Formulae

Discounting

- *Compound Interest.* Future value of C dollars invested for t years at an APR of r% per-year with compounding m times per-year.

$$V_t = C\big(1 + \frac{r}{m}\big)^{mt}$$

- *Continuous Compounding.* Future value of C dollars invested for t years at an APR of r% per-year with continuous compounding

$$V_t = Ce^{rt} = C\exp(rt)$$

- *Effective Annual Interest Rate* (EAR). If r is the APR (from above), then the EAR satisfies

$$\begin{aligned}\big(1 + r_{EAR}\big)^t &= \big(1 + \frac{r}{m}\big)^{tm} \\ \Longrightarrow \big(1 + r_{EAR}\big) &= \big(1 + \frac{r}{m}\big)^{m}\end{aligned}$$

- *Spot Rates and Discount Factors.* The "spot rate" for some date that is t-years in the future is r_t, the EAR on a zero-coupon bond with a discount factor DF_t.

$$\begin{aligned}1 &= DF_t\big(1 + r_t\big)^t \\ \Longrightarrow r_t &= \big(\frac{1}{DF_t}\big)^{1/t} - 1\end{aligned}$$

- *Present Value.* Present value of a stream of cash flows discounted at rate r:

$$V_0 = \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t}$$

- *Perpetuity.* Present value of a perpetual sequence of payments, C:

$$V_0 = \frac{C}{r}$$

- *Growing Perpetuity.* Present value of a perpetuity that grows at rate g with the first payment C, received in one period (*i.e.*, $C_1 = C$, $C_2 = C(1+g)$, $C_3 = C(1+g)^2$, $C_{t+1} = C_t(1+g)$)

$$V_0 = \frac{C}{r - g}$$

- *Annuity*. Present value of an annuity. Receive C each period for t periods.

$$V_0 = \frac{C}{r}(1 - \frac{1}{(1+r)^t})$$

- *Delay*. A perpetuity or annuity in which the first payment is delayed by t periods (so that the first payment is received at period $t+1$) has present value

$$V_0 = V_t \frac{1}{(1+r)^t} ,$$

where V_t is the value at date t, arrived at using the appropriate formula that appears above.

- *Yield-to-Maturity*. A bond with price V_0 and cash flows $c_1, c_2, \ldots c_t$, has a yield-to-maturity defined by

$$V_0 = \frac{c_1}{(1+y)} + \frac{c_1}{(1+y)^2} + \ldots + \frac{c_t}{(1+y)^t} .$$

Value-Based Management

- *Accounting Identity*:

$$V = D + E$$

- *Return on Invested Capital* (ROIC)

$$\text{ROIC} = \frac{\text{NOPAT}}{\text{Invested Capital}} = \frac{\text{NOPAT}}{\text{Sales}} \times \frac{\text{Sales}}{\text{Invested Capital}}$$

NOPAT = "Net Operating Profit After Tax." NOPAT is EBIT ("Earnings Before Interest and Taxes") less estimated taxes. Estimated taxes are the tax rate times EBIT, where we get the tax rate by dividing taxes actually paid by Net Income. "Invested Capital" can be computed as Debt plus Equity less cash, or as Total Assets less Current Liabilities less cash.

Present Value of Growth Options and the P/E Ratio

- *Constant Growth Model*: $g = PLOW \times ROE$

$$\begin{aligned}
BVE_{t+1} &= BVE_1 \left(1+g\right)^t \\
EPS_{t+1} &= EPS_1 \left(1+g\right)^t \\
&= BVE_1 \times ROE \left(1+g\right)^t \\
DIV_{t+1} &= DIV_1 \left(1+g\right)^t \\
&= EPS_1 \times PAY \left(1+g\right)^t \\
&= BVE_1 \times ROE \times PAY \left(1+g\right)^t
\end{aligned}$$

- *Valuation.* Given the dividend-price ratio and the growth rate, we can compute the cost of capital:

$$\begin{aligned} r &= \frac{DIV_1}{P_0} + g \\ &= \frac{DIV_1}{P_0} + PLOW \times ROE \end{aligned}$$

  Or, given the cost of capital (and BVE_1, ROE, and $PLOW$), we can compute the price-per-share:

$$\begin{aligned} P_0 &= \frac{DIV_1}{r - g} \\ &= \frac{BVE_1 \times ROE \times PAY}{r - PLOW \times ROE} \end{aligned}$$

- *Present Value of Growth Options* (PVGO) and the P/E ratio

$$\begin{aligned} PVGO &= P_0 - \frac{EPS_1}{r} \\ \Longrightarrow \frac{P_0}{EPS_1} &= \frac{PVGO}{EPS_1} + \frac{1}{r} \end{aligned}$$