1$  and  $r_2$  are, respectively, the weights of the alpha and the pack influence. Initially,  $r_1$  and  $r_2$  have been defined as random numbers inside the range [0,1] generated with uniform probability. The new social condition is then evaluated:

$$\text{new\_fit}_c^{p,t} = f(\text{new\_soc}_c^{p,t}), \quad (13)$$

and the coyote's cognitive capacity decide if the new social condition is better than the older one to keep it, it means:

$$soc_c^{p,t+1} = \begin{cases} \text{new\_soc}_c^{p,t}, & \text{new\_fit}_c^{p,t} < \text{fit}_c^{p,t} \\ soc_c^{p,t}, & \text{otherwise} \end{cases}. \quad (14)$$

Finally, the social condition of the coyote that best adapted itself to the environment is selected and is used as the global solution of the problem. The pseudo code of the COA is described in Alg. 2, where  $N_c$  can be set as a first guess in the rang [5,10] and  $N_p$  can be subsequently adjusted to defined the total population size of the algorithm.

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**Algorithm 2** Pseudo code of the COA.

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1: Initialize  $N_p$  packs with  $N_c$  coyotes each (Eq. 2).
2: Verify the coyote's adaptation (Eq. 3).
3: while stopping criterion is not achieved do
4:   for each  $p$  pack do
5:     Define the alpha coyote of the pack (Eq. 5).
6:     Compute the social tendency of the pack (Eq. 6).
7:     for each  $c$  coyotes of the  $p$  pack do
8:       Update the social condition (Eq. 12).
9:       Evaluate the new social condition (Eq. 13).
10:      Adaptation (Eq. 14).
11:    end for
12:    Birth and death (Eq.7 and Alg. 1).
13:  end for
14:  Transition between packs (Eq. 4).
15:  Update the coyotes' ages.
16: end while
17: Select the best adapted coyote.
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### III. BENCHMARKS FUNCTIONS AND ALGORITHMS SETUP

A set of 40 benchmark functions has been adopted from the IEEE CEC2005 [22] and from IEEE CEC2015 [23] in order to evaluate the COA performance. These benchmarks have different classification according to the number of local optima, the dimension and the presence of multiple base functions. The benchmark functions from CEC2005 (from F1 to F47) have been tested with different dimensions and the fifteen benchmarks from CEC2015 (from F48 to F92) have been all tested with dimensions 30, 50 and 100, as described in Tab. I (considering the following notation. F: function number in this paper; D: dimension; FS: function number in the source; U: unimodal; SH: shifted; SE: separable; NSE: non-separable; M: multimodal; R: rotated; H: hybrid and C: composition).

These 92 cases have been tested and compared with the results obtained by the Artificial Bee Colony (ABC) [6], the Bat-Inspired Algorithm (BA) [8], the Firefly Algorithm (FA) [10], the Grey Wolf Optimizer (GWO) [12], the Particle Swarm Optimization (PSO) [13] and the Symbiotic Organisms Search (SOS) [14]. In this paper, the COA parameters  $N_p$  and  $N_c$  have been defined as 20 and 5, respectively. The ABC's parameters *trials* and *food sources* have been both setup as 100 (with one scout and 50% of employed bees). The BA' parameters *population size*, *loudness*, *pulse rate*, *minimum frequency* and *maximum frequency* have been setup as 100, 0.5, 0.5, 0 and 2, respectively. The FA's parameters *population size*, *randomness* and *absorption coefficient* have been setup respectively as 100, 0.2 and 0.1. The *number of wolfs* in GWO has been setup as 100 with linearly decreasing  $a$  from 2 to

TABLE I  
BENCHMARKS DEFINITION FROM IEEE-CEC2005 AND IEEE-CEC2015

| F#         | D           | FS | Description         |
|------------|-------------|----|---------------------|
| 1, 2, 3    | 30, 50, 100 | 1  | U, SH, SE           |
| 4, 5, 6    | 30, 50, 100 | 2  | U, SH, NSE          |
| 7, 8       | 30, 50      | 3  | U, SH, R, NSE       |
| 9, 10, 11  | 30, 50, 100 | 4  | U, SH, NSE          |
| 12, 13, 14 | 30, 50, 100 | 5  | U, NSE              |
| 15, 16, 17 | 30, 50, 100 | 6  | SH, M, NSE          |
| 18, 19     | 30, 50      | 7  | SH, R, M, NSE       |
| 20, 21     | 30, 50      | 8  | SH, R, M, NSE       |
| 22, 23, 24 | 30, 50, 100 | 9  | SH, SE, M           |
| 25, 26     | 30, 50      | 10 | SH, R, M, NSE       |
| 27, 28     | 30, 50      | 11 | SH, R, M, NSE       |
| 29, 30, 31 | 30, 50, 100 | 12 | M, NSE              |
| 32, 33, 34 | 30, 50, 100 | 13 | M, NSE              |
| 35, 36     | 30, 50      | 14 | SH, R, M, NSE       |
| 37         | 30          | 15 | SH, SE, C, H, M     |
| 38         | 30          | 16 | SH, R, C, H, M, NSE |
| 39         | 30          | 17 | SH, R, C, H, M, NSE |
| 40         | 30          | 18 | SH, R, C, H, M, NSE |
| 41         | 30          | 19 | SH, R, C, H, M, NSE |
| 42         | 30          | 20 | SH, R, C, H, M, NSE |
| 43         | 30          | 21 | SH, R, C, H, M, NSE |
| 44         | 30          | 22 | SH, R, C, H, M, NSE |
| 45         | 30          | 23 | SH, R, C, H, M, NSE |
| 46         | 30          | 24 | SH, R, C, H, M, NSE |
| 47         | 30          | 25 | SH, R, C, H, M, NSE |
| 48, 49, 50 | 30, 50, 100 | 1  | U, R, NSE           |
| 51, 52, 53 | 30, 50, 100 | 2  | U, R, NSE           |
| 54, 55, 56 | 30, 50, 100 | 3  | SH, R, M, NSE       |
| 57, 58, 59 | 30, 50, 100 | 4  | SH, R, M, NSE       |
| 60, 61, 62 | 30, 50, 100 | 5  | SH, R, M, NSE       |
| 63, 64, 65 | 30, 50, 100 | 6  | SH, R, M, NSE       |
| 66, 67, 68 | 30, 50, 100 | 7  | SH, R, M, NSE       |
| 69, 70, 71 | 30, 50, 100 | 8  | SH, R, M, NSE       |
| 72, 73, 74 | 30, 50, 100 | 9  | SH, R, M, NSE       |
| 75, 76, 77 | 30, 50, 100 | 10 | H, M, NSE           |
| 78, 79, 80 | 30, 50, 100 | 11 | H, M, NSE           |
| 81, 82, 83 | 30, 50, 100 | 12 | H, M, NSE           |
| 84, 85, 86 | 30, 50, 100 | 13 | C, M, NSE           |
| 87, 88, 89 | 30, 50, 100 | 14 | C, M, NSE           |
| 90, 91, 92 | 30, 50, 100 | 15 | C, M, NSE           |

0, while the *ecosystem size* in SOS has been setup as 100. The PSO's parameters *swarm size*, *cognitive constant*, *social constant* and *maximum speed* have been setup as 100, 2, 2 and 20% of the search space. The *inertia weight* has been setup from 0.9 to 0.4 with linear decreasing.

#### IV. EXPERIMENTAL RESULTS AND ANALYSIS

The algorithms have been tested 30 times for each study case with stopping criterion equals to  $10000 \cdot D$  objective function evaluations. The average errors obtained from the global optimum and the standard deviation are shown in Tabs. II, III and IV. As it can be seen, the COA has reached lower errors for most benchmarks tested (although some decimal values are hidden for lack of space in the tables, the best values found are written in boldface). In fact, it has achieved smaller averages in 39.13% of the functions tested against 0%,

28.26%, 19.57%, 4.35%, 8.7% and 0% achieved respectively by PSO, ABC, SOS, GWO, BA and FA.

On the other hand, the COA has achieved best average values only in 30% of the unimodal functions against 60% of the SOS. Considering only the separable functions, the COA has not been able to find the best average for any of the seven cases, while the ABC and the SOS have found 57.14% and 42.86%, respectively. In the case of the composition functions, the COA outperformed the other metaheuristics in terms of average cost in 45% of the cases, against the same 45% of the ABC and 10% of the SOS. For hybrid functions, these numbers are 40% against 45% and 15%, while for multimodal functions the COA found best values in 41.67% of the cases against 33.33% of the ABC, 8.33% of the SOS, 5.56% of the GWO and 11.11% of the BA. Considering shifted and rotated functions, the COA has achieved 37.29% and 53.06%, respectively, while the other algorithms have not achieved more than 23% in both cases.

Separating the benchmarks by its dimensions, the COA has found best averages in 40% (ABC=30%, SOS=17.5%, GWO = 5% and BA=7.5%), 31.03% (ABC=27.59%, SOS=24.14%, GWO = 6.9% and BA=10.34%) and 47.83% (ABC=26.09%, SOS=17.39% and BA=8.7%) for  $D=30$ ,  $D=50$  and  $D=100$ , respectively. Furthermore, considering the CEC2005 and the CEC2015 test problems separated, the COA has achieved best averages in 31.91% (ABC=21.28%, SOS=27.66%, GWO = 8.51% and BA=10.64%) and in 46.67% (ABC=35.56%, SOS=11.11% and BA=6.67%), respectively.

The average rankings separated by the function features have been computed and drawn in the Fig. 1, where it is possible to evaluate the robustness of each algorithm [29]. For hybrid functions, for example, the COA has the smallest average ranking even not been the algorithm that most reached the first place (40% against the ABC's 45%). It happens because when COA has not achieved the first place it has still achieved good rankings and, in general, it has presented better results than the other algorithms. In fact, the COA has obtained the best average ranking in most cases; only for separable and unimodal it has not presented the best results.

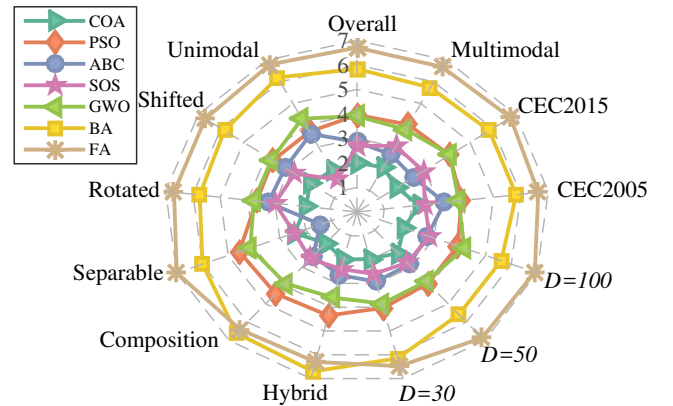


Fig. 1. Average ranking separated by the benchmarks' features.

TABLE II  
AVERAGE ERRORS±STANDARD DEVIATIONS (PART I)

|     | F1                        | F2                        | F3                        | F4                        | F5                        |
|-----|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| COA | 4.32e-07±3.37e-07         | 2.90e-07±1.47e-07         | 9.63e-07±4.59e-07         | 2.14e+02±9.02e+01         | 2.24e+03±5.54e+02         |
| PSO | 1.05e+03±1.08e+03         | 2.33e+03±1.75e+03         | 6.98e+03±2.94e+03         | 5.47e+02±8.64e+02         | 5.79e+03±6.89e+03         |
| ABC | 7.28e-16±9.44e-17         | 1.58e-15±2.06e-16         | 5.39e-15±1.41e-15         | 5.27e+03±1.73e+03         | 2.64e+04±4.21e+03         |
| SOS | <b>0.00e+00±0.00e+00</b>  | <b>0.00e+00 ±0.00e+00</b> | <b>1.13e-24±1.34e-24</b>  | <b>3.95e-05±1.07e-04</b>  | <b>3.20e+01±4.38e+01</b>  |
| GWO | 6.90e+02±6.04e+02         | 2.88e+03±1.40e+03         | 2.07e+04±5.91e+03         | 8.47e+03±3.10e+03         | 2.32e+04±4.27e+03         |
| BA  | 5.97e+04±1.07e+04         | 1.13e+05±1.64e+04         | 2.24e+05±2.79e+04         | 7.53e+04±2.16e+04         | 1.74e+05±4.98e+04         |
| FA  | 8.02e+04±6.78e+03         | 1.73e+05±1.09e+04         | 4.33e+05±1.46e+04         | 8.71e+04±9.56e+03         | 2.41e+05±2.74e+04         |
|     | F6                        | F7                        | F8                        | F9                        | F10                       |
| COA | 1.75e+04±3.34e+03         | 3.10e+06±1.64e+06         | 7.09e+06±3.18e+06         | 2.86e+03±1.18e+03         | 1.71e+04±5.41e+03         |
| PSO | 2.67e+04±1.22e+04         | 4.02e+06±5.82e+06         | 1.80e+07±1.82e+07         | 2.84e+03±3.51e+03         | 2.00e+04±9.05e+03         |
| ABC | 1.27e+05±1.27e+04         | 1.25e+07±2.79e+06         | 2.63e+07±5.13e+06         | 4.38e+04±5.98e+03         | 1.23e+05±1.37e+04         |
| SOS | <b>6.92e+03±2.20e+03</b>  | <b>2.43e+06±1.41e+06</b>  | <b>6.66e+06±2.73e+06</b>  | <b>4.38e+01±4.96e+01</b>  | <b>3.54e+03±1.99e+03</b>  |
| GWO | 9.13e+04±1.41e+04         | 1.62e+07±1.46e+07         | 4.60e+07±1.95e+07         | 1.21e+04±4.67e+03         | 3.18e+04±5.97e+03         |
| BA  | 4.39e+05±1.07e+05         | 8.61e+08±3.80e+08         | 3.57e+09±1.07e+09         | 1.35e+05±4.10e+04         | 4.09e+05±1.45e+05         |
| FA  | 9.75e+05±9.73e+04         | 9.46e+08±1.83e+08         | 4.71e+09±5.64e+08         | 8.81e+04±1.45e+04         | 2.35e+05±3.66e+04         |
|     | F11                       | F12                       | F13                       | F14                       | F15                       |
| COA | 1.02e+05±1.98e+04         | <b>2.13e+03±8.28e+02</b>  | 6.49e+03± <b>9.28e+02</b> | <b>1.67e+04±2.37e+03</b>  | 8.23e+01±5.51e+01         |
| PSO | 1.23e+05±3.86e+04         | 7.48e+03±2.10e+03         | 1.11e+04±2.32e+03         | 3.38e+04±5.11e+03         | 5.54e+07±1.34e+08         |
| ABC | 5.15e+05±4.55e+04         | 9.61e+03±1.17e+03         | 2.36e+04±1.75e+03         | 5.57e+04±3.29e+03         | <b>2.10e+00±2.33e+00</b>  |
| SOS | <b>5.59e+04±1.24e+04</b>  | 2.42e+03±8.48e+02         | <b>6.42e+03±1.45e+03</b>  | 1.83e+04±2.69e+03         | 3.57e+01±4.81e+01         |
| GWO | 1.12e+05±1.67e+04         | 3.75e+03±2.59e+03         | 1.04e+04±2.46e+03         | 2.97e+04±2.48e+03         | 1.78e+07±1.72e+07         |
| BA  | 1.67e+06±4.61e+05         | 3.60e+04±4.37e+03         | 4.13e+04±4.38e+03         | 8.76e+04±5.85e+03         | 3.05e+10±1.09e+10         |
| FA  | 9.91e+05±1.16e+05         | 3.40e+04±2.02e+03         | 4.57e+04±1.74e+03         | 9.98e+04±2.83e+03         | 5.40e+10±6.89e+09         |
|     | F16                       | F17                       | F18                       | F19                       | F20                       |
| COA | 1.85e+02±8.07e+01         | 3.38e+02±8.30e+01         | <b>4.70e+03±3.67e-12</b>  | <b>6.20e+03±3.87e-12</b>  | 2.09e+01±1.72e-01         |
| PSO | 1.83e+08±3.12e+08         | 5.78e+08±7.74e+08         | 5.13e+03±6.15e+02         | 6.90e+03±5.32e+02         | 2.07e+01±1.07e-01         |
| ABC | <b>3.59e+00±5.43e+00</b>  | <b>1.39e-01±1.70e-01</b>  | 4.70e+03±3.20e-09         | 6.20e+03±5.63e-12         | 2.09e+01±6.17e-02         |
| SOS | 8.66e+01±6.75e+01         | 1.53e+02±5.22e+01         | 4.70e+03± <b>6.76e-13</b> | 6.20e+03± <b>3.02e-12</b> | 2.09e+01±5.34e-02         |
| GWO | 1.63e+08±2.21e+08         | 2.74e+09±1.31e+09         | 4.70e+03±1.57e+00         | 6.20e+03±4.75e+00         | 2.10e+01± <b>3.59e-02</b> |
| BA  | 5.03e+10±1.58e+10         | 1.02e+11±2.61e+10         | 1.17e+04±4.96e+02         | 1.61e+04±4.80e+02         | <b>2.01e+01±4.33e-02</b>  |
| FA  | 1.43e+11±1.43e+10         | 4.18e+11±2.81e+10         | 1.05e+04±2.47e+02         | 1.53e+04±3.27e+02         | 2.10e+01±6.59e-02         |
|     | F21                       | F22                       | F23                       | F24                       | F25                       |
| COA | 2.11e+01±3.46e-02         | 3.77e-06±3.02e-06         | 3.16e-06±2.63e-06         | 6.85e-02±3.63e-01         | <b>7.41e+01±2.23e+01</b>  |
| PSO | 2.09e+01±1.05e-01         | 1.02e+02±2.40e+01         | 2.14e+02±2.43e+01         | 5.77e+02±6.09e+01         | 1.44e+02±3.82e+01         |
| ABC | 2.11e+01±3.62e-02         | <b>4.20e-15±1.18e-14</b>  | <b>2.20e-12±1.09e-11</b>  | <b>2.48e-08±9.47e-08</b>  | 3.09e+02±4.02e+01         |
| SOS | 2.11e+01±3.81e-02         | 7.99e+01±1.37e+01         | 1.99e+02±3.82e+01         | 5.08e+02±2.06e+02         | 1.68e+02±3.65e+01         |
| GWO | 2.11e+01± <b>3.35e-02</b> | 7.89e+01±1.65e+01         | 1.96e+02±2.94e+01         | 5.79e+02±5.92e+01         | 1.24e+02±5.88e+01         |
| BA  | <b>2.03e+01±7.37e-02</b>  | 2.35e+02±4.87e+01         | 4.72e+02±6.84e+01         | 9.70e+02±1.19e+02         | 5.08e+02±7.83e+01         |
| FA  | 2.12e+01±3.82e-02         | 4.26e+02±1.81e+01         | 8.62e+02±3.20e+01         | 1.98e+03±4.53e+01         | 6.99e+02±4.05e+01         |
|     | F26                       | F27                       | F28                       | F29                       | F30                       |
| COA | <b>1.69e+02±4.11e+01</b>  | 2.79e+01±3.77e+00         | 5.98e+01±6.45e+00         | 1.35e+04±7.81e+03         | 5.23e+04±2.83e+04         |
| PSO | 3.66e+02±7.92e+01         | 2.54e+01±3.18e+00         | 5.22e+01±4.68e+00         | 2.13e+04±1.65e+04         | 1.32e+05±8.83e+04         |
| ABC | 9.31e+02±8.11e+01         | 2.75e+01± <b>1.39e+00</b> | 5.55e+01± <b>2.04e+00</b> | 1.12e+04± <b>4.51e+03</b> | 5.79e+04±1.33e+04         |
| SOS | 3.63e+02±8.64e+01         | 2.05e+01±6.79e+00         | 5.62e+01±1.21e+01         | 7.49e+03±6.50e+03         | 4.18e+04±1.75e+04         |
| GWO | 2.46e+02±9.47e+01         | <b>1.59e+01±2.54e+00</b>  | <b>3.43e+01±4.38e+00</b>  | 6.84e+04±3.08e+04         | 3.65e+05±1.24e+05         |
| BA  | 1.01e+03±1.13e+02         | 3.82e+01±2.46e+00         | 6.90e+01±2.74e+00         | <b>3.38e+03±6.72e+03</b>  | <b>9.14e+03±8.92e+03</b>  |
| FA  | 1.46e+03±7.40e+01         | 3.21e+01±3.72e+00         | 7.36e+01±3.02e+00         | 1.47e+06±1.39e+05         | 7.42e+06±3.77e+05         |
|     | F31                       | F32                       | F33                       | F34                       | F35                       |
| COA | 3.02e+05±7.36e+04         | <b>1.58e+00±2.91e-01</b>  | <b>2.49e+00±3.72e-01</b>  | <b>4.85e+00±5.14e-01</b>  | 1.31e+01±3.32e-01         |
| PSO | 1.58e+06±1.36e+06         | 5.74e+00±1.78e+00         | 1.71e+01±3.83e+00         | 6.20e+01±1.35e+01         | 1.24e+01±5.25e-01         |
| ABC | 3.81e+05±6.83e+04         | 1.80e+00± <b>1.97e-01</b> | 3.64e+00± <b>3.34e-01</b> | 9.55e+00±5.19e-01         | 1.30e+01±2.56e-01         |
| SOS | 4.45e+05±2.52e+05         | 9.80e+00±1.80e+00         | 2.53e+01±3.17e+00         | 7.24e+01±4.73e+00         | 1.31e+01±2.47e-01         |
| GWO | 3.26e+06±7.81e+05         | 5.20e+00±2.83e+00         | 1.21e+01±4.24e+00         | 6.30e+01±1.58e+01         | <b>1.17e+01±4.71e-01</b>  |
| BA  | <b>2.69e+04±1.82e+04</b>  | 3.02e+02±1.44e+02         | 5.84e+02±1.46e+02         | 1.30e+03±2.36e+02         | 1.40e+01±2.39e-01         |
| FA  | 3.76e+07±1.62e+06         | 6.72e+02±1.84e+02         | 2.90e+03±5.28e+02         | 1.46e+04±2.05e+03         | 1.39e+01± <b>1.10e-01</b> |
|     | F36                       | F37                       | F38                       | F39                       | F40                       |
| COA | 2.31e+01±1.74e-01         | 1.40e+02±1.25e+02         | <b>1.06e+02±3.47e+01</b>  | <b>1.27e+02±3.84e+01</b>  | 9.07e+02± <b>1.55e+00</b> |
| PSO | 2.17e+01±6.35e-01         | 5.60e+02±5.75e+01         | 2.97e+02±1.46e+02         | 3.54e+02±1.59e+02         | 9.39e+02±2.41e+01         |
| ABC | 2.27e+01±1.86e-01         | <b>2.15e+01±1.20e+01</b>  | 3.13e+02±4.34e+01         | 4.33e+02± <b>3.57e+01</b> | <b>8.84e+02±9.59e+01</b>  |
| SOS | 2.29e+01±1.45e-01         | 3.47e+02±1.20e+02         | 2.40e+02±9.94e+01         | 3.01e+02±1.16e+02         | 9.12e+02±4.37e+00         |
| GWO | <b>2.10e+01±6.70e-01</b>  | 4.12e+02±7.94e+01         | 2.40e+02±1.44e+02         | 2.54e+02±1.82e+02         | 9.29e+02±1.11e+01         |
| BA  | 2.37e+01±3.47e-01         | 1.07e+03±2.93e+02         | 8.52e+02±2.42e+02         | 1.16e+03±1.56e+02         | 1.36e+03±5.90e+01         |
| FA  | 2.37e+01± <b>1.12e-01</b> | 1.10e+03± <b>3.78e+01</b> | 8.73e+02±5.28e+01         | 8.52e+02±5.25e+01         | 1.29e+03±2.55e+01         |

TABLE III  
AVERAGE ERRORS±STANDARD DEVIATIONS (PART II)

|     | F41                       | F42                       | F43                       | F44                       | F45                       |
|-----|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| COA | <b>9.07e+02±1.56e+00</b>  | <b>9.07e+02±1.67e+00</b>  | 5.00e+02±4.31e-07         | <b>8.72e+02±2.10e+01</b>  | 5.34e+02±2.05e-03         |
| PSO | 9.43e+02±2.58e+01         | 9.43e+02±2.58e+01         | 1.02e+03±1.79e+02         | 9.24e+02±5.30e+01         | 9.73e+02±1.74e+02         |
| ABC | 9.18e+02±3.23e+00         | 9.14e+02±1.88e+01         | <b>4.89e+02±2.57e+01</b>  | 1.08e+03±2.65e+01         | <b>5.34e+02±2.41e+00</b>  |
| SOS | 9.11e+02±4.62e+00         | 9.11e+02±4.57e+00         | 5.83e+02±1.98e+02         | 9.25e+02±3.02e+01         | 6.02e+02±1.54e+02         |
| GWO | 9.27e+02±9.85e+00         | 9.29e+02±9.42e+00         | 6.61e+02±1.81e+02         | 9.51e+02±2.62e+01         | 7.21e+02±1.34e+02         |
| BA  | 1.36e+03±5.62e+01         | 1.36e+03±5.61e+01         | 1.48e+03±5.23e+01         | 1.64e+03±1.52e+02         | 1.48e+03±4.96e+01         |
| FA  | 1.29e+03±2.75e+01         | 1.29e+03±2.87e+01         | 1.48e+03± <b>2.07e+01</b> | 1.48e+03±5.98e+01         | 1.47e+03± <b>2.16e+01</b> |
|     | F46                       | F47                       | F48                       | F49                       | F50                       |
| COA | 6.23e+02±3.76e+02         | <b>1.62e+03±4.82e+00</b>  | <b>1.02e+06±5.57e+05</b>  | <b>4.92e+06±2.28e+06</b>  | <b>8.68e+06±1.87e+06</b>  |
| PSO | 9.52e+02±5.92e+01         | 1.63e+03±1.73e+01         | 1.83e+07±2.28e+07         | 5.63e+07±7.05e+07         | 6.69e+07±7.32e+07         |
| ABC | 1.07e+03±2.99e+02         | 1.74e+03±1.26e+01         | 4.32e+06±1.08e+06         | 1.40e+07±2.87e+06         | 4.43e+07±5.72e+06         |
| SOS | <b>4.29e+02±3.56e+02</b>  | 1.62e+03± <b>3.97e+00</b> | 1.11e+06±6.99e+05         | 5.23e+06±4.61e+06         | 1.18e+07±4.46e+06         |
| GWO | 4.87e+02±2.91e+02         | 1.63e+03±7.51e+00         | 2.51e+07±1.90e+07         | 5.57e+07±4.22e+07         | 2.64e+08±9.37e+07         |
| BA  | 1.48e+03±3.91e+01         | 1.90e+03±3.79e+01         | 1.43e+09±7.35e+08         | 3.27e+09±1.39e+09         | 3.64e+09±1.32e+09         |
| FA  | 1.46e+03± <b>2.73e+01</b> | 1.84e+03±1.92e+01         | 1.49e+09±2.73e+08         | 5.42e+09±6.92e+08         | 1.47e+10±1.49e+09         |
|     | F51                       | F52                       | F53                       | F54                       | F55                       |
| COA | 2.02e+03±1.98e+03         | 6.65e+03±5.30e+03         | <b>1.42e+03±1.09e+03</b>  | 3.20e+02±5.76e-02         | 3.20e+02±4.55e-02         |
| PSO | 9.44e+08±1.13e+09         | 2.90e+09±2.73e+09         | 6.02e+09±4.47e+09         | 3.20e+02±8.44e-02         | 3.20e+02±1.17e-01         |
| ABC | <b>1.53e+03±7.88e+02</b>  | <b>5.80e+03±3.92e+03</b>  | 5.17e+03±3.53e+03         | 3.20e+02±3.04e-02         | 3.21e+02±2.23e-02         |
| SOS | 3.97e+03±4.77e+03         | 8.97e+03±1.13e+04         | 3.85e+03±4.28e+03         | 3.21e+02±7.20e-02         | 3.21e+02±6.06e-02         |
| GWO | 1.28e+09±1.32e+09         | 4.86e+09±2.75e+09         | 2.90e+10±7.85e+09         | 3.21e+02±5.94e-02         | 3.21e+02±3.28e-02         |
| BA  | 8.56e+10±1.79e+10         | 1.54e+11±2.23e+10         | 2.97e+11±4.12e+10         | <b>3.20e+02±4.82e-06</b>  | <b>3.20e+02±5.41e-06</b>  |
| FA  | 1.12e+11±9.07e+09         | 2.47e+11±1.49e+10         | 5.70e+11±2.98e+10         | 3.21e+02±7.73e-02         | 3.21e+02±2.73e-02         |
|     | F56                       | F57                       | F58                       | F59                       | F60                       |
| COA | 3.20e+02±9.59e-02         | <b>4.54e+02±1.26e+01</b>  | <b>5.46e+02±2.69e+01</b>  | <b>8.03e+02±5.99e+01</b>  | 2.88e+03±4.27e+02         |
| PSO | 3.21e+02±8.66e-02         | 5.20e+02±2.76e+01         | 6.90e+02±5.78e+01         | 1.12e+03±9.11e+01         | 4.06e+03±6.38e+02         |
| ABC | 3.21e+02±1.95e-02         | 4.84e+02± <b>9.76e+00</b> | 6.11e+02± <b>2.06e+01</b> | 1.13e+03± <b>4.17e+01</b> | <b>2.73e+03±2.70e+02</b>  |
| SOS | 3.21e+02±2.48e-02         | 5.07e+02±3.22e+01         | 6.69e+02±9.33e+01         | 1.21e+03±1.96e+02         | 4.52e+03±6.71e+02         |
| GWO | 3.21e+02±2.27e-02         | 4.92e+02±2.20e+01         | 5.96e+02±3.52e+01         | 9.50e+02±6.20e+01         | 3.50e+03±1.25e+03         |
| BA  | <b>3.20e+02±8.56e-06</b>  | 7.03e+02±5.18e+01         | 9.52e+02±7.79e+01         | 1.67e+03±1.29e+02         | 5.06e+03±6.28e+02         |
| FA  | 3.21e+02±2.10e-02         | 9.06e+02±2.99e+01         | 1.46e+03±4.32e+01         | 2.83e+03±6.11e+01         | 8.06e+03± <b>2.17e+02</b> |
|     | F61                       | F62                       | F63                       | F64                       | F65                       |
| COA | <b>4.90e+03±7.26e+02</b>  | <b>1.31e+04±1.22e+03</b>  | <b>2.55e+04±1.99e+04</b>  | <b>2.40e+05±1.54e+05</b>  | <b>1.07e+06±4.87e+05</b>  |
| PSO | 6.88e+03±8.58e+02         | 1.57e+04±1.39e+03         | 1.54e+05±5.01e+05         | 9.43e+05±2.46e+06         | 3.47e+06±3.96e+06         |
| ABC | 4.91e+03± <b>2.40e+02</b> | 1.34e+04± <b>4.72e+02</b> | 1.69e+06±6.53e+05         | 4.29e+06±1.19e+06         | 2.58e+07±3.15e+06         |
| SOS | 1.00e+04±6.63e+02         | 2.64e+04±9.76e+02         | 9.03e+04±6.50e+04         | 5.90e+05±3.45e+05         | 3.50e+06±1.43e+06         |
| GWO | 5.57e+03±6.66e+02         | 1.42e+04±3.17e+03         | 1.06e+06±9.68e+05         | 2.48e+06±2.07e+06         | 1.83e+07±1.01e+07         |
| BA  | 8.75e+03±1.19e+03         | 1.71e+04±1.20e+03         | 7.35e+07±5.40e+07         | 1.53e+08±1.09e+08         | 4.45e+08±3.34e+08         |
| FA  | 1.46e+04±2.92e+02         | 3.22e+04±4.94e+02         | 3.91e+07±1.11e+07         | 2.31e+08±7.26e+07         | 1.78e+09±2.94e+08         |
|     | F66                       | F67                       | F68                       | F69                       | F70                       |
| COA | 7.09e+02±1.52e+00         | 7.42e+02±1.80e+01         | 8.51e+02±2.58e+01         | <b>3.57e+03±1.52e+03</b>  | <b>6.76e+04±4.29e+04</b>  |
| PSO | 7.21e+02±5.51e+00         | 7.84e+02±3.39e+01         | 9.50e+02±9.38e+01         | 6.96e+04±1.69e+05         | 1.75e+05±2.26e+05         |
| ABC | <b>7.08e+02±9.85e-01</b>  | <b>7.18e+02±2.75e+00</b>  | <b>7.99e+02±1.88e+01</b>  | 4.04e+05±1.44e+05         | 3.72e+06±1.27e+06         |
| SOS | 7.12e+02±2.44e+00         | 7.58e+02±1.49e+01         | 8.40e+02±3.04e+01         | 3.01e+04±1.89e+04         | 2.04e+05±1.07e+05         |
| GWO | 7.19e+02±2.50e+00         | 7.85e+02±2.63e+01         | 1.02e+03±6.65e+01         | 1.96e+05±1.50e+05         | 1.96e+06±1.40e+06         |
| BA  | 1.09e+03±1.06e+02         | 1.59e+03±3.05e+02         | 3.59e+03±8.06e+02         | 1.23e+07±1.24e+07         | 4.04e+07±4.24e+07         |
| FA  | 1.15e+03±7.71e+01         | 2.81e+03±3.58e+02         | 1.48e+04±1.89e+03         | 6.96e+06±2.73e+06         | 7.71e+07±2.30e+07         |
|     | F71                       | F72                       | F73                       | F74                       | F75                       |
| COA | <b>3.60e+05±2.02e+05</b>  | 1.00e+03±2.41e-01         | 1.01e+03±3.53e-01         | <b>1.01e+03±6.57e-01</b>  | <b>7.52e+03±4.97e+03</b>  |
| PSO | 7.76e+05±4.55e+05         | 1.03e+03±5.19e+01         | 1.11e+03±1.26e+02         | 1.47e+03±3.31e+02         | 4.50e+05±1.32e+06         |
| ABC | 1.61e+07±3.31e+06         | 1.00e+03± <b>2.17e-01</b> | 1.01e+03± <b>4.31e-01</b> | 1.01e+03± <b>5.77e-01</b> | 7.93e+05±3.71e+05         |
| SOS | 1.35e+06±6.63e+05         | <b>1.00e+03±2.77e-01</b>  | <b>1.01e+03±4.32e-01</b>  | 1.05e+03±1.77e+02         | 2.67e+04±1.81e+04         |
| GWO | 1.22e+07±6.07e+06         | 1.01e+03±3.60e+01         | 1.03e+03±4.49e+01         | 1.15e+03±1.52e+02         | 1.68e+06±1.14e+06         |
| BA  | 8.96e+07±6.64e+07         | 1.40e+03±8.21e+01         | 1.90e+03±1.83e+02         | 3.51e+03±5.24e+02         | 4.42e+07±4.68e+07         |
| FA  | 6.74e+08±1.86e+08         | 1.42e+03±3.72e+01         | 1.95e+03±7.71e+01         | 4.03e+03±2.35e+02         | 2.39e+07±1.17e+07         |
|     | F76                       | F77                       | F78                       | F79                       | F80                       |
| COA | 1.15e+04±1.23e+04         | 6.86e+04±3.21e+04         | 1.58e+03±2.59e+02         | 2.34e+03±2.63e+02         | 3.77e+03±4.75e+02         |
| PSO | 4.72e+05±8.52e+05         | 9.77e+06±1.46e+07         | 1.99e+03±3.26e+02         | 2.77e+03±1.71e+02         | 4.36e+03±9.88e+02         |
| ABC | 1.52e+06±6.15e+05         | 1.83e+06±6.59e+05         | <b>1.41e+03±5.18e+01</b>  | <b>1.44e+03±8.61e+00</b>  | <b>1.52e+03±2.42e+01</b>  |
| SOS | <b>5.63e+03±9.57e+03</b>  | <b>5.69e+03±5.54e+02</b>  | 1.68e+03±1.77e+02         | 2.09e+03±1.36e+02         | 3.51e+03±1.92e+02         |
| GWO | 2.75e+06±2.47e+06         | 3.48e+07±2.84e+07         | 1.78e+03±1.09e+02         | 2.21e+03±9.80e+01         | 3.56e+03±1.72e+02         |
| BA  | 7.56e+07±6.51e+07         | 3.23e+08±2.48e+08         | 3.20e+03±4.53e+02         | 4.45e+03±4.45e+02         | 9.87e+03±1.43e+03         |
| FA  | 1.66e+08±5.45e+07         | 1.64e+09±3.02e+08         | 2.79e+03±9.73e+01         | 3.95e+03±6.57e+01         | 7.87e+03±3.55e+02         |

TABLE IV  
AVERAGE ERRORS±STANDARD DEVIATIONS (PART III)

|     | F81                       | F82                       |
|-----|---------------------------|---------------------------|
| COA | <b>1.31e+03±7.42e-01</b>  | 1.38e+03±4.12e+01         |
| PSO | 1.33e+03±3.00e+01         | 1.39e+03±2.40e+01         |
| ABC | 1.31e+03±3.75e-01         | <b>1.31e+03±5.96e-01</b>  |
| SOS | 1.31e+03±1.72e+01         | 1.38e+03±3.68e+01         |
| GWO | 1.32e+03±2.89e+01         | 1.36e+03±4.53e+01         |
| BA  | 1.41e+03±1.09e+01         | 1.42e+03±7.25e+00         |
| FA  | 1.40e+03±4.69e+00         | 1.43e+03±5.15e+00         |
|     | F83                       | F84                       |
| COA | 1.40e+03±1.53e+01         | 1.30e+03±4.30e-04         |
| PSO | 1.42e+03±1.01e+01         | 1.30e+03±9.88e-01         |
| ABC | <b>1.32e+03±6.71e-01</b>  | <b>1.30e+03±3.24e-04</b>  |
| SOS | 1.39e+03±3.18e+01         | 1.30e+03±1.12e-03         |
| GWO | 1.37e+03±3.55e+01         | 1.30e+03±1.30e-02         |
| BA  | 1.44e+03±1.01e+01         | 1.69e+03±2.35e+02         |
| FA  | 1.51e+03±1.28e+01         | 1.41e+03±4.35e+01         |
|     | F85                       | F86                       |
| COA | <b>1.30e+03±4.36e-03</b>  | <b>1.30e+03±9.88e-04</b>  |
| PSO | 1.30e+03±2.95e+00         | 1.31e+03±2.03e+01         |
| ABC | 1.30e+03±4.55e-03         | 1.30e+03±1.31e-03         |
| SOS | 1.30e+03±8.13e-03         | 1.30e+03±3.37e-03         |
| GWO | 1.30e+03±8.43e-02         | 1.30e+03±1.81e-01         |
| BA  | 2.14e+03±3.99e+02         | 4.61e+03±1.21e+03         |
| FA  | 1.94e+03±1.70e+02         | 4.62e+03±6.36e+02         |
|     | F87                       | F88                       |
| COA | 3.47e+04± <b>6.29e+02</b> | 6.42e+04±5.81e+03         |
| PSO | 4.37e+04±5.95e+03         | 1.09e+05±1.90e+04         |
| ABC | <b>3.18e+04±5.11e+03</b>  | <b>5.10e+04±4.76e+01</b>  |
| SOS | 3.47e+04±1.49e+03         | 6.99e+04±9.20e+03         |
| GWO | 3.54e+04±1.10e+03         | 7.74e+04±5.08e+03         |
| BA  | 1.05e+05±1.74e+04         | 2.25e+05±2.25e+04         |
| FA  | 9.34e+04±4.61e+03         | 2.30e+05±1.49e+04         |
|     | F89                       | F90                       |
| COA | 1.10e+05±4.91e+01         | 1.60e+03±1.88e-03         |
| PSO | 3.27e+05±6.22e+04         | 1.61e+03±5.98e+00         |
| ABC | <b>1.10e+05±1.01e+01</b>  | 1.60e+03±6.87e-13         |
| SOS | 1.14e+05±1.14e+04         | <b>1.60e+03±4.98e-13</b>  |
| GWO | 1.77e+05±8.71e+03         | 1.62e+03±8.20e+00         |
| BA  | 4.79e+05±3.45e+04         | 4.92e+04±3.83e+04         |
| FA  | 7.23e+05±8.65e+04         | 1.42e+05±4.74e+04         |
|     | F91                       | F92                       |
| COA | 1.60e+03±7.28e-01         | <b>1.61e+03±1.15e+00</b>  |
| PSO | 1.61e+03±1.49e+01         | 2.52e+03±4.87e+03         |
| ABC | <b>1.60e+03±3.76e-01</b>  | 1.61e+03± <b>4.52e-01</b> |
| SOS | 1.60e+03±7.88e-01         | 1.61e+03±2.04e+00         |
| GWO | 1.67e+03±8.33e+01         | 2.78e+03±5.30e+02         |
| BA  | 1.01e+05±6.22e+04         | 2.02e+05±9.37e+04         |
| FA  | 8.06e+05±1.38e+05         | 4.64e+06±7.27e+05         |

In order to analyze the significance of the results obtained, the non-parametric statistical significance tests of Wilcoxon (also known as Wilcoxon-Mann-Whitney) and Friedman [30] have been used in this paper. The Tab. V contains the  $p$ -values calculated with the COA as the control method using, where the corrected values have been obtained using the Holm *post-hoc* procedure [31]. It can be seen that there is significant difference in all cases for an  $\alpha$  level of 0.05, which means that the COA's performance is statistically better than the other algorithms for the employed experimental design.

Regarding the exploration and exploitation balance of the COA, an example of the convergence and the diversity be-

TABLE V  
NON-PARAMETRIC SIGNIFICANCE STATISTICAL TESTS

| vs. | Wilcoxon |           | vs. | Friedman |           |
|-----|----------|-----------|-----|----------|-----------|
|     | $pvalue$ | Corrected |     | $pvalue$ | Corrected |
| ABC | 2.21E-04 | 2.21E-04  | ABC | 3.71E-02 | 3.71E-02  |
| SOS | 9.85E-05 | 1.97E-04  | SOS | 6.71E-03 | 1.34E-02  |
| GWO | 4.31E-18 | 1.29E-17  | GWO | 1.21E-14 | 3.63E-14  |
| PSO | 4.04E-20 | 1.61E-19  | BA  | 2.31E-15 | 9.23E-15  |
| BA  | 5.62E-25 | 2.81E-24  | PSO | 4.22E-16 | 2.11E-15  |
| FA  | 1.22E-33 | 7.31E-33  | FA  | 8.67E-22 | 5.20E-21  |

haviour has been plotted on Fig. IV and Fig. IV, respectively. The curves (collected after the first iteration, re-sampled and in log scale for a better view) represent the average values of all experiments and the diversity has been computed as the average distance from the solutions to the mean of all solutions divided by the maximum diagonal of the search space [32]. This example has been extracted from the function F71, which is a non-separable and multimodal benchmark named "Shifted and Rotated Expanded Griewank's plus the Rosenbrock's Function" with dimension equals to 100. It is important to highlight the COA's ability of keeping higher diversity values and simultaneously converge to the smallest cost among all metaheuristics.

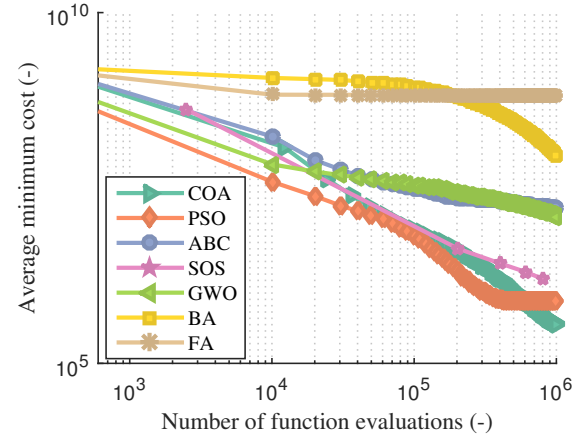


Fig. 2. Convergence curve of function F71.

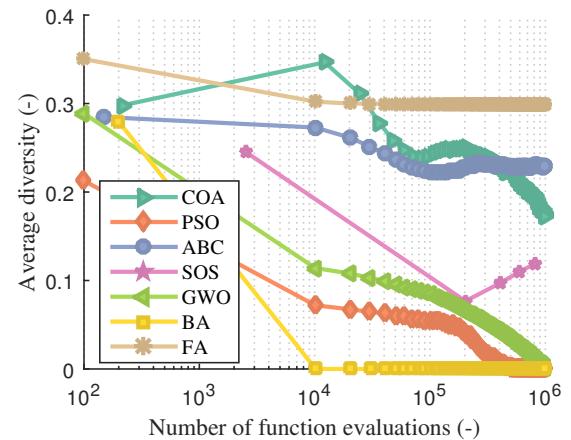


Fig. 3. Diversity curve of function F71.

## V. CONCLUSION AND FUTURE RESEARCH

In this paper, the new bio-inspired Coyote Optimization Algorithm (COA) has been proposed to solve boundary con-



strained global optimization problems with continuous variables. This new method has been inspired on the social life of the *canis latrans* species and it has only two parameters, the number of packs and the number of coyotes per pack, which defines the population size.

The performance of the COA has been evaluated under 40 benchmark functions with different features as multimodality, separability and the number of optimized variables. The total 92 case studies have supported the non-parametric statistical significance tests of Wilcoxon-Mann-Whitney and Friedman to show that the COA has outperformed other bio-inspired metaheuristics from literature.

This paper is a preliminary study opening up a wide range of possibilities for further improvement and extension. In the future research, the COA structure and its parameters will be analyzed to improve its performance. Moreover, an adaptive version of the COA and the multiobjective version of this metaheuristic might be tested for large-scale global optimization problems.

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