

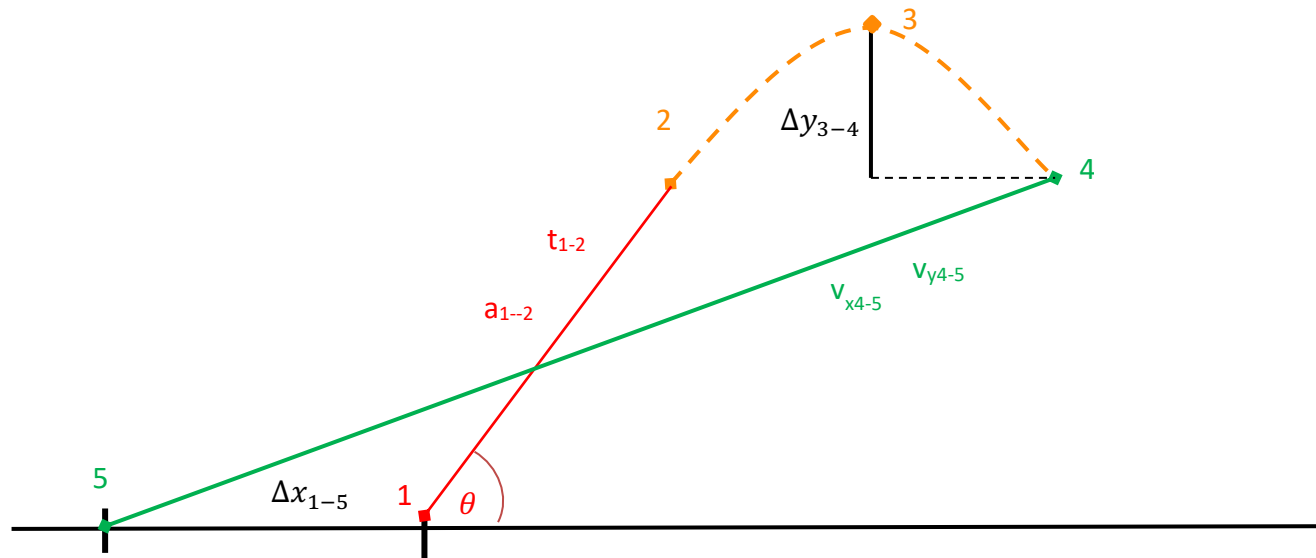
Über Rocket: Hamster Huey and Algebra Alex

Description: One breezy afternoon Algebra Alex decides to launch Hamster Huey into the air using a model rocket. The rocket is launched over level ground, from rest, at a specified angle above the East horizontal. The rocket engine is designed to burn for specified time while producing a constant net acceleration for the rocket. Assume the rocket travels in a straight-line path while the engine burns. After the engine stops the rocket continues in projectile motion. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens the rocket instantly changes speed and descends at a constant vertical speed. A horizontal wind blows the rocket, with parachute, from the East to West at the constant speed of the wind. Assume the wind affects the rocket only during the parachute stage.

Goal: To calculate the total x-displacement of Hamster Huey.

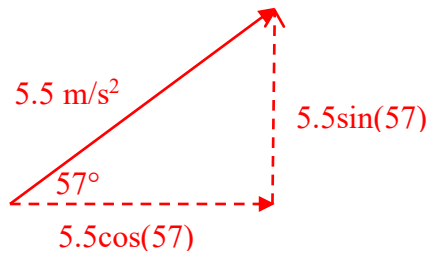
Givens and Diagram:

Launch Angle:	$\theta = 57^\circ \text{ N of E}$
Engine Burn Time:	$t_{1-2} = 8.2 \text{ sec}$
Net acceleration of rocket while engine burns:	$a_{1-2} = 5.5 \text{ m/s}^2 \text{ diagonally up and to the right}$
Vertical Distance rocket falls from max height before parachute opens:	$\Delta y_{3-4} = 71 \text{ m down}$
Rocket with parachute constant vertical speed:	$v_{y4-5} = 7 \text{ m/s down}$
Wind and rocket with parachute constant horizontal speed:	$v_{x4-5} = 17 \text{ m/s left}$
Gravity:	$a = -9.8 \text{ m/s}^2 \text{ down}$



Strategy: I will begin by calculating the displacement from the initial position in both the x and y directions from 1-2 so that different values such as the maximum height can be found later in the calculations. Then, the velocity of the rocket after the engine stops burning at 2 will be found and can be separated into the x and y component velocities. The x velocity at 2 will be used to calculate the displacement from 2-4 as it is constant. The next step would be to find the time it takes to go from 2-3. This can be done by finding how long it will be until the y velocity becomes 0, as this is the apex of the free fall movement. This time will then also be used to find the maximum height, and as the distance from max height to 4 is given the time from 3-4 can also be calculated. The height at 4 can also be calculated using this time, and the time from 2-4 is known and the x displacement during that time period can be calculated. Finally, the x and y velocities at 4 are given and so time until $y=0$ can be found (as this is the final y position), and the x displacement during that time can be calculated. This would final x displacement would give the final answer as each of the x displacement equations have factored in the previous x displacements and have thus been tracking the change in x position overall.

First the y_2 and x_2 are found to be used in further calculations such as the total displacement and so that the maximum height can be found:



$$y_2 = \frac{1}{2}a_{1-2}\Delta t_{1-2}^2 + v_{y2}\Delta t_{1-2} + y_1$$

$$y_2 = \frac{1}{2}(5.5\sin(57))(8.2)^2 + (0) + (0)$$

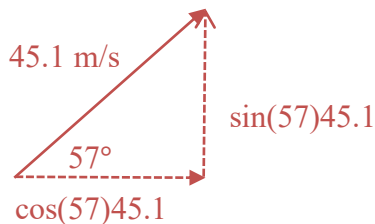
$$y_2 = 155.079 \text{ m}$$

$$x_2 = \frac{1}{2}a_{1-2}\Delta t_{1-2}^2 + v_{x2}\Delta t_{1-2} + x_1$$

$$x_2 = \frac{1}{2}(5.5\cos(57))(8.2)^2 + (0) + (0)$$

$$x_2 = 100.709 \text{ m}$$

From here the v_2 will be found for use in the free fall section calculations and can be separated into the x and y components.



$$v_2 = a_{1-2}\Delta t_{1-2} + v_1$$

$$v_2 = (5.5)(8.2) + (0)$$

$$v_2 = 45.1 \text{ m/s}$$

Then the t_{2-3} component can be found by setting v_{y3} to 0 and this can be used for the max height and to find the x displacement during that time. From here the calculation can be done to find y_3 for use in the next section and x_3 can also be calculated.

$$v_{y3} = a_{2-3}\Delta t_{2-3} + v_{2y}$$

$$0 = -9.8\Delta t_{2-3} + (45.1\sin(57))$$

$$\Delta t_{2-3} = 3.85959 \text{ s}$$

$$y_3 = \frac{1}{2}a_{y2-3}\Delta t_{2-3}^2 + v_{y2}\Delta t_{2-3} + y_2$$

$$y_3 = \frac{1}{2}(-9.8)(3.86)^2 + (45.1\sin(57))(3.86) + 155$$

$$y_3 = 228.072 \text{ m}$$

$$x_3 = \frac{1}{2}a_{x2-3}\Delta t_{2-3}^2 + v_{x2}\Delta t_{2-3} + x_2$$

$$x_3 = (0) + (45.1\cos(57))(3.86) + (100.71)$$

$$x_3 = 195.513 \text{ m}$$

The displacement of 3-4 can be found knowing that the hamster falls 71 meters from the max height so, the t_{3-4} can be found and used to find x displacement during that time. The y_4 position can also be calculated so that the t_{3-4} can be found. In this case the initial height does not matter for the y -dir. equation, one can simply set delta y to be -71m. Using that time, the y_4 position and x_4 position can be found.

$$y_f = \frac{1}{2}a_{2-4}\Delta t_{3-4}^2 + v_{y3}\Delta t_{3-4} + y_3$$

$$-71 = \frac{1}{2}(-9.8)\Delta t_{3-4}^2 + (0)$$

$$-71/-4.9 = \Delta t_{3-4}^2$$

$$3.80655 \text{ s} = \Delta t_{3-4}$$

$$y_4 = \frac{1}{2}a_{3-4}\Delta t_{3-4}^2 + v_{y3}\Delta t_{3-4} + y_3$$

$$y_4 = \frac{1}{2}(-9.8)(3.80655)^2 + (0) + (228.072)$$

$$y_4 = 157.072 \text{ m}$$

$$x_4 = \frac{1}{2}a_{3-4}\Delta t_{3-4}^2 + v_{x3}\Delta t_{3-4} + x_3$$

$$x_4 = (0) + (45.1\cos(57))(3.80655) + 195.513$$

$$x_4 = 289.014 \text{ m}$$

Finally, using the height at 4 and knowing that the hamster goes down to $y=0$, I can find the t_{4-5} and from there use that time to find the final position/total displacement x_5 given the constant parachute velocities.

$$y_5 = \frac{1}{2}a_{4-5}\Delta t_{4-5}^2 + v_{y4}\Delta t_{4-5} + y_4$$

$$0 = (0) + -7\Delta t_{4-5} + 157.072$$

$$-157.072/-7 = \Delta t_{4-5}$$

$$22.4389 \text{ s} = \Delta t_{4-5}$$

$$x_5 = \frac{1}{2}a_{4-5}\Delta t_{4-5}^2 + v_{x4}\Delta t_{4-5} + x_4$$

$$x_5 = (0) + (-17)(22.4389) + (289.014)$$

$$x_5 = -92.45 \text{ m west}$$