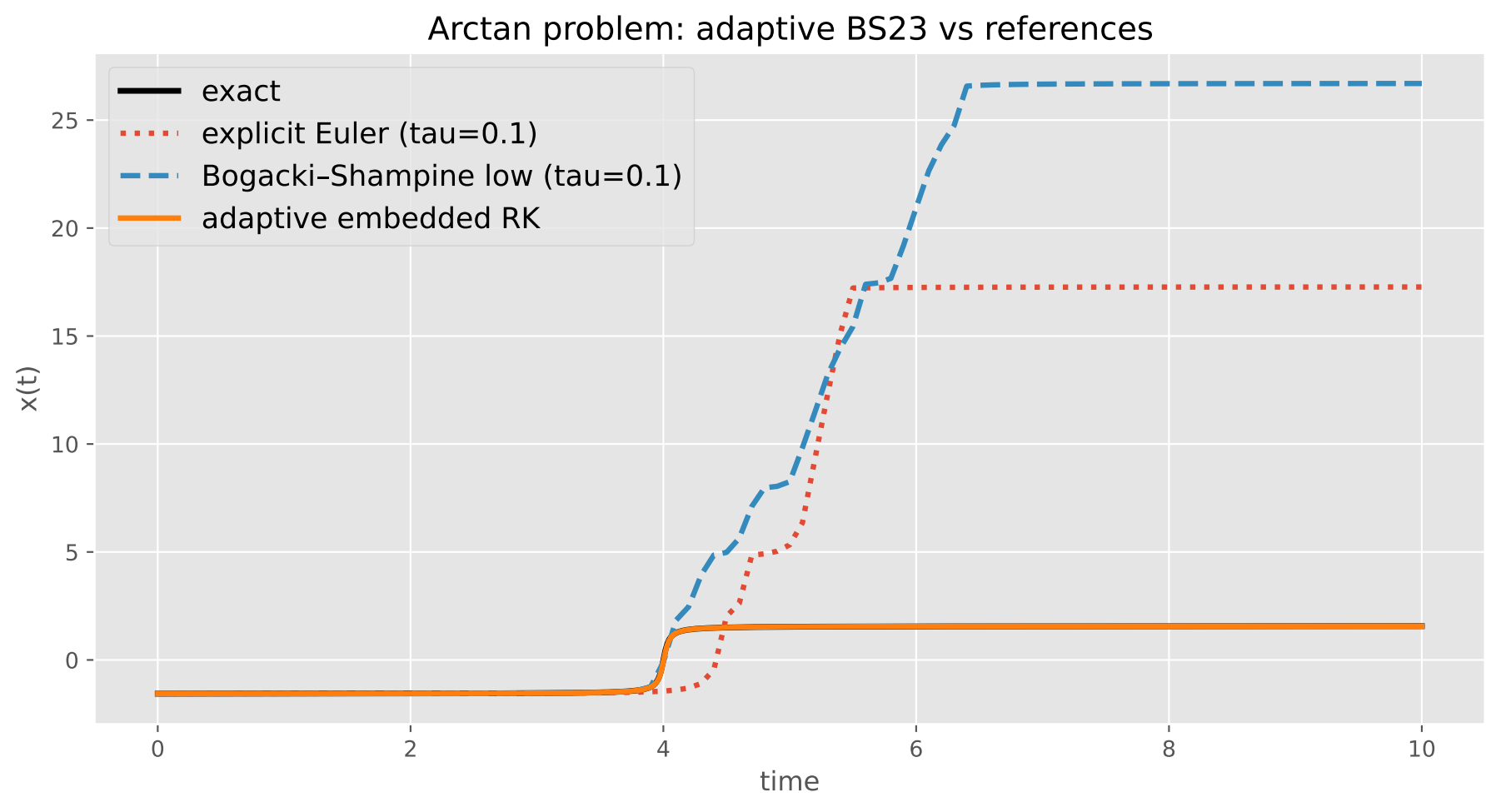
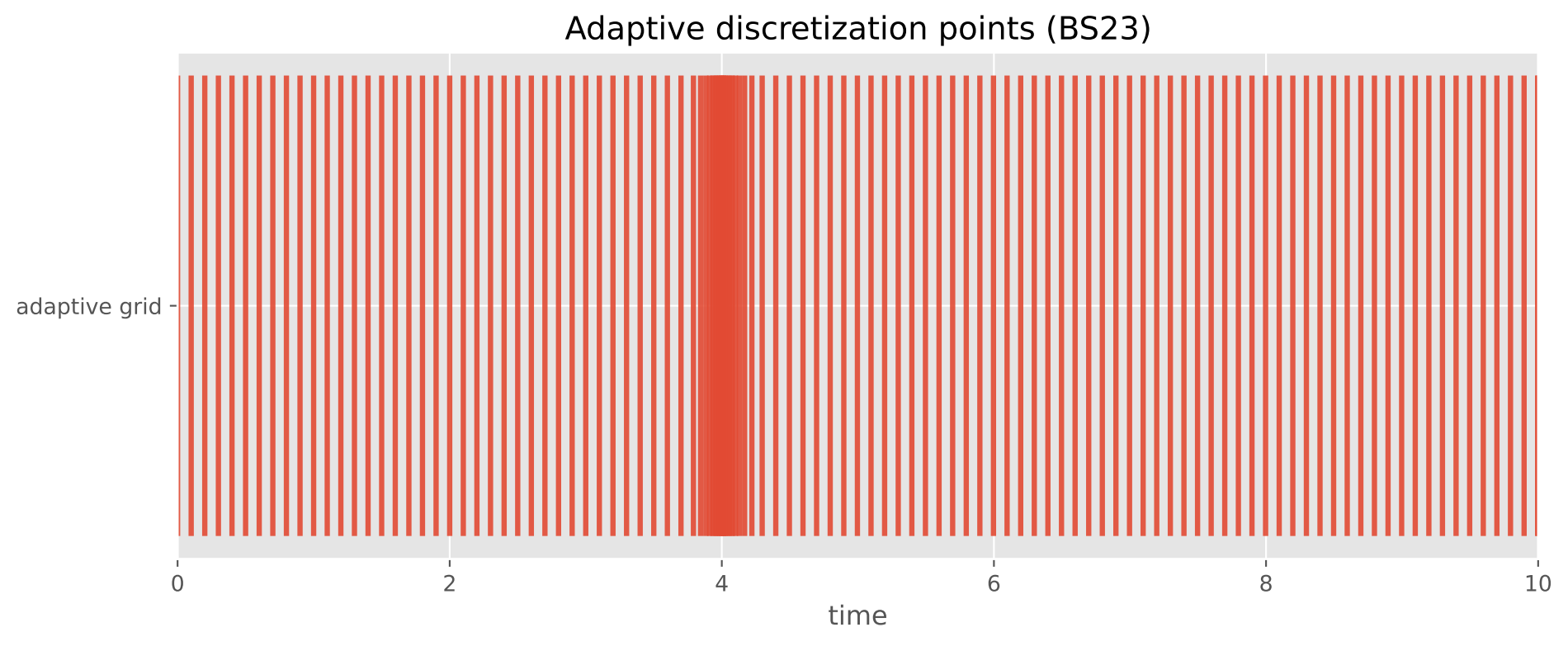
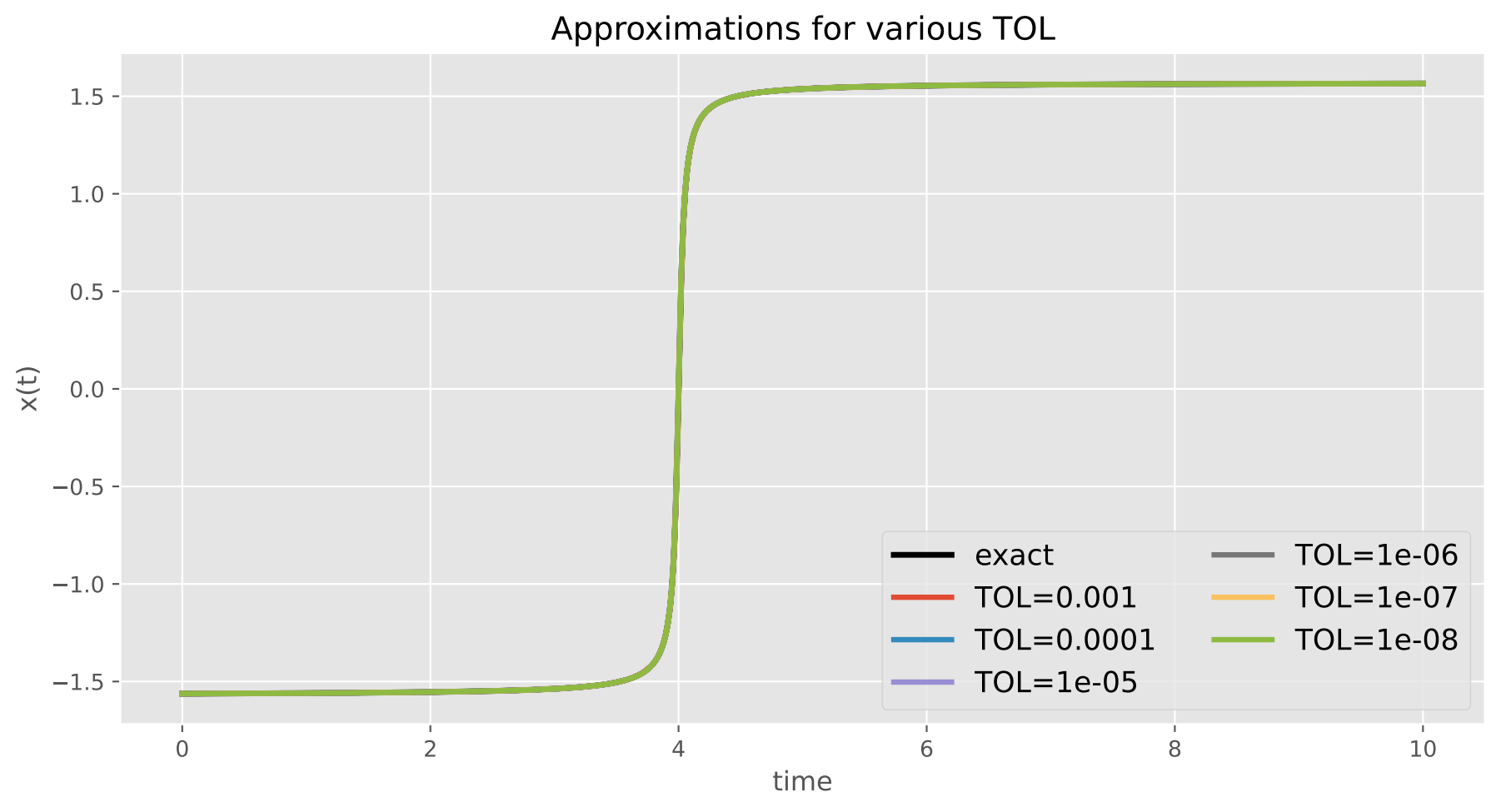
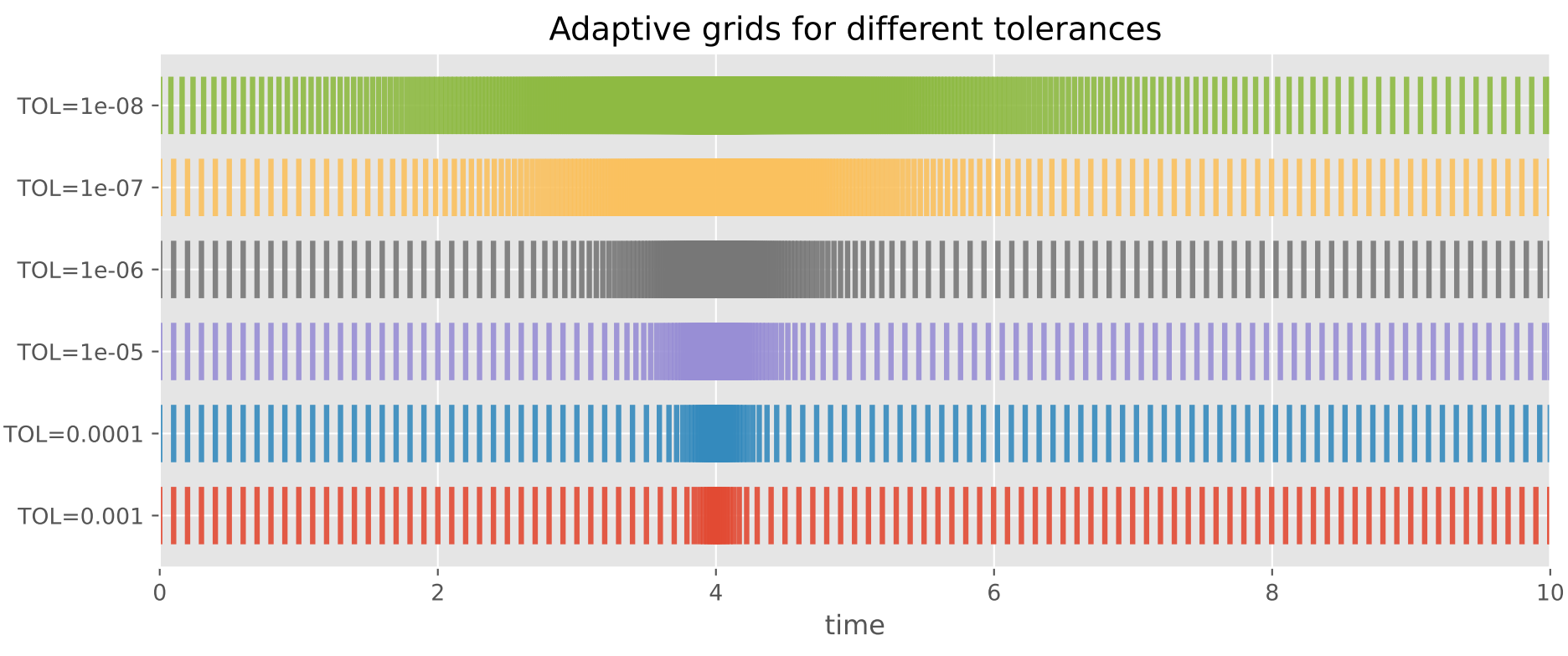
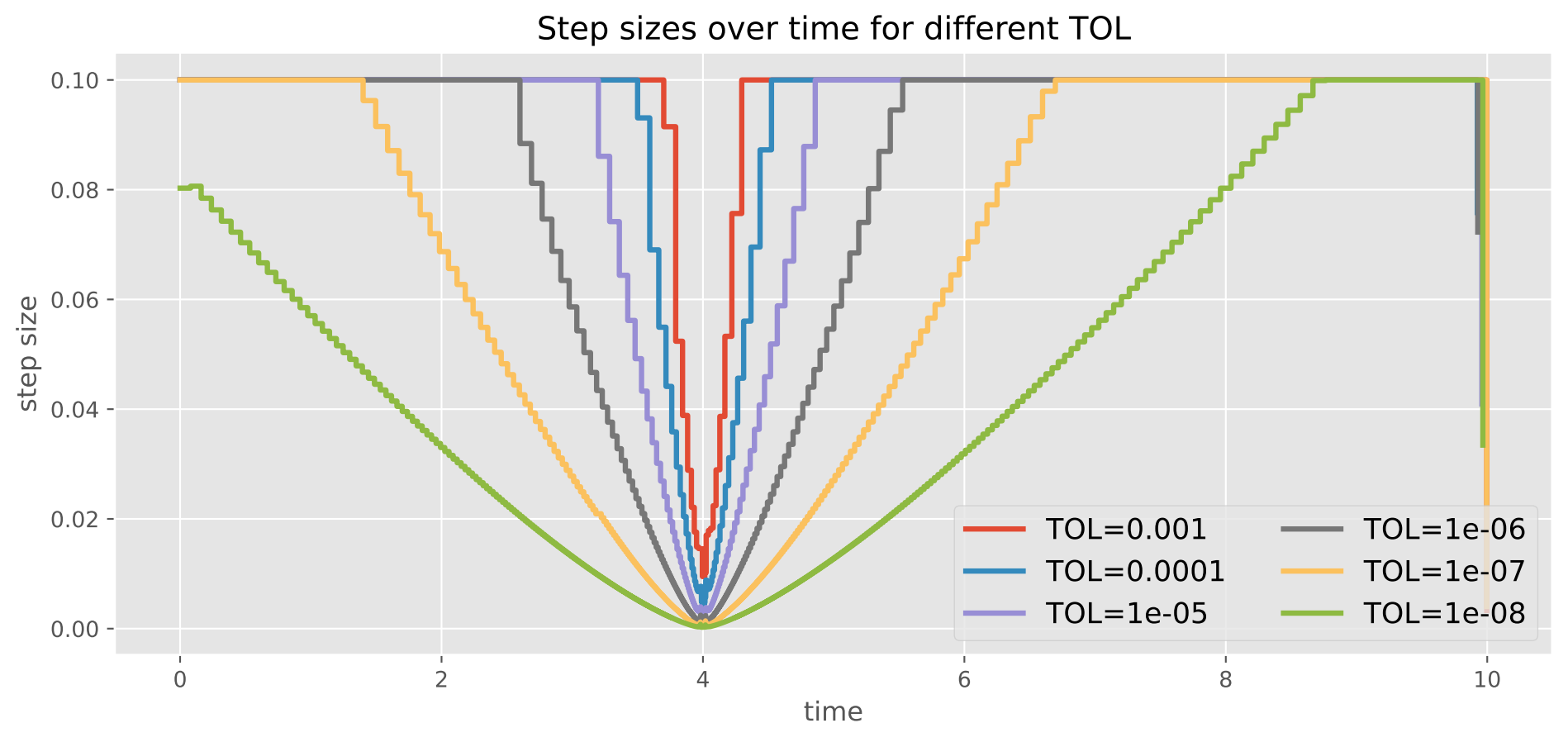
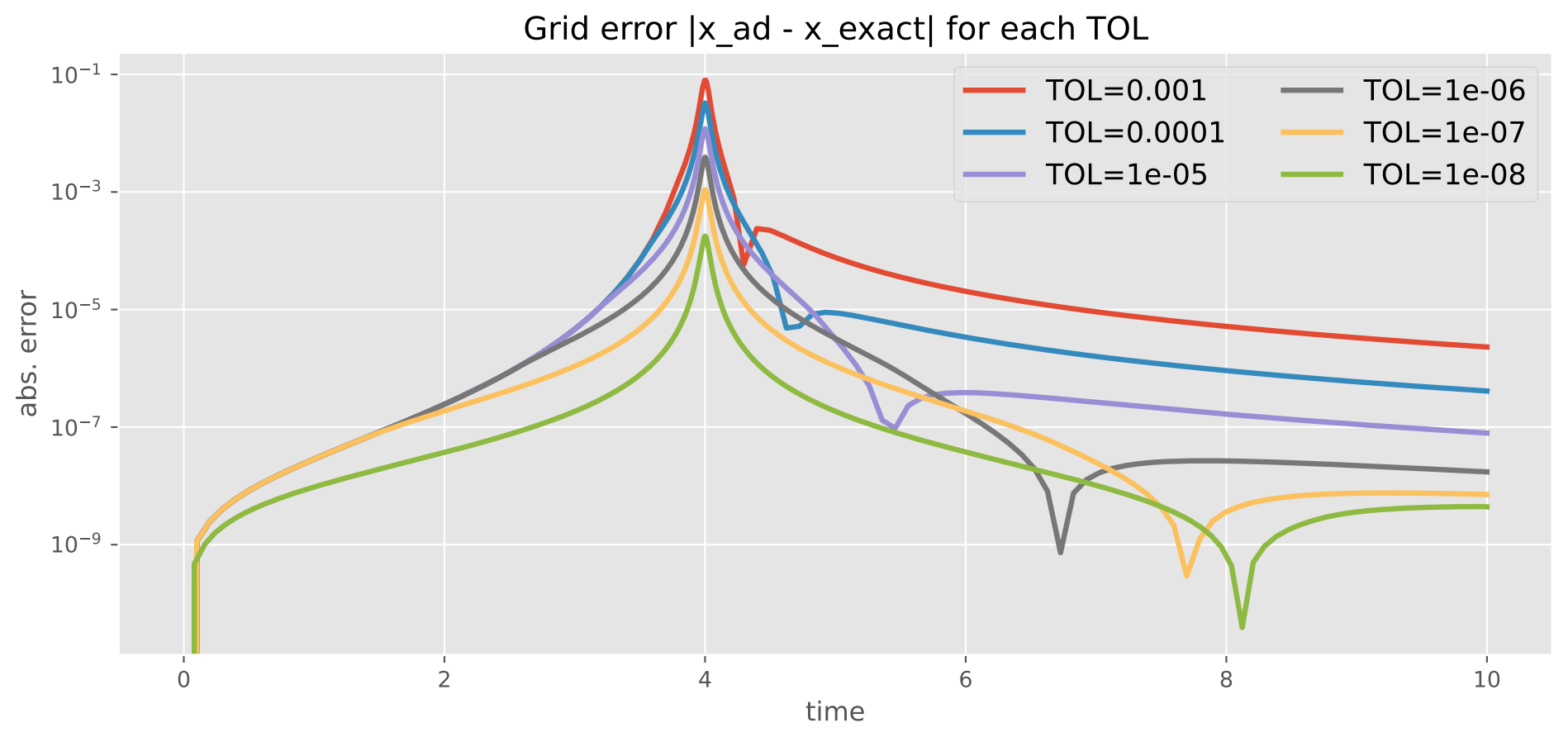

Arctan problem: adaptive BS23 vs references
exact
explicit Euler (tau=0.1)
Bogacki–Shampine low (tau=0.1)
adaptive embedded RK
x(t)
time
25
20
15
10
5
0
0
2
4
6
8
10

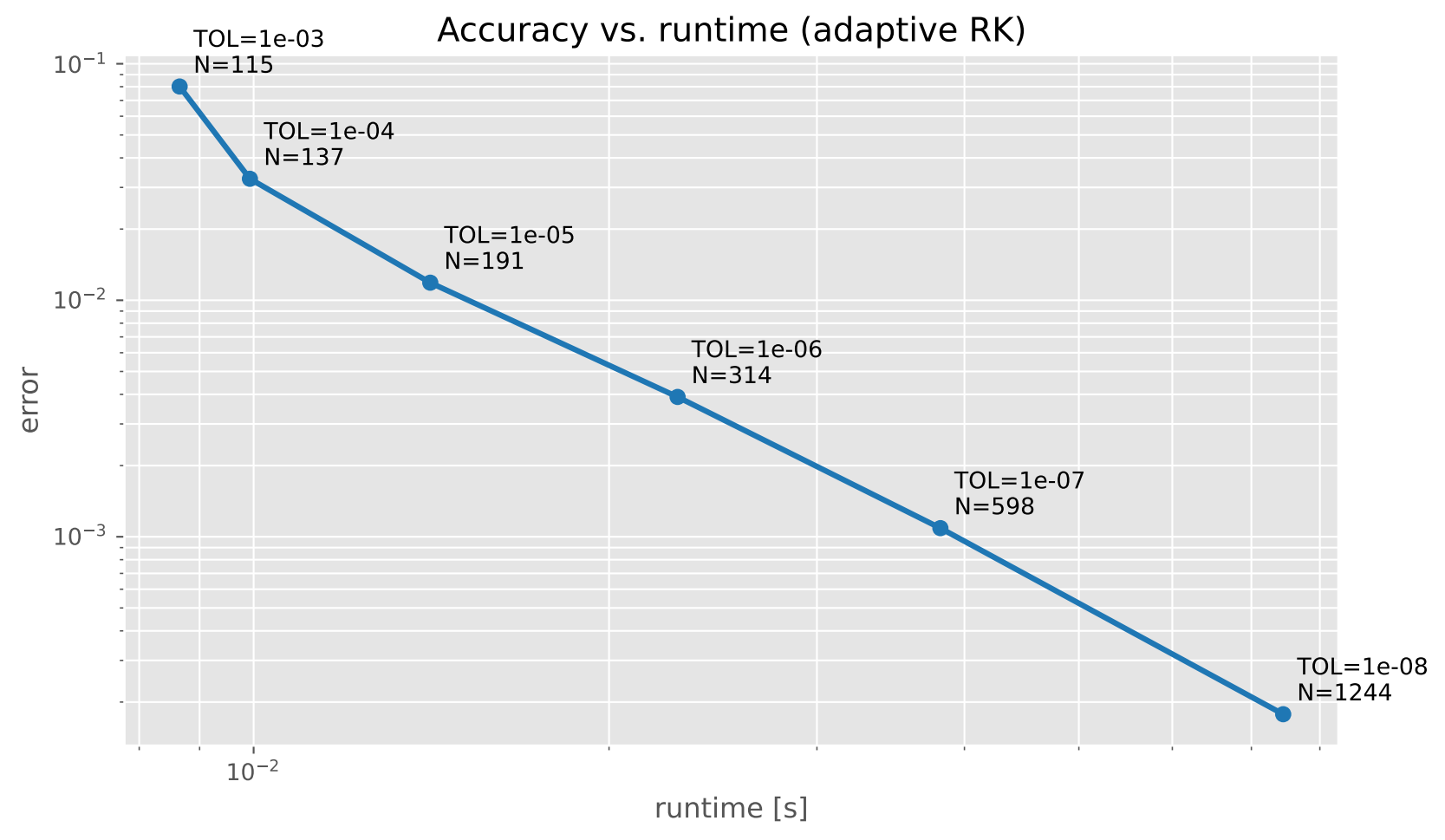

Adaptive discretization points (BS23)
adaptive grid
0
2
4
6
8
10
time


Approximations for various TOL
x(t)
time
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
0
2
4
6
8
10
exact
TOL=0.001
TOL=0.0001
TOL=1e-05
TOL=1e-06
TOL=1e-07
TOL=1e-08


Adaptive grids for different tolerances
TOL=1e-08
TOL=1e-07
TOL=1e-06
TOL=1e-05
TOL=0.0001
TOL=0.001
0
2
4
6
8
10
time

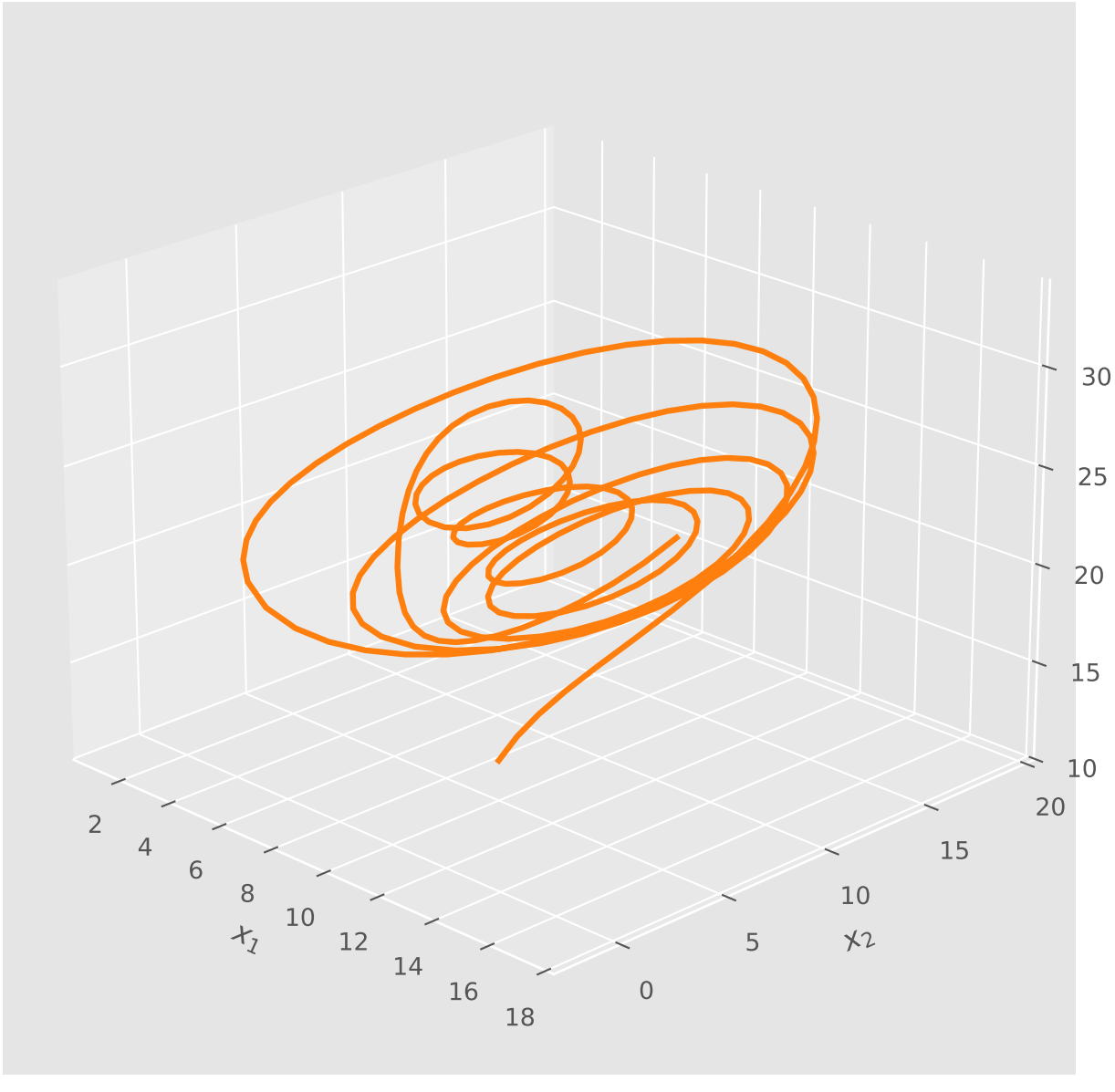

Step sizes over time for different TOL
step size
time
0.10
0.08
0.06
0.04
0.02
0.00
0
2
4
6
8
10
TOL=0.001
TOL=0.0001
TOL=1e-05
TOL=1e-06
TOL=1e-07
TOL=1e-08

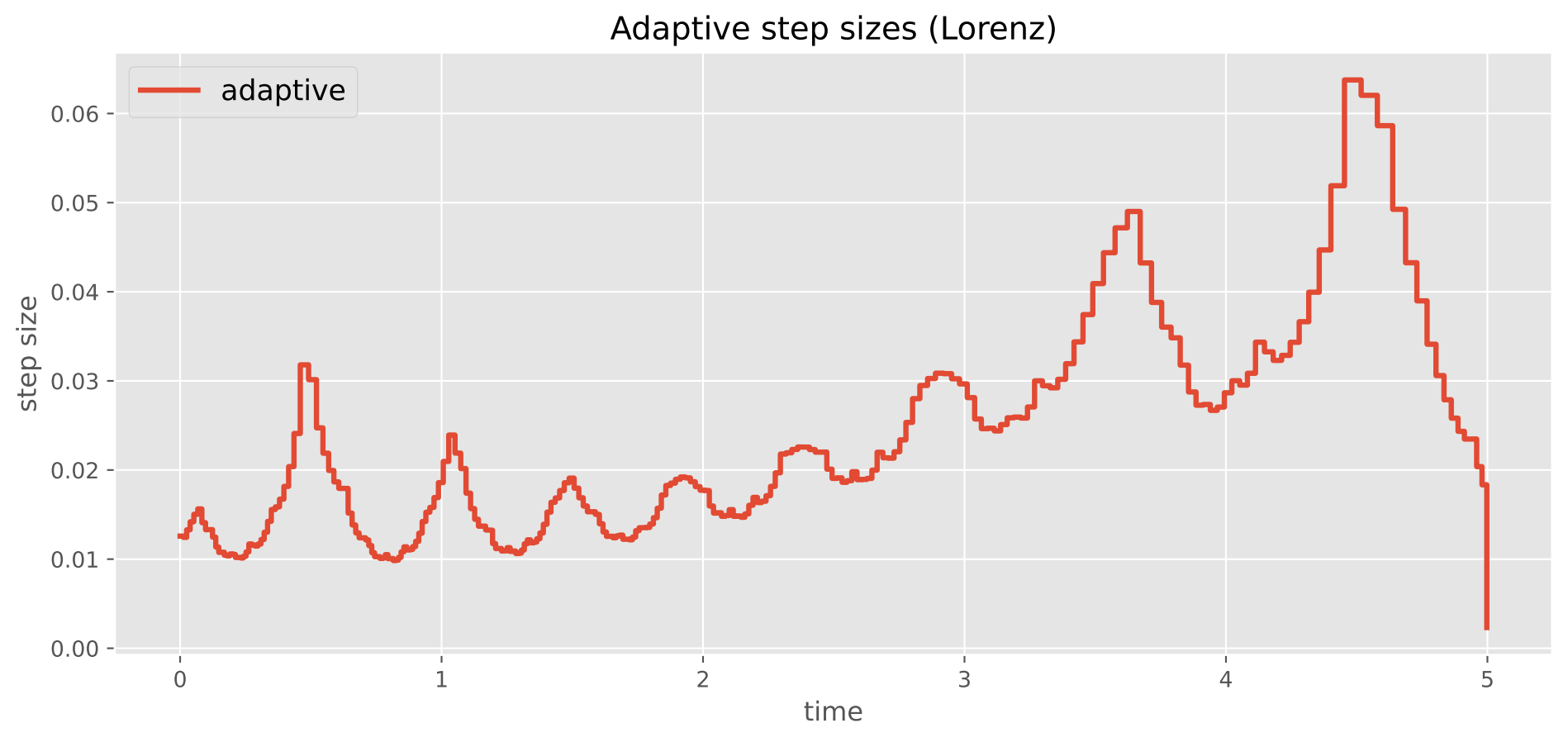

Grid error |x_ad - x_exact| for each TOL
TOL=0.001
TOL=0.0001
TOL=1e-05
TOL=1e-06
TOL=1e-07
TOL=1e-08
abs. error
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
0
2
4
6
8
10
time


Accuracy vs. runtime (adaptive RK)
TOL=1e-03
N=115
TOL=1e-04
N=137
TOL=1e-05
N=191
TOL=1e-06
N=314
TOL=1e-07
N=598
TOL=1e-08
N=1244
10^{-1}
10^{-2}
10^{-3}
error
10^{-2}
runtime [s]

TOL trade-off metrics (adaptive RK)

TOL	runtime [s]	N steps	max error	final error
1e-03	0.008656	115	8.015e-02	2.305e-06
1e-04	0.009928	137	3.264e-02	4.114e-07
1e-05	0.014113	191	1.187e-02	7.921e-08
1e-06	0.022862	314	3.901e-03	1.729e-08
1e-07	0.038168	598	1.088e-03	7.149e-09
1e-08	0.074482	1244	1.777e-04	4.398e-09


Forced Lorenz (adaptive BS23)
30
25
20
15
10
20
15
10
5
0
x_2
2
4
6
8
10
12
14
16
18
x_1


Adaptive step sizes (Lorenz)
adaptive
step size
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0
1
2
3
4
5
time