Epsilon's Enigma

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The problem:

A school called the Epsilon School is increasing its student population from 490 to 630 for the following year, and is thus receiving 7 new teachers. In the past the incoming sophomore class has been as big as the graduating senior class. Below is a table of the teacher distribution and the enrollments for the current year:

- Mathematics: 6
- Chemistry: 3
- Physics: 3
- Biology: 4
- Social Studies: 5
- English: 5
- Foreign language: 3
- Music: 1
- Art: 1

Departmental enrollment Totals: September 2021

Department	10th	11th	12th	Total
Art:	31	33	35	99
Biology:	198	95	26	319
Chemistry:	59	126	109	294
English:	183	155	152	490
French:	41	32	49	122
German:	19	22	10	51
Spanish:	51	26	33	110
Mathematics:	184	201	262	647
Music:	50	56	49	155
Physics:	50	58	183	291
Social Studies:	183	131	59	373

What will be done:

We will develop a model that will allow us to figure out where the teacher will be most needed given the data about the incoming 2022-2023 class.

Assumptions:

For our model we assumed that:

- That the dropout rate is the same for the move from sophomore-junior and from junior-senior
- Each class type requires the same amount of work from the teacher
- Each of the teachers will require the same amount of resources from the school once they are hired
- Every student is required to take one English course (needed for calculating class sizes)
- The students of the 2022-2023 year will have the same ratio of enrollment for each grade, for example, if 10/100 students took art in 10th grade the year of 2021-2022, then the same ratio of 10/100*#students will take it the following year
- No new students are joining junior or senior year
- Foreign language teachers can teach all three languages

Analysis of problem

- The problem gives you the amount of teachers, the amount of students in total before and after the incoming increase, the amount of enrollment for each class, and the amount of teachers for each subject.
- Using this information we can assume that each student takes exactly one english class a year. From here we can assume that the amount of students taking english is the size of the grade. This can be used to determine enrollment ratios by stating that the **ratio = class enrollment/total class size**
- This problem does not give any information on why the ratios of class enrollment are what they are and whether or not the ratios are dependent on the grade, the needs of certain people, or if they are random from year to year.
- Since there can only be seven faculty added it makes sense that the problem wants them to go to where they are most needed as there are more than seven subjects.

Hypothesis:

- We can assume that the dropout rate is constant over the time that the students spend at the school, and the size of the senior and junior classes for the next year will be less than the previous year by that dropout rate.
- We can write an equation that calculates the dropout rate knowing this information, knowing that the amount of seniors is 140 less than the amount of sophomores, and knowing that the total class size is 630.
- If one can find the enrollment rates for the previous year this allows you to calculate the per grade subject enrollment using new class size values.
- If you find the total enrollment per subject for the next year, you can calculate the student to teacher ratio assuming no new teachers are added, and thus we can find what subjects need the new teachers most.

Justification of Model:

- In this model we assume that we can write the equation: **630=(155r+140)+r(183)+r(155)**, because we know the total amount of students in each grade for the previous year, we can set the amount of seniors and juniors in the 2022-2023 school year equal to the amount of seniors and juniors in the previous year, respectively, multiplied by an unknown percent of students left after dropping out, **r**. The number of sophomores in the 2022-2023 school year would be equal to 140 greater than the amount of seniors.
- We also in this model will assume that each subject requires the same amount of work from the teacher and that teacher teacher will be paid the same, etc. because we have no information on the structure or workload of different subjects in this school
- Assuming that each grades class enrollment to total student ratios remain the same each year, we can find the number of students in each class by multiplying each ratio by the predicted number of students for each grade in the 2022-2023 school year
- Given the previous statement, it is reasonable to assume that the subjects that will have the highest student to teacher ratios the next year assuming no teachers are added will require the additional teachers, with the seven highest student to teacher subjects getting the seven teachers. This is because assuming each subject requires the same amount of work from the teacher teaching the largest amount of students would require the largest amount of work.

Our Model:

- The equation for the amount of sophomores (x) is: 630=(155r+140)+r(183)+r(155).
- The dropout rate is r, the amount of seniors is 155r, the amount of juniors is 183r, and the amount of sophomores is the amount of seniors (155r) + 140.
- The model then finds the amount of students taking each class for the 2022-2023 year for each grade and subject using the equation: number students in grade *(number of students in grade for previous year/number of students in that class and grade previous year)
- Then the model adds up the enrollment of students in the subjects over all grades
- The model then divides this enrollment per subject total by the amount of teachers for that subject
- The model then assigns the teachers to the classes with the seven highest values for the previously calculated student teacher ratio value

How model can be tested and prediction:

- In order to test this model the amount of students in the following grades can be found, from there the enrollments can be found using the ratios that were previously calculated, from here these can be applied to the total enrollment and teacher counts to find which subjects have the least amount of teachers per student. Thus the data will be found to indicate where teachers are needed most, and those subjects will be assigned teachers.
- A prediction can be made looking that previous year's enrollment and knowing that the amount of sophomores will be the greatest the following year and thus the amount of swing that grade has will be the most, that the teachers will be assigned to the classes the largest amount of sophomores take, in this case the biology, english, math, social studies, and foreign language (combined) classes.

Our Solution:

Finding the amount of students in the following year per grade:

630=(155r+140)+r(183)+r(155)

630=155r+140+183r+155r

630=140+493r

490=493r

r≈0.9939

Sophomores = 0.9939(155)+140**≈294**

Juniors = 0.99(183)**≈182**

Seniors= 0.99(155)**≈154**

Finding enrollment for 2022-2023 year (rounded):

Sophomores:

Art:31/183*294**≈50**
Chemistry: 59/183*294**≈95**
French:41/183*294**≈66**
Spanish:51/183*294**≈82**
Music:50/183*294**≈80**
Soc.Stud.:183/183*294**≈294**
Biology:198/183*294**≈318**
English:**294**
German:19/183*294**≈31**
Math:184/183*294**≈295**
Physics:50/183*294**≈80**

Juniors:

Art:33/155*182**≈39**
Chemistry: 126/155*182**≈148**
French:32/155*182**≈38**
Spanish:26/155*182**≈31**
Music:56/155*182**≈66**
Soc.Stud.:131/155*182**≈154**
Biology:95/155*182**≈112**
English:**182**
German:22/155*182**≈26**
Math:201/155*182**≈236**
Physics:58/155*182**≈68**

Seniors:

Art:35/152*154**≈35**
Chemistry:109/152*154**≈110**
French: 49/152*154**≈49**
Spanish: 33/152*154**≈33**
Music: 49/152*154**≈50**
Social Studies: 59/152*154**≈60**
Biology:26/152*154**≈26**
English: **154**
German: 10/152*154**≈10**
Math: 262/152*154**≈265**
Physics: 183/152*154**≈185**

Our Solution Continued:

Finding total enrollment:	Finding student:teacher ratio values (bolded means one of 7 highest):	Student teacher ratio with new teachers:
Art:50+39+35=**124**	Ratio= total enrollment 2022-2023/#teachers 2021-2022	Art: 124/2= **62**
Biology:318+112+26=**456**	Art: 124/1=**124**	Biology: 456/4=114
Chemistry:95+148+110=**353**	Biology: 456/4=114	Chemistry: 353/4=**88.25**
English:294+182+154=**630** ✓	Chemistry: 353/3=**177.6…**	English:630/6=**105**
French:66+38+49=**153**	English:630/5=**126**	Foreign Lang.:366/4=**91.5**
German:31+26+10=**67**	Foreign Lang.:366/3=**122**	Math:796/7=**113.7…**
Spanish:82+31+33=**146**	Math:796/6=**132.6…**	Music:196/2=**98**
Foreign Lang.:153+67+146=366	Music:196/1=**196**	Physics: 333/3=111
Math:295+236+265=**796**	Physics: 333/3=111	Social Studies: 508/5=101.6
Music:80+66+50=**196**	Social Studies: 508/5=101.6	
Physics:80+68+185=**333**		
Social Studies:294+154+60=**508**		

Our Solution Continued cont.:

Old teacher ratio:	Student teacher ratio with new teachers:
Art: 124/1=**124**	Art: 124/2= **62**
Biology: 456/4=114	Biology: 456/4=114
Chemistry: 353/3=**177.6…**	Chemistry: 353/4=**88.25**
English:630/5=**126**	English:630/6=**105**
Foreign Lang.:366/3=**122**	Foreign Lang.:366/4=**91.5**
Math:796/6=**132.6…**	Math:796/7=**113.7…**
Music:196/1=**196**	Music:196/2=**98**
Physics: 333/3=111	Physics: 333/3=111
Social Studies: 508/5=101.6	Social Studies: 508/5=101.6

- After calculating the the student-teacher ratios for each class in the 2022-2023 school year, we found that the subjects art, chemistry, english, foreign language, math, and music had the highest seven student-teacher ratios. Then, we calculated the ratios with the additional seven teachers, and found that the highest new ratio was still less than lowest ratio with the old allocations.
- Therefore, we concluded that our model should allocate the faculty on each to the **art, chemistry, english, foreign language, math, and music** departments where they were most needed.

Strength and Weaknesses:

- One of the weaknesses of our model is that we assumed that the dropout rate was constant across the grades and this is not necessarily the case. It also did not equal to 5% but was closer to 1% per year at the beginning of the academic year.
- The largest weakness lies in our assumption that we had to make to complete our calculations, which is that we assumed that the enrollment ratios were tied to the grade and not to people's 'random' choices. This is also not necessarily the case.
- The errors can be seen in the dropout rate of 1%, however this can be accounted by for saying that it will rise to 5% over the following academic year and will compound on itself for classes like the 2022-2023 junior class.
- Our class size was equal to 630 with all of the grade sizes combined so this constant dropout rate assumption did give us correct class sizes assuming that rate.
- The model is once again not very stable because of the assumption that teachers require the same amount of payment, and have to do the same amount of work across subjects. This might not be true as in the Epsilon school perhaps a math teacher does much more work than a physics teacher, or vice versa.
- This model does not account for the years past the next year and only counts for the first year when the bump first exists and this does not account for the class sizes of the following years.

Thank You!

Any Questions?

P.S., our proof was too big for the margin so please forgive this presentation