Home Work 2

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We must prove that $f(n) = n^2 + 3n^3 \in (n^3)$

Let $g(n) = n^3$. First we prove that $f(n) = O(g(n))$. Thus for some c and all $n \geq n_0$, $f(n) \leq cg(n)$. If we choose $c = 4$ and $n_0 = 1$ then

$$g(n) = 4n^3 = 3n^3 + 1n^3 \geq 3n^3 + 1n^2 = f(n) \tag{1}$$

Essentially this equation boils down to $n^3 \geq n^2$, which means $n \geq 1$ (if n is positive). Thus we see $f(n) = O(g(n))$.

Now we prove that $f(n) = \Omega(g(n))$. This means $f(n) \geq cg(n)$ for some c, and for all n such that $n \geq n_0$. This is true for $n_0 = 1$ and $c = 1$. we know that as long as n is positive then

$$\begin{aligned}
&3 * (1n^3) \geq 1n^3 \\
&3n^3 + n^2 \geq 1n^3 \ (\textit{because n is positive}) \\
&f(n) \geq cg(n)
\end{aligned}$$

Prove 2:

Let $g(n) = 2n^{10}$. We prove that $2n^{n+10} = O(g(n))$. Thus for some c and all $n \geq n_0$, $f(n) \leq cg(n)$. If we rewrite 2^{n+10} as $2^n + 2^{10}$ then the equation holds true for $c = 2^{10}$ and $n_0 = 1$. This is because

$$\begin{aligned}
f(n) = 2^{n+10} =& 2^n * 2^{10} \leq 2^{10} * 2^n \\
& f(n) \leq cg(n) \ (every\ number\ is\ \leq\ than\ itself)
\end{aligned}$$