Redistribution with a continuum of agents

1 Introduction

We study optimal affine taxation in an economy with continuum of agents that face (uninsurable) shocks to their idiosyncratic labor productivities. We first specify the problem without aggregate risk.

2 Ramsey planner's problem

Given a joint distribution $\Gamma_0(x_0, m_0, e_0)$ over marginal utilities adjusted assets, scaled marginal utilities and past productivities, the Ramsey allocation solves the following problem from t>0.

$$\max_{c_t^i, l_t^i, x_t^i, m_t^i} \sum_t \beta^t \int \omega^i u(c_t^i, l_t^i) di \tag{1}$$

subject to

$$\frac{x_{t-1}^i u_{c,t}^i}{\beta \mathbb{E}_{t-1} u_{c,t}^i} = u_{c,t}^i (c_t^i - T_t) + u_{l,t}^i l_t^i + x_t^i \tag{2a}$$

$$\alpha_t^1 = m_t^i \mathbb{E}_t u_{c,t+1}^i \tag{2b}$$

$$\alpha_t^2 = m_t^i u_{c,t}^i \tag{2c}$$

$$-u_{l,t}^i = (1 - \tau_t) u_{c,t}^i e_t^i \tag{2d}$$

$$e_t^i = (1 - \nu)\bar{e} + \nu e_{t-1}^i + q \epsilon_t^i \tag{2e}$$

$$\int m_t^i di = 1 \tag{2f}$$

$$\int l_t^i e_t^i di = \int c_t^i di + g \tag{2g}$$

Equations (2b) and (2c) ensure that individual Euler equations hold. The FOCs of this problem are summarized below:

$$\mu_{t-1}^i = \mathbb{E}_{t-1} \frac{u_{c,t}^i}{\mathbb{E}_{t-1} u_{c,t}^i} \mu_t^i \tag{3a}$$

$$\omega^i u^i_{l,t} - \mu^i_t[u^i_{ll,t} l^i_t + u^i_{l,t}] - \phi^i_t u^i_{ll,t} + \lambda_t e^i_t = 0 \tag{3b}$$

$$\rho^i_{2,t} u^i_{c,t} + \rho^i_{1,t} \mathbb{E}_t u^i_{c,t+1} + \eta_t = 0 \tag{3c}$$

$$\omega^i u^i_{c,t} + x^i_{t-1} \frac{u^i_{cc,t}}{\beta \mathbb{E}_{t-1} u^i_{c,t}} \left[\mu^i_t - \mu^i_{t-1}\right] - \mu^i_t [u^i_{cc,t} c^i_t + u^i_{c,t}] + \beta^{-1} \rho^i_{1,t-1} m^i_{t-1} u^i_{cc,t} + \rho^i_{2,t} m^i_t u^i_{cc,t} - \phi^i_t e^i_t (1-\tau_t) u^i_{cc,t} - \lambda_t = 0 \tag{3d}$$

$$\int \phi^i_t u^i_{c,t} e^i_t di = 0 \tag{3e}$$

$$\int u^i_{c,t} \mu^i_t di = 0 \tag{3f}$$

$$\int \rho^i_{j,t} di = 0 \quad j = 1, 2 \tag{3g}$$

The solution to these FOC can be expressed recursively using individual states $\xi^i_{t-1} = [\mu^i_{t-1}, m^i_{t-1}, e^i_{t-1}]$, shocks ϵ^i_t and aggregate state $\Gamma_{t-1}(\xi^i_{t-1})$.

3 Ricardian Equivalence

Note that, using equation (2b), equation (3f) can be rewritten as

$$\int \frac{\mu^i_t}{m^i_t} = 0$$

Moreover, equation (3a) can be expressed as

$$\frac{\alpha^1_{t-1} \mu^i_{t-1}}{m^i_{t-1}} = \alpha^2_t \mathbb{E}_{t-1} \frac{\mu^i_t}{m^i_t}$$

By integrating both sides with respect to i, we obtain

$$\alpha^1_{t-1} \int \frac{\mu^i_{t-1}}{m^i_{t-1}} di = \alpha^2_t \int \frac{\mu^i_t}{m^i_t} di$$

This implies that if

$$\int \frac{\mu^i_{t-1}}{m^i_{t-1}} = 0$$

is satisfied for time $t-1$ we have then

$$\int \frac{\mu^i_t}{m^i_t} = 0$$

thus equation (3f) is redundant and we can choose T to be any value. For simplicity we set $T = 0$.

4 Optimal Taxes

In this section we derive a simple expression for taxes and use it to examine how the Ramsey plan uses distortionary taxes to redistribute inview of inequality in earnings and wealth.

The FOC for labor (3b) can be re-written as

$$\omega^i u^i_{c,t} \frac{u^i_{l,t} e^i_t}{u^i_{ll,t}} - \mu^i_t u^i_{c,t} e^i_t [l^i_t + \frac{u^i_{l,t}}{u^i_{ll,t}}] + \frac{\lambda_t u^i_{c,t} (e^i_t)^2}{u^i_{ll,t}} = \phi^i_t u^i_{c,t} e^i_t$$

Suppose $u(c,l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\gamma}}{1+\gamma}$. The above expression simplifies to

$$\frac{\omega^i u^i_{c,t} y^i_t}{\gamma} - \mu^i_t u^i_{c,t} y^i_t [1 + \frac{1}{\gamma}] - \lambda_t \gamma^{-1} \frac{y^i_t}{1-\tau_t} = \phi^i_t u^i_{c,t} e^i_t$$

Define $w_{i,t} = u^i_{c,t}[\omega^i - \mu^i_t(1+\gamma)]$ and $\bar{w}_t = \int u^i_{c,t} \omega^i di$ and $\bar{w}_{i,t} = \frac{w_{i,t}}{\bar{w}_t}$.
Integrating over i's we have

$$\frac{1}{1-\tau_t} = \frac{\hat{Y}_t}{Y_t} \frac{\bar{w}_t}{\lambda_t} \tag{4}$$

Where $Y_t = \int y^i_t di$ and $\hat{Y}_t = \int y^i_t \bar{w}_{i,t} di$

4.1 Special cases

With quasilinear utility and absent non-negativity constraints, we have a simple expression for how taxes depend on the distribution of initial Pareto weights.

$$\frac{1}{1-\tau} = \frac{\hat{Y}}{Y} \tag{5}$$

where $\bar{w}^i_t$ weights used in computing $\hat{Y}$ are given by $1 + \gamma(\omega^i - 1)$. We can quickly draw some implications from this.

- Taxes are constant through time.
- If $\omega^i = 1$, taxes are zero.
- Suppose productivities are iid over time, taxes will be close to zero.
- Suppose productivities have a permenant component, taxes will be positive if agents with high productivities have low pareto weights and vice versa

4.2 Risk aversion

With risk aversion we have spreads in marginal utilities. This generates a desire to use transfers even if $\omega^i = 1$.To examine this in detail conisder two extreme economies. The first has iid productivity shocks ($\rho = 0$) and the second one has permenant productivity shocks ($\rho = 1$)[1].

The term $\frac{\bar{w}_t}{\lambda_t}$ captures increase in the welfare from transfers relative to the value of resources (note λ is a multiplier on the resource constraint.) Thus a higher value for this term would imply larger role of taxes and consequent transfers.

[1]XXX stationarity

The first term $\frac{\hat{Y}_t}{Y_t}$ is increasing in the covariance between $\bar{w}_t^i$ and y_t^i labor earnings. Notice that these weights $\bar{w}_t^i$ are decreasing in the multiplier on the budget constraint of agent i : μ_t^i.

Consider the case where shocks to productivities are predominatly transitory. Agents save most of the shock for precautionary reasons. Overtime the agents with a unlucky stream of shocks will have lower wealth and low consumption and lower leisure. This will imply a lower $\bar{w}_t^i$ and higher y_t^i for such agents. The optimal response of government is to lower taxes and possibly give labor tax subsidies.

In the case where productivities have a permenant component, the output y^i will be higher for productive agents. These agents will also have higher wealth shares. This would make y_t^i and $\bar{w}_t^i$ positively correlated and call for higher taxes.

The next two sections describe the computational algorithm for computing an approximation for the Ramsey allocation for this economy.

5 Deterministic Steady State

We first look at an economy with $q = 0$. Given a joint distribution $\Gamma_0(\xi)$ the non-stochastic steady state is described by an allocation $c(\xi), l(\xi), x(\xi)$ and aggregate labor taxes τ.

Suppose $\Gamma_0(\xi)$ satisfies the following two properties

$$\int m^i di = 1 \tag{6a}$$

$$\int \frac{\mu^i}{m^i} di = 0 \tag{6b}$$

Given this we can conclude that the FOC can be satisfied with $\alpha^1 = \alpha^2$, $\eta = 0$ and $\rho_1^i = -\rho_2^i$. To obtain the deterministic steady state we add a few auxiliary variables $\phi(\xi)$,$\rho_1(\xi)$ and aggregates λ and α. The steady state is given by the following set of equations. The first set solves for individual policies for a guess of (α, λ, τ) and the second set solves the aggregates

$$\alpha = m^i u_c^i \tag{7a}$$

$$-u_l^i = (1-\tau) u_c^i e^i \tag{7b}$$

$$\gamma \phi^i = l^i[\omega^i - \mu^i(1+\gamma)] + \frac{\lambda e^i}{u_{ll}^i} \tag{7c}$$

$$u_c^i[\omega^i - \mu^i(1-\sigma)] + \rho_1^i m^i u_{cc}^i(\beta^{-1} - 1) - \phi_t^i e^i (1-\tau) u_{cc}^i - \lambda = 0 \tag{7d}$$

The aggregates (α, τ, λ) solve

$$\int l_t^i e_t^i di = \int c_t^i di + g \tag{8a}$$

$$\frac{1}{1-\tau_t} = \frac{\hat{Y}_t}{Y_t} \frac{\bar{w}_t}{\lambda_t} \tag{8b}$$

$$\int \rho_1^i di = 0 \tag{8c}$$

6 Approximation

Collect the individual policy rules in $y(\xi, \epsilon|\Gamma, q)$ and the aggregates as $Y(\Gamma)$. The laws of motion for the state variables are defined by $\xi'(\xi, \epsilon|\Gamma, q)$ and $\Gamma' = \Phi(\Gamma, q)$.

1. What is the goal ?

   The goal is to obtain an approximation of the individual policy rules and aggregate variables using pertubation around $q = 0$ for an aribrary choice of Γ. Thus even with a first order exapansion, how agents respond to shocks will depend both on the aggregate distribution Γ and individual state variables ξ

2. What are the challanges

   - Agggregating across agents
   - Forecasting future aggregate variables
   - Accounting for GE effects of higher order terms

3. What are the "tricks" ?

4. How is it actually implemented ?

5. Remarks - what are the class of problems this can work? extensions etc

7 Numerical results

We use the methods describe to compute solutions to two extreme economies.

Calibration

Graphs

- Taxes (IID,Persistent) - quantiles for consumption, output and labor earnings - time paths for covariance in consumption and output

Discussion

Extentions

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