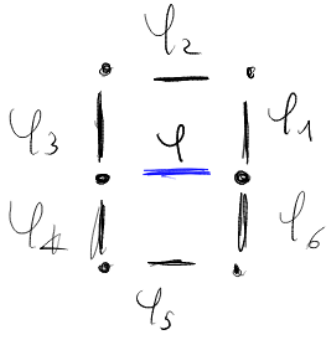
Fact sheet: Metropolis step

Metropolis proposal: $\varphi^n \in [0, 2\pi]$

$$S = \beta \sum_{\square}' (1 - \mathrm{Re}\,\square)$$

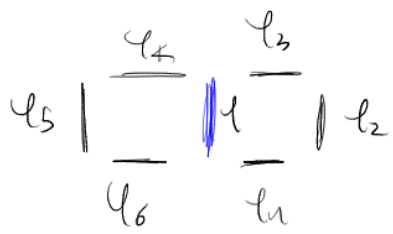


$$-\frac{\Delta S}{\beta} = \cos\left(\varphi^n + \varphi_1 - \varphi_2 - \varphi_3\right) + \cos\left(\varphi_5 + \varphi_6 - \varphi^n - \varphi_4\right) +$$
$$-\cos\left(\varphi + \varphi_1 - \varphi_2 - \varphi_3\right) - \cos\left(\varphi_5 + \varphi_6 - \varphi - \varphi_4\right)$$
$$= 2\left[\cos\left(\varphi^n + \underbrace{\frac{\varphi_1 - \varphi_2 - \varphi_3 + \varphi_4 - \varphi_5 - \varphi_6}{2}}_{\bar{\varphi}}\right) - \cos\left(\varphi + \bar{\varphi}\right)\right]$$

$$= 2\left[\cos\left(\varphi^n + \bar{\varphi}\right) - \cos\left(\varphi + \bar{\varphi}\right)\right] \qquad \bar{\varphi} = \frac{\varphi_1 - \varphi_2 - \varphi_3 + \varphi_4 - \varphi_5 - \varphi_6}{2}$$

Accept: $\xi < e^{2\beta\left[\cos\left(\varphi^n + \bar{\varphi}\right) - \cos\left(\varphi + \bar{\varphi}\right)\right]}$ $\qquad \xi \in [0, 1]$

$$-\frac{\Delta S}{\beta} = \cos\left(\varphi_1 + \varphi_2 - \varphi_3 - \varphi^n\right) + \cos\left(\varphi_6 + \varphi^n - \varphi_4 - \varphi_5\right) +$$
$$-\cos\left(\varphi_1 + \varphi_2 - \varphi_3 - \varphi\right) - \cos\left(\varphi_6 + \varphi - \varphi_4 - \varphi_5\right)$$



$$= 2\left[\cos\left(\varphi^n - \underbrace{\frac{\varphi_1 + \varphi_2 - \varphi_3 + \varphi_4 + \varphi_5 - \varphi_6}{2}}_{\bar{\varphi}}\right) - \cos\left(\varphi - \bar{\varphi}\right)\right]$$

$$= 2\left[\cos\left(\varphi^n - \bar{\varphi}\right) - \cos\left(\varphi - \bar{\varphi}\right)\right] \qquad \bar{\varphi} = \frac{\varphi_1 + \varphi_2 - \varphi_3 + \varphi_4 + \varphi_5 - \varphi_6}{2}$$

Accept: $\xi < e^{2\beta\left[\cos\left(\varphi^n - \bar{\varphi}\right) - \cos\left(\varphi - \bar{\varphi}\right)\right]}$ $\qquad \xi \in [0, 1]$