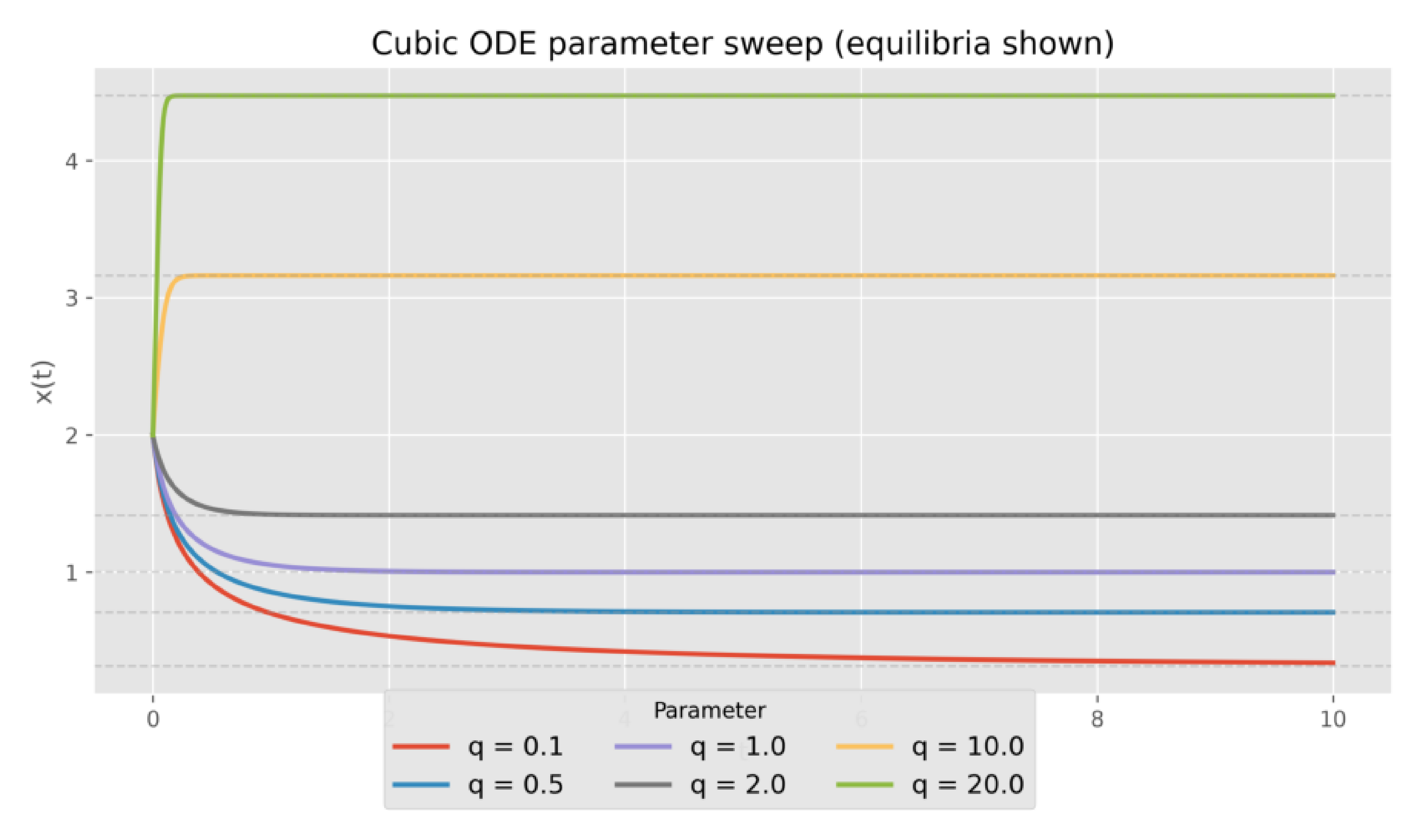
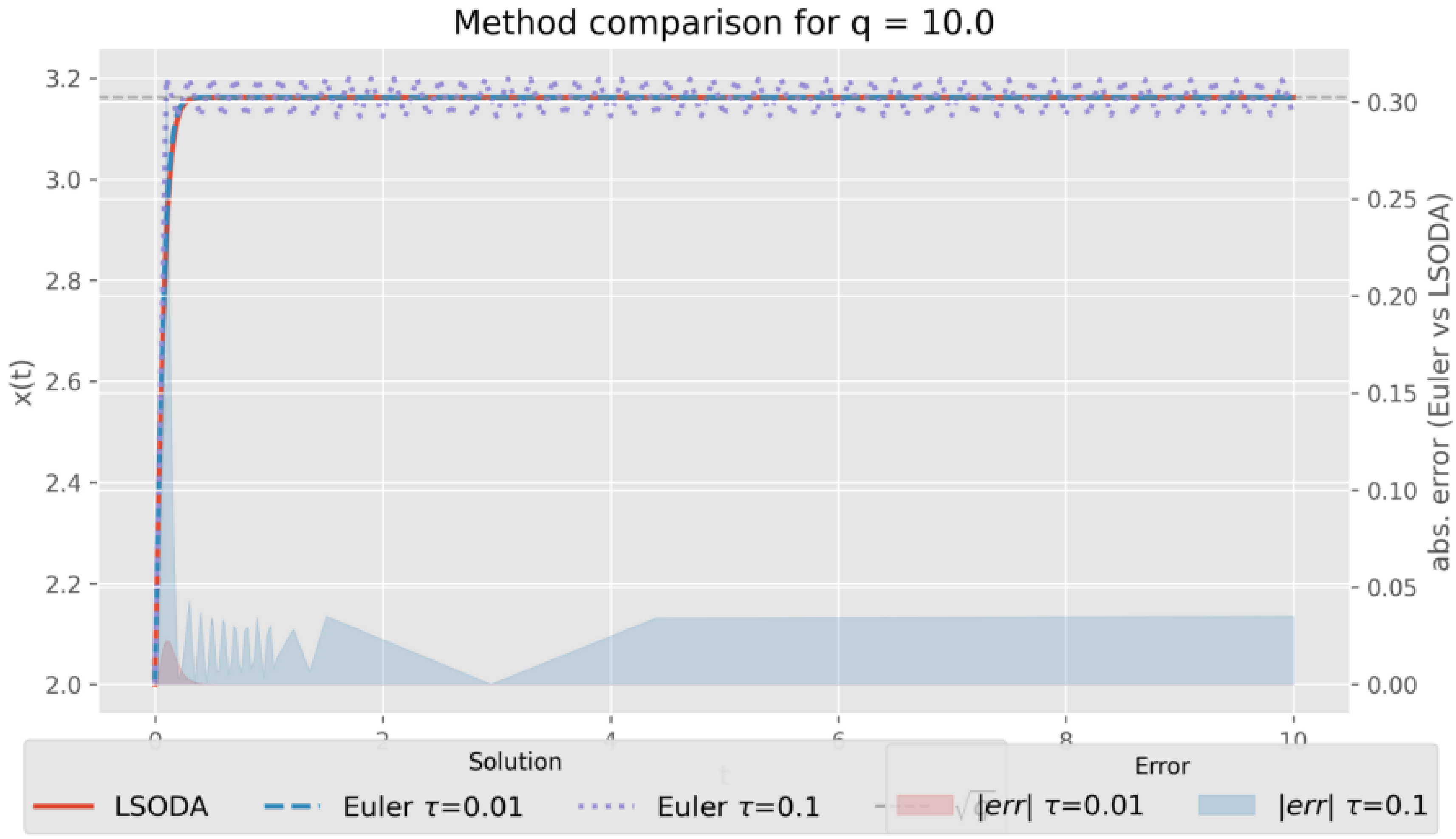
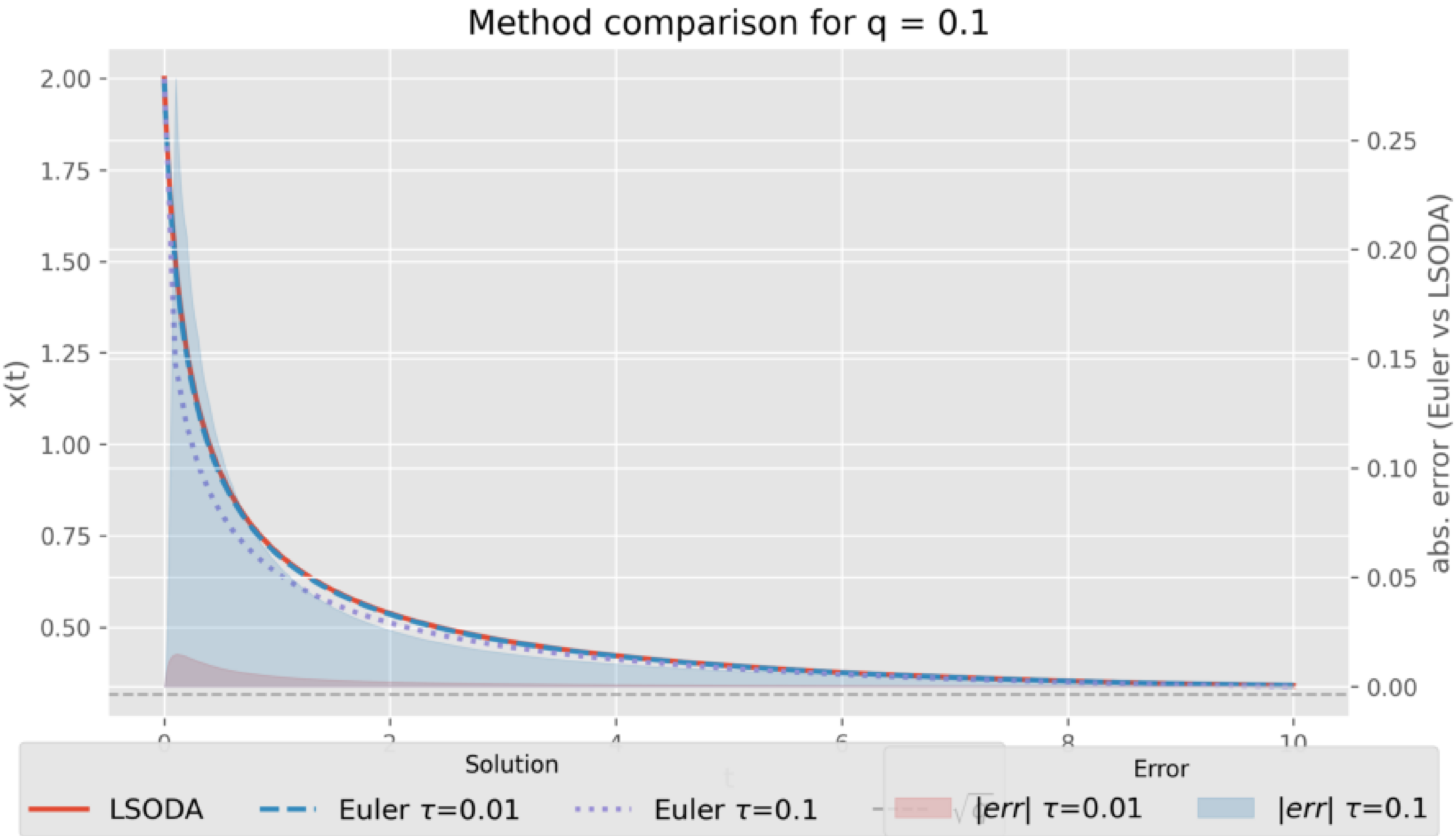
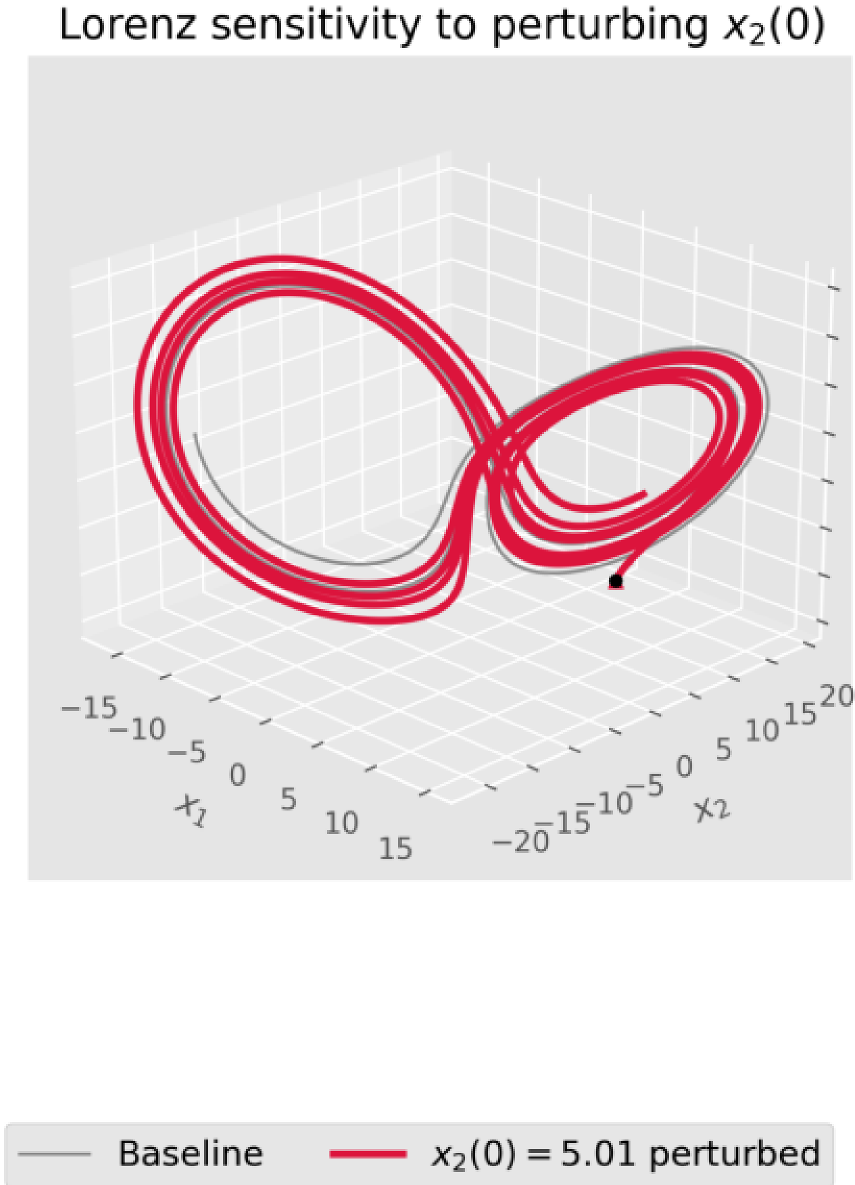


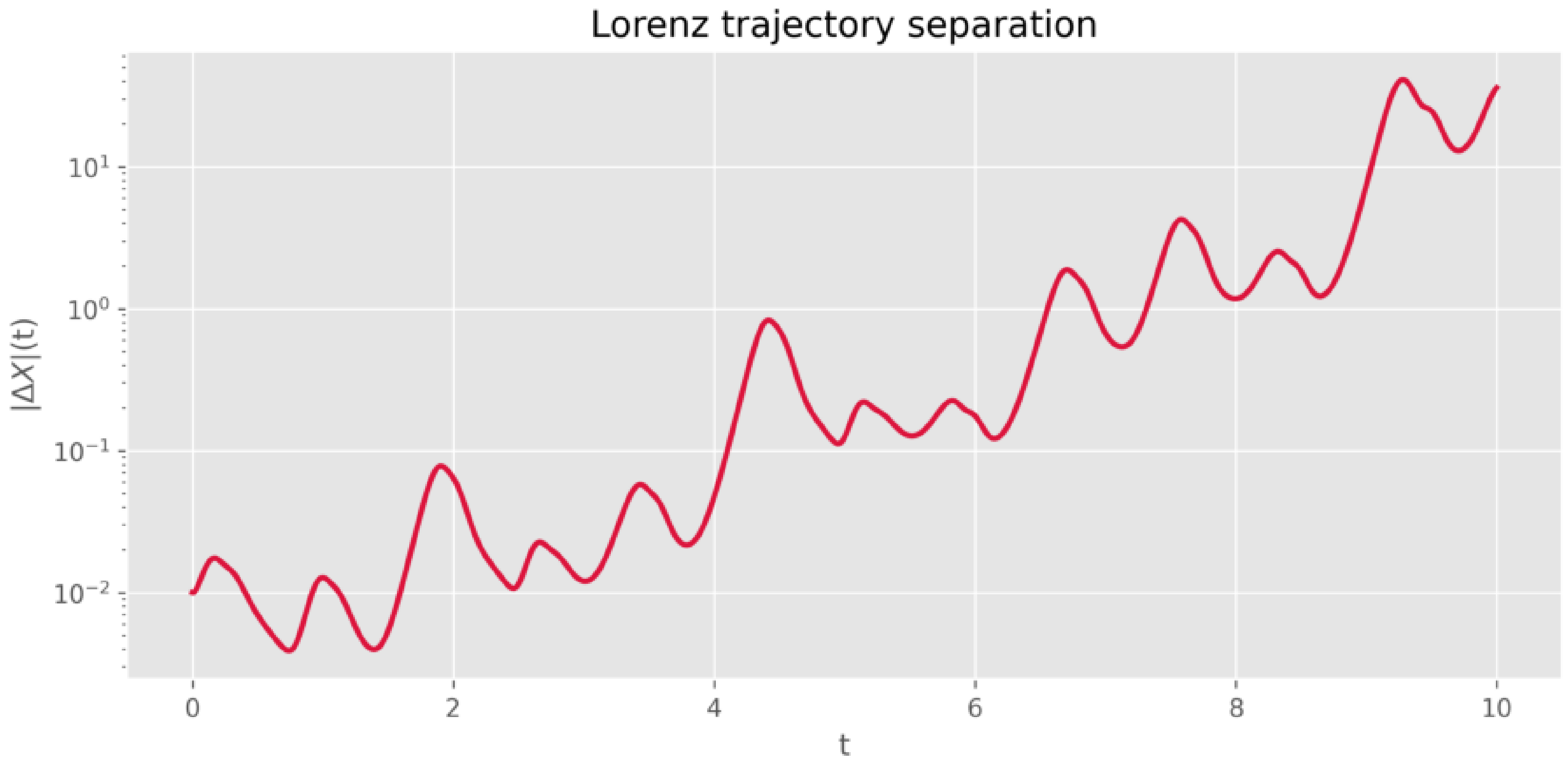
Cubic ODE parameter sweep (equilibria shown)
x(t)
4
3
2
1
0
2
4
6
8
10
t
Parameter
q = 0.1
q = 0.5
q = 1.0
q = 2.0
q = 10.0
q = 20.0



Method comparison for q = 10.0
x(t)
3.2
3.0
2.8
2.6
2.4
2.2
2.0
abs. error (Euler vs LSODA)
0.30
0.25
0.20
0.15
0.10
0.05
0.00
0
2
4
6
8
10
t
Solution
LSODA
Euler τ=0.01
Euler τ=0.1
√q
Error
|err| τ=0.01
|err| τ=0.1


Method comparison for q = 0.1
2.00
1.75
1.50
1.25
1.00
0.75
0.50
x(t)
0.25
0.20
0.15
0.10
0.05
0.00
abs. error (Euler vs LSODA)
0
2
4
6
8
10
t
Solution
LSODA
Euler τ=0.01
Euler τ=0.1
Error
|err| τ=0.01
|err| τ=0.1


Lorenz sensitivity to perturbing $x_2(0)$
40
35
30
25
20
15
10
5
−15
−10
−5
0
5
10
15
x_1
−20
−15
−10
−5
0
5
10
15
20
x_2
Baseline
$x_2(0) = 5.01$ perturbed


Lorenz trajectory separation
10^{1}
10^{0}
10^{-1}
10^{-2}
$|\Delta X|(t)$
0
2
4
6
8
10
t

(b) Long-term behavior vs q
For x = q x x^3 with x(0)=2 and q>0, the solution tends to the stable equilibrium sqrt(q). If q<4 (sqrt(q)<2), it decreases from 2 down to sqrt(q). If q=4, it stays constant at 2 (add this case and the curve is flat at 2). If q>4 (sqrt(q)>2), it increases from 2 up to sqrt(q). This matches the plot; for small q the approach is slow so at T=10 it can still be above sqrt(q).

(c) Observations; change from q=10 to q=0.1
Euler is first order (global error ~ O(tau)). Near the stable equilibrium, the linearization gives y = 2 q y; explicit Euler is stable only if tau < 1/q.
For q=10, tau=0.1 is at the stability boundary, so Euler(tau=0.1) shows large error and oscillatory artifacts vs LSODA, while tau=0.01 is accurate.
For q=0.1, both tau=0.1 and tau=0.01 are deep inside the stable region and the dynamics are slow, so both Euler curves are close to LSODA and look very similar.
This is exactly what the plots show.

(d) Sensitivity in the Lorenz system
Yes, it changes significantly. With x2(0)=5 vs 5.01, the two trajectories start close but then diverge clearly, consistent with sensitive dependence on initial conditions (positive Lyapunov exponent). The 3D plot shows the divergence as expected.