Question 1

1.1 The speed of light is exactly 299792456 m/s, which rounded to 1% give us $c \approx 3.00 \times 10^8$. We also know $\hbar \approx 1.05 \times 10^{-34}\,\mathrm{m^2kg/s} = 6.58 \times 10^{-16}\,\mathrm{eV}\cdot\mathrm{s}$ up to 1% error.

1.2

(1.2.1) The given mass is only in units of energy, and we want a dimension of mass. So, we recall that energy is $\frac{[M][L]^2}{[T]^2}$, where $[M], [L], [T]$ are dimensions of mass, length and time respectively. Then, we see that we only need to get rid of the length/time dimensions twice, which is just our dimensions for c, so

$$938\,\mathrm{MeV} \to \frac{938\,\mathrm{MeV}}{c^2}$$

will be the true mass.

(1.2.2) We recall that a unit of energy is the eV, so to get a length from this quantity that has units of energy, we recognize $\hbar$ has units of energy-time and we can get length from the speed of light. That is,

$$\lambda = \frac{2\pi}{E_\gamma} \to \frac{2\pi}{E_\gamma} \cdot \frac{\hbar}{c}$$

will be the true wavelength.

(1.2.3) We recall that the dimensions of the inverse square-root gravitational constant are $\frac{[M]^{1/2}[T]}{[L]^{3/2}}$, and we want dimensions of $[M]$.

Question 2

Question 3

Question 4

4.1 To show that $\mathbf{O}(n)$ is a group under multiplication, we need only show the definition of a group is satisfied. In particular, if $M, N \in \mathbf{O}(n)$, notice

$$MN(MN)^t = MN(N^tM^t) = MNN^tM^t = MM^t = I \implies MN \in \mathbf{O}(n)\,,$$

which is closure (Notice we don't have to show $(MN)^tMN = I$ since we showed the inverse of MN is it's transpose and inverses are unique from linear algebra). Next, since $II^t = II = I$, we have an identity $I \in \mathbf{O}(n)$. Matrix multiplication is associative, and since $\mathbf{O}(n) \subset M_{n\times n}(\mathbb{R})$, we have associativity for free. Finally, we show inverses are also orthogonal. We know they exist, since

$$\det(MM^t) = det(I) \implies (\det(M))^2 = 1 \implies \det(M) = \pm 1\,.$$

But, since M is orthogonal, by definition $M^{-1} = M^t$, so

$$M^{-1}(M^{-1})^t = M^t(M^t)^t = M^tM = I \implies M^{-1} \in \mathbf{O}(n)\,.$$

So, we can conclude that $\mathbf{O}(n)$ is indeed a group.

4.2 To show that $\mathbf{SO}(n)$ is a group, we need only show that it is a subgroup, so our criterion aren't as restrictive. In particular, we get associativity for free, since $\mathbf{SO}(n) \subset \mathbf{O}(n)$, and since $\det(I) = 1$, $I \in \mathbf{SO}(n)$, and so we have the identity as well. All we need is closure and inverses. Well, notice if $M, N \in \mathbf{SO}(n)$, then

$$\det(MN) = \underbrace{\det(M)}_{1}\underbrace{\det(N)}_{1} = 1 \implies MN \in \mathbf{SO}(n)\,.$$

For inverses, we note

$$\det(M^{-1}) = \det(M^t) = \det(M) = 1 \implies M^{-1} \in \mathbf{SO}(n)\,.$$

Thus, we have shown $\mathbf{SO}(n)$ is indeed a subgroup of $\mathbf{O}(n)$.

4.3