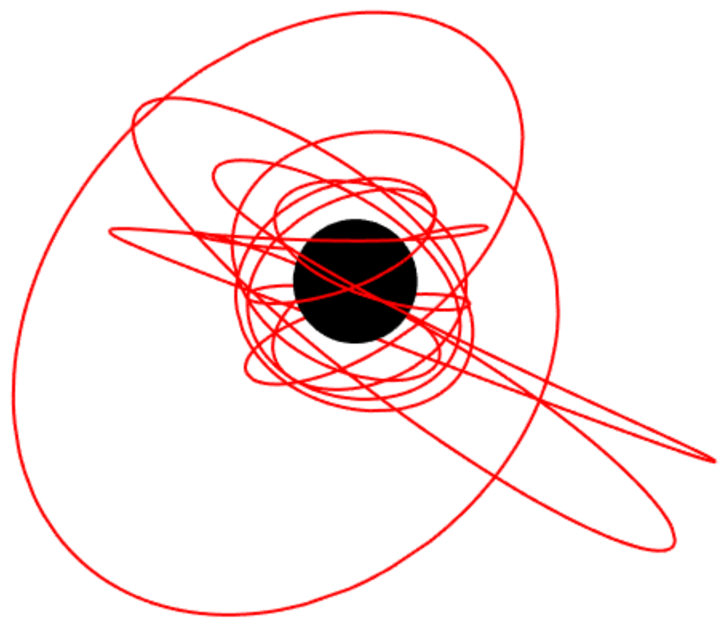
Extreme mass-ratio inspirals



Niels Warburton
Royal Society - SFI University Research Fellow
University College Dublin

Gravitational-wave Excellence through Alliance Training (GrEAT) school at the University of Birmingham

THE ROYAL SOCIETY

Science Foundation Ireland sfi For what's next

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Outline

- EMRIs and what makes them unique
- Science deliverables with EMRIs
- Modelling approaches
- Black Hole Perturbation Theory

Extreme mass-ratio inspirals

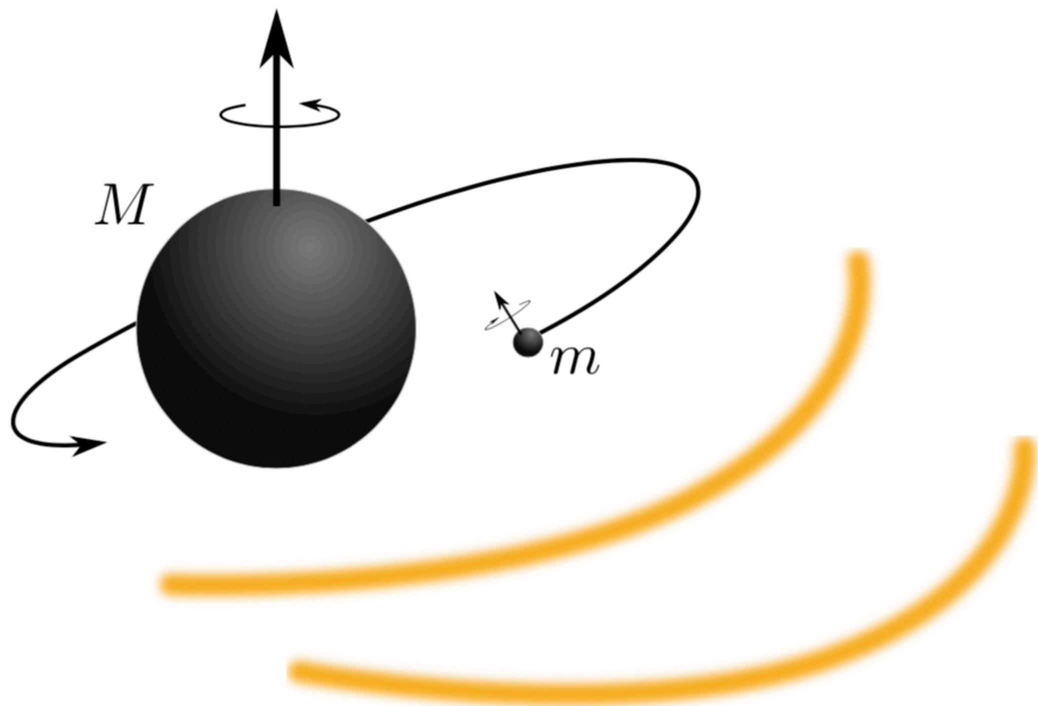
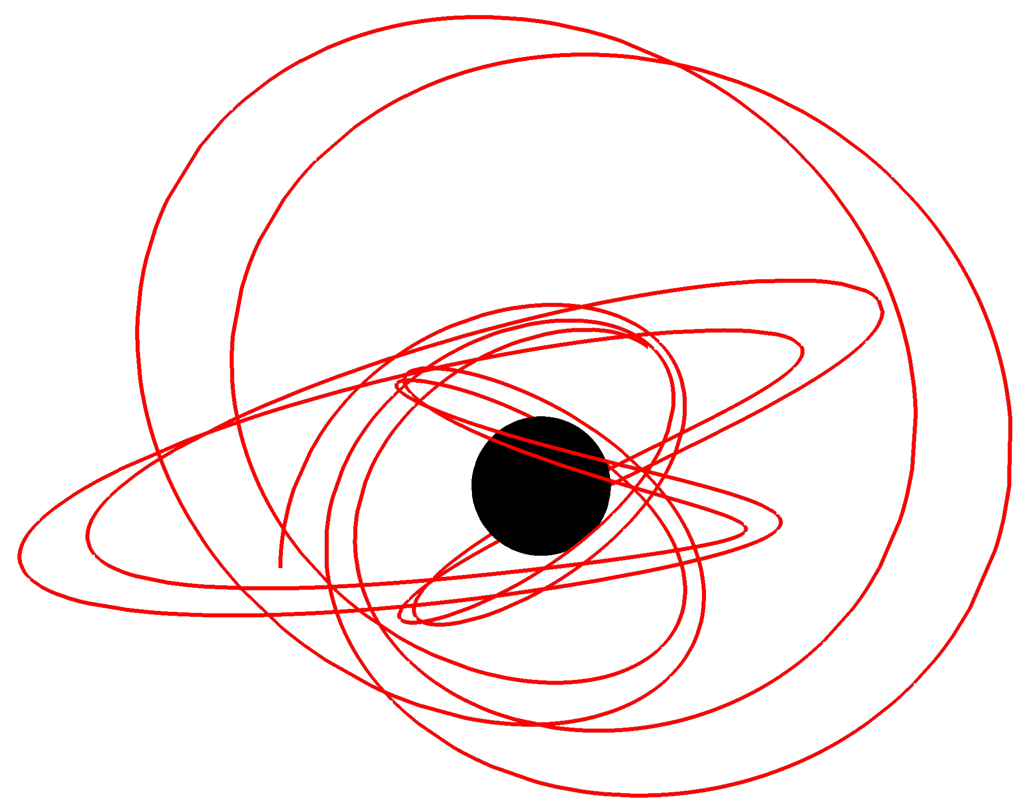


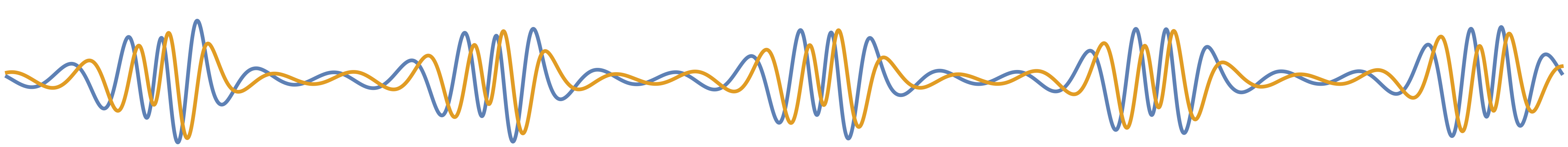
Image credit: A. Pound

- Binary with an extremely small mass ratio $\epsilon = m/M << 1$
- Primary: massive black hole
- Secondary: compact object such stellar-mass black hole, neutron star
- For LISA EMRIs: $\epsilon = 10^{-4} - 10^{-7}$



Key Features:

- Millihertz gravitational-wave source
- Over 100,000+ orbits in strong field
- Visible for months to years in LISA band
- No spin alignment expected
- Considerable eccentricity
- Rich waveform phenomenology



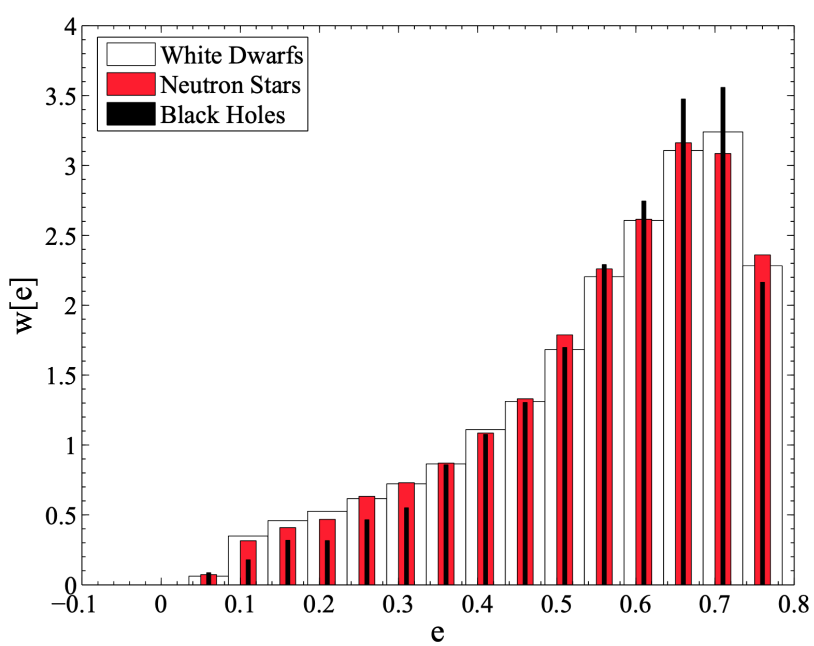
EMRIs: formation channels

Two key ingredients needed:

- Massive black holes
- Compact objects

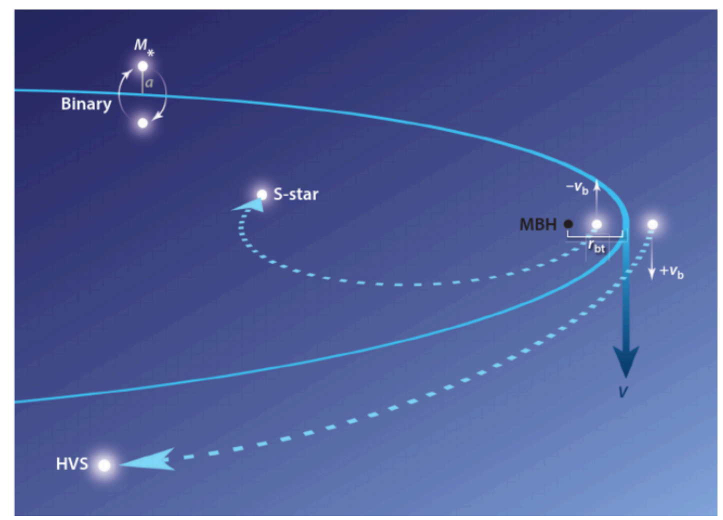
Direct capture

High eccentricity when entering LISA band e ~ 0.7



Significant eccentricity at plunge e ~ 0.1-0.3

Tidal disruption of binaries

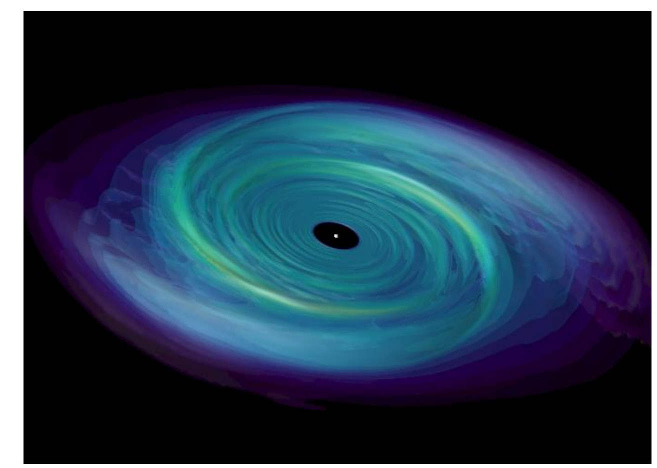


Evidence from S-stars and hyper-velocity stars near SgrA*

Circular orbit upon entering LISA band

Star formation in accretion disk

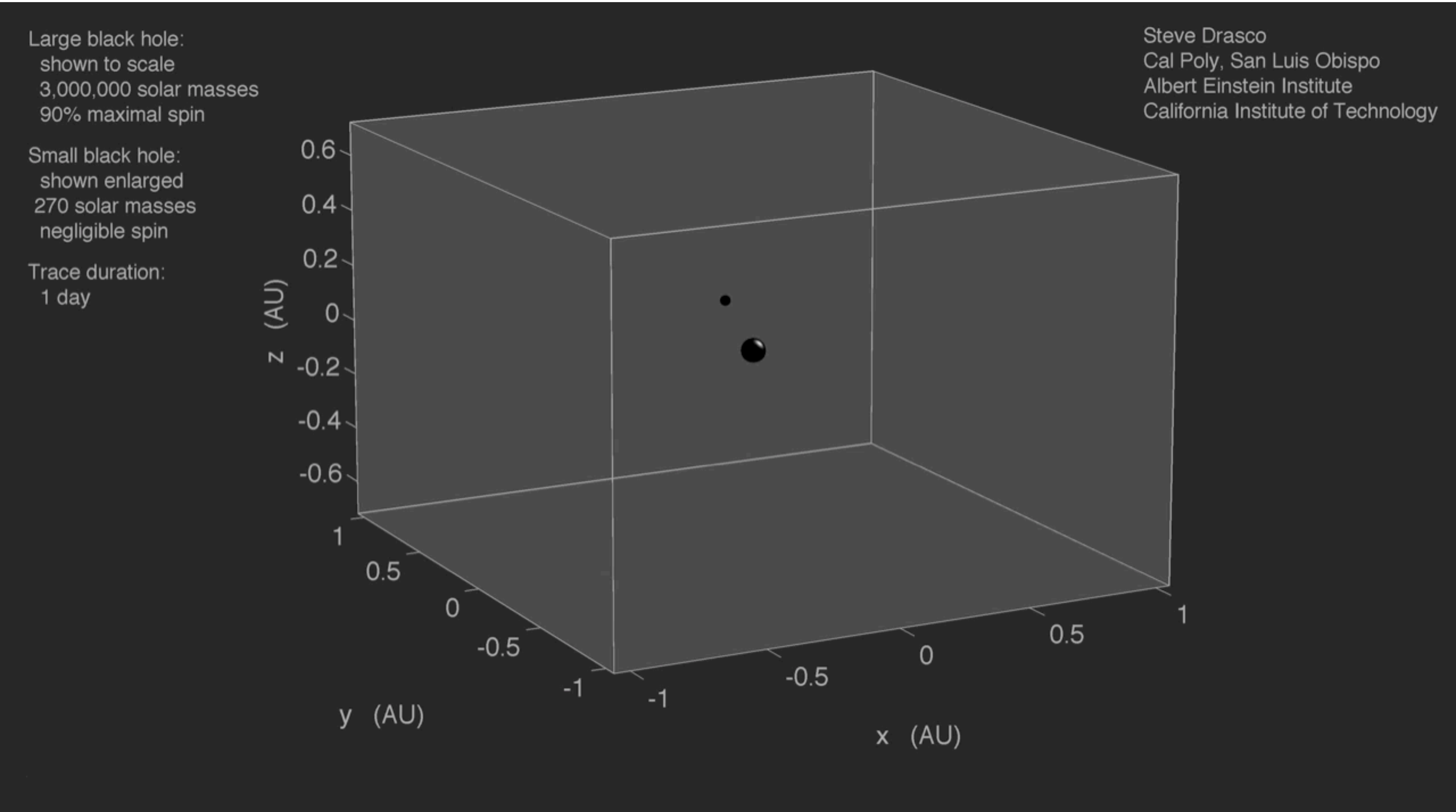
Outer disk fragments and collapses to massive star



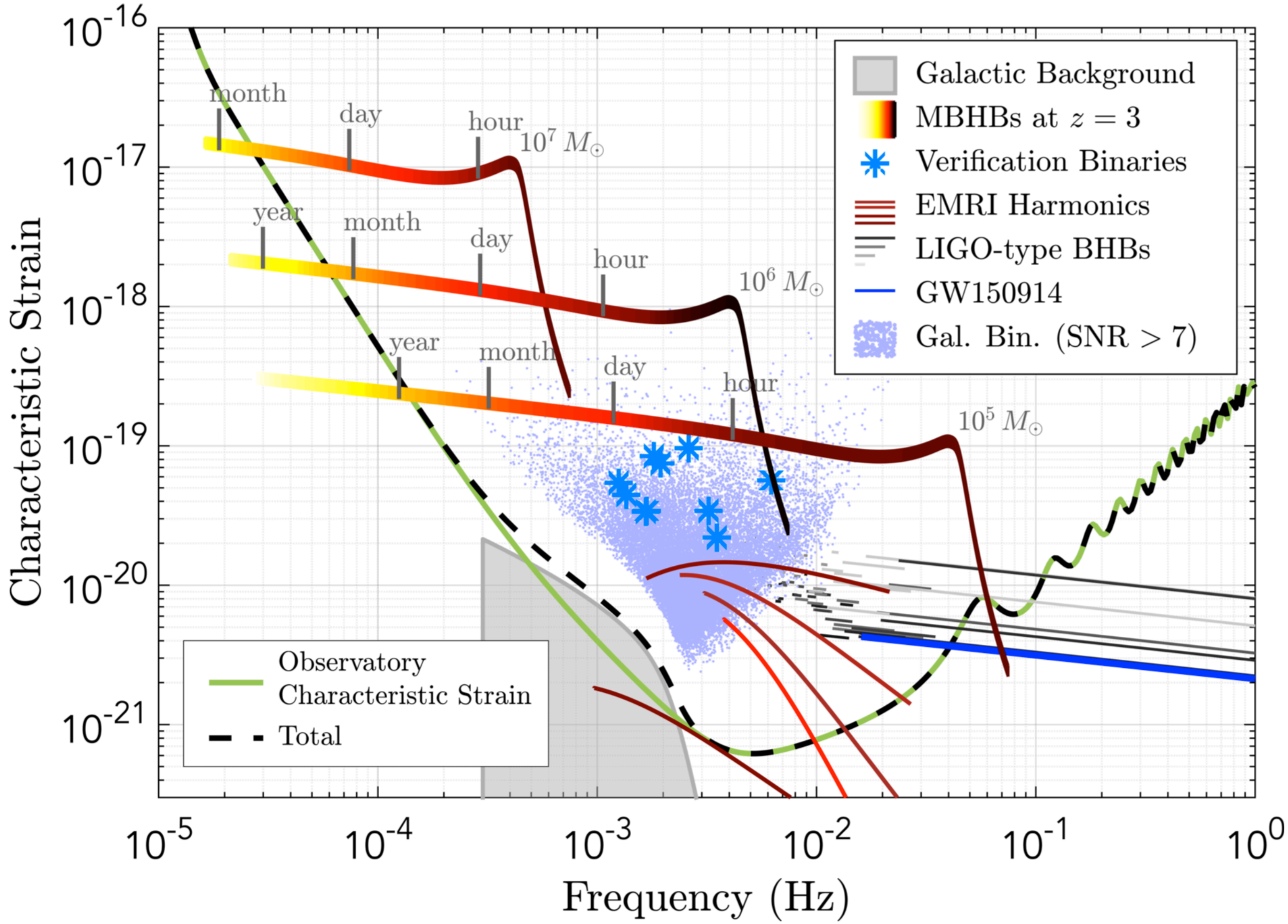
Circular and equatorial orbits upon entering LISA band

None of these methods is well understood so event rate estimates for EMRIs have large uncertainties: expect 1 - 1000 EMRIs over LISA mission

Extreme mass-ratio inspirals



EMRIs as LISA sources

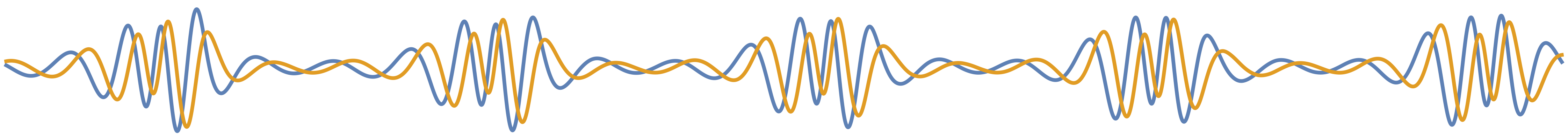


Goal: detect and estimate parameters for extreme mass-ratio inspirals (EMRIs)

The need for accurate waveforms

Instantaneous signal-to-noise ratio (SNR) is very low $\longrightarrow$

With matched filtering we expect SNRs of 10-50



Can write waveform as: $$h_+(t) + ih_\times(t) = \underline{A(t)}e^{i\underline{\Phi(t)}}$$

Phase

Amplitude

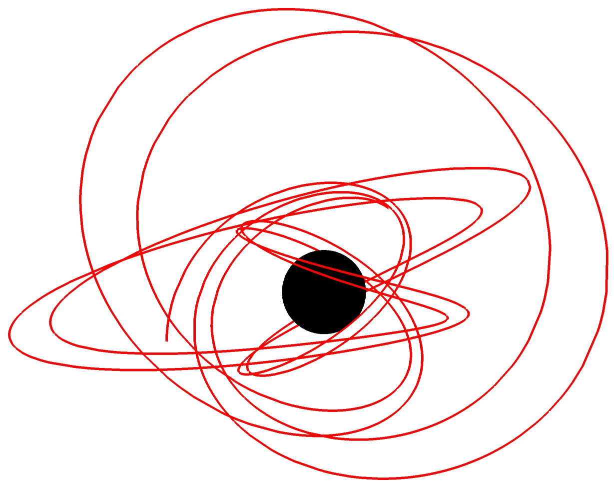
Phase accumulates as: $$\Phi(t) \propto \epsilon^{\underline{-1}}$$

Check: does this make sense? Must be a negative power as in geodesic limit the particle never plunges

Typical EMRI accumulates tens of thousands of radians of phase whilst in LISA band

Over this long waveform our templates cannot lose more than a fraction of a radian for matched filtering to work

The EMRI parameter space



Extrinsic parameters:

(only changes how the GWs projects on to the detector)

- Position on sky (2)
- Orientation (2)
- Azimuthal phase at t_0

Intrinsic parameters:

(fundamental changes the GWs)

- Distance to source
- Mass of primary, M
- Spin magnitude of primary, a
- Mass of secondary, m
- Spin magnitude of secondary
- Spin orientation of secondary (2)
- radial/polar phase at t_0 (2)
- constants of motion at t_0 (3)

Very large parameter space (17 parameters - 12 intrinsic)

Not only do we need waveforms accurate to a fraction of a radian over 100,000+ radians of accumulated phase, we also need billions of waveforms

Huge open data analysis task

Science deliverables with EMRIs

Supposing that:

- EMRIs exist
- LISA is flying
- We have accurate waveform models

What can we learn about the universe?

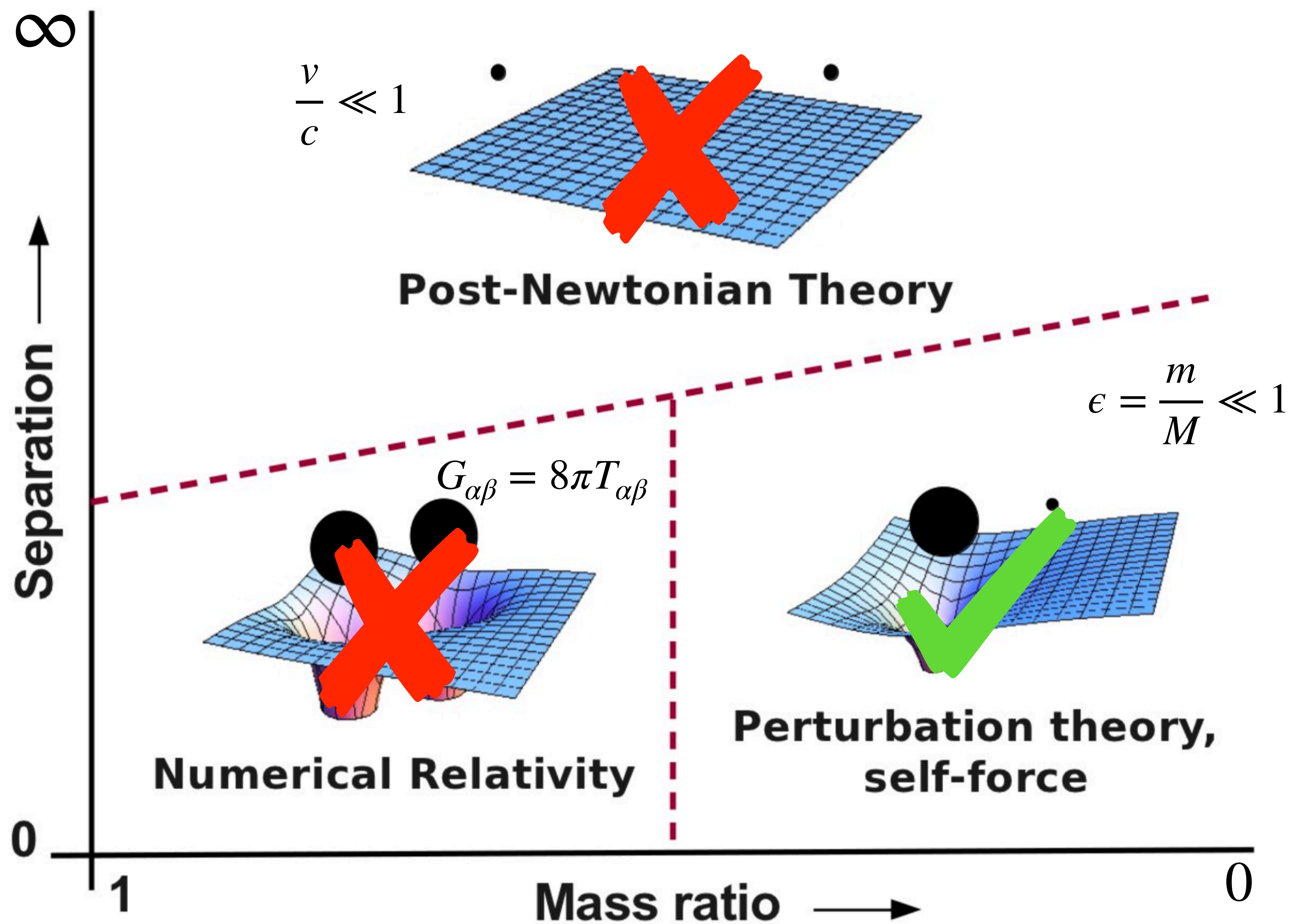
Astrophysics

- Measure massive black hole properties out to a redshift $z \sim 4$: $\delta M/M < 10^{-4}$, $\delta m/m < 10^{-3}$, $\delta a/a < 10^{-3}$
- Learn about dense stellar environment around massive black holes
- Survey massive black holes in nearby universe: informs structure formation models

Fundamental physics

- Measure the quadrupole moment of the central object $\delta Q/Q < 10^{-2}$. This allows the Kerr hypothesis to be tested
- Test propagation of GWs
- Test for massive fields around MBHs
- Measure Hubble parameter (if the EMRI can be localized)

Modelling approaches



Credit: Leor Barack

Black Hole Perturbation Theory: what is it?

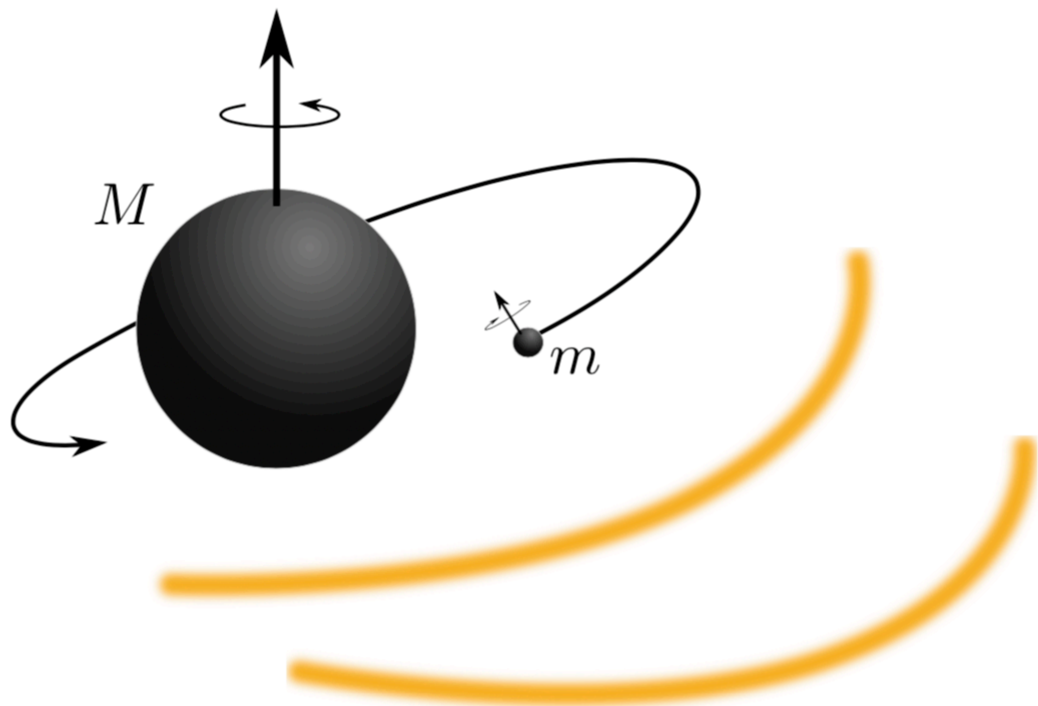


Image credit: A. Pound

Use mass ratio, ϵ = m/M, as a mass parameter

$$g_{\alpha\beta} = \bar{g}_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta} + \mathcal{O}(\epsilon^3)$$

Schwarzschild or Kerr

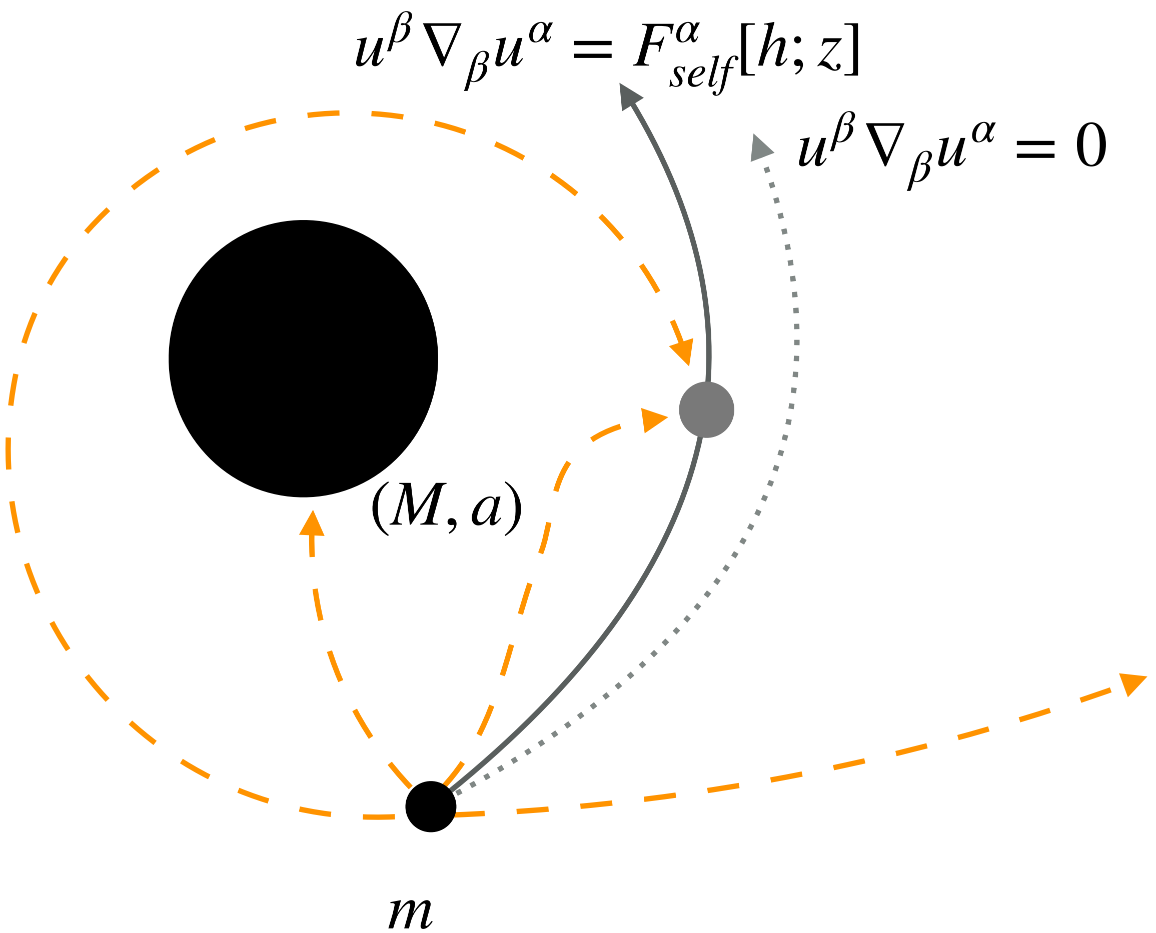
Model secondary as a point particle

$$T_{\alpha\beta} = m \int_{\infty}^{\infty} |\,\bar{g}\,|^{-1/2} \delta^4(x^\mu - z^\mu) u_\alpha u_\beta \, d\tau$$

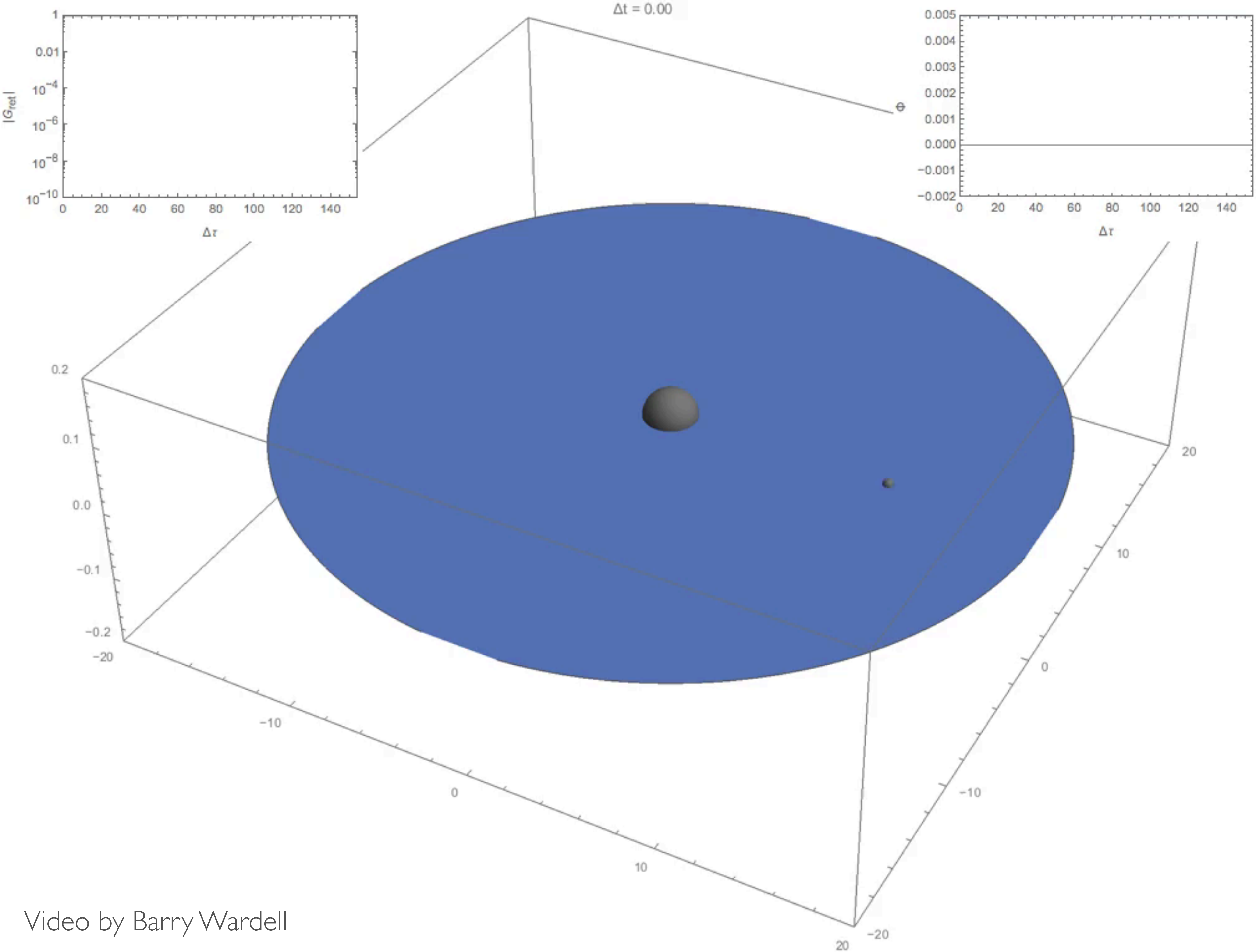
Substitute into the Einstein equation $G_{\alpha\beta}[g] = 8\pi T_{\alpha\beta}$ and expand order-by-order

Equations of motion $u^\beta \nabla_\beta u^\alpha = F^\alpha_{self}[h; z]$

Black Hole Perturbation Theory: what is it?

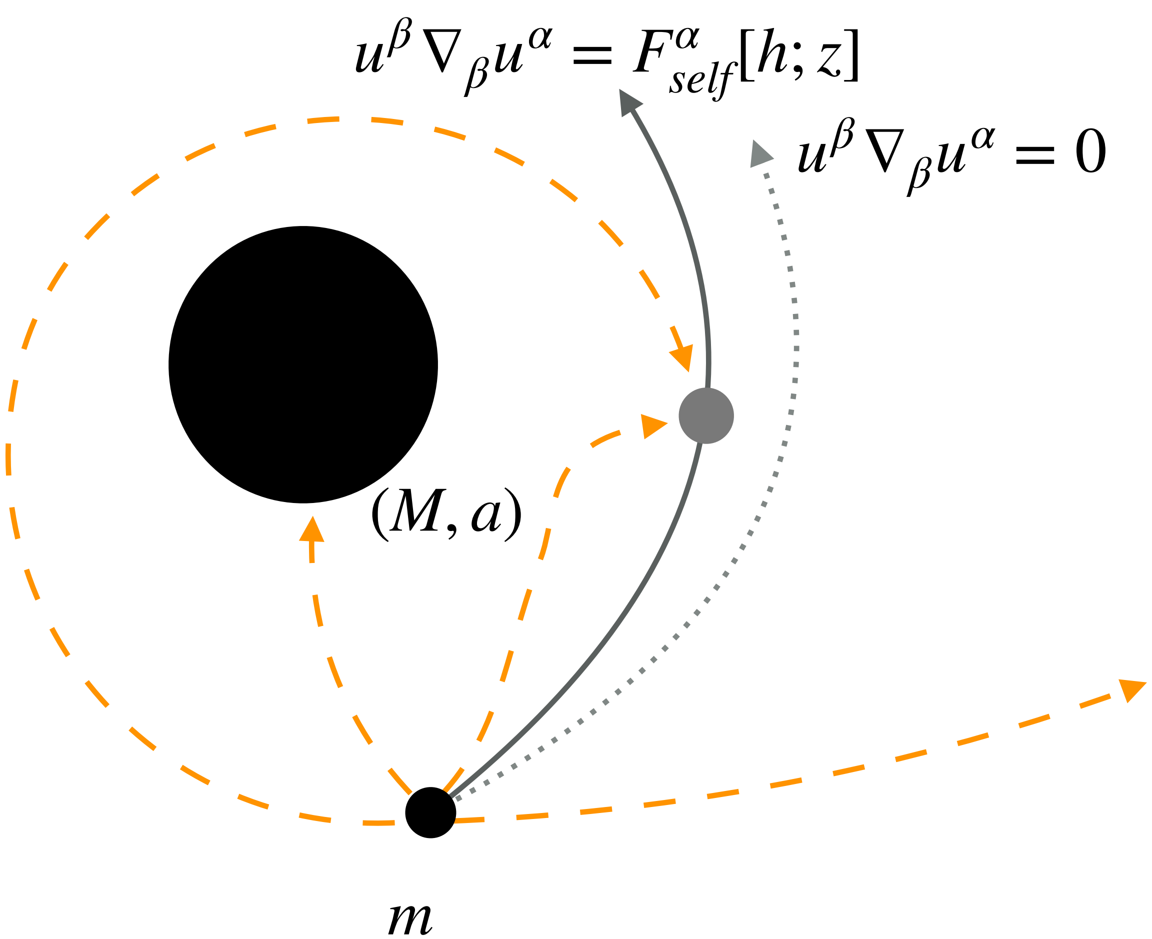


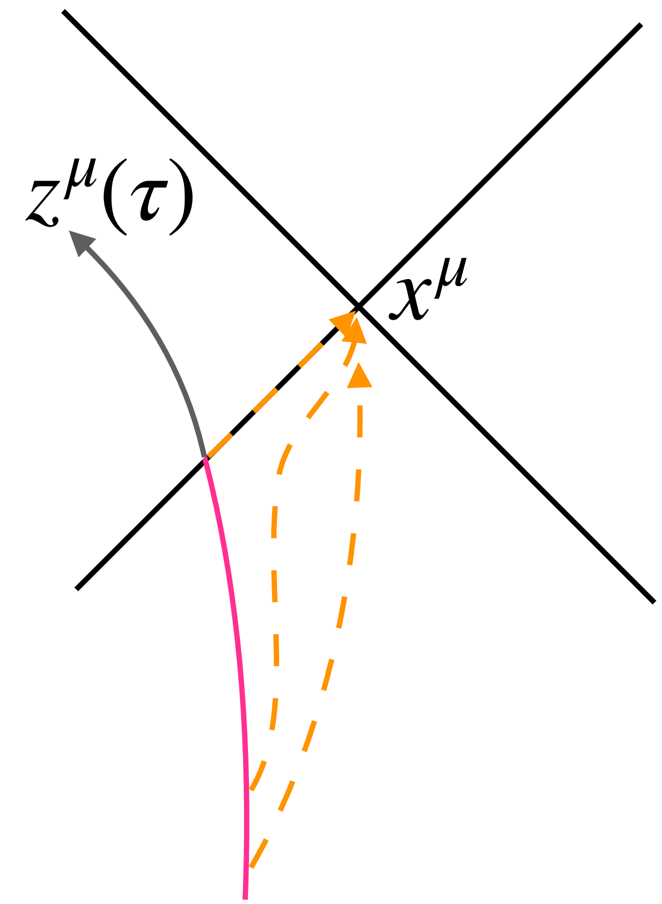
The force at a given spacetime point depends upon the local metric perturbation, which is a functional of the entire past history of the particle



Video by Barry Wardell

Black Hole Perturbation Theory: what is it?





The force at a given instance depends upon the local metric perturbation, which is a functional of the entire past history of the particle

$$F^\alpha_{self}[z_\mu(\tau)] = \lim_{x^\mu \to z^\mu(\tau)} F[\nabla^\alpha h(x^\mu)]$$

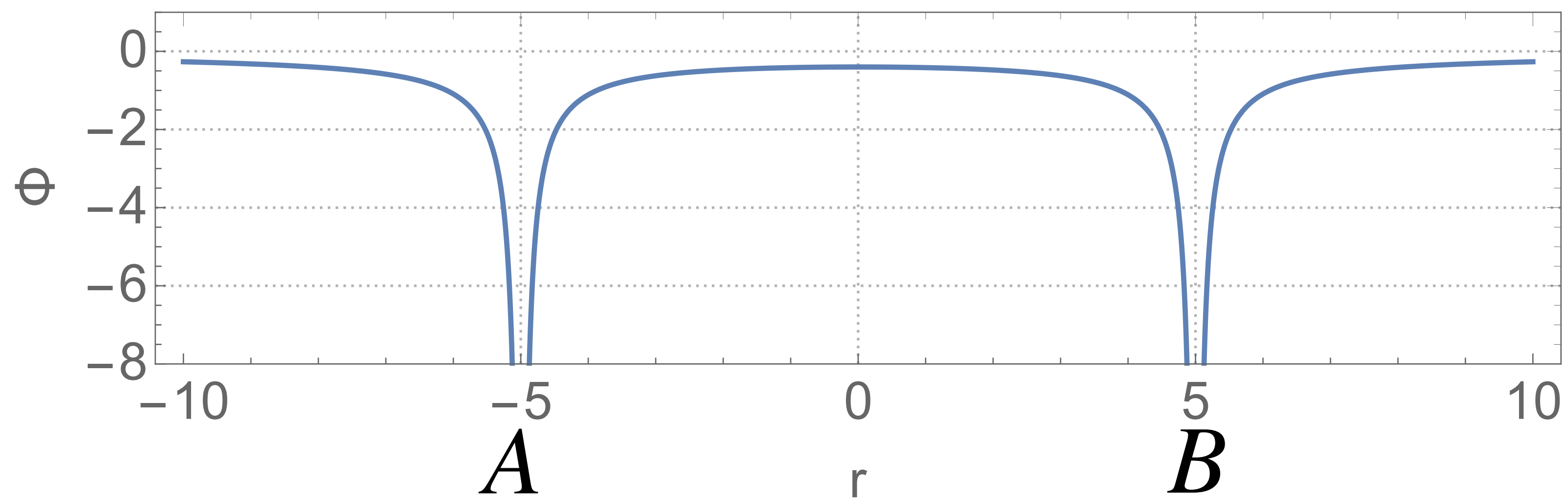
As defined, this diverges in the limit. Thus we need to regularize

Defining the self-force: regular/singular split

We model the secondary as a point particle
This necessitates regularisation

Example: electrostatics

$$F_{\overrightarrow{BA}} = k_e \frac{q_A q_B}{r^2} = \partial_r \Phi$$



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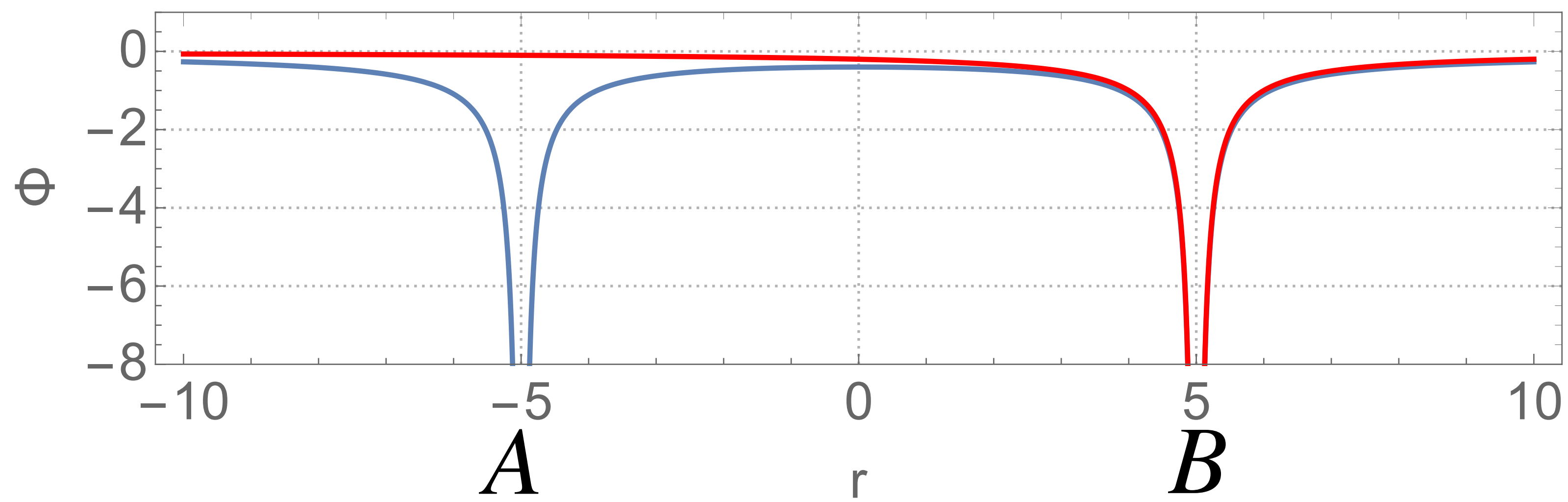
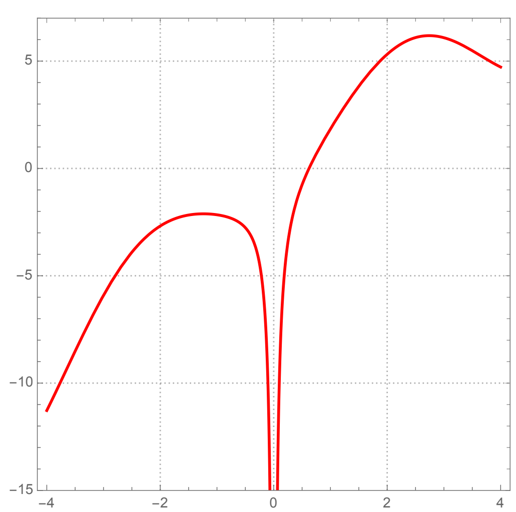
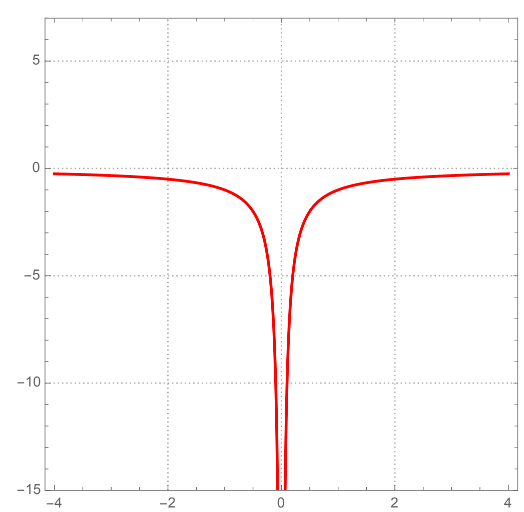


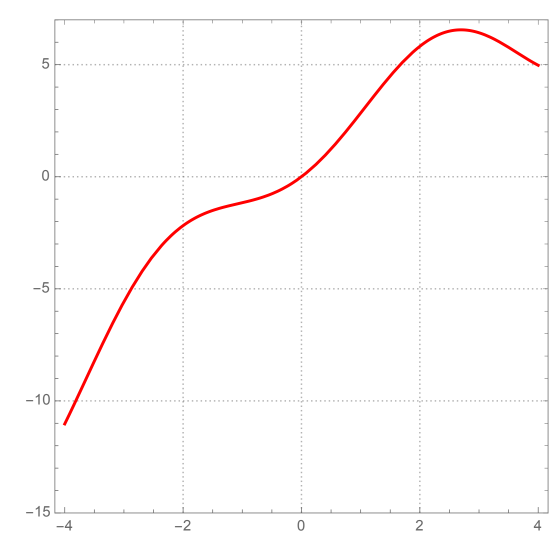
Do the same for the metric perturbation (locally)



h^{ret} $=$



h^S $+$



h^R

The self-force depends on the derivative of the regular metric perturbation

$$F^{\alpha}_{self}[z_\mu(\tau)] = \lim_{x^\mu \to z^\mu(\tau)} F[\nabla^\alpha h^R]$$

Black Hole Perturbation Theory: field equations

$$G_{\alpha\beta}[\bar{g}_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta}] = 8\pi T_{\alpha\beta}$$

Field equations from ϵ^n coefficients:

$$\epsilon^0: \qquad G_{\alpha\beta}[\bar{g}] = 0$$

$$\epsilon^1: \qquad G^1_{\alpha\beta}[h^1] = 8\pi T_{\alpha\beta}$$

$$\epsilon^2: \qquad G^1_{\alpha\beta}[h^2] + G^2_{\alpha\beta}[h^1, h^1] = 0$$

Black Hole Perturbation Theory: field equations

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Field equations from ϵ^n coefficients:

$$\epsilon^0: \qquad G_{\alpha\beta}[\bar{g}] = 0$$

$$\epsilon^1: \qquad G^1_{\alpha\beta}[h^{1S} + h^{1R}] = 8\pi T_{\alpha\beta}$$

$$\epsilon^2: \qquad G^1_{\alpha\beta}[h^{2S} + h^{2R}] = - G^2_{\alpha\beta}[h^1, h^1]$$

Black Hole Perturbation Theory: field equations

$$G_{\alpha\beta}[\bar{g}_{\alpha\beta} + \epsilon h^{(1)}_{\alpha\beta} + \epsilon^2 h^{(2)}_{\alpha\beta}] = 8\pi T_{\alpha\beta}$$

Field equations from ϵ^n coefficients:

Mino, Sasaki, Tanaka 1997
Quinn and Wald 1997
MiSaTaQuWa equations

$$\epsilon^0: \quad G_{\alpha\beta}[\bar{g}] = 0$$

$$\epsilon^1: \quad G^1_{\alpha\beta}[h^{1R}] = 8\pi T_{\alpha\beta} - G^1_{\alpha\beta}[h^{1S}]$$

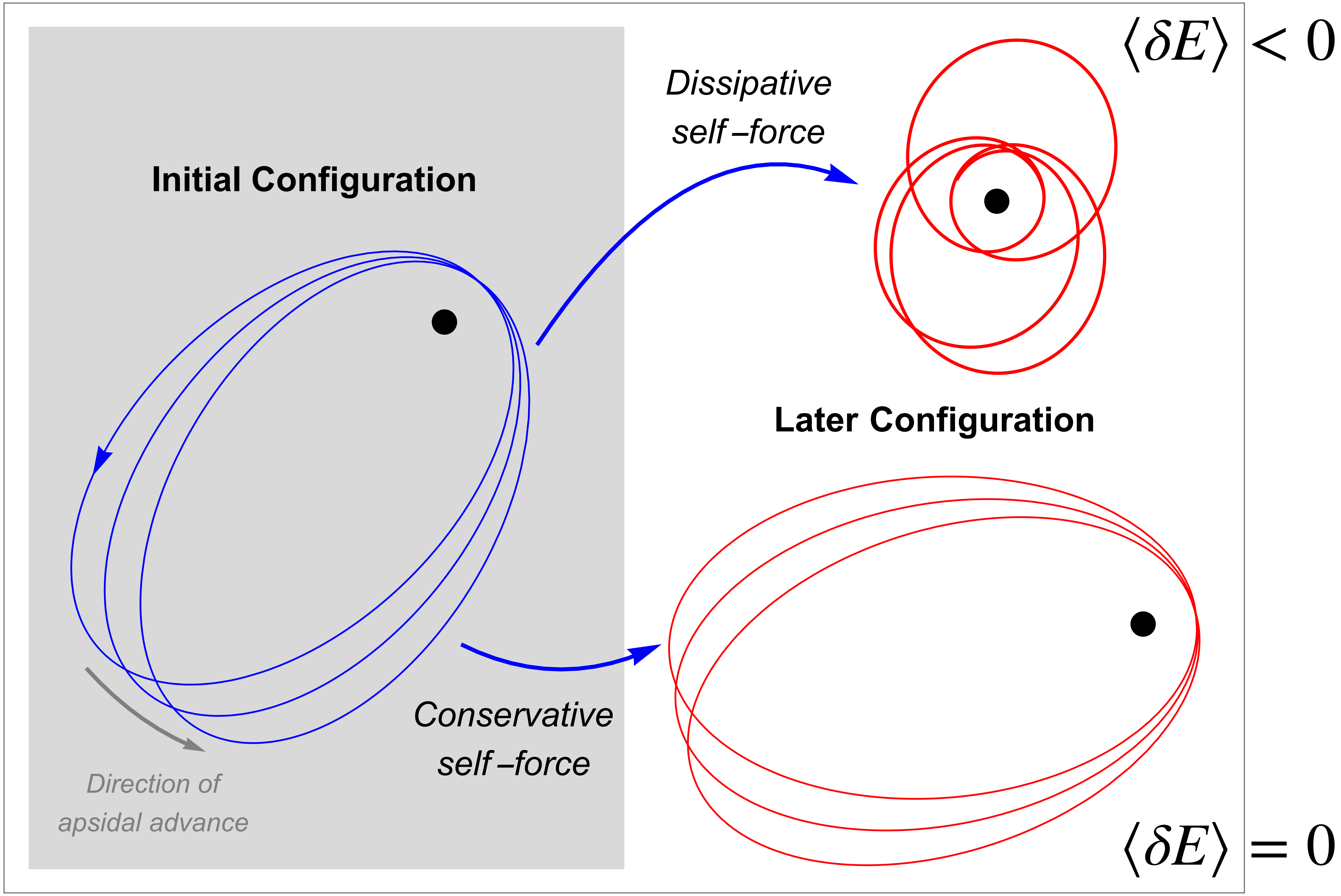
$$\epsilon^2: \quad G^1_{\alpha\beta}[h^{2R}] = -G^2_{\alpha\beta}[h^1, h^1] - G^1_{\alpha\beta}[h^{2S}]$$

Pound 2012
Gralla 2012

Equations of motion

$$u^\beta \nabla_\beta u^\alpha = F^\alpha_{self}[\nabla h^{1R}, \nabla h^{2R}]$$

Effects of the self-force



Black Hole Perturbation Theory: what is needed?

Crucial question: how high in the expansion in ε do we need to go?

Two time-scale analysis shows [1]:

$$\Phi = \underbrace{\epsilon^{-1}\Phi_0\left[\langle h^1_{\textbf{diss}}\rangle\right]}_{\text{Adiabatic order}} + \underbrace{\epsilon^{0}\Phi_1\left[h^1_{\textbf{diss,osc}} + h^1_{\textbf{cons}} + \langle h^2_{\textbf{diss}}\rangle\right]}_{\text{Post-adiabatic order}} + \mathcal{O}(\epsilon)$$

$\langle h^1_{\textbf{diss}}\rangle$ can be obtained from flux balance

All 3 require calculation of local, regular metric perturbation

[1] Hinder and Flanagan Phys.Rev.D78:064028 (2008)
here ignoring resonances and the spin of the secondary

Black Hole Perturbation Theory: state of the art

$$\Phi(t) = \epsilon^{-1}\Phi_0\left[\langle h^1_{\textbf{diss}}\rangle\right] + \Phi_1\left[\underbrace{h^1_{\textbf{diss,osc}} + h^1_{\textbf{cons}}} + \langle h^2_{\textbf{diss}}\rangle\right] + \mathcal{O}(\epsilon)$$

Flux for Circular, equatorial orbit in Kerr (Detweiler 1970s)

Fluxes for fully generic orbits in Kerr (Hughes +, 2006) But no systematic exploration of parameter space

Circular orbit in Schwarzschild (Barack, Sago 2007)

Generic orbits in Kerr spacetime (van de Meent, 2017) but code very slow and only run for 4 points in the parameter space

Recent breakthrough: Pound, Wardell, NW, Miller (2019)

First dissipative results in 2020? (for circular orbit in Schwarzschild)

Vast amount of remaining theoretical and practical work

Black Hole Perturbation Theory: computational approaches

In practice you have to choose a gauge. Common choices are:

Lorenz gauge: $\nabla^\alpha \bar{h} = 0$

- Solve for 10 coupled components of the metric perturbation
- Regularisation well understood
- Challenging in Kerr spacetime

Teukolsky formalism and radiation gauge:

- Solve Eqs. for the perturbed Weyl scalars
- Single scalar field to solve for
- Good for fluxes
- Hard to regularize

Numerical approaches:

Always decompose into modes

$$h_{\alpha\beta}(t,r,\theta,\phi) = \sum_{ilm} h_{lm}(t,r) Y^{(i)lm}_{\alpha\beta}(\theta,\phi)$$

PDEs:

- 1+1D equations in Schwarzschild
- 2+1D equations in Kerr

$$h_{\alpha\beta}(t,r,\theta,\phi) = \sum_{ilm\omega} h_{lm\omega}(r) Y^{(i)lm}_{\alpha\beta}(\theta,\phi) e^{-i\omega t}$$

Fourier decompose to get ODEs

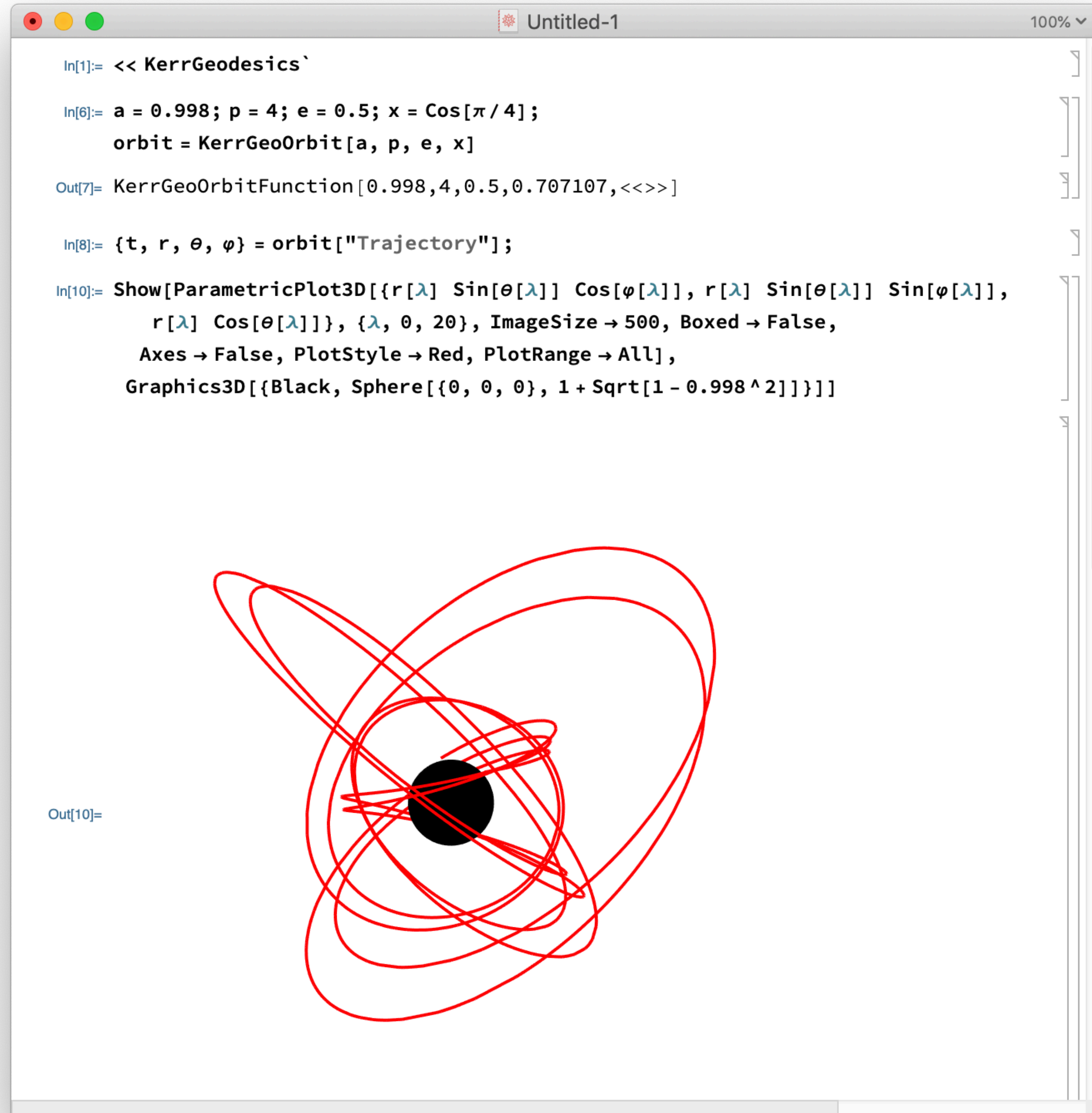
The Black Hole Perturbation Toolkit

That all sounds very interesting, but there are a lot of moving parts. Where would I get started?

The Black Hole Perturbation Toolkit (bhptoolkit.org)

Free and open-source repository of code and data

Example: bound timeline geodesics in Kerr spacetime



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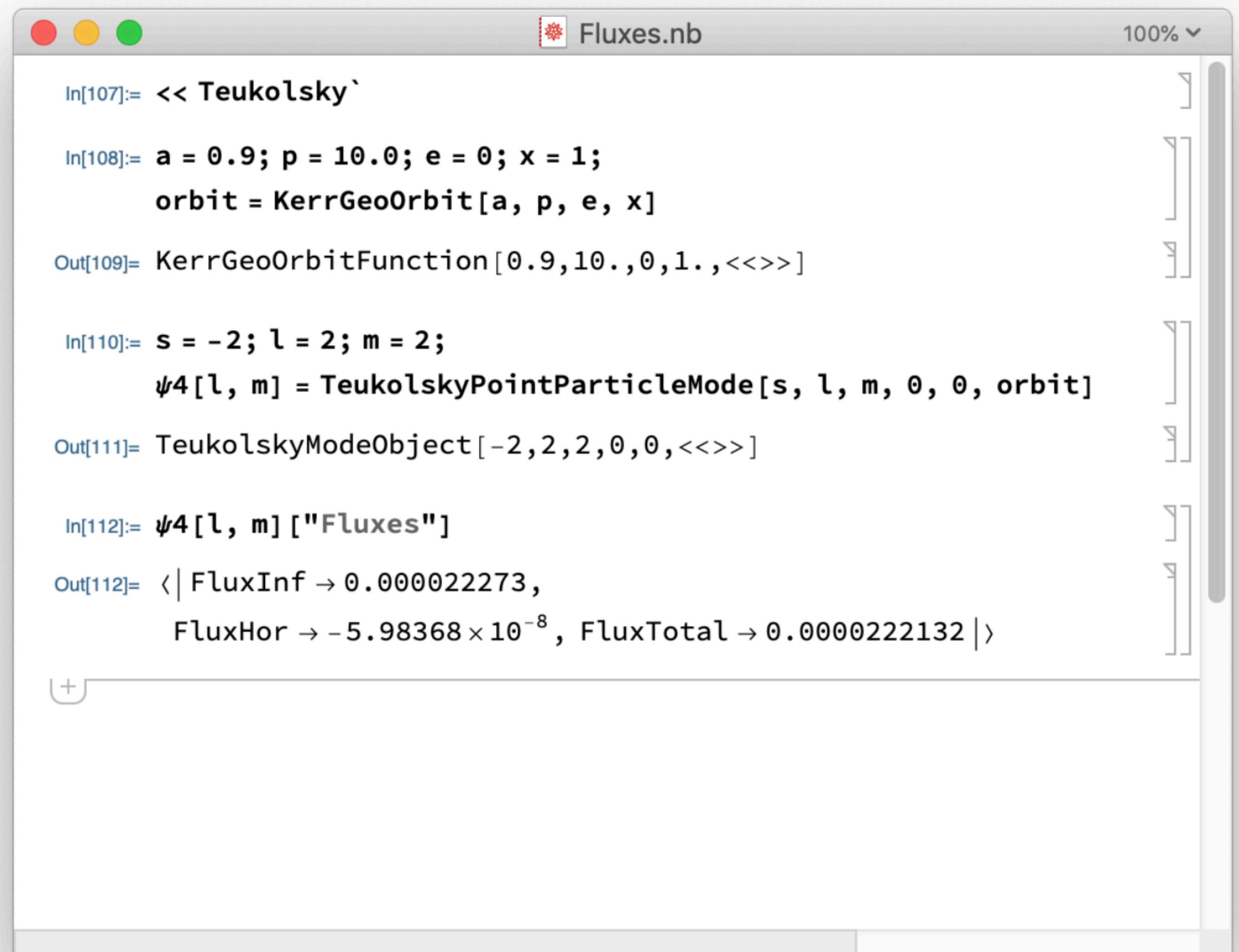
Example: bound timeline geodesics in Kerr spacetime

Example: compute the radiated flux for a particle on a circular orbit in Kerr spacetime



Also have code in C++/python/SageMath and repositories of data (post-Newtonian series, regularisation parameters, numerical results)

Recap

- EMRIs and what makes them unique
- Science deliverables with EMRIs
- Modelling approaches
- Black Hole Perturbation Theory
    - Field equations
    - Self-forced equations of motion
    - Regular/singular split
    - Computational methods

Useful references

Introductory review to self-force:
- L. Barack, A. Pound, https://arxiv.org/abs/1805.10385

More advanced review of self-force
- E. Poisson, A. Pound, I. Vega, https://arxiv.org/abs/1102.0529

Kerr bound geodesics in Kerr spacetime:
- W. Schmidt, https://arxiv.org/abs/gr-qc/0202090
- R. Fujita, W. Hikida, https://arxiv.org/abs/0906.1420

Solving Teukolsky equation:
- S. Drasco, S. Hughes, https://arxiv.org/abs/gr-qc/0509101
- M. van de Meent, https://arxiv.org/abs/1711.09607

Two-timescale analysis of EMRIs:
- T. Hinderer, E. Flanagan, https://arxiv.org/abs/0805.3337

Second-order self-force:
- A. Pound, B. Wardell, N. Warburton, J. Miller, https://arxiv.org/abs/1908.07419