6.4 a) We know $J_2 = \frac{1}{2}\sigma_2$, $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, so, since we have $\sigma_2^{2k} = \mathbb{1}$, $\sigma_2^{2k+1} = \sigma_2$, we get,

$$R_2(\theta_2) = \exp\{ i/_2 \, \theta_2 \sigma_2 \} = \sum_{k=0}^{\infty} \frac{1}{k!} (\tfrac{i}{2})^k \theta_2^k \sigma_2^k$$

$$= \mathbb{1} \sum_{k=0}^{\infty} \frac{(-1)^k}{2^{2k}(2k)!} \theta_2^{2k} + i \sum_{k=0}^{\infty} \frac{(-1)^k \sigma_2}{2^{2k+1}(2k+1)!} \theta_2^{2k+1}$$

$$= \cos(\theta_2/2)\,\mathbb{1} + i\sigma_2 \sin(\theta_2/2)$$

$$= \begin{pmatrix} \cos(\theta_2/2) & \sin(\theta_2/2) \\ -\sin(\theta_2/2) & \cos(\theta_2/2) \end{pmatrix}$$

b) Same as above, but now, we use θ_3 & $\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, so,

$$R_3(\theta_3) = \exp\{ i/_2 \, \theta_3 \sigma_3 \} = \ldots = \begin{pmatrix} \cos(\theta_3/2) + i\sin(\theta_3/2) & 0 \\ 0 & \cos(\theta_3/2) - i\sin(\theta_3/2) \end{pmatrix} = \begin{pmatrix} e^{i\theta_3/2} & 0 \\ 0 & e^{-i\theta_3/2} \end{pmatrix}$$