

6.5 a) $\vec{k} = (k_1, k_2, k_3)^T$, $k_i = -i\frac{1}{2}\sigma_i$, $i\sigma_0$,

$$B_1(\psi_1) = \exp\left\{\frac{1}{2}\psi_1\sigma_1\right\} = \sum_{k=0}^{\infty} \frac{1}{k!} \frac{1}{2^k} \psi_1^k \sigma_1^k = \sum_{k=0}^{\infty} \left(\frac{1}{2^k (2k)!} \psi_1^{2k} \mathbb{I} + \frac{1}{2^{2k+1} (2k+1)!} \psi_1^{2k+1} \sigma_1 \right)$$

$$= \mathbb{I} \cosh\left(\frac{\psi_1}{2}\right) + \sigma_1 \sinh\left(\frac{\psi_1}{2}\right) = \begin{pmatrix} \cosh(\psi_1/2) & \sinh(\psi_1/2) \\ \sinh(\psi_1/2) & \cosh(\psi_1/2) \end{pmatrix}$$

b) Using the above, we can get,

$$B_2(\psi_2) = \mathbb{I} \cosh\left(\frac{\psi_2}{2}\right) + \sigma_2 \sinh\left(\frac{\psi_2}{2}\right) = \begin{pmatrix} \cosh(\psi_2/2) & -i \sinh(\psi_2/2) \\ i \sinh(\psi_2/2) & \cosh(\psi_2/2) \end{pmatrix}$$

c) Again, we have that,

$$B_3(\psi_3) = \mathbb{I} \cosh\left(\frac{\psi_3}{2}\right) + \sigma_3 \sinh\left(\frac{\psi_3}{2}\right) = \begin{pmatrix} \cosh(\psi_3/2) + \sinh(\psi_3/2) & 0 \\ 0 & \cosh(\psi_3/2) - \sinh(\psi_3/2) \end{pmatrix}$$

$$= \begin{pmatrix} e^{\psi_3/2} & 0 \\ 0 & e^{-\psi_3/2} \end{pmatrix}$$