6.2 a) $J_1 = -i\hbar \frac{d}{d\theta_1} R_1(\theta_1)\Big|_{\theta_1=0} = -i\hbar \frac{d}{d\theta_1}\begin{pmatrix}1&0&&\\0&1&&\\&&\cos\theta_1&-\sin\theta_1\\&&\sin\theta_1&\cos\theta_1\end{pmatrix}\Bigg|_{\theta_1=0}$

$$= -i\hbar \begin{pmatrix}0&0&&\\0&0&&\\&&-\sin\theta_1&-\cos\theta_1\\&&\cos\theta_1&-\sin\theta_1\end{pmatrix}\Bigg|_{\theta_1=0} = -i\hbar\begin{pmatrix}0_{2\times2}&&\\&0&-1\\&1&0\end{pmatrix}$$

$$K_1 = -i\hbar \frac{d}{d\varphi_1} B_1(\varphi_1)\Big|_{\varphi_1=0} = -i\hbar \frac{d}{d\varphi_1}\begin{pmatrix}\cosh(\varphi_1)&-\sinh(\varphi_1)&&\\-\sinh(\varphi_1)&\cosh(\varphi_1)&&\\&&1&0\\&&0&1\end{pmatrix}\Bigg|_{\varphi_1=0}$$

$$= -i\hbar\begin{pmatrix}\sinh(\varphi_1)&-\cosh(\varphi_1)&\\-\cosh(\varphi_1)&\sinh(\varphi_1)&\\&&0_{2\times2}\end{pmatrix} = -i\hbar\begin{pmatrix}0&-1&\\-1&0&\\&&0_{2\times2}\end{pmatrix}$$

b) $[J_1, K_1] = J_1K_1 - K_1J_1 = -\hbar^2\begin{pmatrix}0_{2\times2}&&\\&0&-1\\&1&0\end{pmatrix}\begin{pmatrix}0&-1&\\-1&0&\\&&0_{2\times2}\end{pmatrix} + \hbar^2\begin{pmatrix}0&-1&\\-1&0&\\&&0_{2\times2}\end{pmatrix}\begin{pmatrix}0_{2\times2}&&\\&0&-1\\&1&0\end{pmatrix}$

$$= -\hbar^2\begin{pmatrix}0_{2\times2}&\\&0_{2\times2}\end{pmatrix} + \hbar^2\begin{pmatrix}0_{2\times2}&\\&0_{2\times2}\end{pmatrix} = 0$$