# PHYS 434: Assignment 3 Q2

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In [2]:
```
# 2
# This is the solution from Assignment 2 Q3(b).
N = 4
for Ket = 0:(2^N - 1)
    println("For Ket", Ket)
    for SpinIndex = 0:N - 1
        Spin = 2*((Ket>>SpinIndex) & 1) - 1
        print(Spin, " ")
    end
    println()
end
```

```
For Ket0
-1 -1 -1 -1
For Ket1
1 -1 -1 -1
For Ket2
-1 1 -1 -1
For Ket3
1 1 -1 -1
For Ket4
-1 -1 1 -1
For Ket5
1 -1 1 -1
For Ket6
-1 1 1 -1
For Ket7
1 1 1 -1
For Ket8
-1 -1 -1 1
For Ket9
1 -1 -1 1
For Ket10
-1 1 -1 1
For Ket11
1 1 -1 1
For Ket12
-1 -1 1 1
For Ket13
1 -1 1 1
For Ket14
-1 1 1 1
For Ket15
1 1 1 1
```

In [26]:
```
# We need to implement our nearest neighbour hamiltonian
# This Hamiltonian isn't particularly complicated:
# We just need to take a ket and take the product of the i-th eigenvalue
# element with the i-th+1 eigenvalue and then take a sum!
# In particular, note that for N particles, we expect N-1 interactions.
# Further, the diagonals of our matrix will just be the sum over
# each ket.
# I update the above loop with this:
N = 4
# (a) Allocate the Hamiltonian Array
Ham = zeros(Int8, 2^N, 2^N)
for Ket = 0:(2^N - 1)
    Diagonal::Int8 = 0
    for SpinIndex = 0:N - 2 # Since we expect N-1 interactions
        # (b) Fill the Diagonal Element for this particular Ket
        Spin1 = 2*((Ket>>SpinIndex) & 1) - 1
        Spin2 = 2*((Ket>>(SpinIndex+1)) & 1) - 1
        Diagonal = Diagonal + Spin1*Spin2
    end
    # (c) Fill the Hamiltonian with the diagonal component in the i=Ket and j=Ket compo
    Ham[Ket+1,Ket+1] = Diagonal
    #println("Hamiltonian for Ket = ", Ket, " is " ,-Diagonal)
end
print(Ham)
```

```
Int8[3 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
     0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
     0 0 -1 0 0 0 0 0 0 0 0 0 0 0 0 0
     0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
     0 0 0 0 -1 0 0 0 0 0 0 0 0 0 0 0
     0 0 0 0 0 -3 0 0 0 0 0 0 0 0 0 0
     0 0 0 0 0 0 -1 0 0 0 0 0 0 0 0 0
     0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
     0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
     0 0 0 0 0 0 0 0 0 -1 0 0 0 0 0 0
     0 0 0 0 0 0 0 0 0 0 -3 0 0 0 0 0
     0 0 0 0 0 0 0 0 0 0 0 -1 0 0 0 0
     0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
     0 0 0 0 0 0 0 0 0 0 0 0 0 -1 0 0
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
     0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 3]
```

After running this algorithm and building the Hamiltonian for a few $N$, we notice that the Hamiltonian diagonals behave like so:

$$\begin{aligned} N = 2 &\implies \{e_i\} = \{1, -1\} \\ N = 3 &\implies \{e_i\} = \{2, 0, -2\} \\ N = 4 &\implies \{e_i\} = \{3, 1, -1, -3\} \\ N = 5 &\implies \{e_i\} = \{4, 2, 0, -2, -4\} \\ &\vdots \end{aligned}$$

So we see that the energy eigenvalues correspond with spin systems. In particular, for odd $N$, the groundstate energy will be either $-1$ or $1$ due to the nature of the nearest neighbour interaction working with pairs. If $N$ is even, then we see that pairing will result in the groundstate energy being $0$ as we would expect.

In particular, we see that the eigenstates with the groundstates are Kets that occur most frequently, as we would expect with a statistical model. This agrees with the theoretical expectation since this is the form we would expect the lowest energy eigenstate to take with a strictly statistical approach.