HW10
Daniel Ginsburg
1. We prove that for any given set of boxes with specified weights, the greedy algorithm currently in place minimizes the amount of trucks needed. We do this by proving that for any given number of trucks, the greedy solution transports at least as much weight as any other solution (including the optimal) and therefore will require no more trucks than any other solution. So if the optimal solution uses n trucks to transfer p packages, the greedy solution will transfer at least p packages with n trucks so it will definitely transfer p packages with n trucks. This proves that the greedy solution is just as good as the optimal solution.
2.We know that no truck can carry more than the max load W (given by the problem).
Let G represent the greedy solution and O represent some other solution.
Let Tot_g represent the total weight G has transferred, and Tot_o represent the total weight O has transferred.

Greedy Stays Ahead:
Base Case:
let us assume that G and O are equal until Truck R.
Until Truck R, Tot_g and Tot_o are the same. For Truck R, Tot_g increases by X_r, and Tot_o increases by $X_r - w$ (The weight of the packages that O didn't pack.) Thus $Tot_g \geq Tot_o$.

Inductive Step:
At the next truck R+1, either O packs less/equal weight than G or it packs more weight.
Case 1- O packs less/equal weight than G:
In this case Tot_g is obviously still $\geq Tot_o$.
Case 2- O packs more weight than G:
Recall w is all the weight of all the packages "underpacked" by O. It must pack those packages first because it must pack packages in order (stated in problem) and they are the earliest arrivals left .
Let G pack weight X_{r+1}. O can pack AT MOST X_{r+1} + w. (This is the case where O packs ALL the packages it underpacked before). If O could pack anymore, it would mean that G could ALSO pack more. This isn't possible because G by definition always packs until there isn't enough available space/weight left in the truck. Thus if we compare O to G we get
$X_r - w + X_{r+1} + w \leq X_r + X_{r+1}$
$X_r + X_{r+1} \leq X_r + X_{r+1}$
$Tot_o \leq Tot_g$
Thus, for all trucks, the total weight transferred by G will be $\geq$ the total weight of O, for all O (including optimal). Thus if O uses n trucks to transfer W weight, G will transfer at least W with n trucks. Meaning it may use fewer trucks to transfer the same weight.
So Total Trucks of G, $TT_g \leq TT_o$ and greedy is optimal.