Question 1

Question 2

Question 3

(a) We recall that

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \; Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \; X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Then, we see that

$$HXH = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix} = Z$$

as expected.

(b) To avoid drawing out the diagrams in tikz, I label them as 1, 2, and 3 from top to bottom. So, circuit 1 is the one with 4 hadamard matrices.

(1) We just need to expand the tensor expression, that is the first diagram can equivalently be written as

$$(H \otimes H)(CU)(H \otimes H) = (H \otimes H) \left(|0\rangle \langle 0| \otimes I + |1\rangle \langle 1| \otimes X \right) (H \otimes H)$$

where CU is the $CNOT$ gate. We notice that it will be convenient to use BraKet notation. In that notation, we recognize that $X = |1\rangle \langle 0| + |0\rangle \langle 1|$ and $H = \frac{1}{\sqrt{2}} \left((|0\rangle + |1\rangle) \langle 0| + (|0\rangle - |1\rangle) \langle 1| \right)$