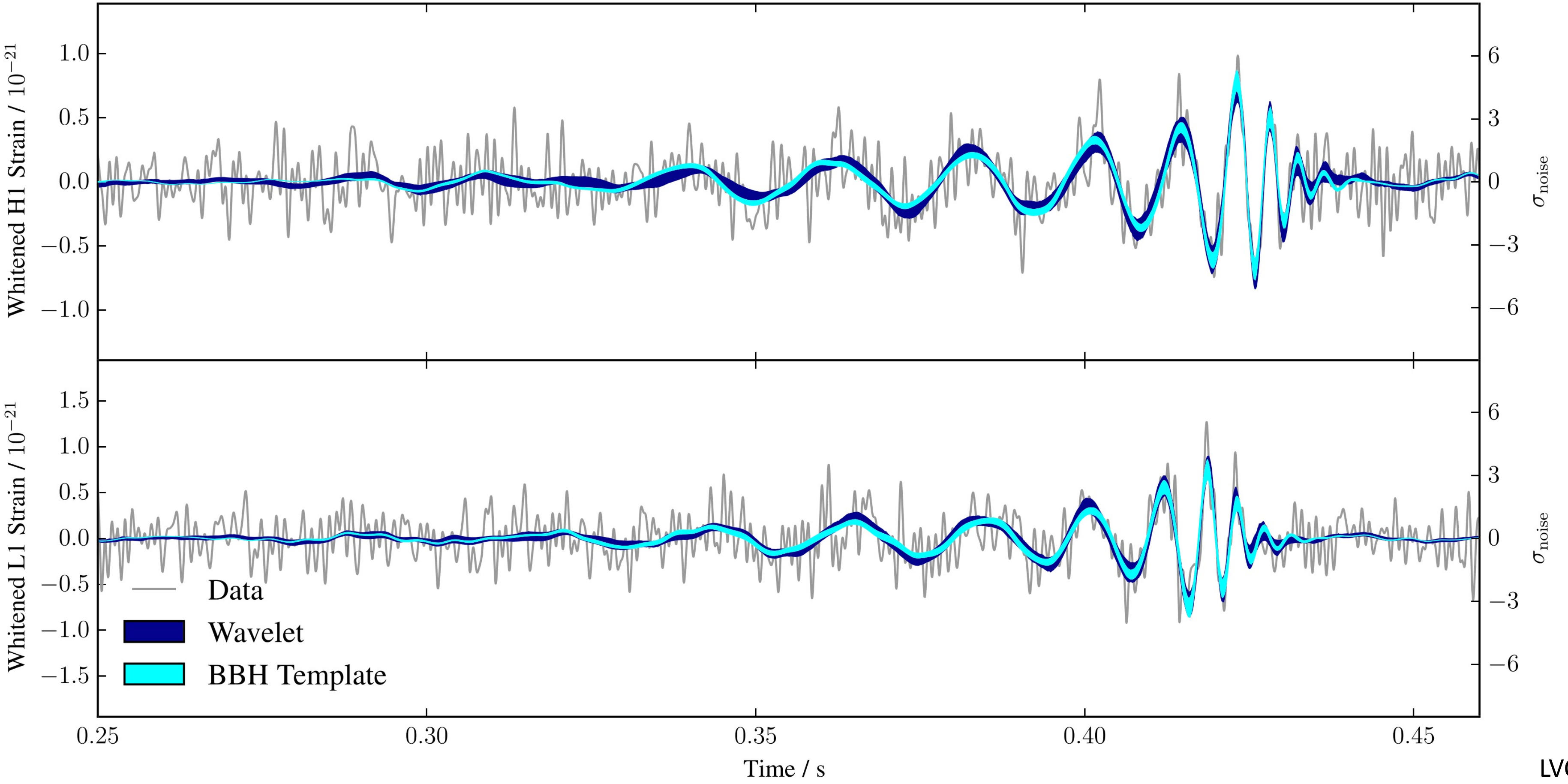
An overview of gravitational wave surrogate modelling

CARL-JOHAN HASTER

SCIENTIFIC MACHINE LEARNING FOR GRAVITATIONAL WAVE ASTRONOMY

3 JUNE 2025

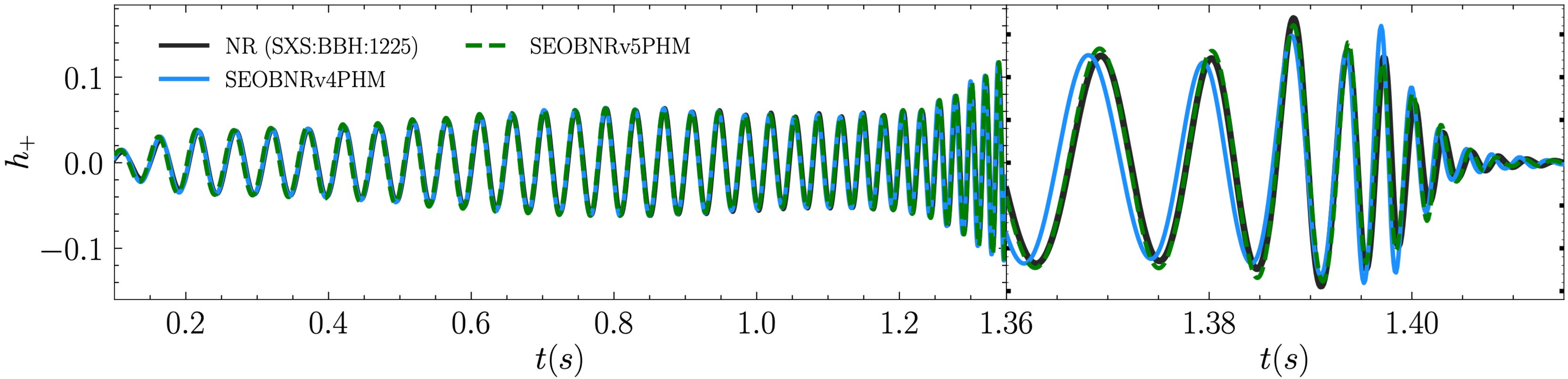
A gravitational wave



LVC, PRL 116, 241102 (2016)

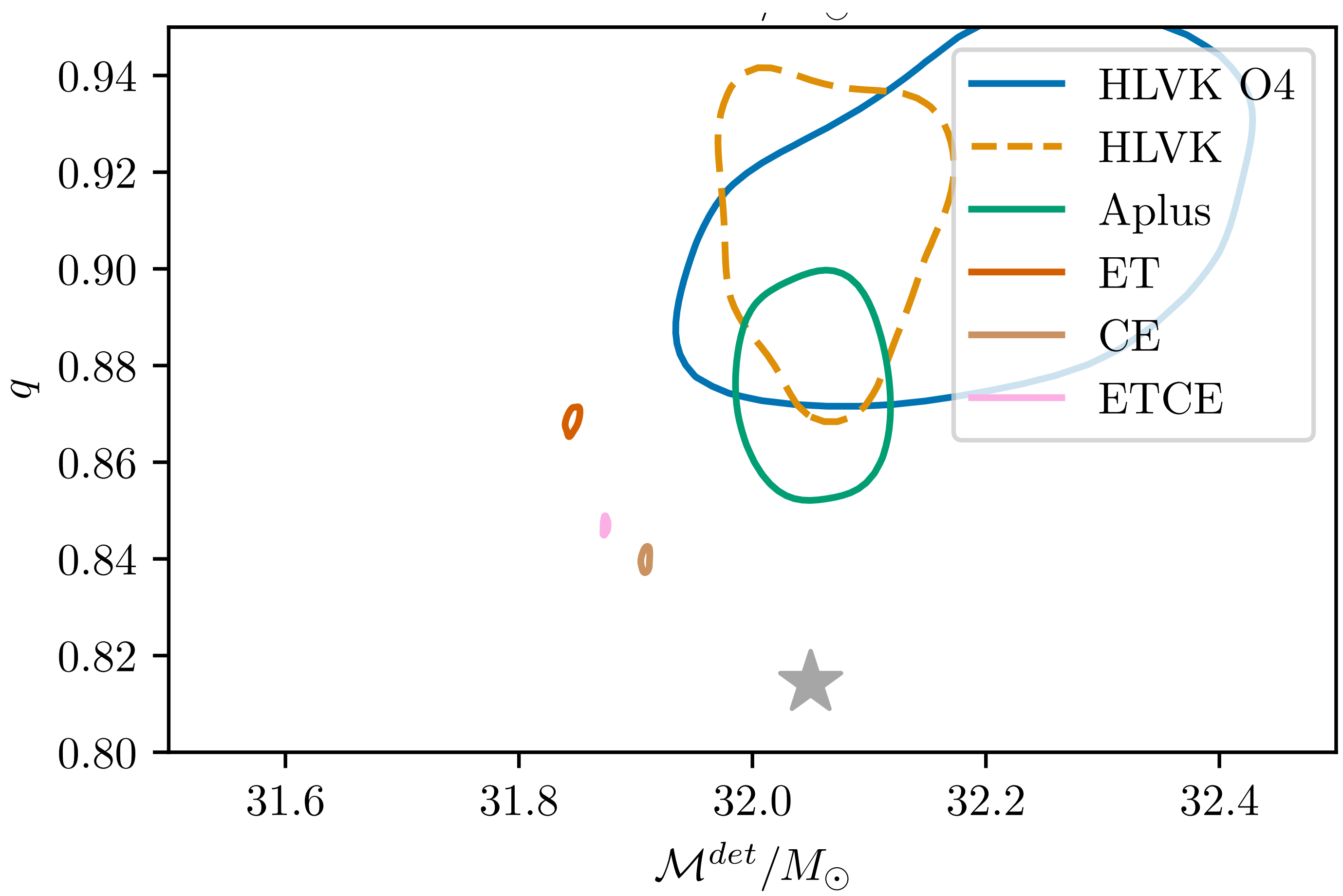
Infer physics

NEED TO KNOW HOW TO CONNECT INFERRED WAVEFORM TO PHYSICAL DESCRIPTION OF SOURCE



How well do we need to know the physics → waveform mapping?

Ramos-Buades et al., PRD 108, 124037 (2023)

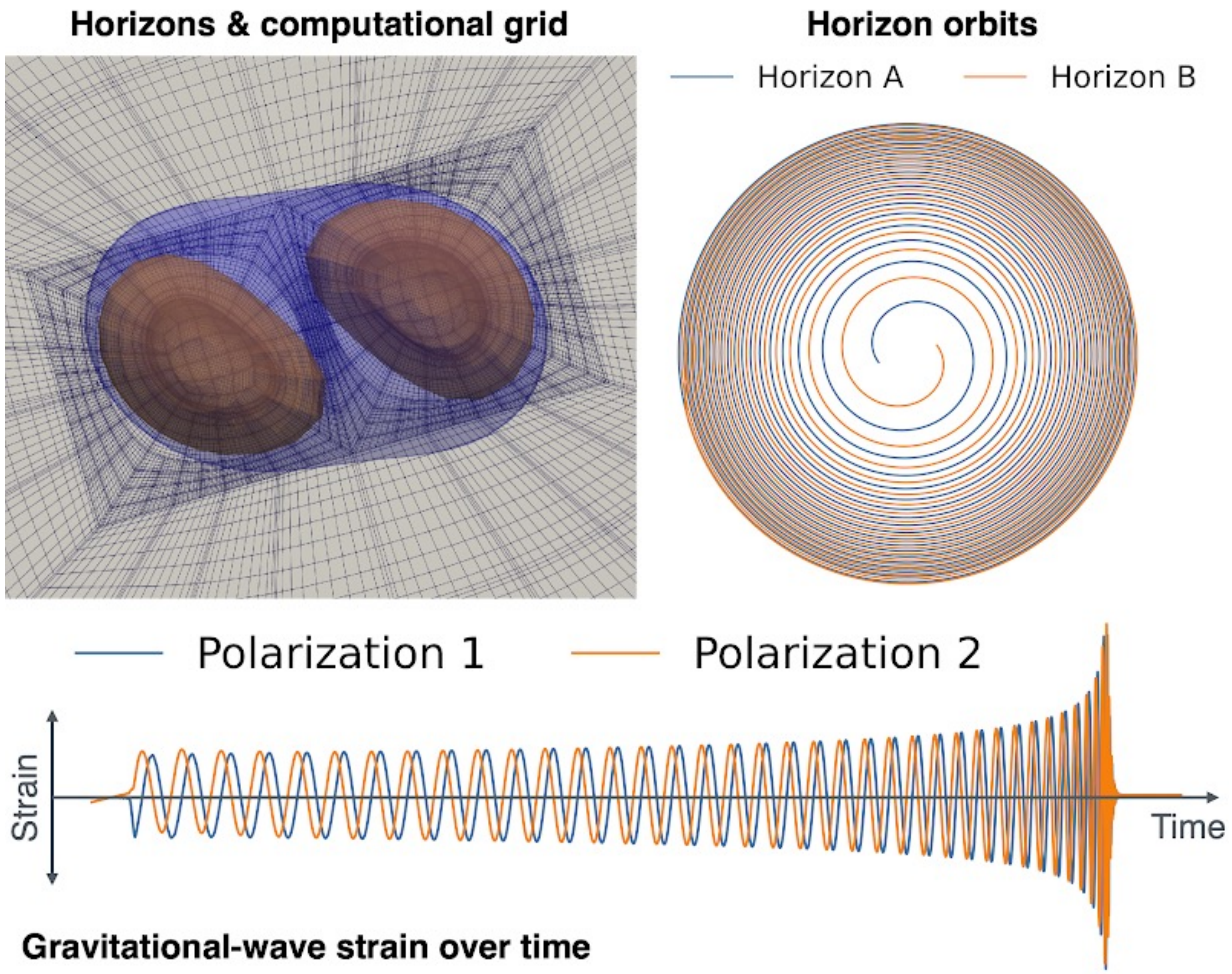


How well do we need to know the physics → waveform mapping?

Langgin, Pürrer, Haster (in prep)

We need good waveforms

AND WE NEED THEM NOW



Numerical Relativity

Image: Geoffrey Lovelace, CSUF

GWs encode lots of exciting physics

Assuming GR is correct:

- All physics describing the complete observable configuration of a Black Hole Binary is encoded in the emitted gravitational waves (and nothing more)
- If we can resolve the complete waveform structure, we've learned everything we need

- Spend **ALL** computing on making NR waveforms for all binary parameters!

Surrogates

ALL THE GOOD STUFF, BUT FOR CHEAP!

Surrogate modelling

Get as much physics (aka waveform structure) from the fewest number of computational operations.

Most convenient method:

- Construct some set of basis functions
- Multiply them by some relevant coefficients
- Sum it up

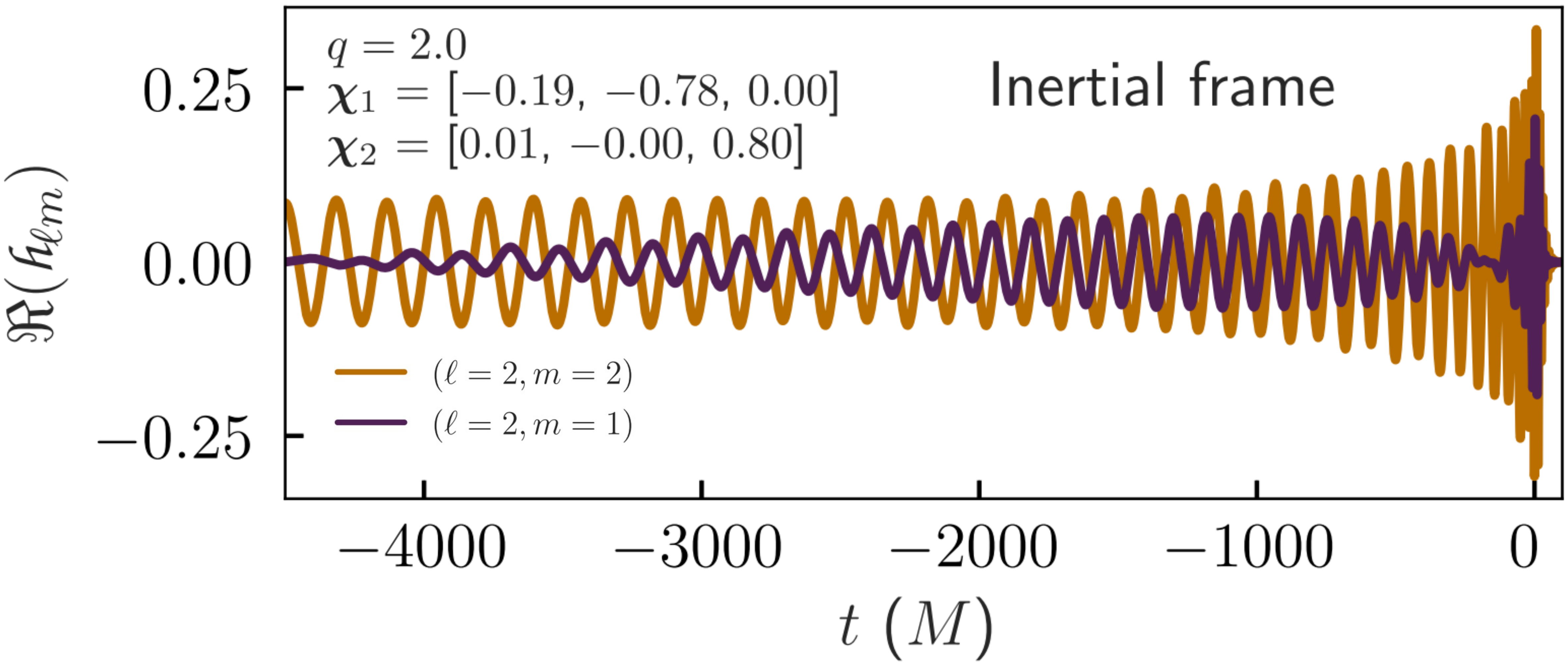
→ Science

Worth trying to be a bit clever in how to setup the problem though…

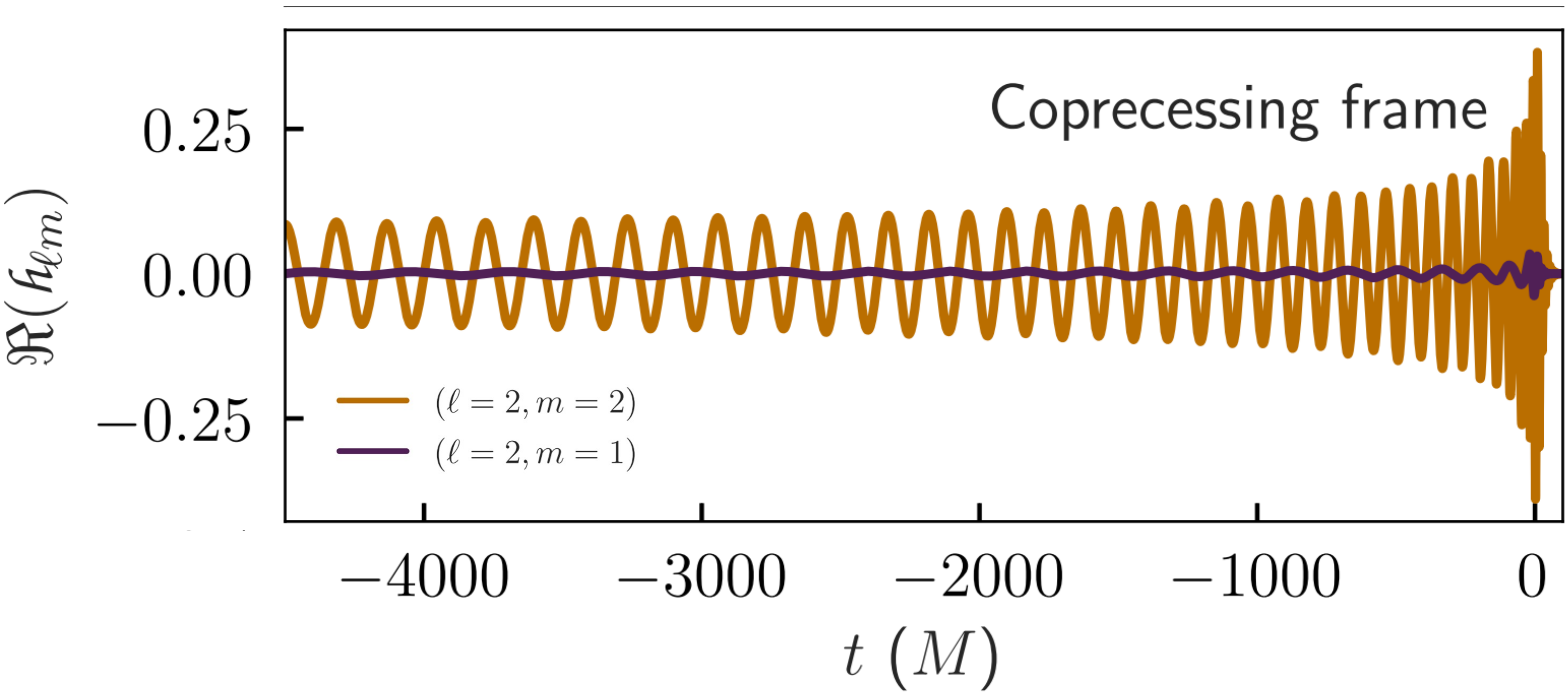
Some notation conventions

$$\mathcal{h} = h_+ - i h_\times$$

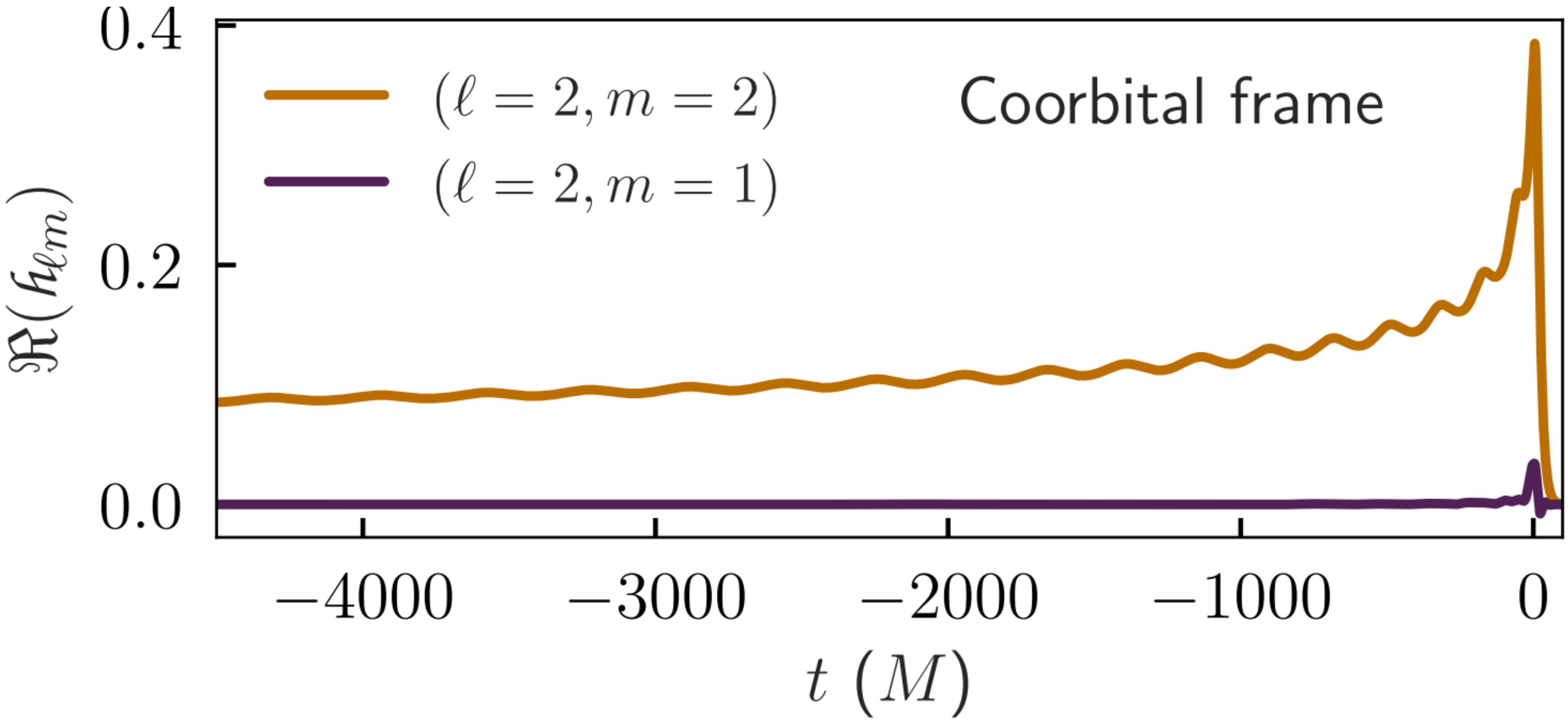
$$\mathcal{h}(t, \iota, \varphi_0) = \sum_{\ell=2}^{\infty} \sum_{m=-l}^{l} \mathcal{h}_{\ell m}(t) \, {}_{-2}Y_{\ell m}(\iota, \varphi_0)$$



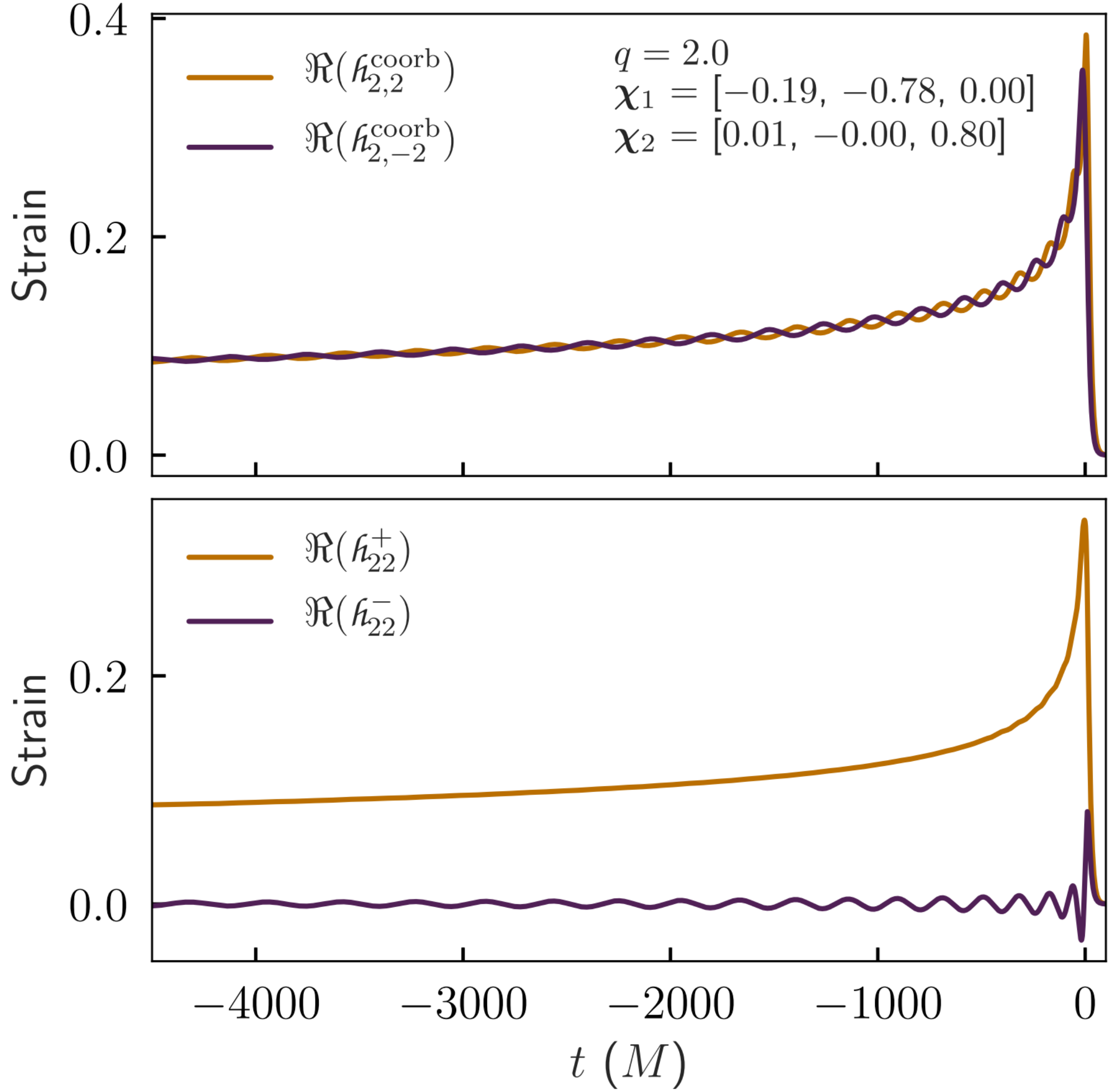
Varma et al. PRR 1, 033015 (2019)



Varma et al. PRR 1, 033015 (2019)



Varma et al. PRR 1, 033015 (2019)

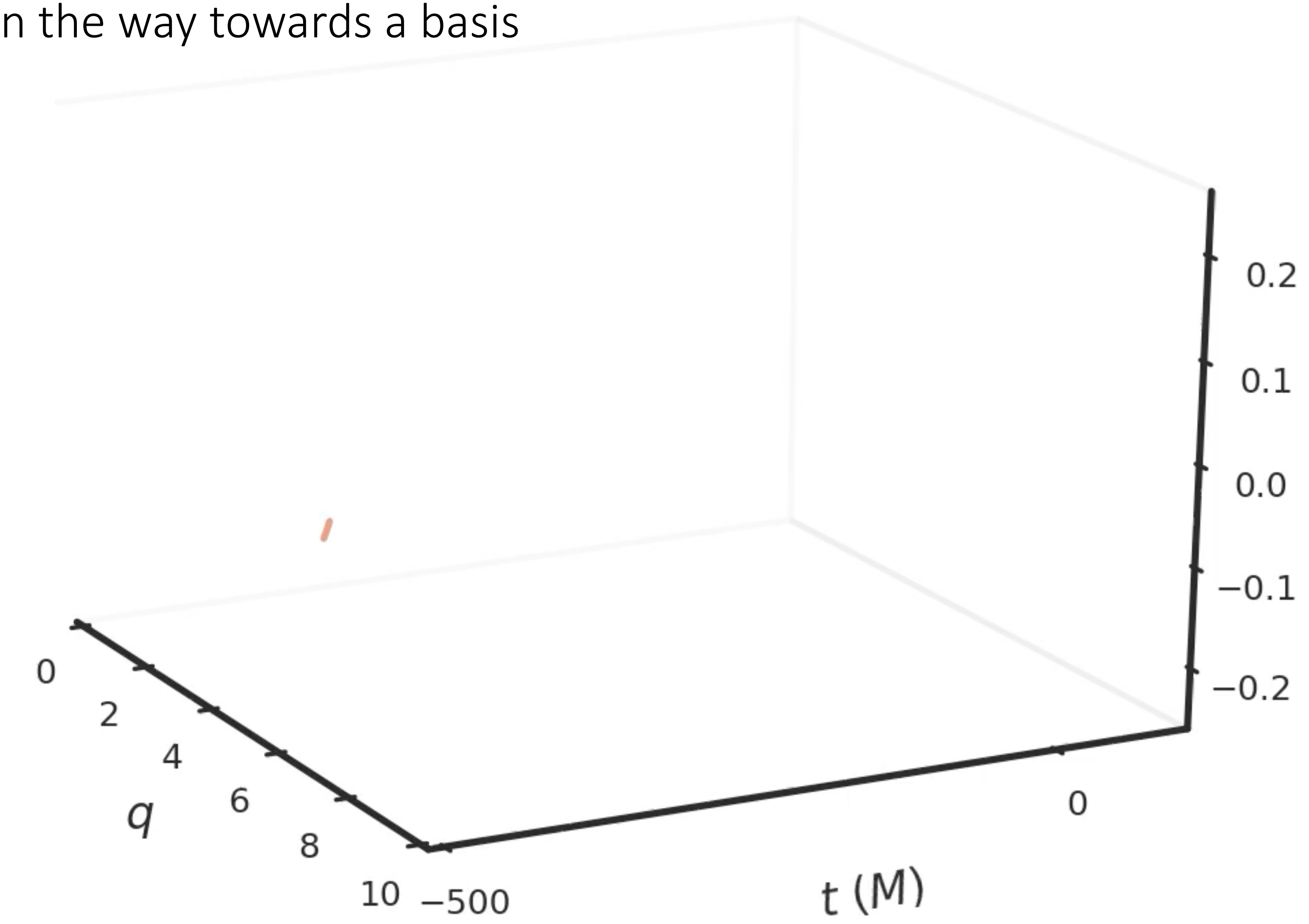


$$h^{\pm}_{\ell m} = \frac{h^{\rm coorb}_{\ell,m} \pm h^{\rm coorb\,*}_{\ell,-m}}{2}$$

The different parts of a surrogate model

VIDEOS COURTESY OF VIJAY

Waveforms – on the way towards a basis



Vijay 🙂 - https://vijayvarma392.github.io/SurrogateMovie/

Gram–Schmidt your way to success!

$$\mathrm{proj}_{\mathbf{u}}(\mathbf{v}) = \frac{\langle \mathbf{v}, \mathbf{u} \rangle}{\langle \mathbf{u}, \mathbf{u} \rangle} \mathbf{u}$$

$$\mathbf{u}_1 = \mathbf{v}_1,$$

$$\mathbf{u}_2 = \mathbf{v}_2 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_2),$$

$$\mathbf{u}_3 = \mathbf{v}_3 - \mathrm{proj}_{\mathbf{u}_1}(\mathbf{v}_3) - \mathrm{proj}_{\mathbf{u}_2}(\mathbf{v}_3),$$

How to pick your training set of waveforms

Greedy algoritm!

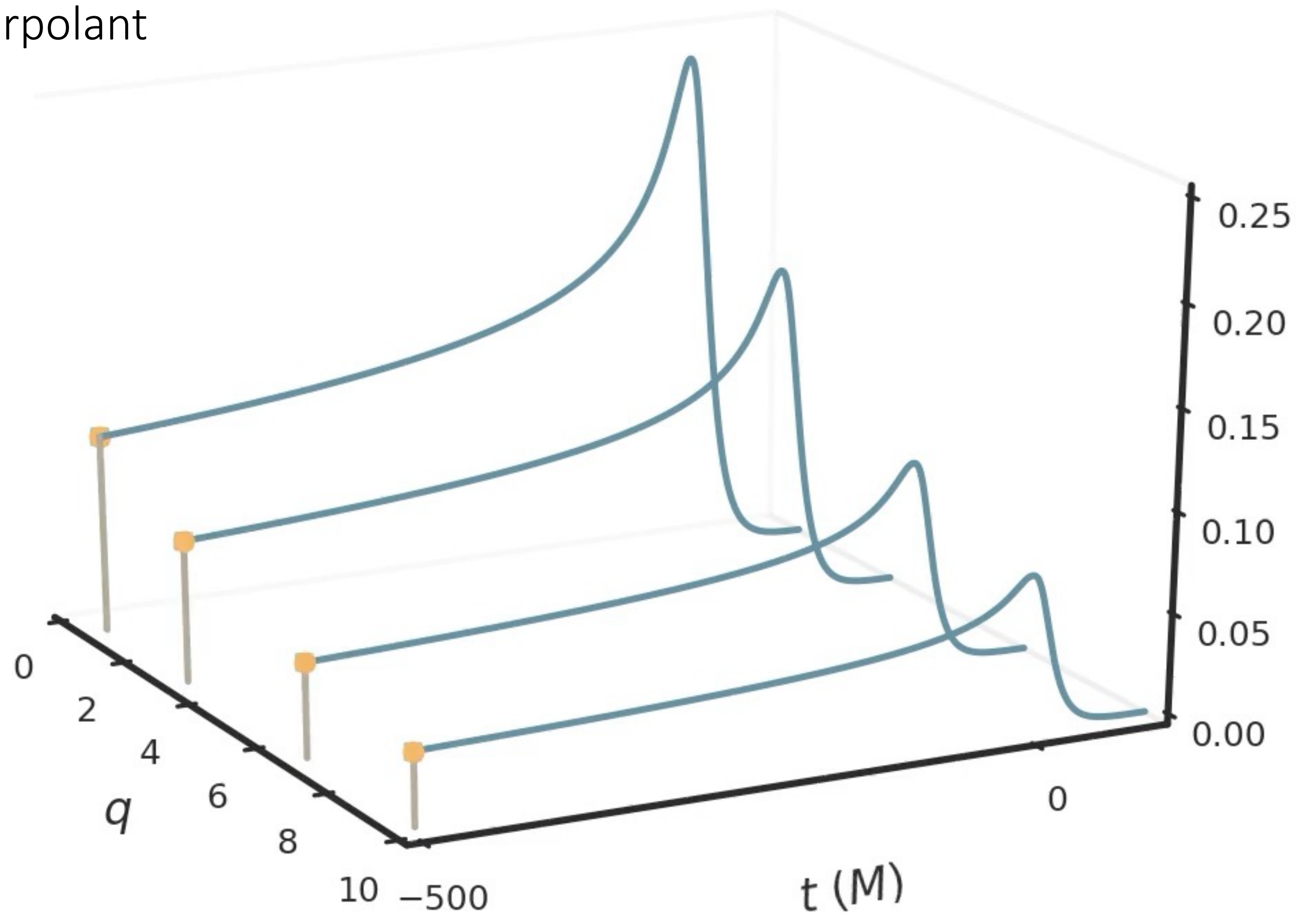
Build it iteratively (if you can)

The next point you add should be the one that provides the most new information

Stop adding more points when your basis is accurate enough

[but, consider redoing everything again after a verification step]

Empirical interpolant



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Algorithm 2 The Empirical Interpolation Method

1: **Input:** $\{\vec{e}_i\}_{i=1}^{m}$, $\{t_i\}_{i=1}^{L}$

2: $i = \text{argmax}|\vec{e}_1|$ (argmax returns the largest entry of its argument).

3: Set $T_1 = t_i$

4: **for** $j = 2 \to m$ **do**

5: Build $\mathcal{I}_{j-1}[e_j](\vec{t})$

6: $\vec{r} = \mathcal{I}_{j-1}[e_j](\vec{t}) - \vec{e}_j$

7: $i = \text{argmax}|\vec{r}|$

8: $T_j = t_i$

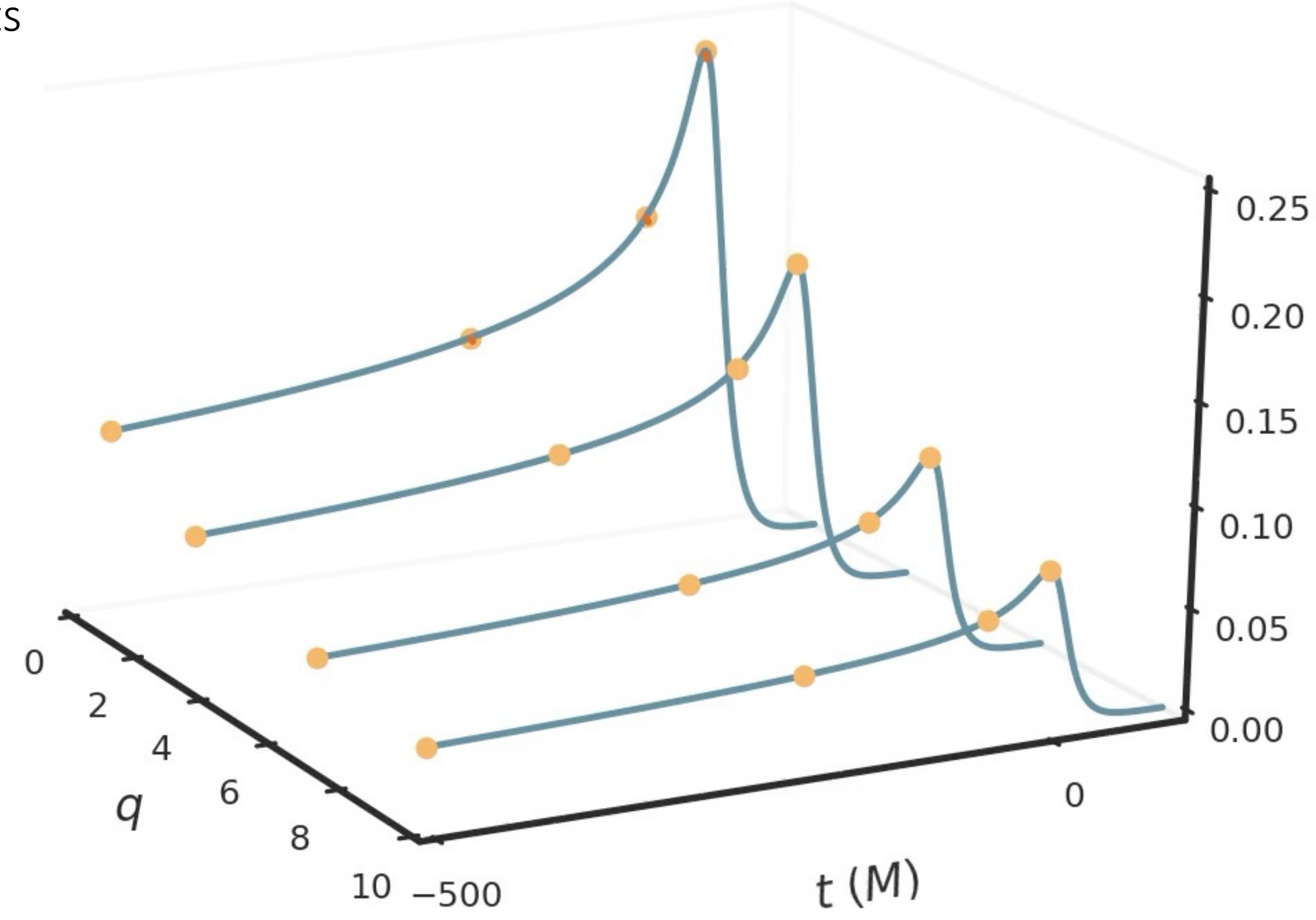
9: **end for**

10: **Output:** EIM nodes $\{T_i\}_{i=1}^{m}$ and interpolant $\mathcal{I}_m$

Algorithm time!

Field et al. PRX 4, 031006 (2014)

Parametric fits



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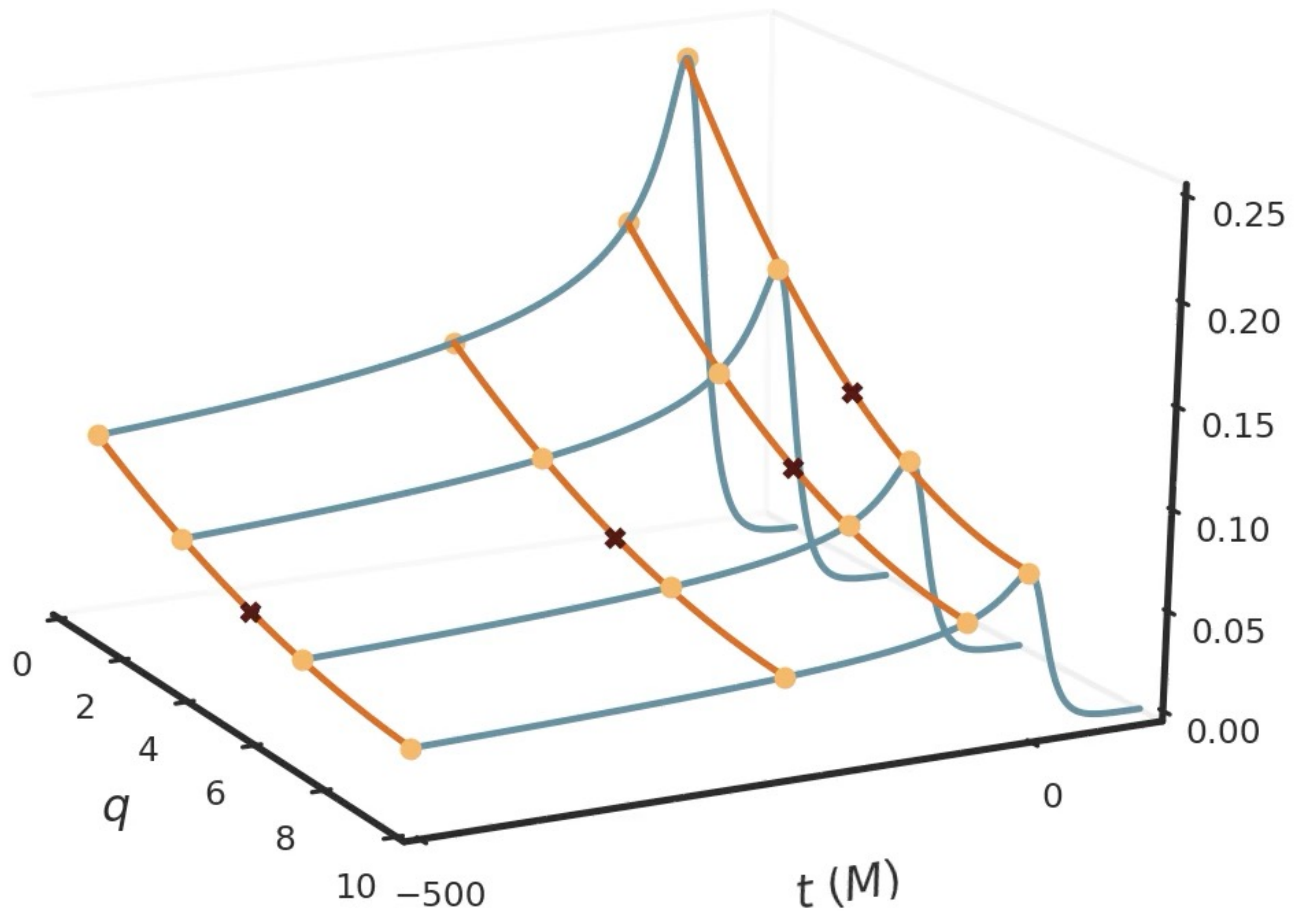
Parametric fits

Know the “basis coefficients” which for the EI-nodes will give you the waveforms from your training set

Find a (hopefully) smooth set of functions that describe these coefficients for a general set of binary parameters

- Polynomials/Splines
- Gaussian Process
- Neural Network

Pros and cons with all approaches



Vijay 🙂 - https://vijayvarma392.github.io/SurrogateMovie/

Science!

NOW WE HAVE A BASIS FOR THE AMPLITUDE OF THE (2,2)-MODE

MAKE OTHER BASES FOR REMAINING BITS OF THE WAVEFORM + ROTATIONS

Surrogates for other things?

NOT JUST WAVEFORMS

Likelihood of data given parameters

$$L(d|\theta) = e^{-\frac{\langle n|n\rangle}{2}}$$

$$\langle n|n\rangle = \langle d-h|d-h\rangle$$

$$\langle n|n\rangle = \langle d|d\rangle + \langle h|h\rangle - 2\langle d|h\rangle$$

$$\langle a|b\rangle = \sum_{i=1}^{M} \Delta f \frac{a^*(f_i)b(f_i)}{S_n(f_i)}$$

Reduced order quadrature likelihood

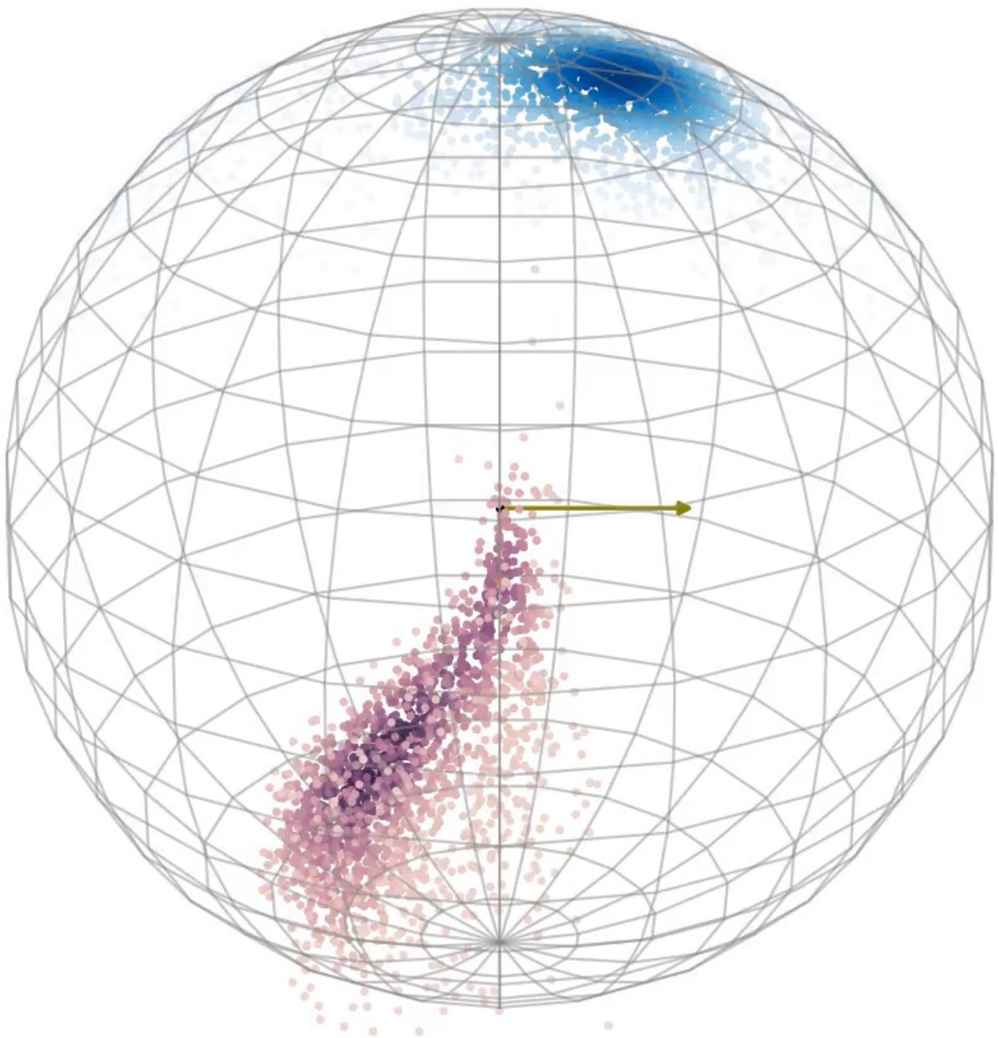
$$\langle d|h\rangle = \sum_{i=1}^{M} \Delta f \frac{d^*(f_i)}{S_n(f_i)} h(f_i)$$

$$\omega_j = \sum_{i=1}^{M} \Delta f \frac{d^*(f_i)}{S_n(f_i)} e_j(f_i)$$

N << M

$$\langle d|h\rangle_{\mathrm{ROQ}} \approx \sum_{j=1}^{N} \omega_j h(\nu_j)$$

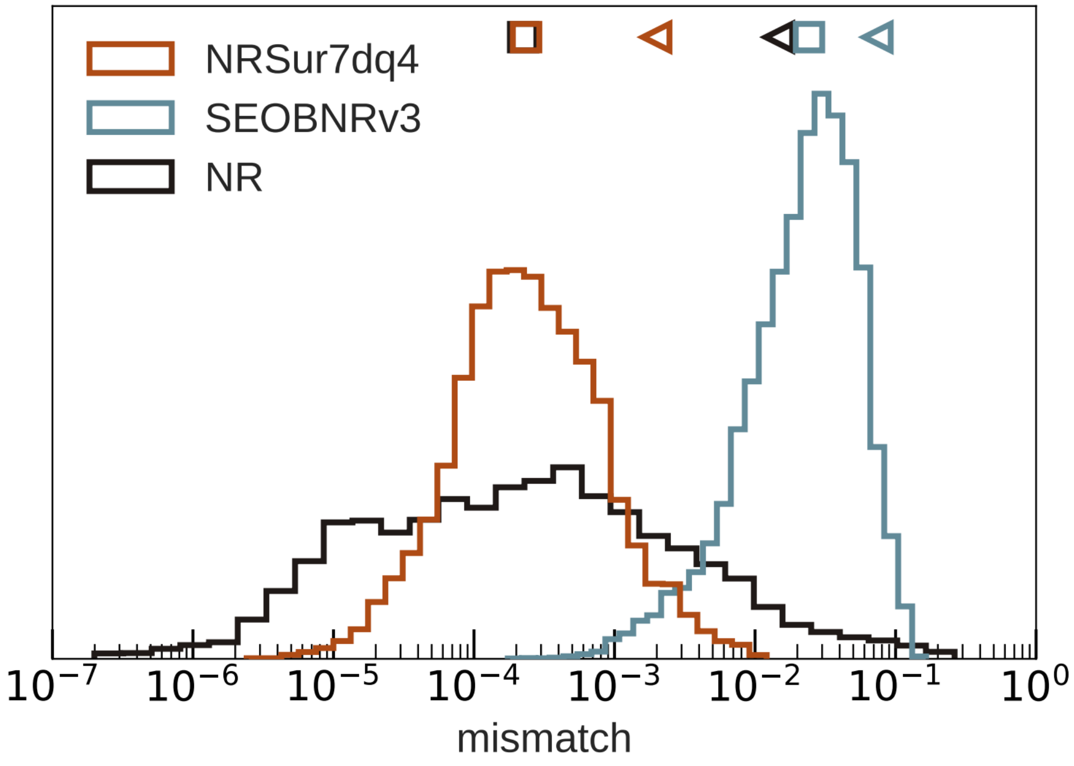
Smith et al.:
Phys. Rev. D 94, 044031 (2016)

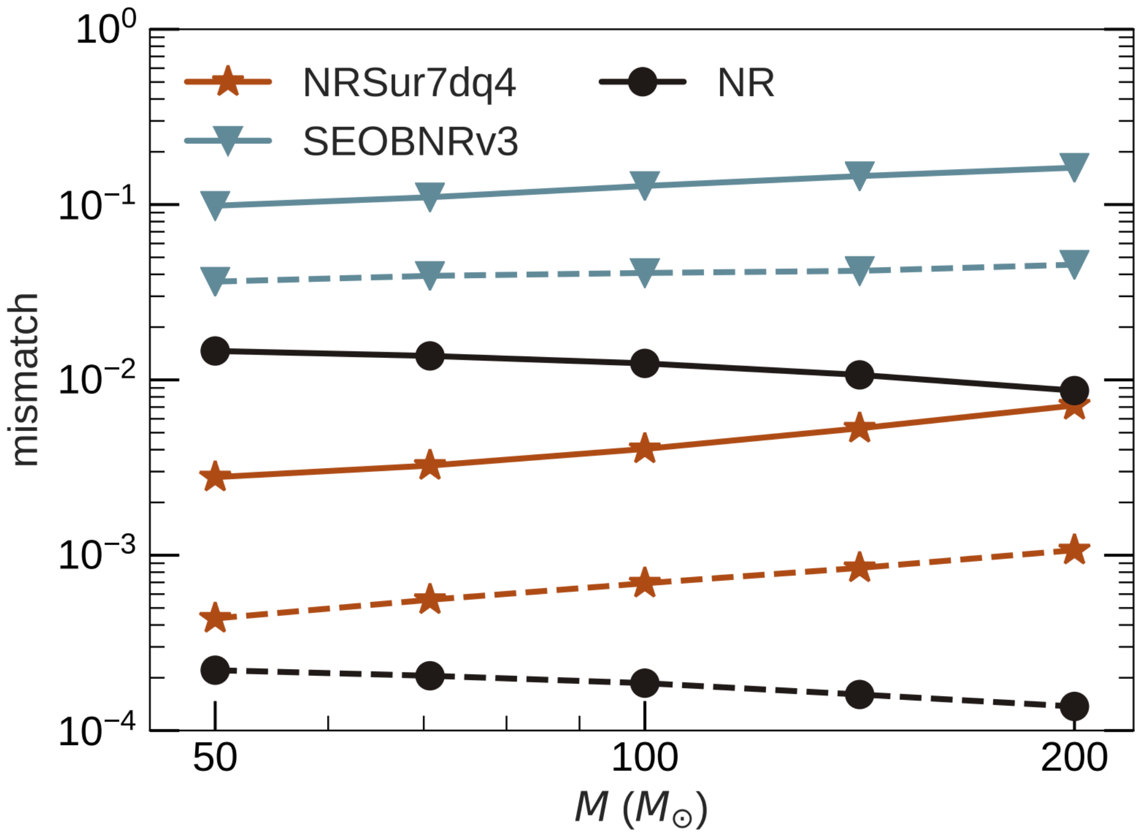


BBH remnant properties

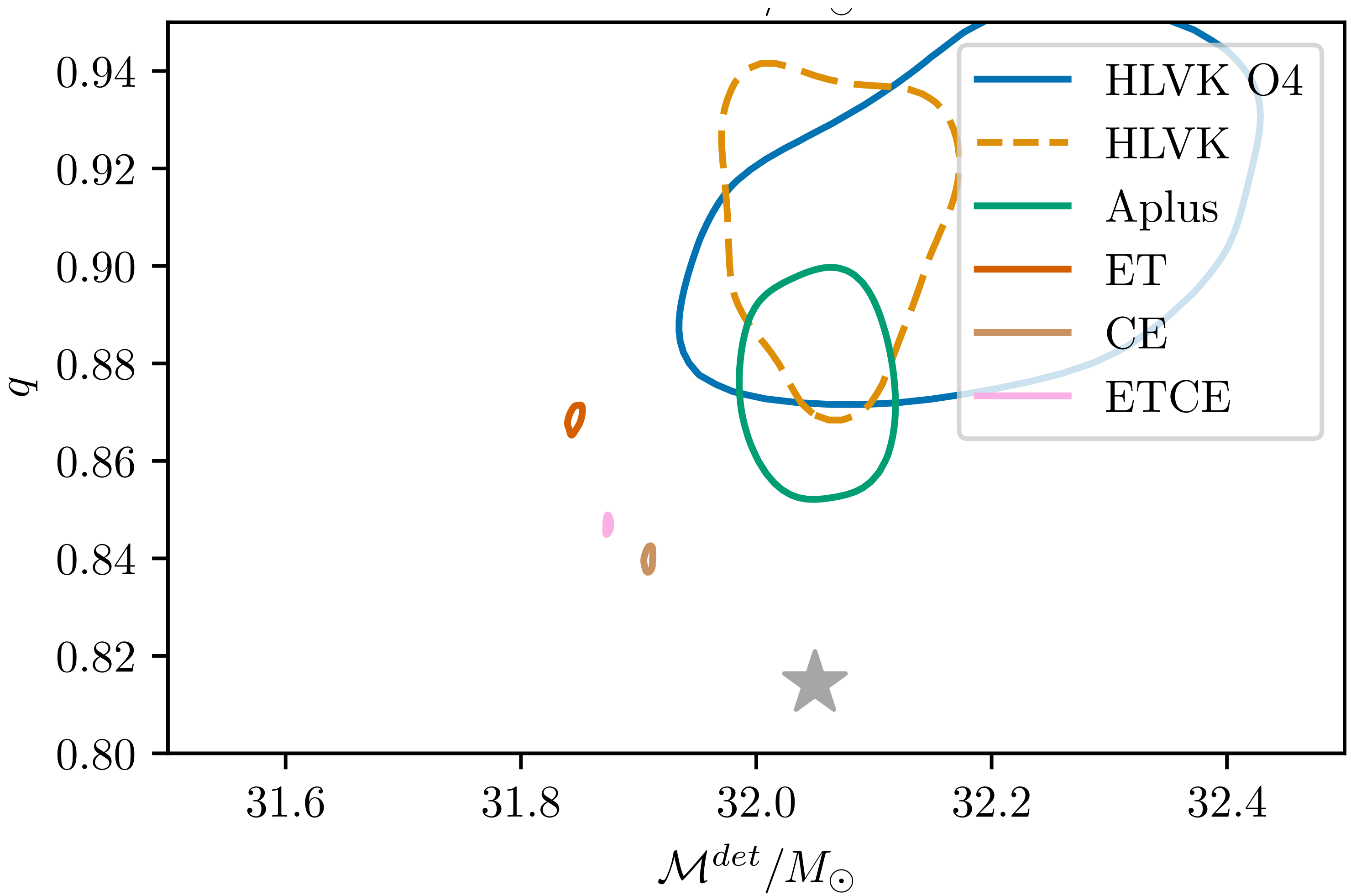
Varma et al. PRL 128, 191102 (2022)

Surrogates are good, but not perfect





Varma et al. PRR 1, 033015 (2019)



Langgin, Pürrer, Haster (in prep)

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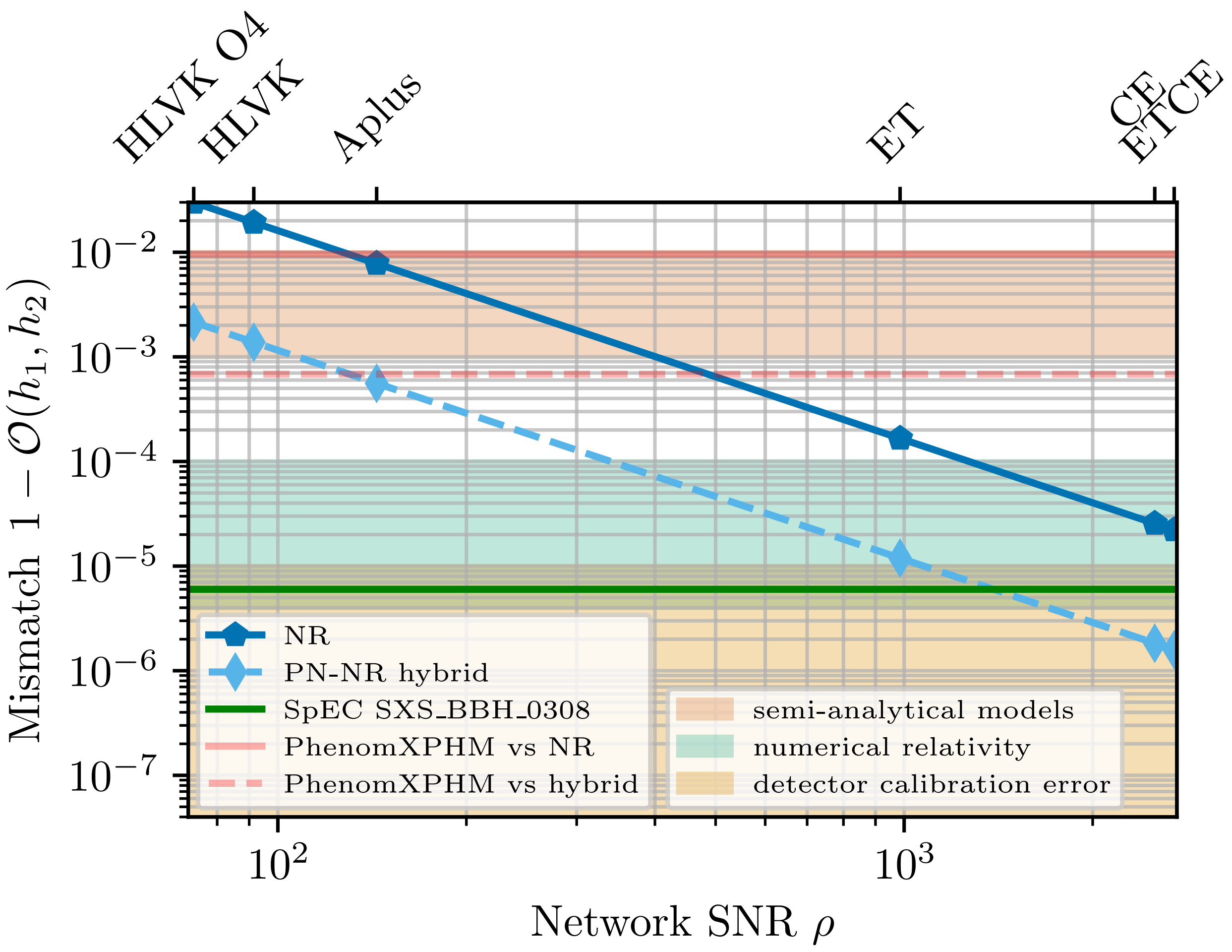

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How well do we need to know the physics → waveform mapping?

No model is perfect – marginalize over imperfections

Inaccuracies in the assumed model *will* lead to biased astrophysics

- Do we care?

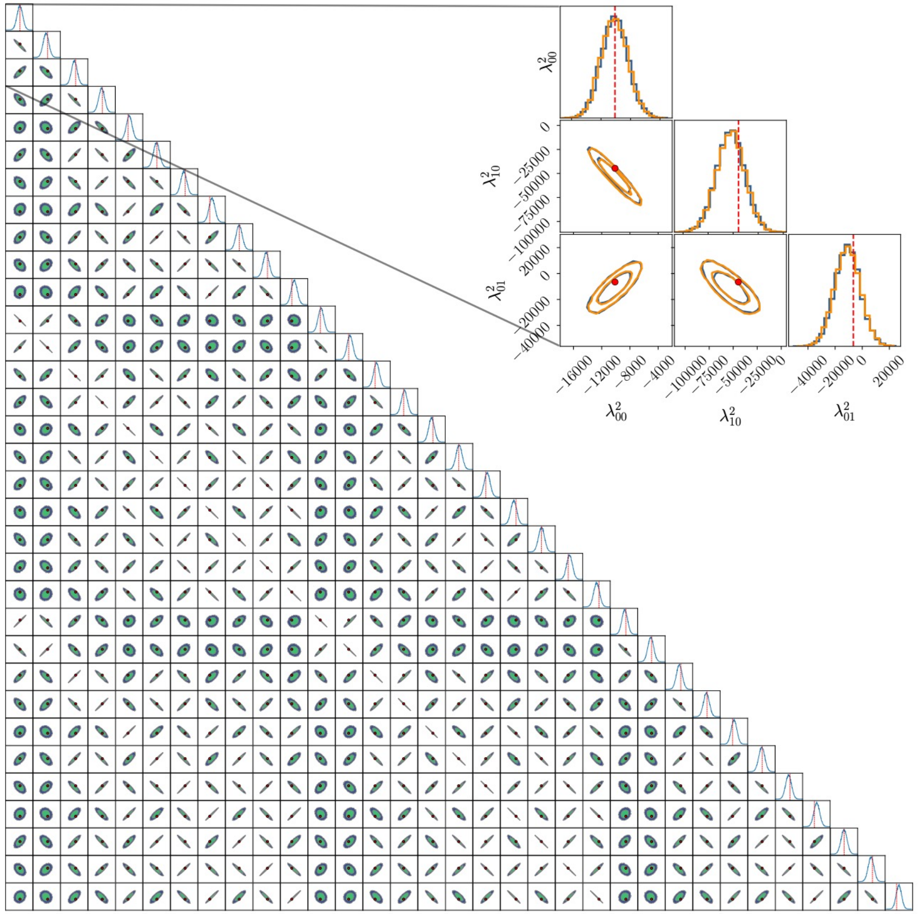
Quantify the impact on those inaccuracies → marginalize over them

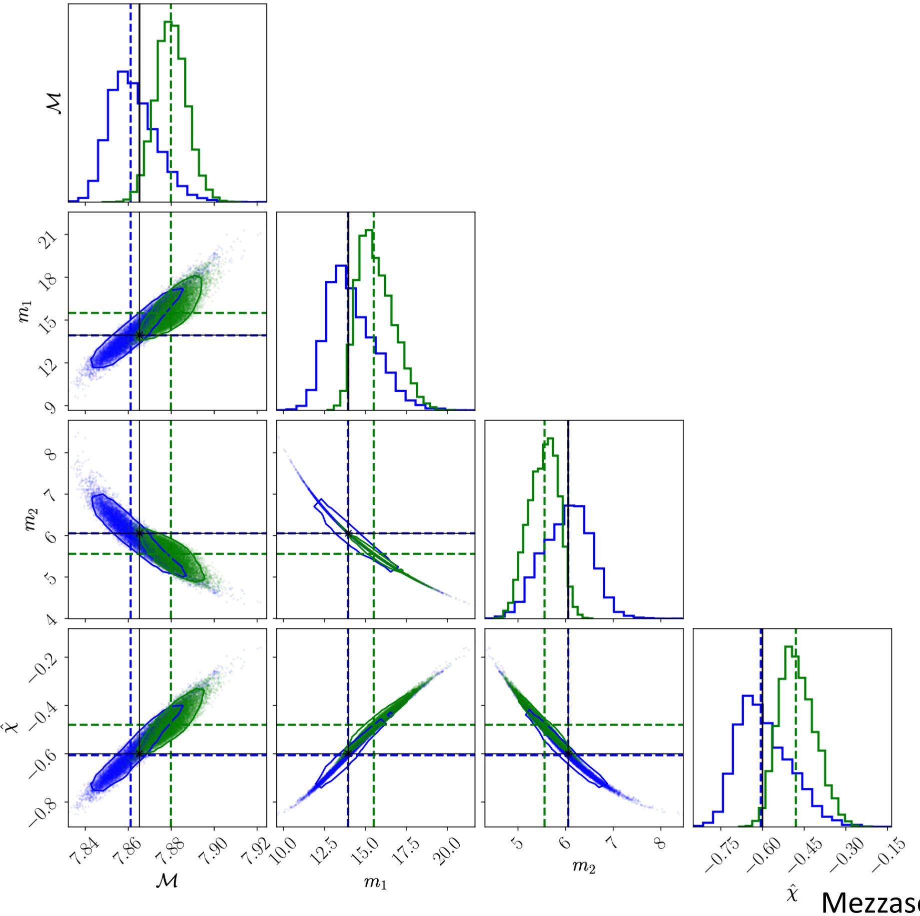
- Variability in parametric fits
- GPRs – does this by construction

One person's model bias is another person's beyondGR detection…

- Topic for discussion session later 😉

Shameless plug





Mezzasoma et al. - 2503.23304

Surrogates the whole way down?

COMPRESSED ROQ BASED OFF A SURROGATE WAVEFORM MODEL

(PLUS ACCOUNTING FOR MODEL UNCERTAINTY I GUESS)