

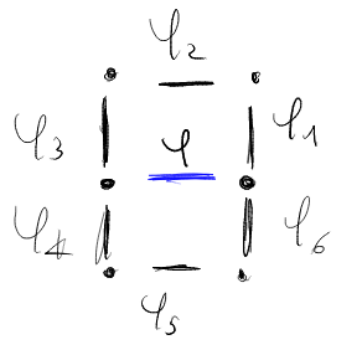
Fact sheet: Metropolis steps

Metropolis proposal: $\varphi^n = \varphi + \delta \quad \delta \in (-1, 1)$

$$S = \beta \sum_{\square} (1 - \text{Re } \square)$$

$$-\frac{\Delta S}{\beta} = \cos(\varphi^n + \varphi_1 - \varphi_2 - \varphi_3) + \cos(\varphi_5 + \varphi_6 - \varphi^n - \varphi_4) +$$

$$- \cos(\varphi + \varphi_1 - \varphi_2 - \varphi_3) - \cos(\varphi_5 + \varphi_6 - \varphi - \varphi_4)$$



$$= 2 \left[\cos \left(\varphi^n + \underbrace{\frac{\varphi_1 - \varphi_2 - \varphi_3 + \varphi_4 - \varphi_5 - \varphi_6}{2}}_{\bar{\varphi}} \right) - \cos(\varphi + \bar{\varphi}) \right]$$

$$= 2 \left[\cos(\varphi^n + \bar{\varphi}) - \cos(\varphi + \bar{\varphi}) \right]$$

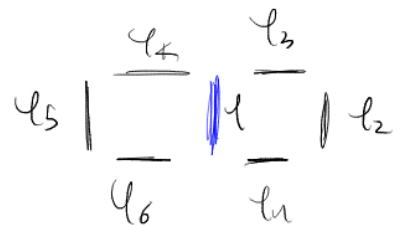
$$\bar{\varphi} = \frac{\varphi_1 - \varphi_2 - \varphi_3 + \varphi_4 - \varphi_5 - \varphi_6}{2}$$

Accept: $\xi < e^{2\beta [\cos(\varphi^n + \bar{\varphi}) - \cos(\varphi + \bar{\varphi})]}$

$$\xi \in [0, 1]$$

$$-\frac{\Delta S}{\beta} = \cos(\varphi_1 + \varphi_2 - \varphi_3 - \varphi^n) + \cos(\varphi_6 + \varphi^n - \varphi_4 - \varphi_5) +$$

$$- \cos(\varphi_1 + \varphi_2 - \varphi_3 - \varphi) - \cos(\varphi_6 + \varphi - \varphi_4 - \varphi_5)$$



$$= 2 \left[\cos \left(\varphi^n - \underbrace{\frac{\varphi_1 + \varphi_2 - \varphi_3 + \varphi_4 + \varphi_5 - \varphi_6}{2}}_{\bar{\varphi}} \right) - \cos(\varphi - \bar{\varphi}) \right]$$

$$= 2 \left[\cos(\varphi^n - \bar{\varphi}) - \cos(\varphi - \bar{\varphi}) \right]$$

$$\bar{\varphi} = \frac{\varphi_1 + \varphi_2 - \varphi_3 + \varphi_4 + \varphi_5 - \varphi_6}{2}$$

Accept: $\xi < e^{2\beta [\cos(\varphi^n - \bar{\varphi}) - \cos(\varphi - \bar{\varphi})]}$

$$\xi \in [0, 1]$$