Consider the following of the stochastic growth model, in its social planner version:

$$V(a,k) = \max_{c,h,i,k'} \theta \log(c) + (1-\theta)\log(1-h) + \beta \mathbb{E}[V(a',k')]$$

subject to

$$c + i = \exp(a) k^{\alpha} h^{1-\alpha}$$
$$k' = (1-\delta)k + i$$

1. Begin by assuming that $a = 0$ is deterministic. The model then has the following parameters $\{\beta, \theta, \alpha, \delta\}$. Set $\beta = 0.98$ (to target a quarterly frequency). Solve for the steady state of this model and calibrate the model to match (i) average hours worked $h^* = 0.3$; (ii) capital share of income of 0.3; (iii) an investment to output ratio of $i^*/y^* = 0.16$.

2. Write a program to solve for the perfect foresight transition path $\{k_t\}_{t=0}^{\infty}$.

3. Choose a grid over capital and write a computer program to solve for the optimal policies in the non-stochastic environment using Value Function Iteration. Check that the steady state of your approximated policy rules corresponds to the steady state you computed in 1. and the path you computed in step 2.

4. Solve for the policy rules with the additional restriction $i \geq 0$ (repeat steps 2 and 3). When do the policy rules differ?

5. Now introduce uncertainty. Download the "logtfp_detrended.csv"[1] from canvas. Estimate the following process for a_t:

   $$a_t = \rho a_{t-1} + \sigma \epsilon_t$$

   with ϵ_t standard normal. Approximate the AR(1) process using the Rouwenhorst method with $N = 25$ grid points.[2]

6. Start from the steady state and simulate for 10,000 a timeseries for output, consumption, investment and hours worked. Drop the first 3,000 realizations. On the remaining 7,000 compute the standard deviations, autocorrelations, and cross-corellations for all of these variables. (Don't impose that $i_t \geq 0$).

7. Repeat 6. with the additional restriction of $i \geq 0$. Is there a difference? Why or why not? In what regions of the state space does the constraint bind? How frequently are they visited?[3]

[1] I used the TFP series from `http://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/` which was then logged and detrended with an hp filter (this is somewhat questionable) with scale parameter 1600.

[2] The QuantEcon package has a built in function to do this.

[3] You might find it helpful to simulate a variable which is one when the constraint is binding and zero otherwise.