GrEAT Synergy School

Data analysis: search and parameter estimation

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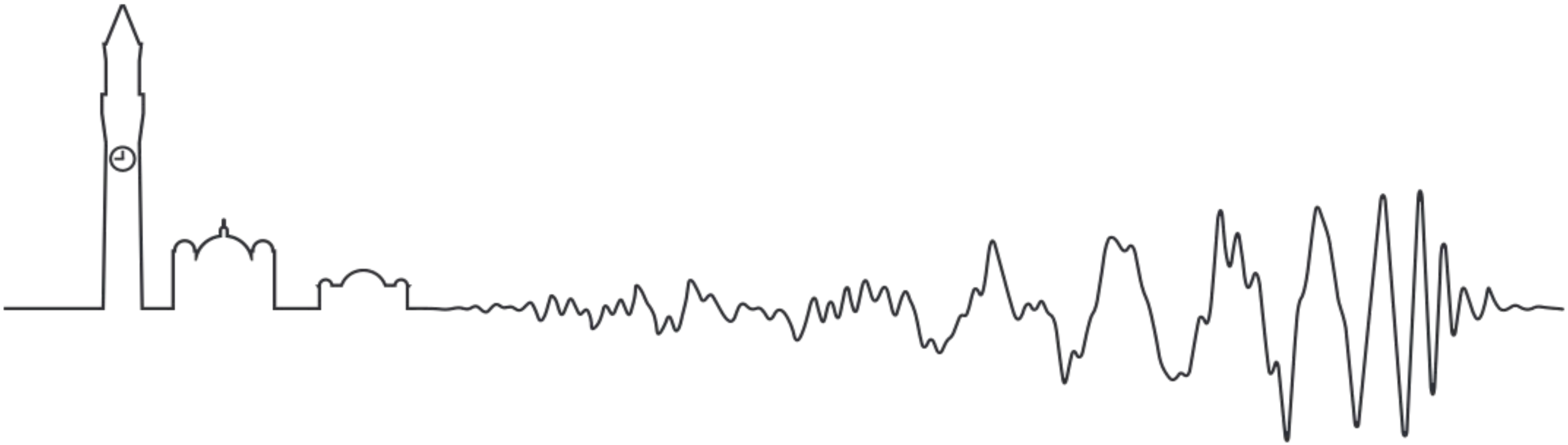
GRAVITATIONAL WAVE ASTRONOMY

Introduction

- What does GW data look like?   E.g. LIGO data
- Data analysis basics
- **(1)** Search, and **(2)** Characterisation of sources in data from ground-based detectors
- Extra challenges that come from trying this in space



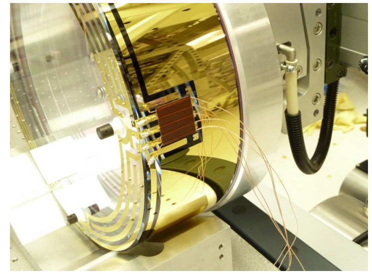
Ground-Based Detectors

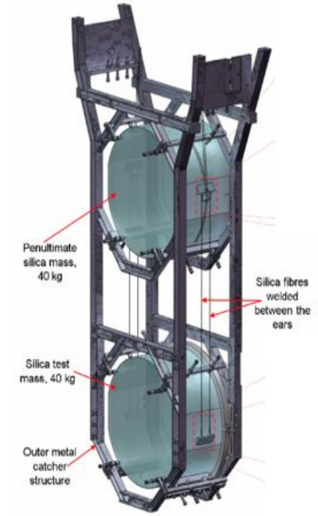
Ground-based detectors work using **laser interferometry**

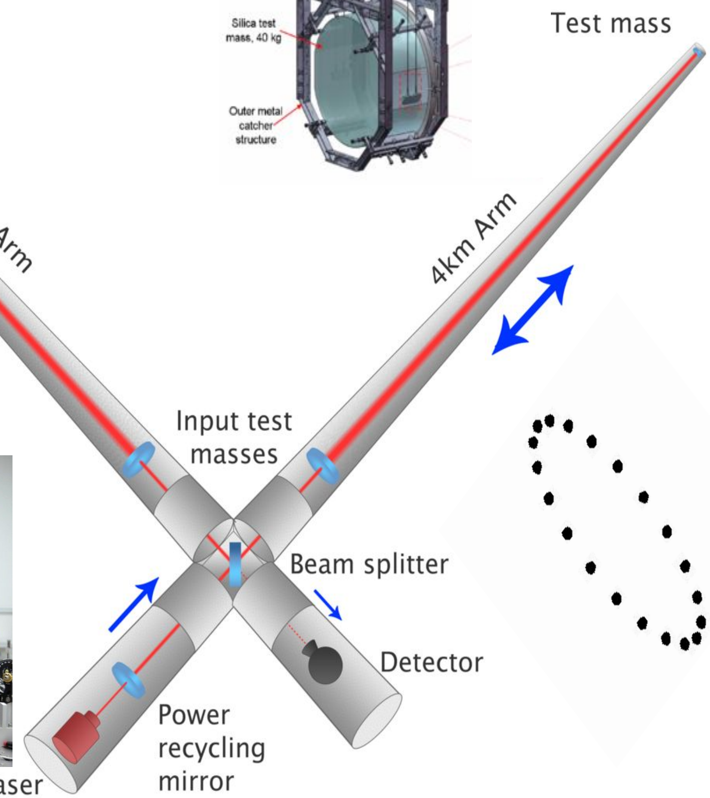
Derived from the classic **Michelson-Morley** interferometer

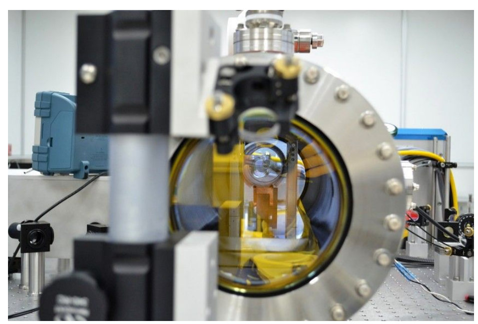
Several collaborations worldwide have developed kilometer scale interferometers

E.g. **LIGO** and **Virgo**









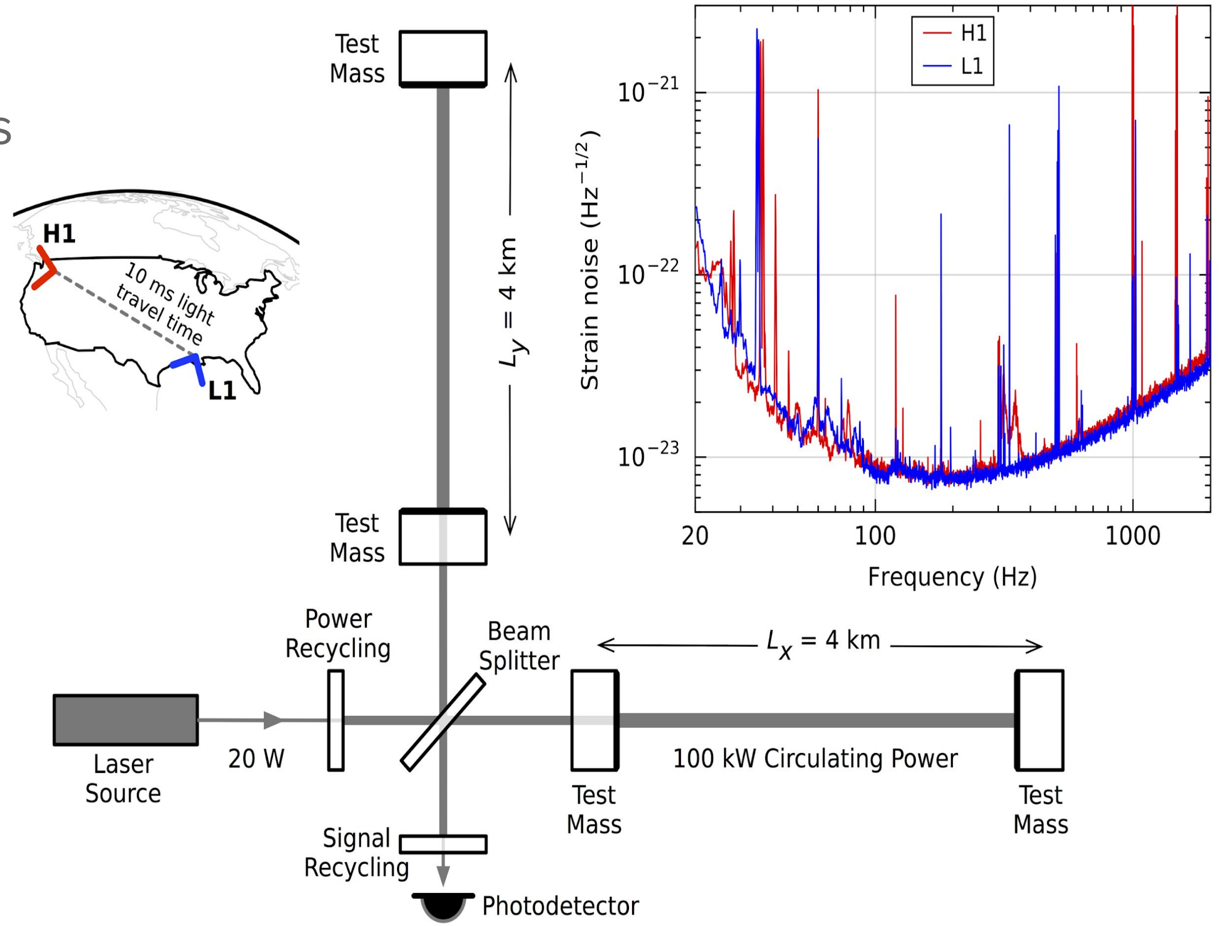
Ground-Based Detectors: LIGO

LIGO consists of two separated interferometers

4km long laser arms

Designed to minimise the impact of various noise sources

- Photon shot noise at high frequencies
- Thermal noise at mid frequencies
- Seismic noise at low frequencies



Data Analysis Basics

GW data is (several) time series

$$s_\alpha(t) = \{s_\alpha(t_1),\ s_\alpha(t_1),\ \ldots,\ s_\alpha(t_{N_{\mathrm{times}}})\}$$
$$\text{where} \quad \alpha \in \{1, 2, N_{\mathrm{detectors}}\}$$

Signals are buried in noise

$$s_\alpha(t) = h_\alpha(t) + n_\alpha(t)$$

Noise is usually assumed to be **stationary**, and **Gaussian**, and **uncorrelated** between different detectors

$$\langle n_\alpha(f) n_\alpha(f') \rangle = \frac{1}{2} S_\alpha(f) \delta(f - f') \delta_{\alpha\alpha'}$$

Data Analysis Basics

The noise **PSD** gives a natural definition of a inner product between signals

$$\langle A|B\rangle = 4\mathcal{R}\left\{\int_0^\infty \mathrm{d}f\,\frac{\tilde{A}(f)\tilde{B}(f)}{S(f)}\right\}$$

Sum over different α channels if necessary

This allows us to give a precise measure of how loud a signal, *h*, is: **signal-to-noise ratio**

$$\rho^2 = \langle h|h\rangle$$

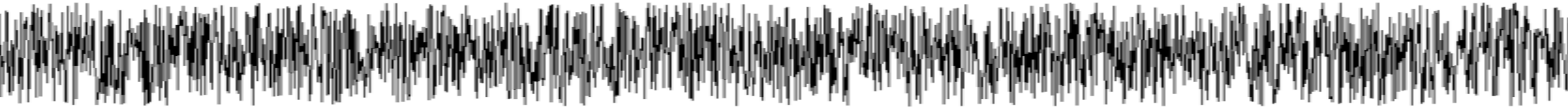
Allows us to give a measure of how similar two signals, h_1 and h_2, are: **match**

$$\mathcal{M} = \frac{\langle h_1, h_2\rangle}{|h_1||h_2|}$$

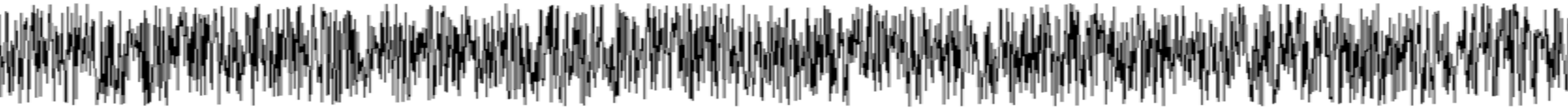
Ground-Based Data Analysis: detection

Ground-based detector data, at least initially, consists of long stretches of noise populated with short signal(s)

Data (detector 1)

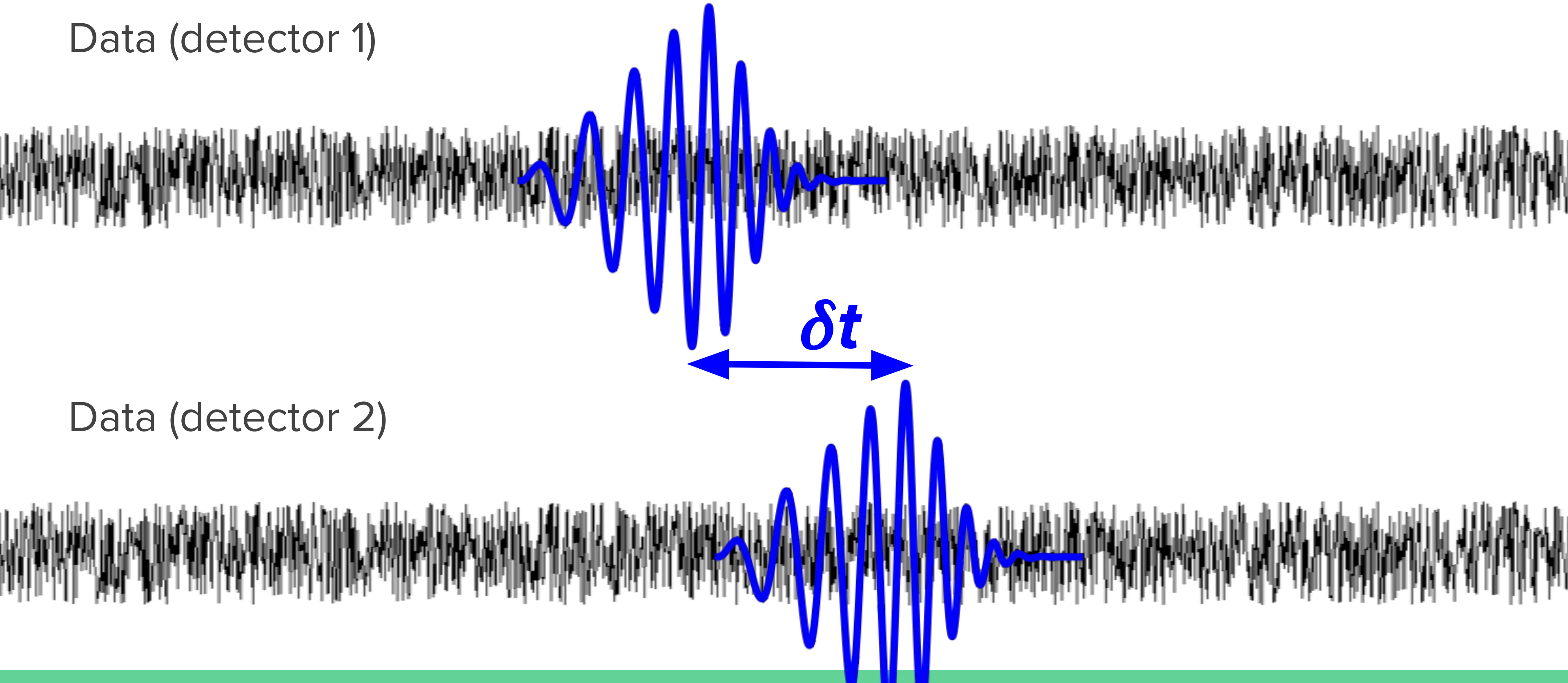


Data (detector 2)



Ground-Based Data Analysis: detection

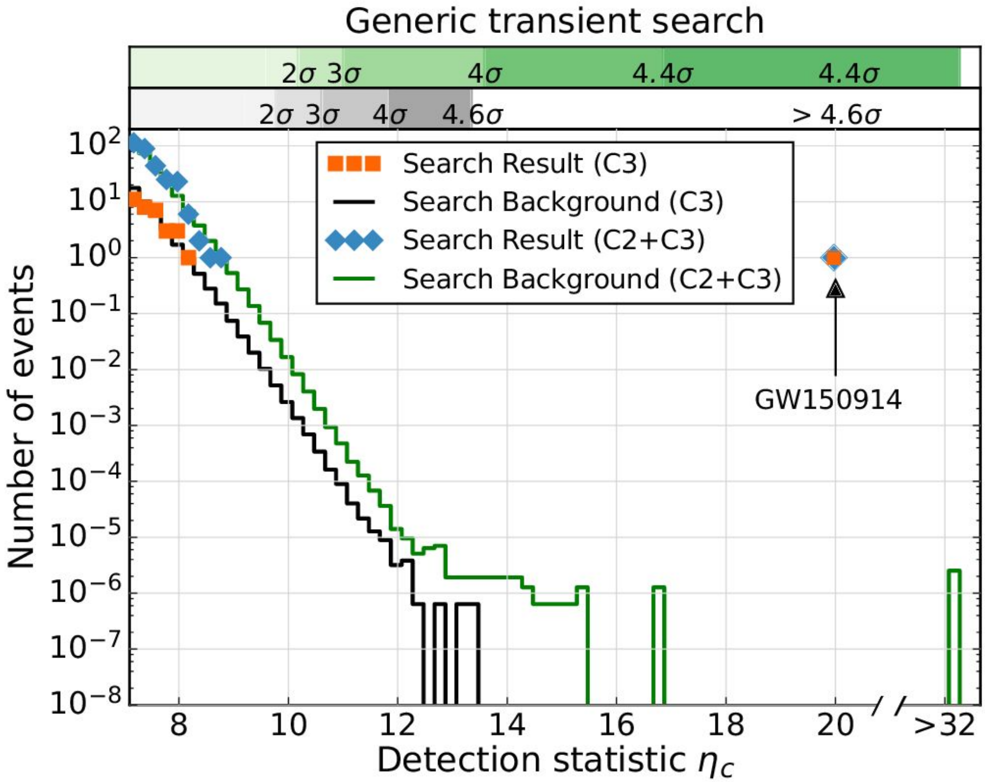
Ground-based detector data, at least initially, consists of long stretches of noise populated with short signal(s)



Ground-Based Data Analysis: detection

Ground-based detector data, at least initially, consists of long stretches of noise populated with short signal(s)

Detection can be tackled using **time slides**



Ground-Based Data Analysis: parameter estimation

Once we have identified a short (several seconds) of data which contains a signal we then turn to characterising what type of source it is from

We must have a theoretical model of the source signal $h(\theta)$ - for example this might come from post-Newtonian theory, or numerical relativity

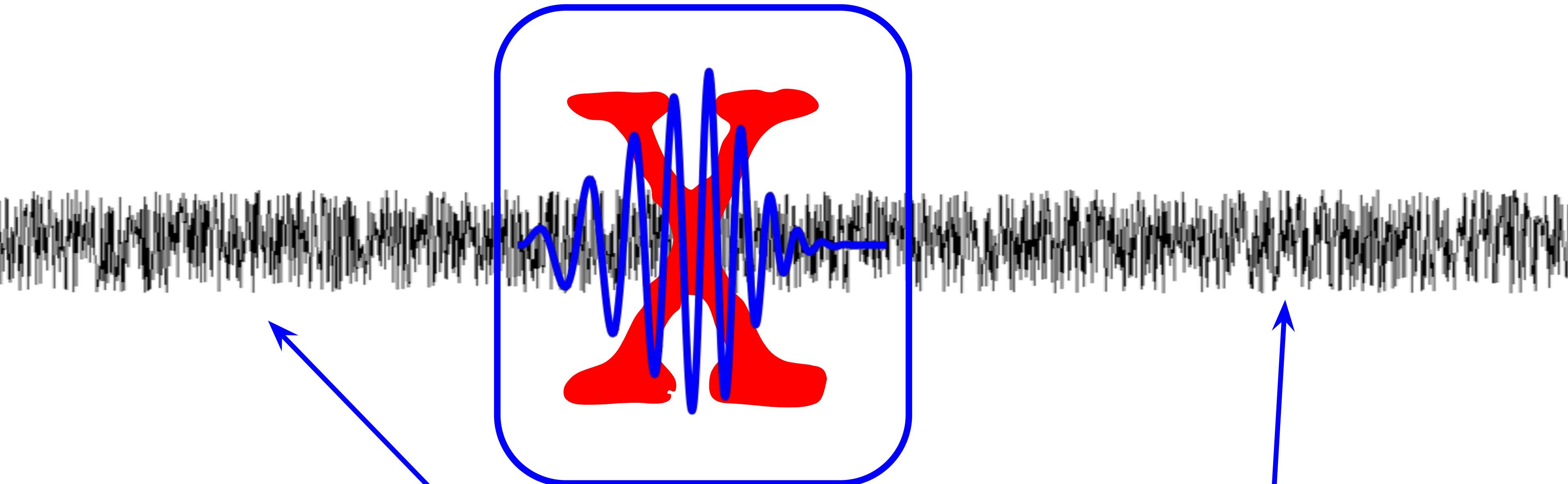
$$\mathcal{L}(\theta) = \exp\left(\frac{-1}{2}\langle s - h(\theta)|s - h(\theta)\rangle\right)$$

The likelihood takes a particularly simple form, because we assumed regularly sampled data, with stationary and Gaussian noise

Ground-Based Data Analysis: parameter estimation

What do we use for the noise PSD? $S(f)$?

Fortunately, the signal is short and surrounded by lots of “clean” noisy data. We can use this clean data to learn about the noise.



We can use this “clean” data to learn $S(f)$

Ground-Based Data Analysis: parameter estimation

Bayes' theorem

$$P(\theta) \propto \mathcal{L}(\theta)\Pi(\theta)$$

LHS: Posterior probability distribution on the source parameters

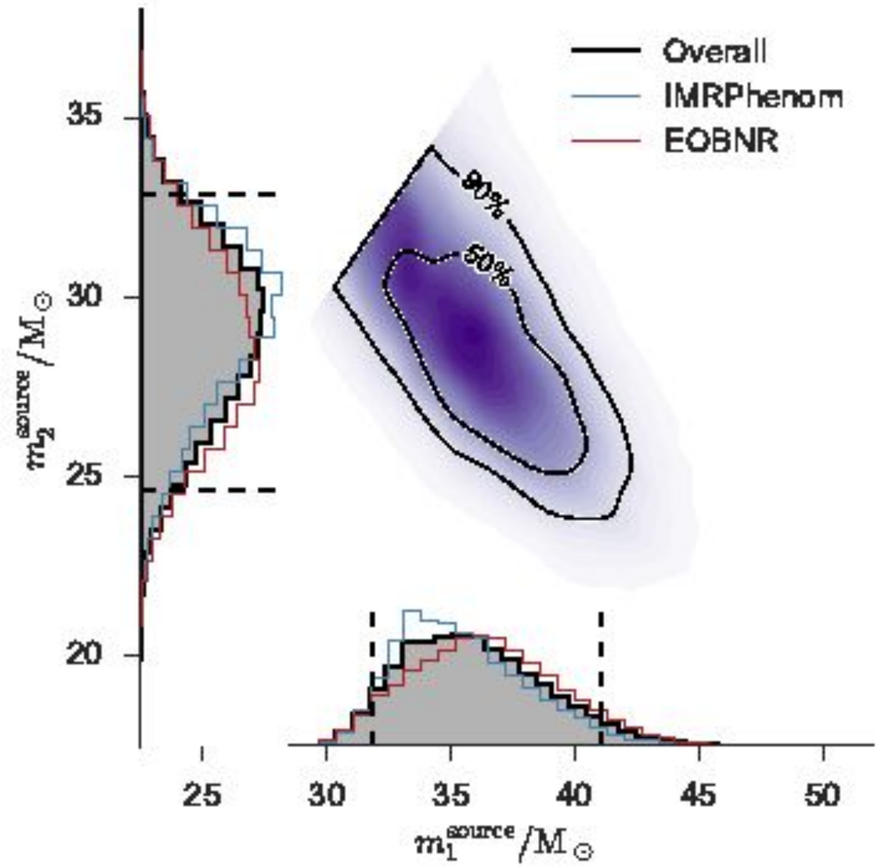
RHS: Likelihood time the Prior distribution on the source parameters

Ground-Based Data Analysis: parameter estimation

Bayes' theorem

$$P(\theta) \propto \mathcal{L}(\theta)\Pi(\theta)$$

Mass object 1
Mass object 2

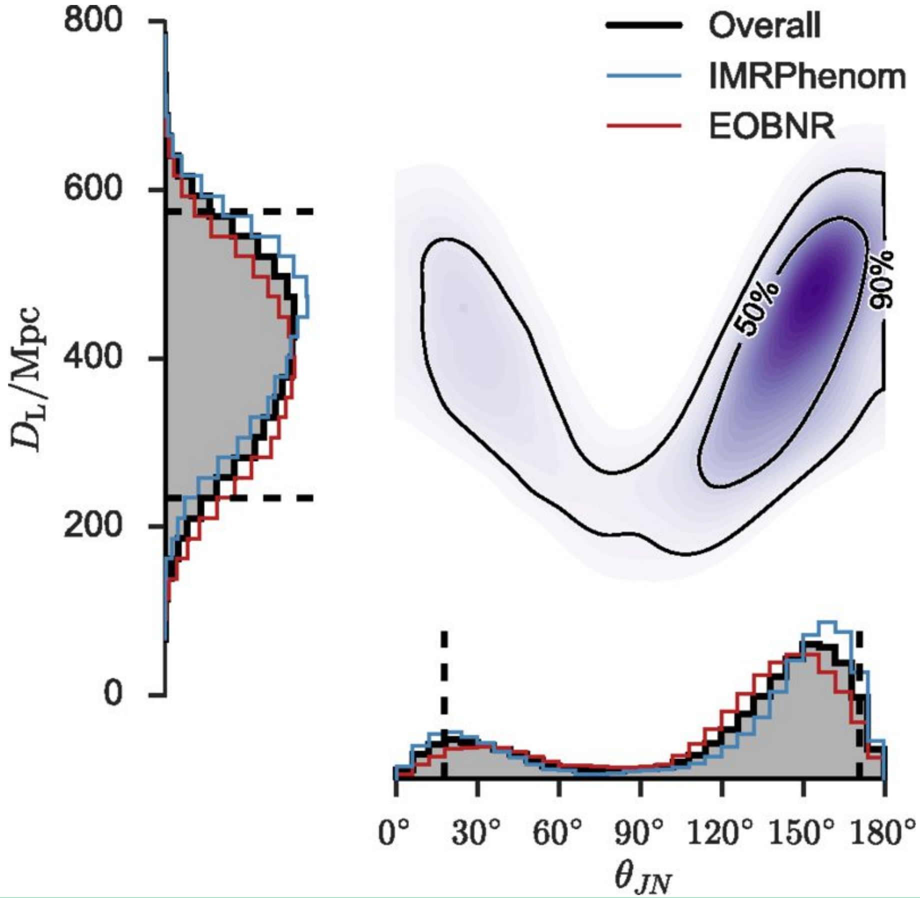


Ground-Based Data Analysis: parameter estimation

Bayes' theorem

$$P(\theta) \propto \mathcal{L}(\theta)\Pi(\theta)$$

Inclination angle
Distance to source

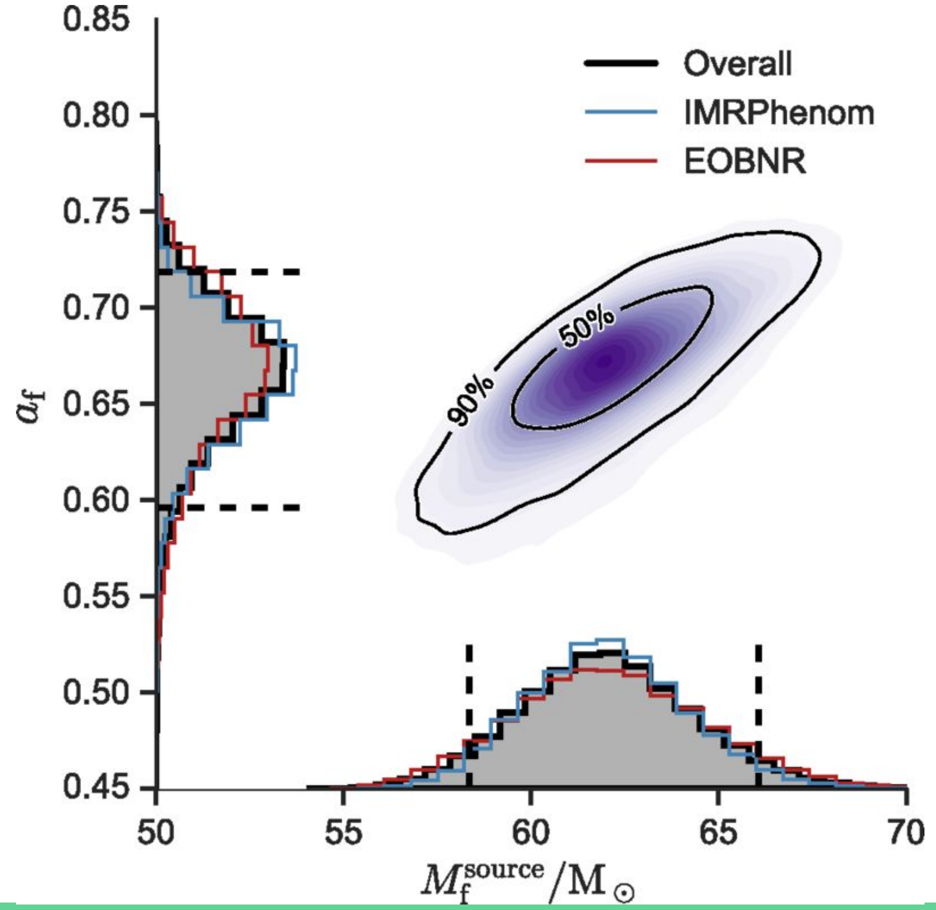


Ground-Based Data Analysis: parameter estimation

Bayes' theorem

$$P(\theta) \propto \mathcal{L}(\theta)\Pi(\theta)$$

Final object mass
Final object spin



Space-Based Data Analysis: challenges

In the data from space-based detectors there are a great many additional complications.

- Signals are long-lived (many years in some cases)
- There are a great many of overlapping sources

This means that we can't use time slides for detection, and we can't estimate *S*(*f*) from "clean" noisy data

$$S(f) \to S(f;\theta)$$

Space-Based Data Analysis: challenges

In the data from space-based detectors there are a great many additional complications.

- Always trying to estimate the parameter of several sources at once
- We don't know how many source there are before we start

This means that our expression for the likelihood becomes much more complicated. Very high dimensional problem, and also variable number of dimensions.

$$h = \sum_{i=1}^{N_{\mathrm{WD}}} h_{\mathrm{WD}}(\theta) + \sum_{i=1}^{N_{\mathrm{SMBH}}} h_{\mathrm{SMBH}}(\theta) + \sum_{i=1}^{N_{\mathrm{EMRI}}} h_{\mathrm{EMRI}}(\theta) + \ldots$$

Space-Based Data Analysis: challenges

In the data from space-based detectors there are a great many additional complications.

- Finally, the data will not be regularly sampled
- Noise will not be stationary, or perfectly Gaussian
- There will be gaps, glitches and other issues to be dealt with

There is lots of work to be done! That’s where you come in.

Space-Based Data Analysis: challenges

In the data from space-based detectors there are a great many additional complications.

- We don’t know the noise *a priori*
- As well as the detector noise, there is also a stochastic background of GWs that looks quite similar to other noise sources

This means that our signal model must also describe the noise