Dimensional Analysis

Question 1

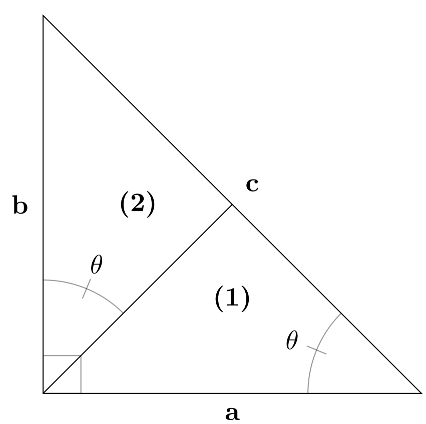
Simple exercise of comparing dimensions. In particular we know that period, $P \sim [T]$, must be related to the two physical quantities of gravity, $g \sim [L][T]^{-2}$, and length of the string, $\ell \sim [L]$. Thus, supposing some constant, α, we can suppose

$$\begin{aligned} P &\sim \alpha g^a \ell^b \sim \alpha \left([L][T]^{-2}\right)^a [L]^b \\ [T] &\sim \alpha [L]^{a+c}[T]^{-2a} \\ \implies a &= -\frac{1}{2} \qquad \implies \qquad b = \frac{1}{2} \end{aligned}$$

and thus, $P \sim \alpha\sqrt{\frac{\ell}{g}}$.

Question 2

We start by splitting our right angle triangle into two parts:



Note by geometric properties, the two smaller triangles (**(1)** and **(2)**) are actually similar (infact, all 3 are similar!). Thus, as is shown with θ being consistent with them both, we can apply our rule of area:

$$\begin{aligned} A_{(0)} &= A_{(1)} + A_{(2)} \\ c^2 f(\theta) &= a^2 f(\theta) + b^2 f(\theta) \\ &\boxed{c^2 = a^2 + b^2} \end{aligned}$$

Question 3

First, we need to relate the number of rowers with the area of the boat, A. This effectively reduces to the square-cubed law; the depth of the boat scales with n so the area must go as $a^{\frac{2}{3}}$. Now, the problem is straight forward if we assume maximum velocity of the boat:

$$F \sim \rho A v^2$$
$$P \sim \rho a^{\frac{2}{3}} v^3$$
$$n \sim \rho a^{\frac{2}{3}} v^3$$
$$\boxed{v \sim n^{\frac{1}{9}} \rho^{-\frac{1}{3}}}$$

Question 4

This is simple; we know that distance is inversely proportional to mass in natural units, so we can approximate r as such:

$$r \sim \frac{1}{m} = \frac{1}{0.938\,\text{GeV}} \cdot 2 \times 10^{-14}\,\text{Gev} \cdot \text{cm} = 2.132 \times 10^{-14}\,\text{cm}$$

as required.