Mössbauer Neutrino Oscillations

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1 Introduction

In 1958, Mössbauer discovered the recoil-less emmission gammas in crystal structures. This behaviour allowed for certain resonance energies to be carried by the photons that would otherwise not be observed in the standard emmission process. A natural progression would thus be to question whether this recoil-less process can be observed with other emission products such neutrinos, in crystal structures. A potential process that could induce such an interaction would be the decay :

$$^3\mathrm{H} \to {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

and the detection using the inverse process,

$$^3\mathrm{He} + e^- + \bar{\nu}_e \to {}^3\mathrm{H}\,.$$

2 External Wave-Packet Formalism

In essence, the benefit of the *external wave-packet formalism* is to treat the larger nuclei as external to the key internal interactions, like the anti-neutrino and electron in our case. For this reason, we can imagine a Feynman diagram that looks like figure 1.

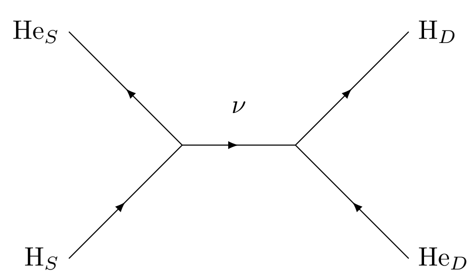

Figure 1: A simplified Feynman diagram of the interaction we are interested in.

2.1 Quantum Approach

We have to first go through a simple formalism where we use quantum mechanics to get an estimate on the production rate of neutrinos without accounting for oscillations. We see in the paper that they reference the following hamiltonian for the tritium source decay

$$H_S^+ = \int d^3x \frac{1}{\sqrt{2}} G_F \cos\theta_c \langle {}^3\mathrm{He}| \, J^\mu \, |{}^3\mathrm{H}\rangle \, \bar{\psi}_{e,S} \gamma_\mu (1-\gamma^5)\psi_\nu, \tag{1}$$

and the helium detection process,

$$H_D^- = \int d^3x \frac{1}{\sqrt{2}} G_F \cos\theta_c \langle {}^3\mathrm{H}| J^\mu |{}^3\mathrm{He}\rangle \bar{\psi}_\nu \gamma_\mu (1-\gamma^5)\psi_{e,D} \,. \tag{2}$$

To calculate the rate of neutrino production, we need to use these hamiltonians in combination with the ground state function(s):

$$\psi_{A,B,0}(\boldsymbol{x},t) = \left[\frac{m_A, \omega_{A,B}}{\pi}\right]^{\frac{3}{4}} \exp\left[-\frac{1}{2} m_A \omega_{A,B} |\boldsymbol{x} - \boldsymbol{x}_B|^2\right] e^{-iE_{A,B}t} \,, \tag{3}$$

for $A \in \{\mathrm{H}, \mathrm{He}\}$ and $B \in \{S, D\}$ for the atom type and location respectively. Applying Fermi's golden rule,

$$\begin{aligned}\Gamma_{i\to f} &= 2\pi |\langle f| H_I |i\rangle|^2 \rho(E_f) \\ &= 2\pi |\langle {}^3\mathrm{He}_S,\, {}^3\mathrm{H}_D| H_S^+ |{}^3\mathrm{H}_S,\, {}^3\mathrm{He}_D\rangle|^2 \rho(E_f)\end{aligned}$$

where we need to replace the nuclear states with their corresponding ground state functions described in equation 3. This is going to get ugly fast as everything is expanded, and this is before we evaluate the β^- decay processes. In order to avoid becoming over encumbered in this first section (especially since it is before the QFT approach), we note that the leading coeffecients to the Γ_0 term begin to appear as we factor out the G_F and $\cos\theta_c$ terms. That being said, the final result takes the form

$$\Gamma_p = \Gamma_0 X_S \,, \tag{4}$$

where

$$\Gamma_0 = \frac{G_F^2 \cos^2\theta_c}{\pi} |\psi(R)|^2 m_e^2 (|M_V|^2 + g_A^2 |M_A|^2) \left(\frac{E_{S,0}}{m_e}\right)^2 \kappa_S \,. \tag{5}$$

To avoid repeating the paper reviewed, I won't explain the meaning of each term here other than the ones of interest to the derivation. The nuclear matrix elements M_V and M_A arise from the two different β^- decay processes, Fermi and Gamow-Teller. The X_S term carries the energy and mass terms (and hence the momentum) from the introduced groundstate wavefunctions for the source,

$$X_S = 8\left(\eta_S + \frac{1}{\eta_S}\right)^{-3} e^{-\frac{p^2}{\sigma_{pS}^2}} \tag{6}$$

where

$$\eta_S = \sqrt{\frac{m_\mathrm{H} \omega_{\mathrm{H},S}}{m_\mathrm{He} m_{\mathrm{He},S}}}, \quad \sigma_{pS}^2 = m_\mathrm{H}\omega_{\mathrm{H},S} + m_\mathrm{He}\omega_{\mathrm{He},S} \,. \tag{7}$$

For the cross section, we consider the detection process and recall that the simplest relation of the differential cross-section is related to the scattering amplitude,

$$\frac{d\sigma}{d\Omega}(\theta,\phi) = |f(\theta,\phi)|^2 \,. \tag{8}$$

We can follow along in [**?**, **?**] to get the final result as

$$\sigma(E) = B_0 X_D \delta(E - E_{D,0}) \,, \tag{9}$$

where

$$B_0 = 4\pi G_F^2 \cos^2\theta_c |\psi_e(R)|^2 (|M_V|^2 + g_A^2 |M_A|^2) \kappa_D \,. \tag{10}$$

Here X_D is the detection analogue of the source X_S. The text continues to then combine the production, propogation and detection processes to reach the total rate,

$$\Gamma = \frac{1}{4\pi L^2} \int_0^\infty \rho(E)\sigma(E) dE \approx \frac{\Gamma_0 B_0}{4\pi L^2} X_S X_D \frac{(\gamma_S + \gamma_D)/2\pi}{(E_{S,0} - E_{D,0})^2 + (\gamma_S + \gamma_D)^2/4} \,, \tag{11}$$

where γ_S and γ_D are the energy widths assoiciated with production and detection. This result is used primarily as a comparison with the QFT treatment that is following.

2.2 QFT Approach